1) 2 lens cloaking System (1)

- image is not inverted.
- afocal condition. in that total system should not have a focal point.
- afocal system front | back focal length of 2 lens system $= \infty$

$$f.f.l = \frac{f_1 (d - f_2)}{d - (f_1 + f_2)}$$

only if distance b/w lens is $d = f_1 + f_2$, we will have afocal system

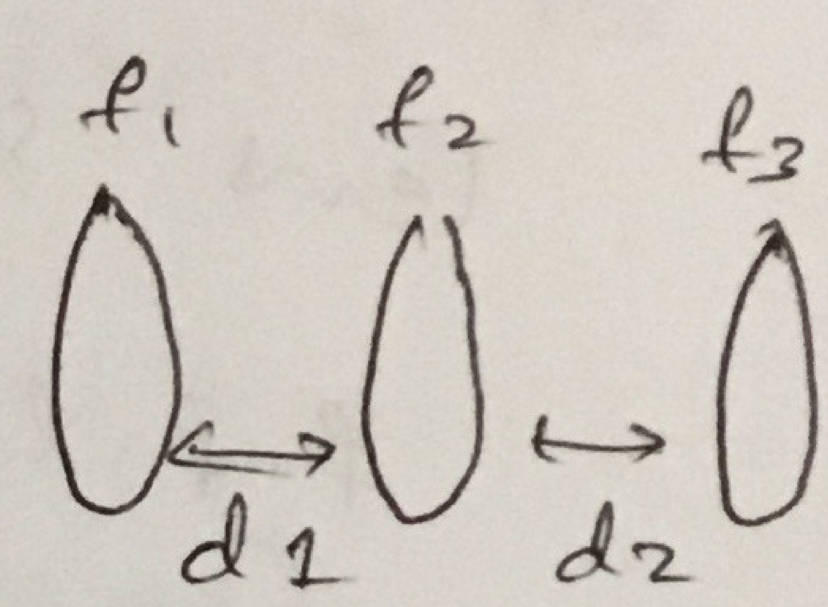
$$bfl = \frac{f_2 (d - f_1)}{d - (f_1 + f_2)}$$

- Magnification is also maintained due to similar triangle arguement.
- as the object to be cloaked gets closer to the focal point, the position of the entrance pupil needs to go far away.

- Design the 3 lens Cloaking System.

- To design afocal 3 lens system from b.f.l & f.f.l techniques, treate 3 lens system as 2 lens at a time

- first two lens: consider only b.f.l.



$$b.f.l_{1,2} = \frac{f_2(d_1 - f_1)}{d_1 - (f_1 + f_2)}$$

$$b.f.l_{total} = \frac{(b.f.l_{1,2})(d_2 - f_2)}{d_2 - (f_2 + (b.f.l_{1,2}))}$$

need $b.f.l_{total} = \infty$ for a focal system.

$$\therefore d_2 = f_2 + \frac{f_2(d_1 - f_1)}{d_1 - (f_1 + f_2)}$$

$$= \frac{(\cancel{d_1 f_2} - f_1 f_2 - f_2 f_2) + \cancel{f_2 d_1} - f_1 f_2}{d_1 - (f_1 + f_2)}$$

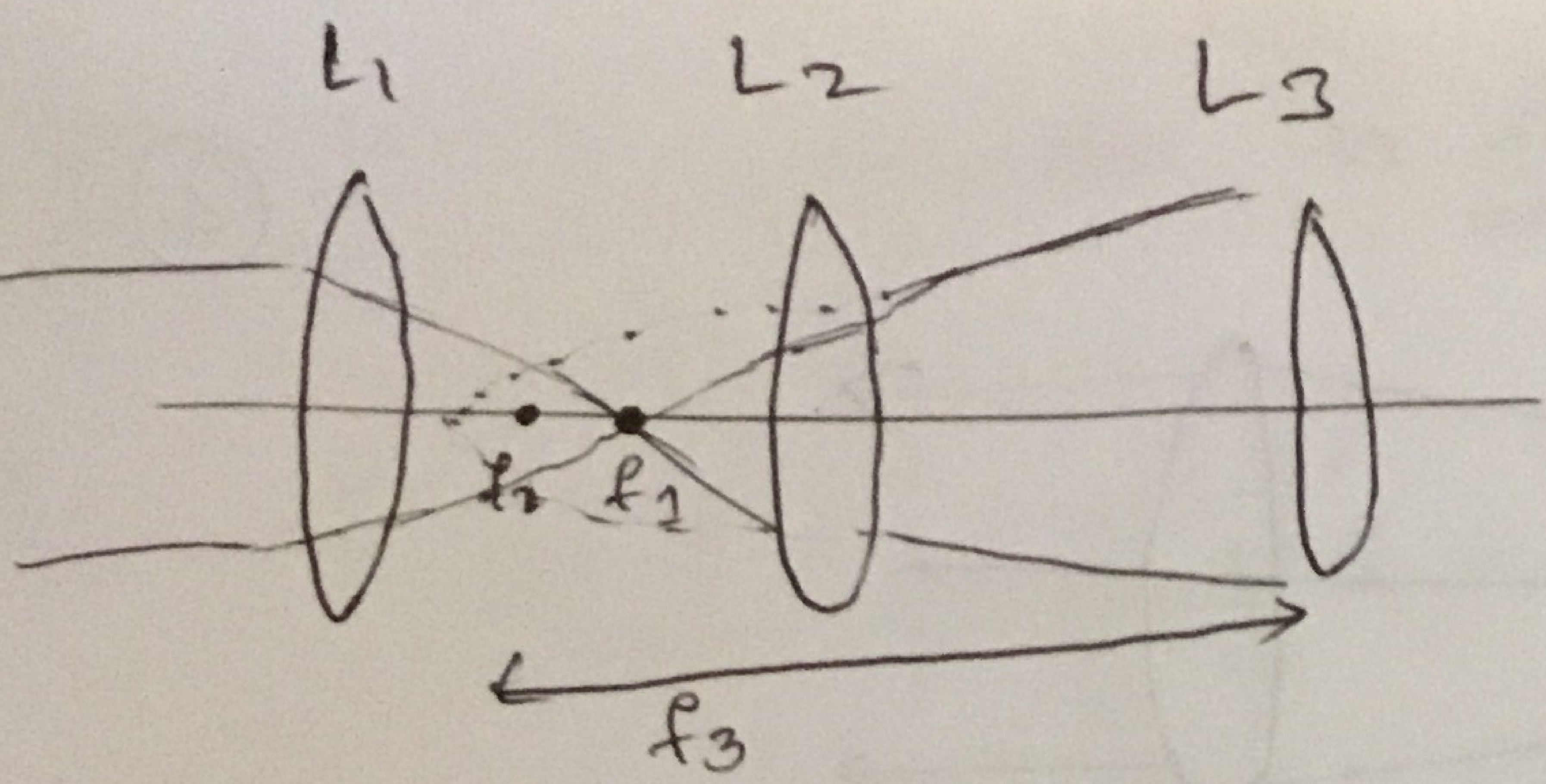
$$= \frac{-2f_1 f_2 - f_2 f_2}{d_1 - (f_1 + f_2)}$$

$$d_2 = \frac{2f_1 f_2 + f_2 f_2}{(f_1 + f_2) - d_1}$$

$\therefore$ Conclusion:

(1) $\therefore \boxed{f_1 + f_2 > d_1}$

(2) $d_2 \to \infty$ if $(f_1 + f_2) \approx d_1$.



Since, the object for L_2 is within the f_2, rays from output of L_2 will diverge / converge and
which one?
when back projected, looks like the object for the L_3 is @ it's f_3.

i.e. image of the L_2 is at f_3. (virtual).

This means, first 2 lens combined

object $S_0 = \infty$

$f =$ b.f.l.

$S_i = ?$

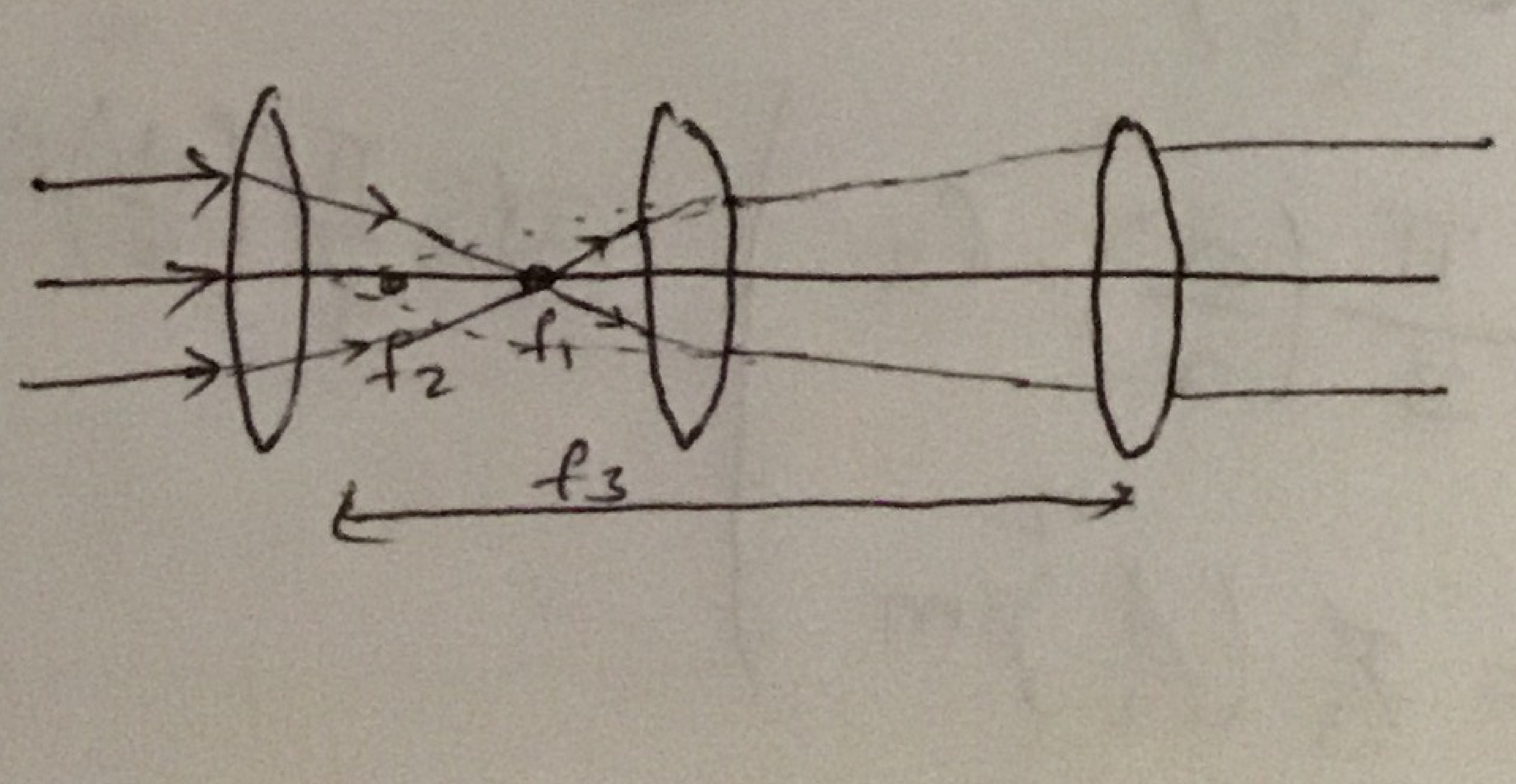
$$\frac{1}{\infty} + \frac{1}{S_i} = \frac{d_1 - (f_1 + f_2)}{f_2(d_1 - f_1)}$$

$$S_i = \frac{f_2(d_1 - f_1)}{d_1 - (f_1 + f_2)}$$

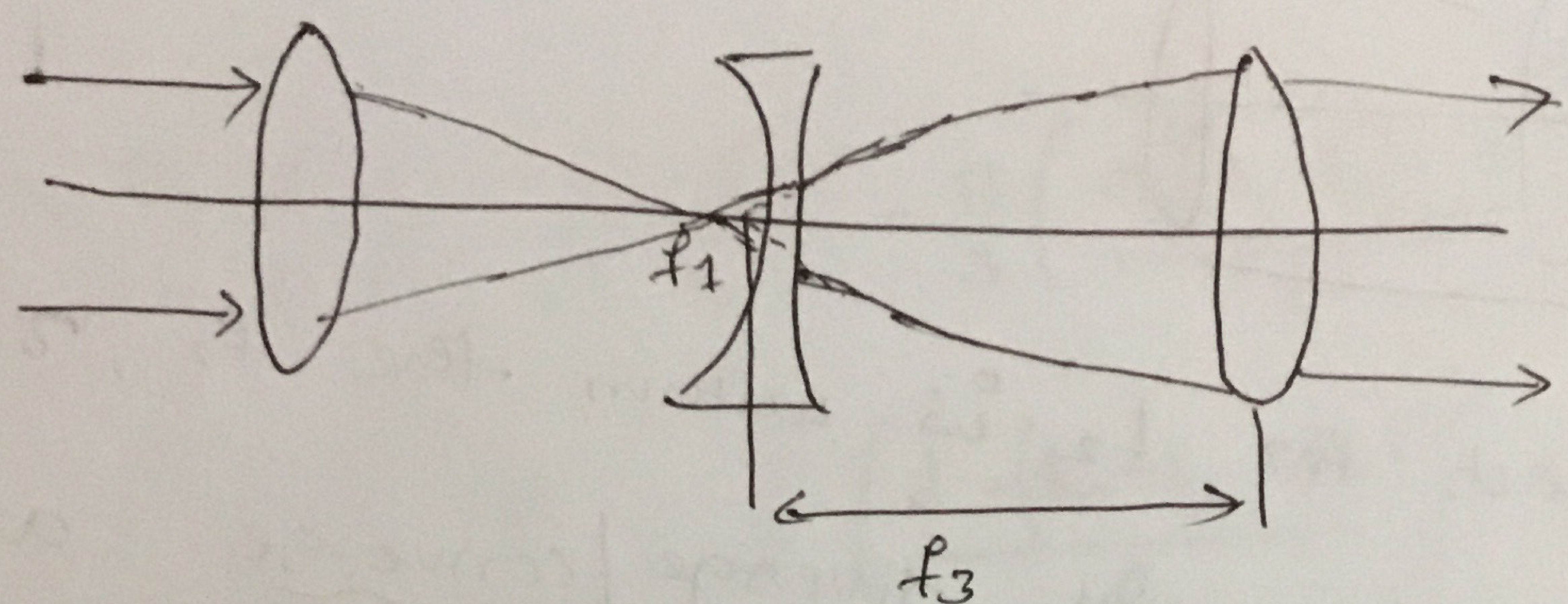
This S_i should be -ive so that

$$\boxed{S_i + d_2 = f_3}$$

Case I: Converging:



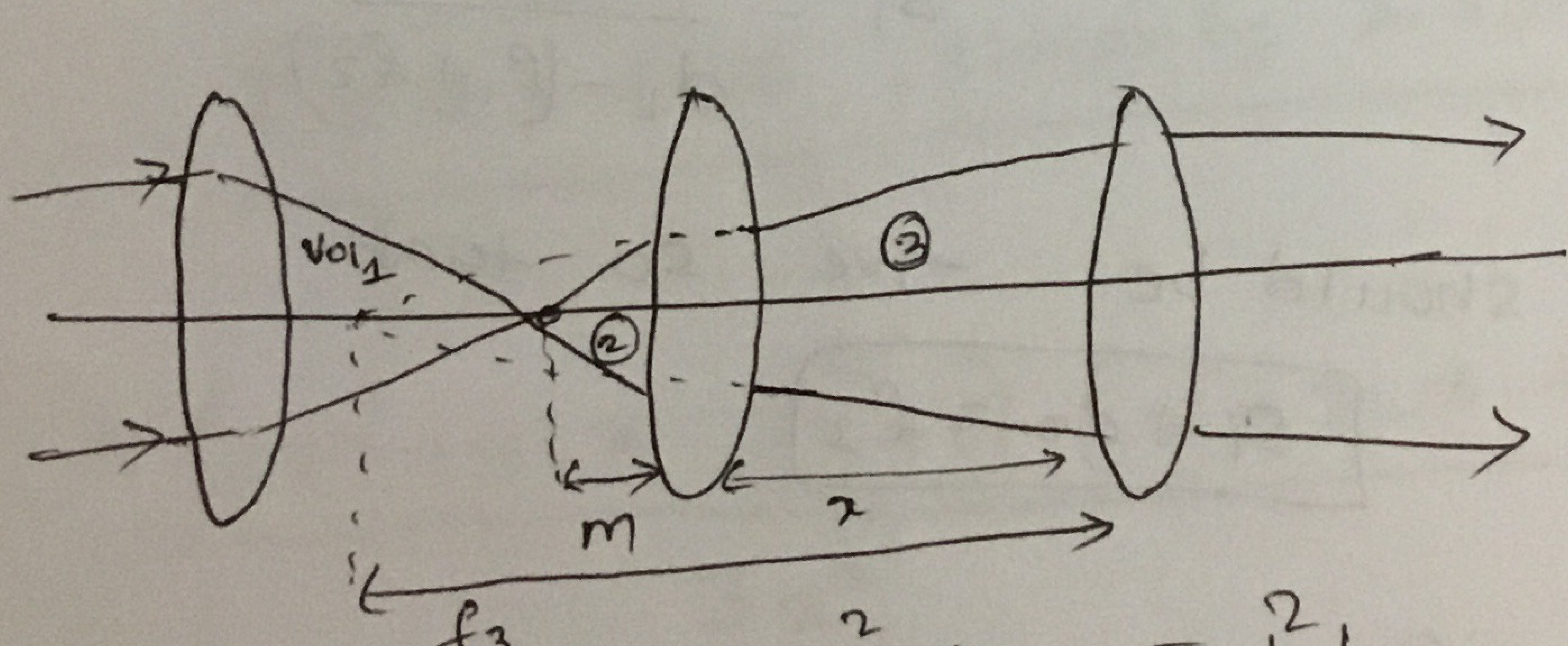
Case II : diverging lens:



- diverging lens (f_3) image of the L_2 will form in front of f_1 as the rays diverge

- if the image is within vertex & f point,
  rays will diverge for the diverging lens (concave)
  " " converge " " converging lens (convex).
  "

Volume formulation 3 lens: (for case I.) only



all diameters
$d_1 = d_2 = d_3 = d$

Volume:

Total volume $= \pi\left(\frac{d}{2}\right)^2 \times L = \pi \frac{d^2}{4} L$

volume covered by L1 rays Vol1 $= \frac{\pi}{3}\left(\frac{d}{2}\right)^2 \times f_1$

② $= \frac{\pi}{3}\left(\frac{d}{2}\right)^2 m$

③ $= \frac{\pi}{4}\left(\frac{d}{2}\right)^2 f_3 - \frac{\pi}{4}\left(\frac{d}{2}\right)^2 (f_3 - x)$

now, only variable is $\underline{f}$. $(f = f_1 = f_2 = f_3)$ (5)

$$V_{cloak} = \pi \frac{d^2}{4} L - \cancel{\frac{\pi}{3} \frac{d^2}{4} f} - \frac{\pi}{3}\left(\frac{d((L-S_2)-f)}{f}\right)^2 (L-S_2-f)$$

$$+ \cancel{\frac{\pi}{3} \frac{d^2}{4} f} - \frac{\pi}{3}\left(\frac{d-(L-S_2-f_1)}{f_1}\right)^2 (f-S_2).$$

$$= \pi \frac{d^2}{4} L - \frac{\pi}{3}\left(\frac{d-(L-S_2-f)}{f}\right)^2 (L-S_2-f+f-S_2)$$

$$= \pi \frac{d^2}{4} L - \frac{\pi}{3}\left(\frac{d-(L-S_2-f)}{f}\right)^2 (L-2S_2).$$

$$= \pi \frac{d^2}{4} L - \frac{\pi}{3}\left(\frac{d-(L-L^2+Lf\times 2-f^2-f)}{f}\right)^2 (L-2L^2+2Lf-f^2).$$

$\boxed{\text{put } f=x \text{, simplify \& solve online}}$

*

Simplify:

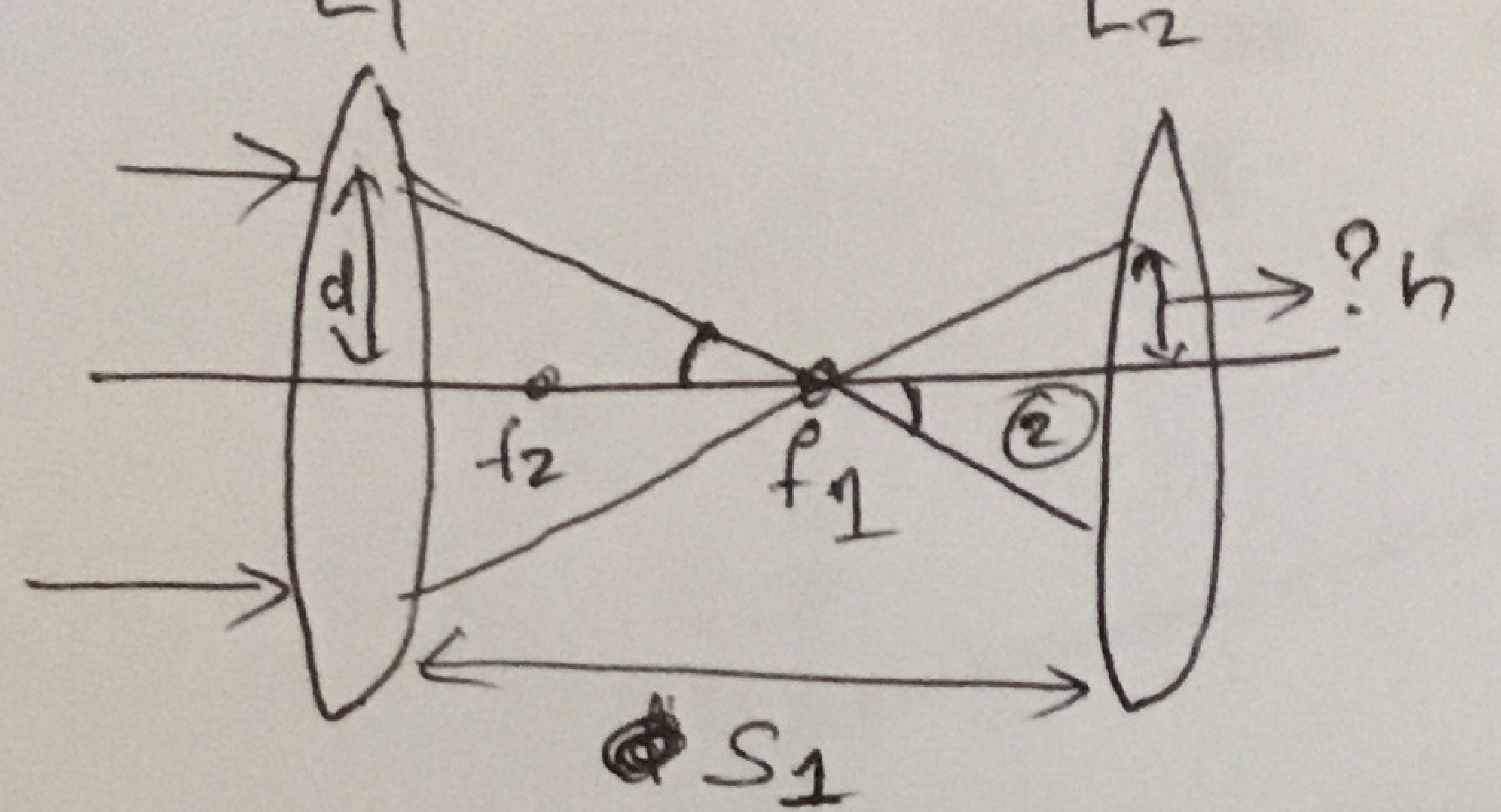
$$f(x) = \frac{\pi d^2 L}{4} - \frac{\pi(-2x^2+4Lx-2L^2+L)(x^2-2Lx+x+L^2-L+d)^2}{3x^2}$$

Soln: $x = \frac{2L-1}{2}$.

$$\boxed{f = \frac{2L-1}{2}}$$

2 volme part 2

$S_1 \Rightarrow$ Separation 1.

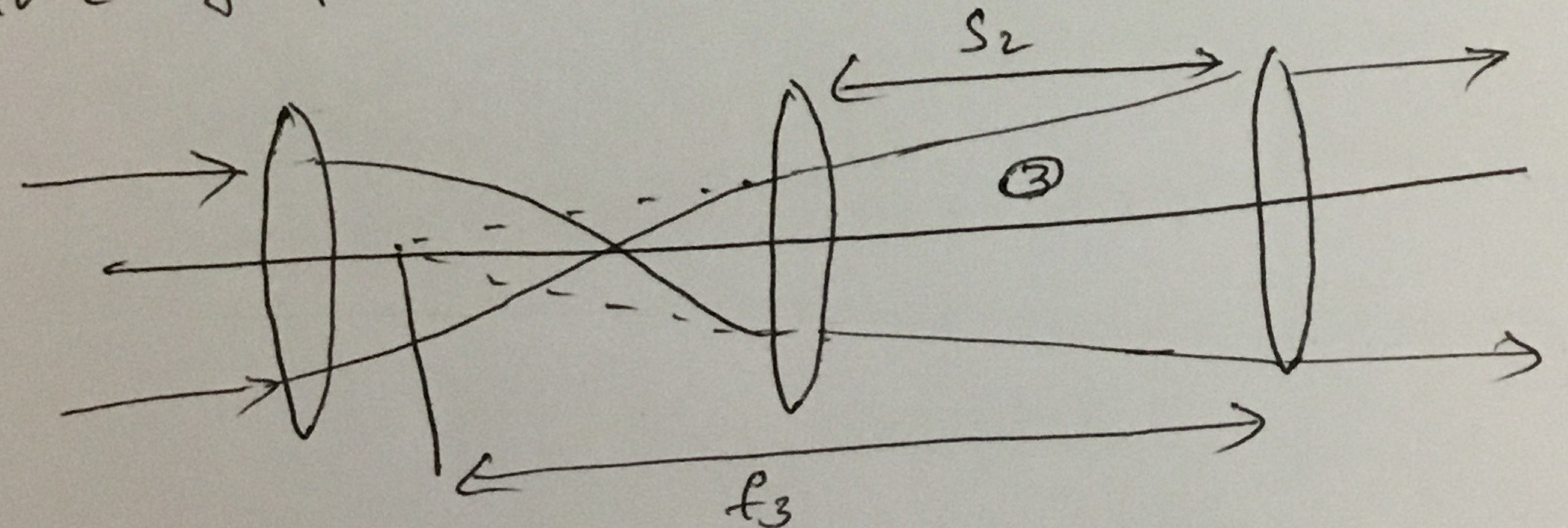


$$\frac{f_1}{S_1 - f_1} = \frac{d}{h}$$

$$h = \frac{d\,(S_1 - f_1)}{f_1}$$

$$\text{Volme } ② = \frac{\pi}{3}\left(\frac{d\,(S_1 - f_1)}{f_1}\right)^2 (S_1 - f_1)$$

3 volme of part 3



volme of 3

$$= \frac{\pi}{3}\left(\frac{d}{2}\right)^2 f_3 - \frac{\pi}{3}\left(\frac{d\,(S_1 - f_1)}{f_1}\right)^2 (f_3 - S_2).$$

$\therefore$ cloaking volme =

$$\pi \frac{d^2}{4} L - \frac{\pi}{3}\frac{d^2}{4} f_1 - \frac{\pi}{3}\left(\frac{d\,(S_1 - f_1)}{f_1}\right)^2 (S_1 - f_1) + \frac{\pi}{3}\frac{d^2}{4} f_3 - \frac{\pi}{3}\left(\frac{d(S_1 - f_1)}{f_1}\right)^2 \times (f_3 - S_2).$$

variables: L, f_1, d, S_1, S_2, f_3

constants: L, d.

and : $S_1 + S_2 = L$ (remaining variables: (S_2, f_1, f_3)).