Volumetric Bounds of a Paraxial Ray-Optics Cloak

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Cloaking Introduction

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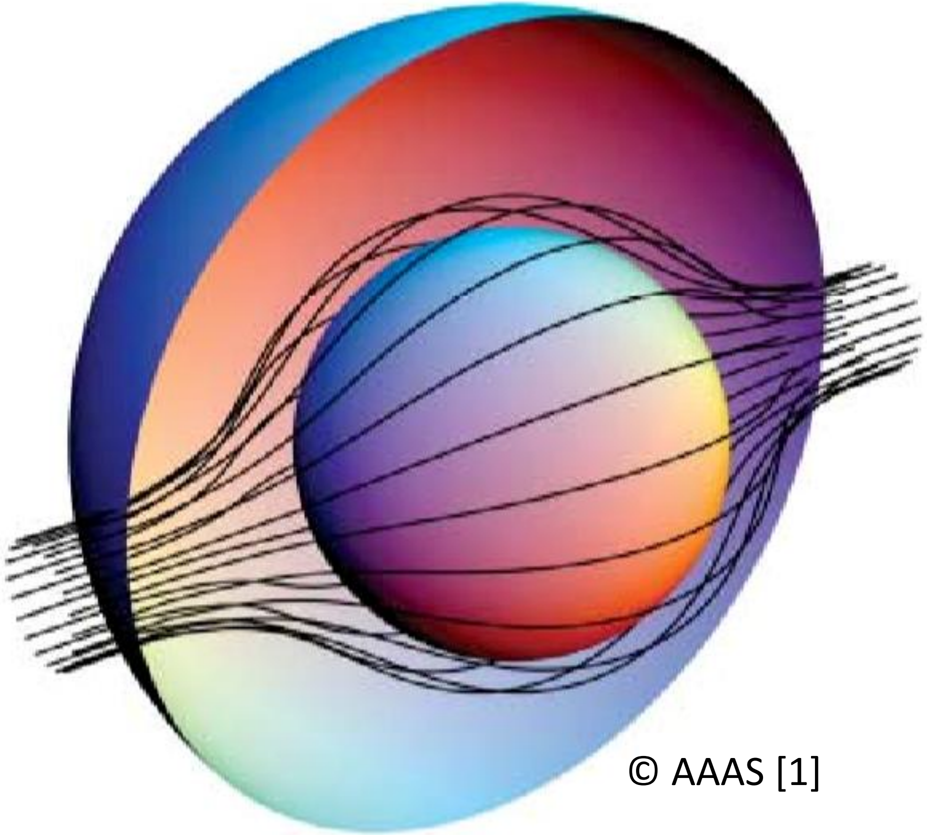

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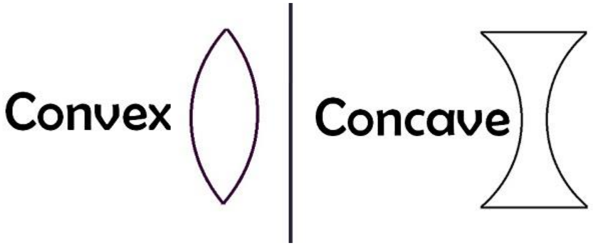
[1] Pendry, J. B., Shurig, D. & Smith D. R. Controlling electromagnetic fields. *Science* **312,** 1780–1782 (2006).

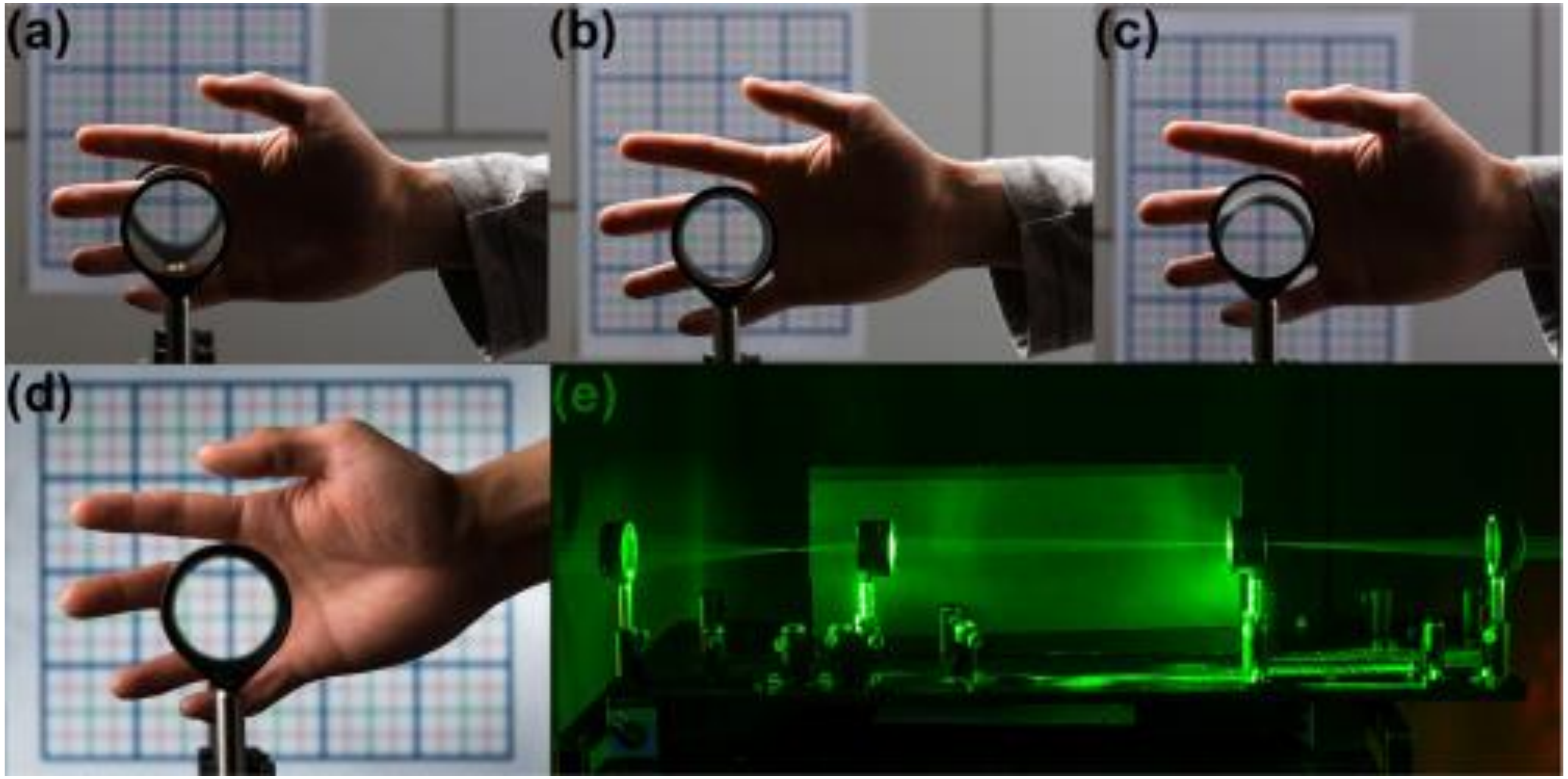
Cloaking System Requirements

1. The background image should remain normal: No Magnification and Inversion

2. The background image should remain normal for all the distance: Afocal

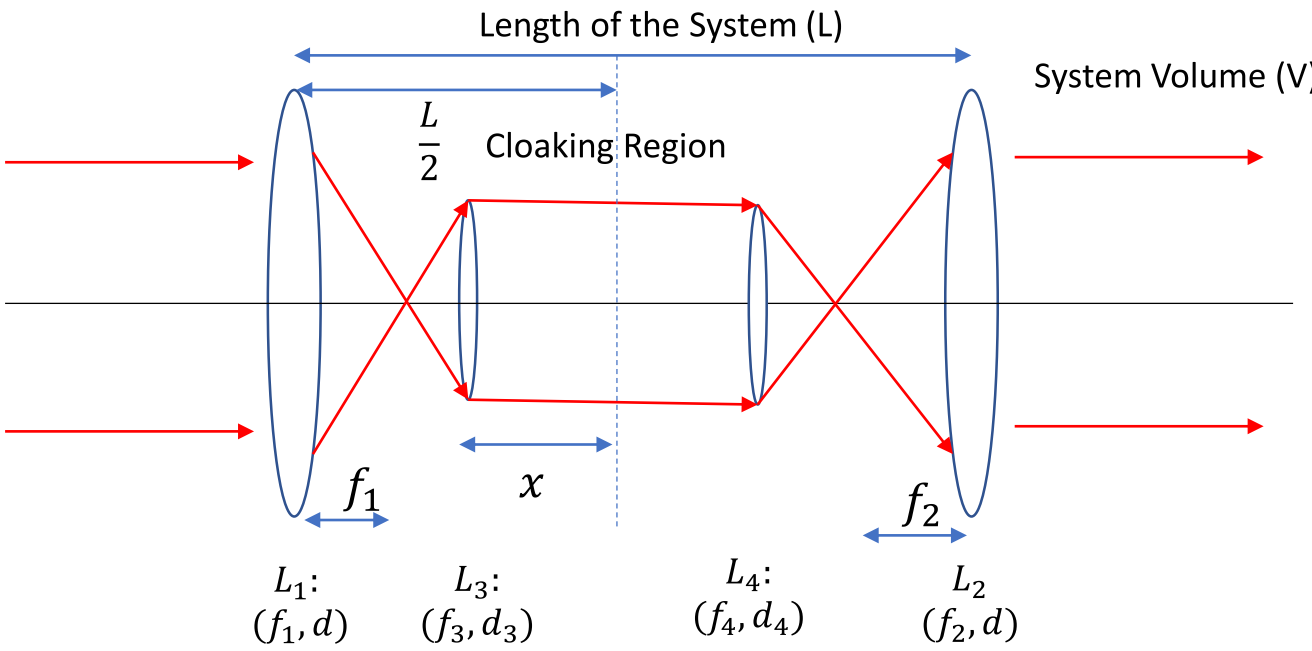


Paraxial Cloaking in the literature

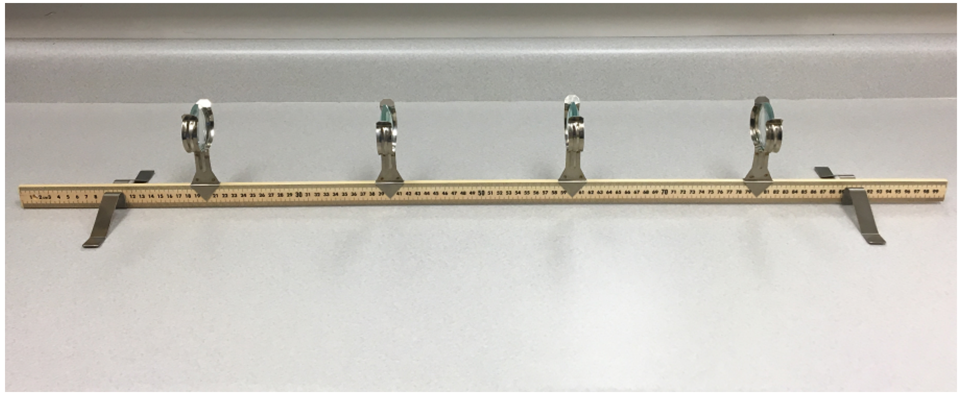


[1] Joseph S. Choi and John C. Howell, "Paraxial ray optics cloaking," Opt. Express 22, 29465-29478 (2014)

Paraxial Cloaking in the literature



System Volume (V) = $\frac{\pi d^2}{4} L$



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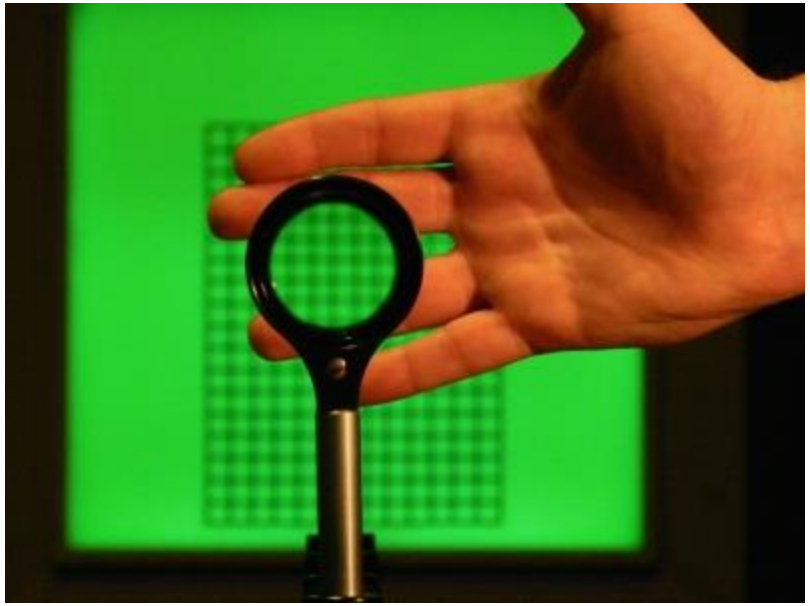
[1] Joseph S. Choi and John C. Howell, "Paraxial ray optics cloaking," Opt. Express 22, 29465-29478 (2014)

Paraxial Cloaking in the literature



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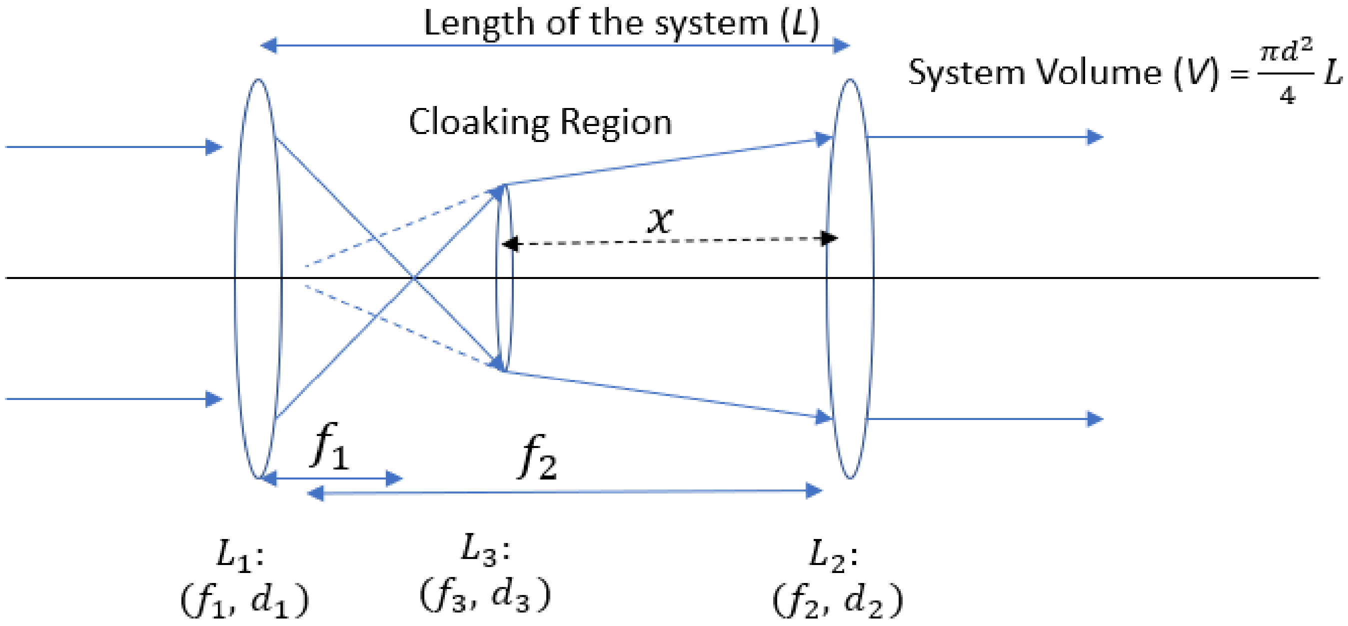
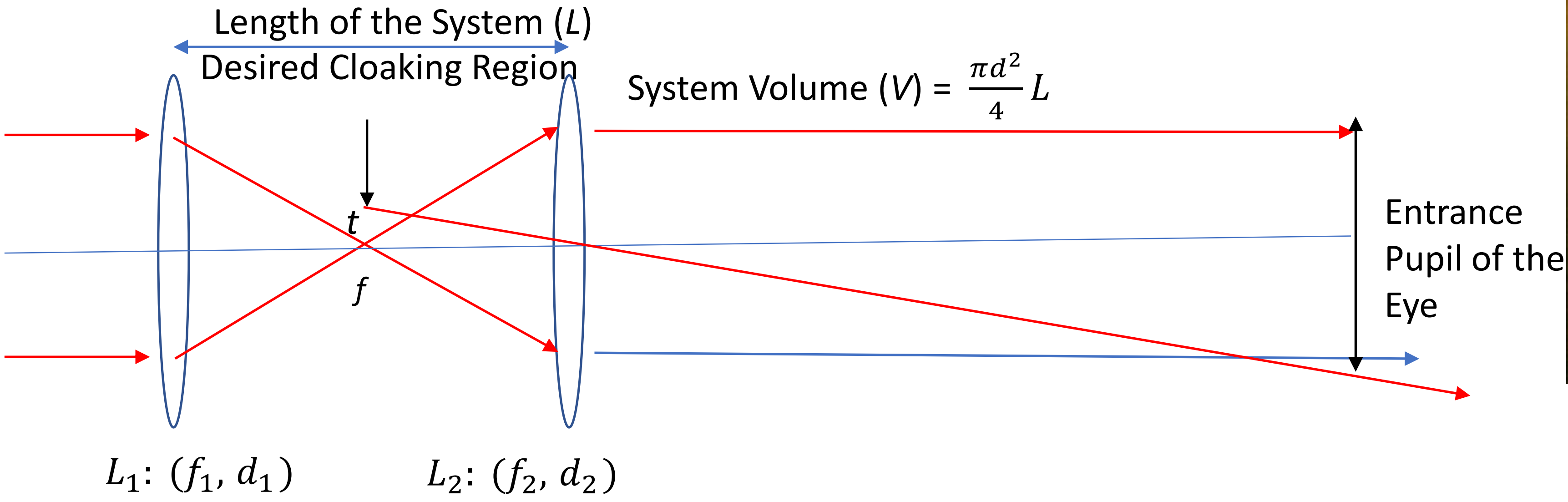
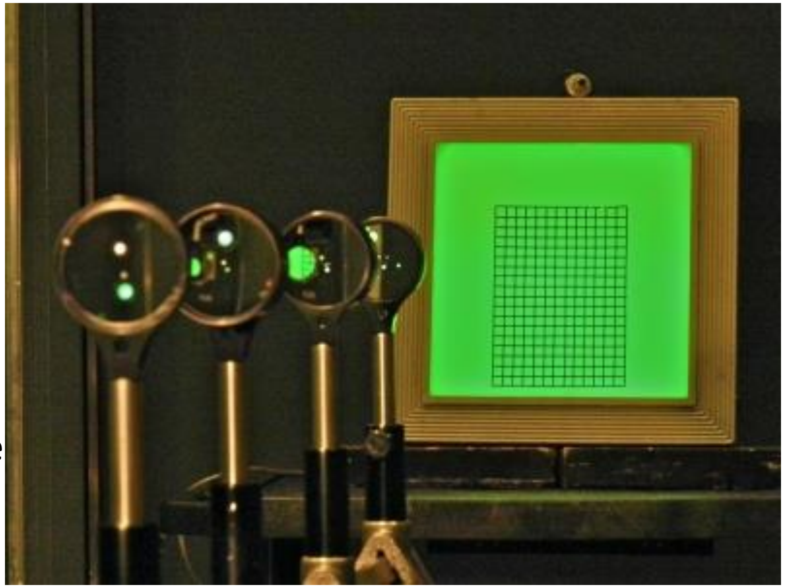


[1] Joseph S. Choi and John C. Howell, "Paraxial ray optics cloaking," Opt. Express 22, 29465-29478 (2014)

Paraxial Cloaking in the literature: Identifying the Opportunities

1. Matrix Method.
2. Cloaking Volume Quantification
3 2-lens and 3-lens cloaking systems.





$$\begin{bmatrix} 1 & 0 \\ -1/f_3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & t_2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1/f_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & t_1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix} .$$

[1]

[1] Joseph S. Choi and John C. Howell, "Paraxial ray optics cloaking," Opt. Express 22, 29465-29478 (2014)

Two Lens Cloaking System: Design

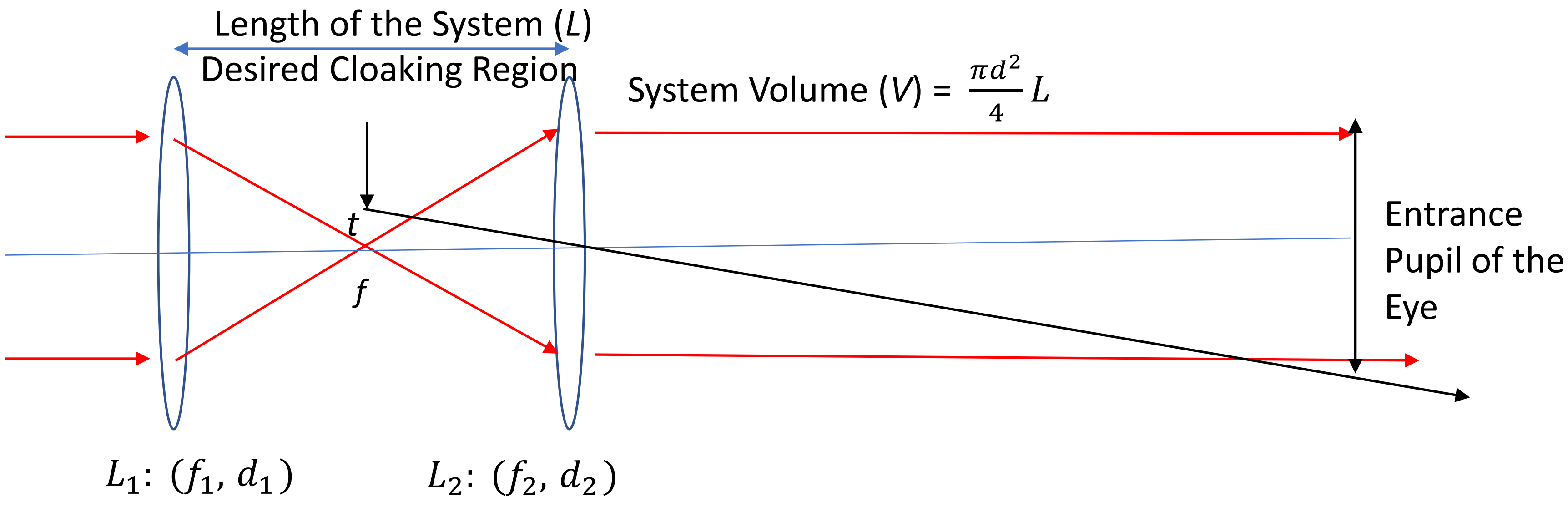


Image is inverted

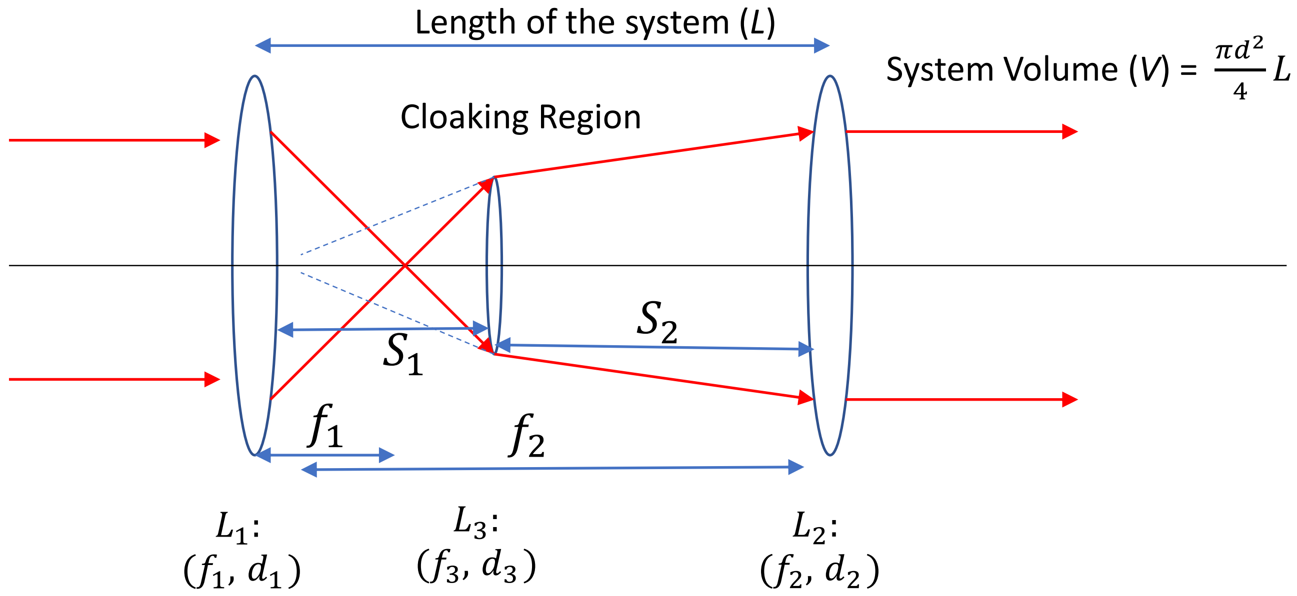
Afocal system requirement b. f. l and f. f. l

$$b.f.l. = \frac{f_2\,(d - f_1)}{d - (f_1 - f_2)} \qquad d = (f_1 - f_2)$$

Similar triangle argument

The object to be cloaked gets closer to the focal point, entrance pupil moves way.

Three Lens Cloaking System : Design



Assumptions:

1. Thin lens formulations
2. Volume contribution of lenses
3. Entrance pupil is placed appropriately
4. No aberrations

Design:

Afocal requires image of L_3 be at f_2

Image due to L_1 be object to L_3 at $\frac{f_3}{2}$

Choose:

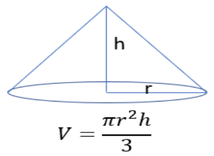
$$f_3 = \frac{f_2}{2} \qquad f_1 = f_2 = f$$

Solution leads to

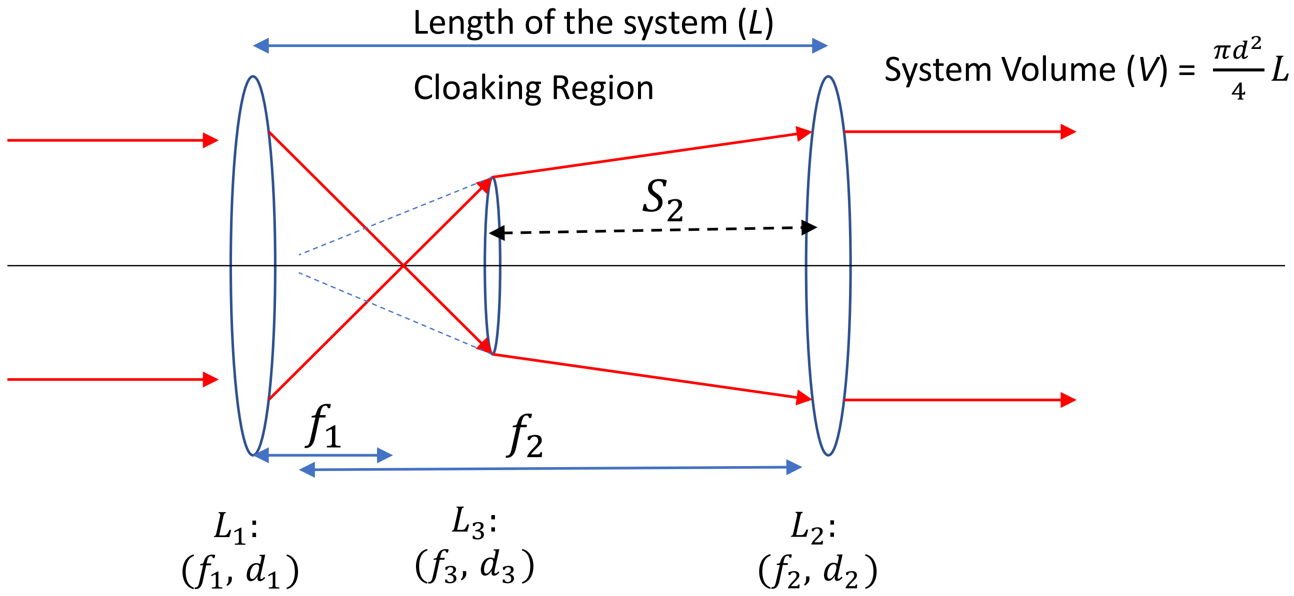
$$L = \frac{7}{4} \qquad S_2 = \frac{1}{2} \qquad f_3 = \frac{1}{2}f$$

$$V_{cloaking} = \pi\left(\frac{d^2L}{4} - \frac{d^2f}{6} + \frac{1}{3}\left(d * \frac{S_1 - f}{2f}\right)^2 * (2 * f - L)\right) \qquad f_1 = f_2 = f$$

Volume of a cone equation



Three Lens Cloaking System : Design



Design:

Afocal requires image of L_3 be at f_2

Image due to L_1 be object to L_3 at $\frac{f_3}{2}$

Choose:

$$f_3 = \frac{f_2}{2} \qquad f_1 = f_2 = f$$

Solution leads to

$$L = \frac{7}{4} \qquad S_2 = \frac{1}{2} \qquad f_3 = \frac{1}{2} f$$

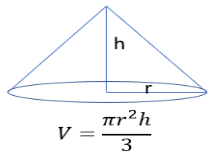
$$V_{cloaking} = \pi\left(\frac{d^2 L}{4} - \frac{d^2 f}{6} + \frac{1}{3}\left(d * \frac{S_1 - f}{2f}\right)^2 * (2 * f - L)\right)$$

$$V_{cloaking} = \pi\left(\frac{d^2 (S_1 - x)^2 (x - L + f_2)}{4x^2} - \frac{d^2 x}{6} + \frac{d^2 L}{4}\right)$$

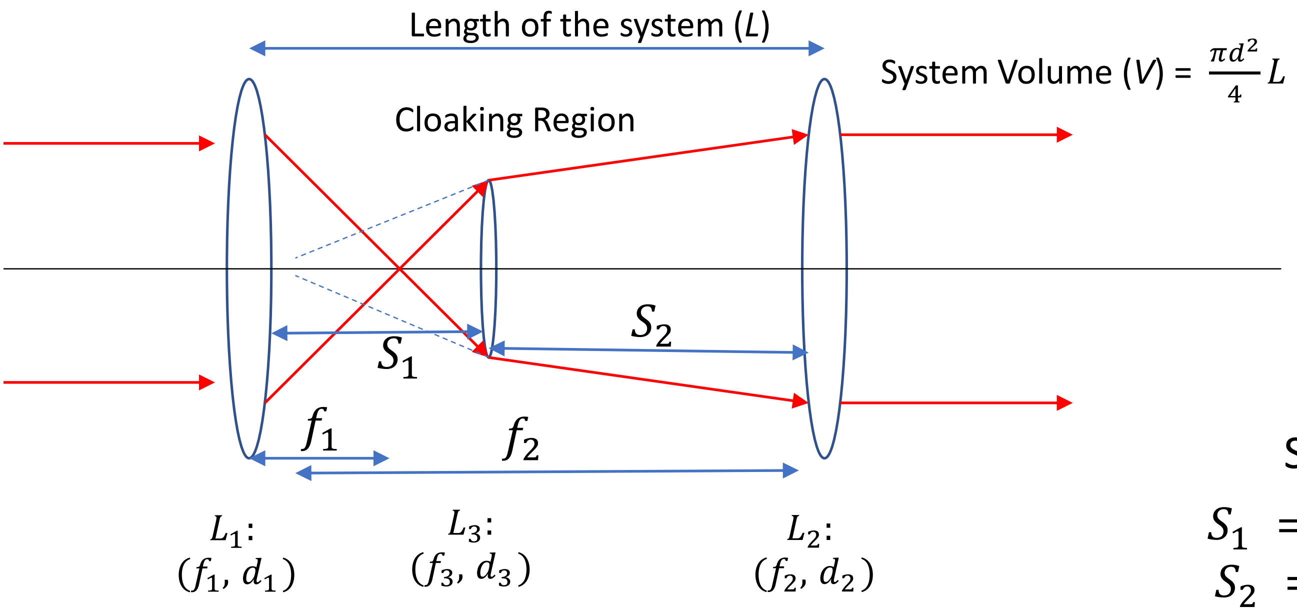
$Put\ f_1 = f = x, \quad L = 7/4, \quad S_1 = 0.5.$

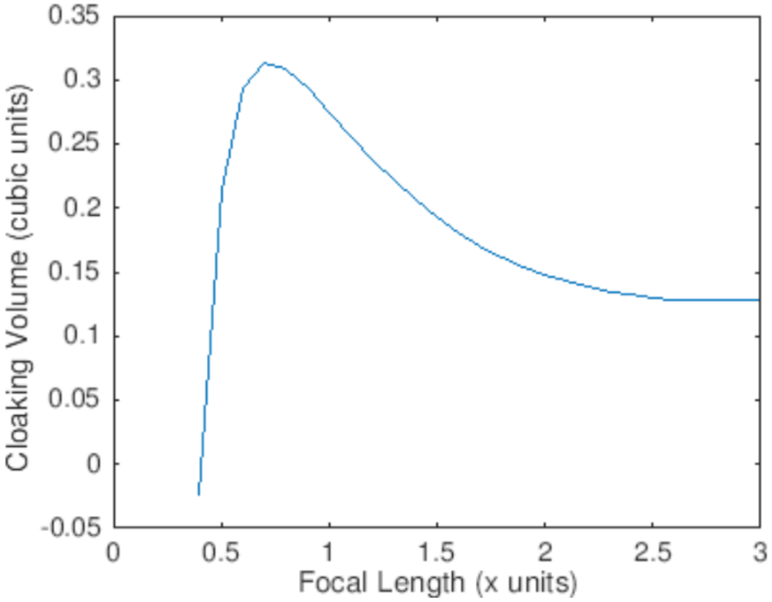
Solve cubic equation, find $x(f_1)$ to maximize the volume and then find $f_3 = S_1 - f$

Volume of a cone equation



Three Lens Cloaking System : Design





Solution:

$$S_1 = 1.25\ units,$$
$$S_2 = 0.5\ units,$$
$$L = 1.75\ units,$$
$$f_1 = 0.6\ units,$$
$$f_2 = 0.6\ units$$
$$f_3 = 0.3\ units$$

Design:

Afocal requires image of L_3 be at f_2

Image due to L_1 be object to L_3 at $\frac{f_3}{2}$

Choose:

$$f_3 = \frac{f_2}{2} \qquad f_1 = f_2 = f$$

Solution leads to

$$L = \frac{7}{4} \qquad S_2 = \frac{1}{2} \qquad f_3 = \frac{1}{2}f$$

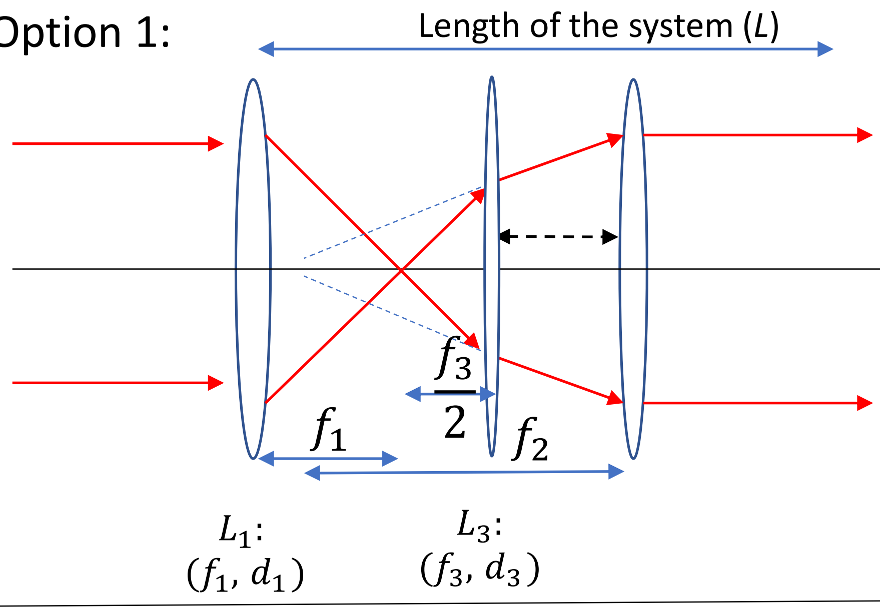
$$f = 0.6\ from\ volume\ plot$$

$$V_{cloaking} = \pi\left(\frac{d^2(S_1 - x)^2(x - L + f_2)}{4x^2} - \frac{d^2x}{6} + \frac{d^2L}{4}\right)$$

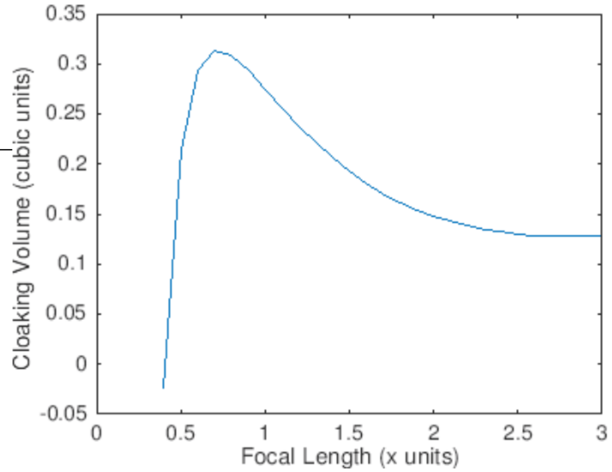
$$x = \frac{s_1\sqrt{s_1^2 - 2ls_1 + 3l^2} + s_1^2 + ls_1}{4s_1 - 2l}$$

Three Lens Cloaking System : Design Options

Option 1:



System Volume (V) = $\frac{\pi d^2}{4} L$



Design:

Afocal requires image of L_3 be at f_2

Image due to L_1 be object to L_3 at $\frac{f_3}{2}$

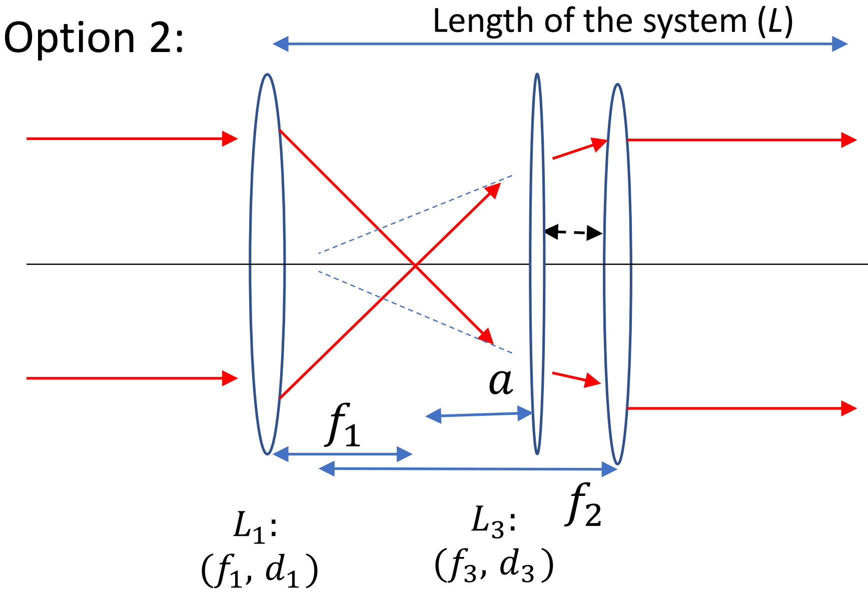
Choose:

$$f_3 = \frac{f_2}{2} \qquad f_1 = f_2 = f$$

Solution leads to

$$L = \frac{7}{4} \qquad S_2 = \frac{1}{2} \qquad f_3 = \frac{1}{2} f$$

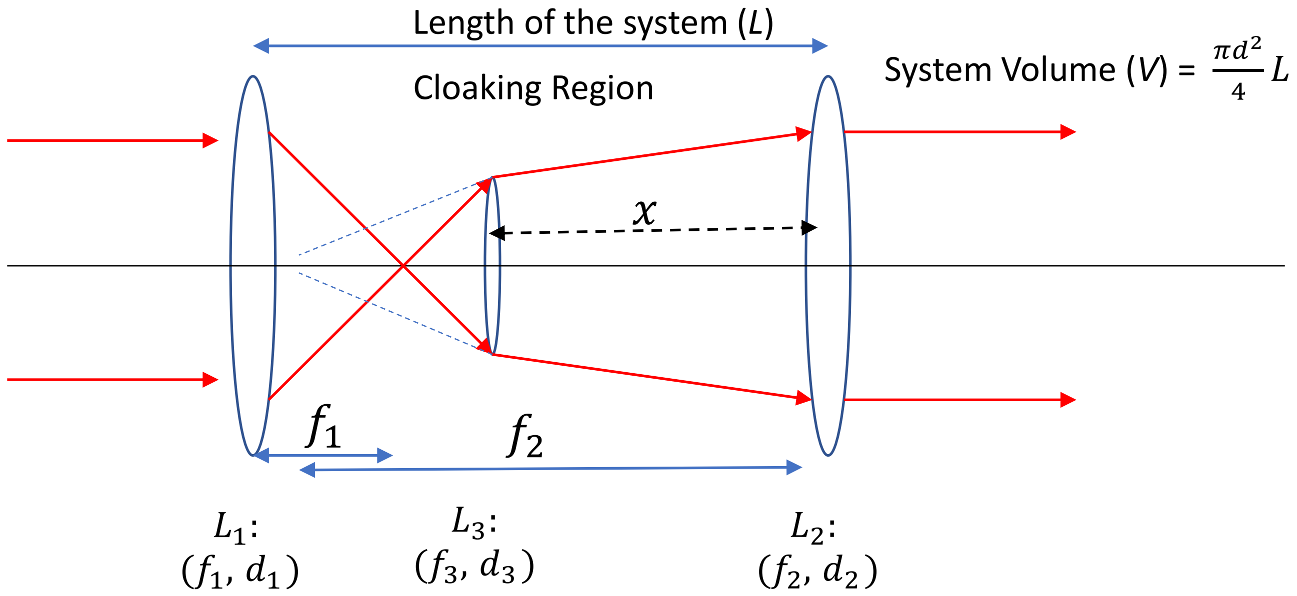
Option 2:

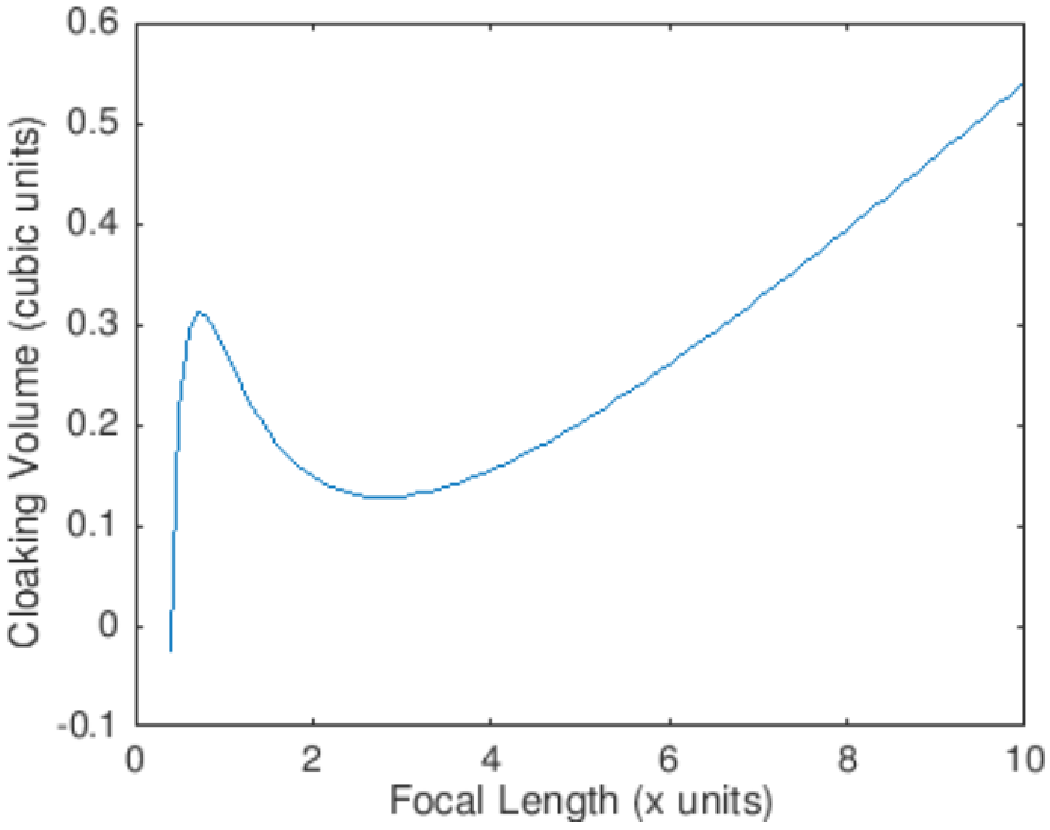


System Volume (V) = $\frac{\pi d^2}{4} L$

$$a = \frac{3}{4} f_1$$

Three Lens Cloaking System : Caution

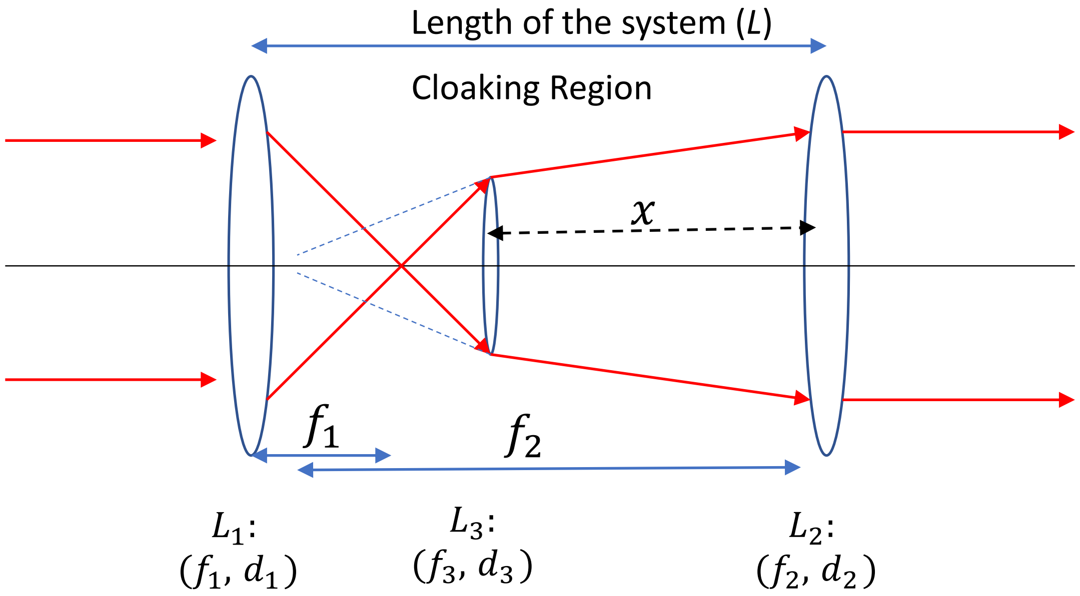


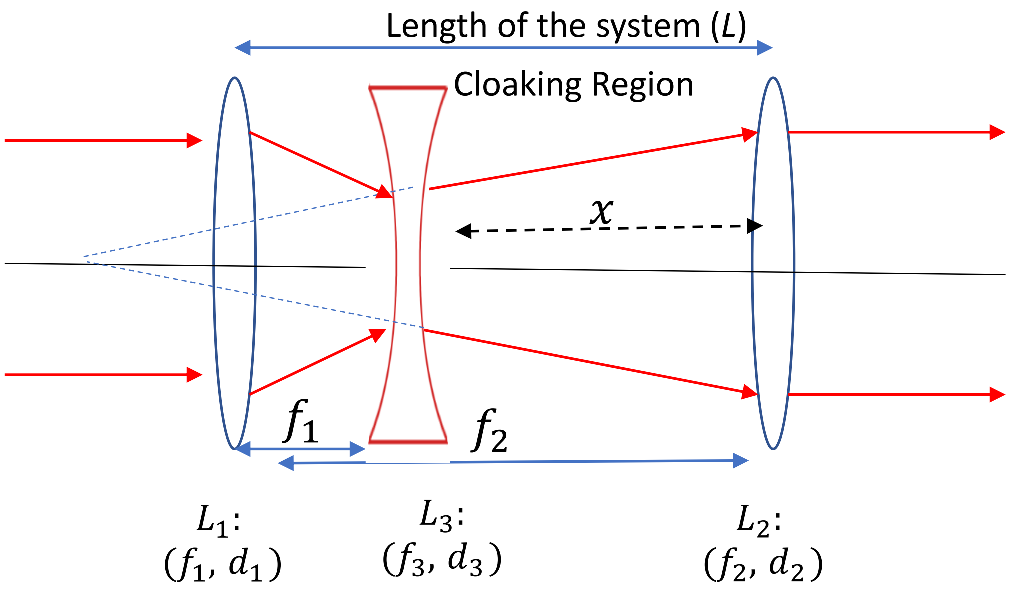


Other Drawbacks?

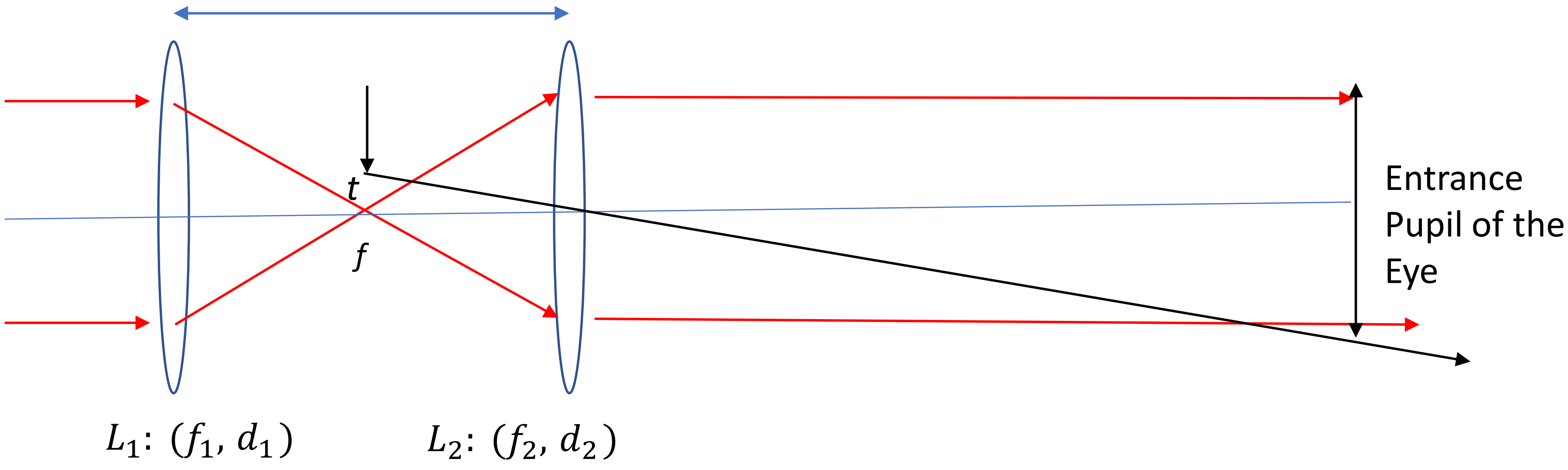
$$V_{cloaking} = \pi\left(\frac{d^2(S_1 - x)^2(x - L + f_2)}{4x^2} - \frac{d^2 x}{6} + \frac{d^2 L}{4}\right)$$

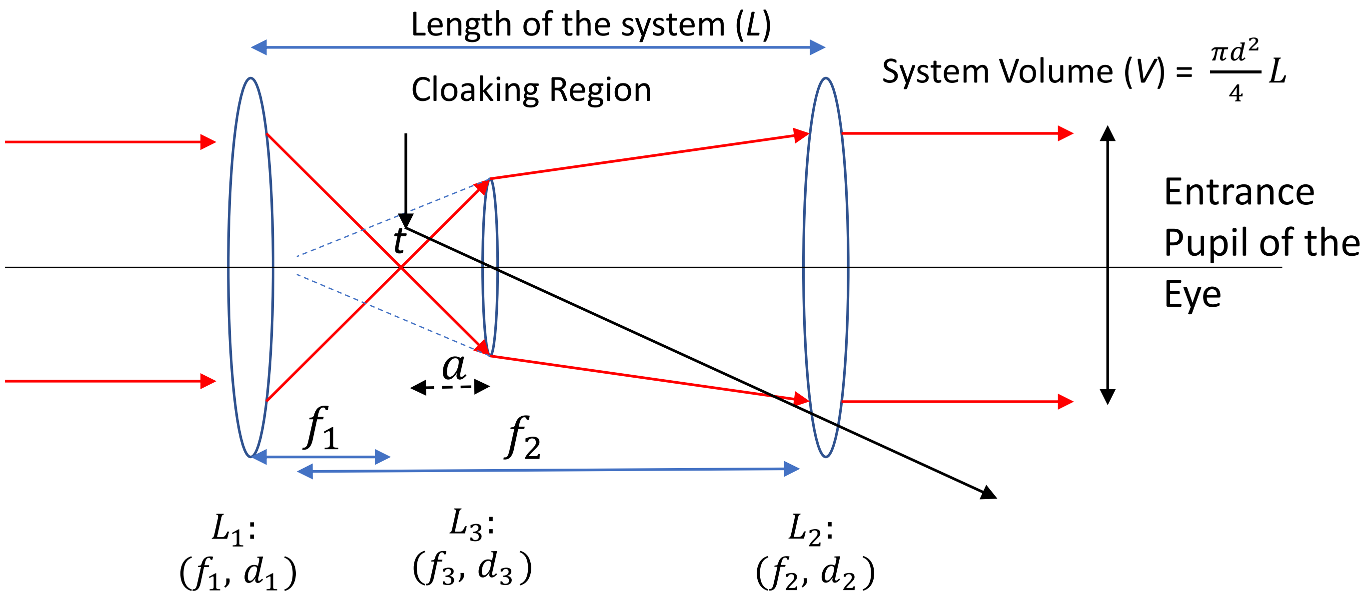
Three Lens Cloaking System : Image Inversion





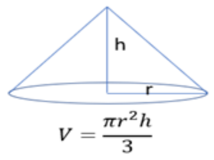
Three Lens Cloaking System : Entrance Pupil



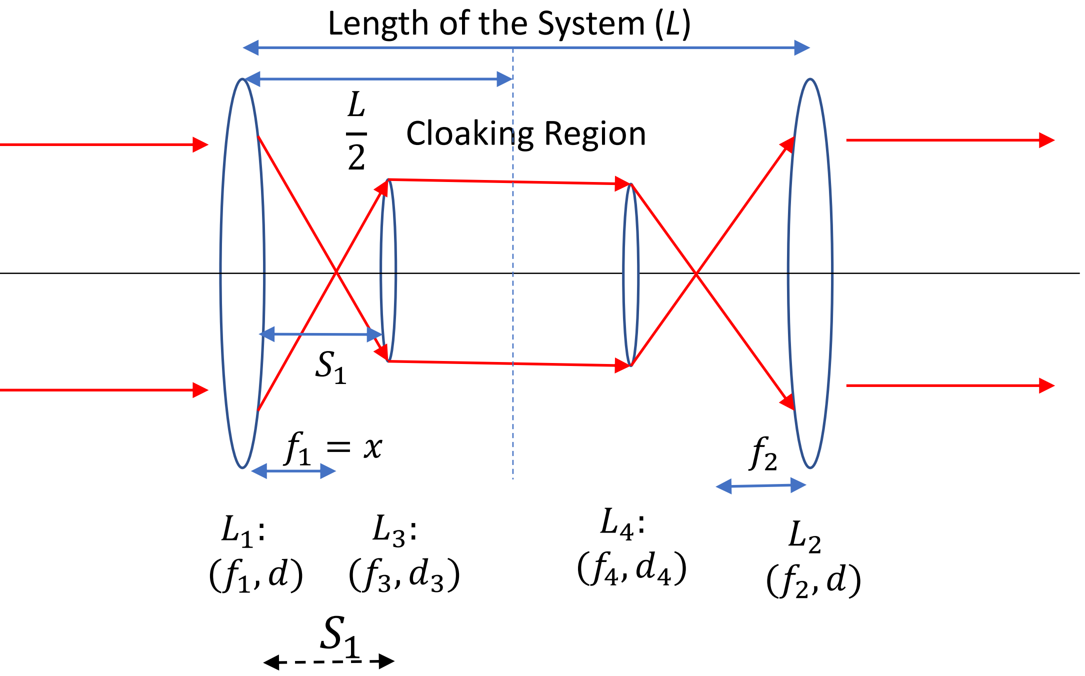


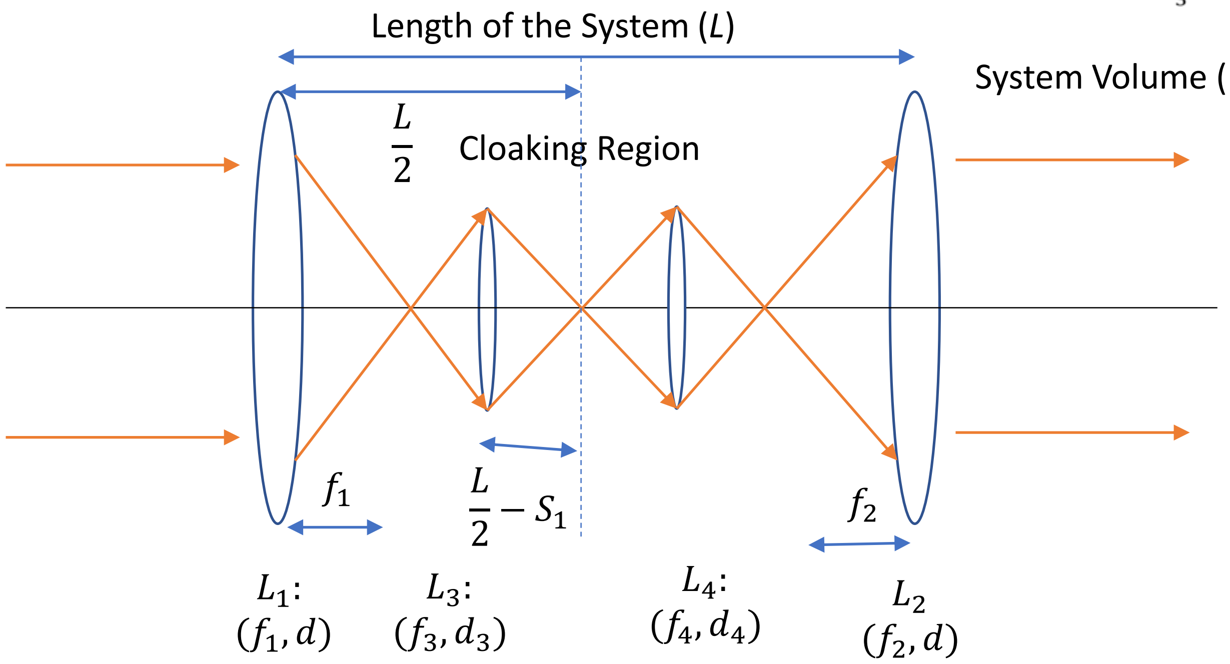
Four Lens Cloaking System

Volume of a cone equation



System Volume (V) = $\frac{\pi d^2}{4} L$





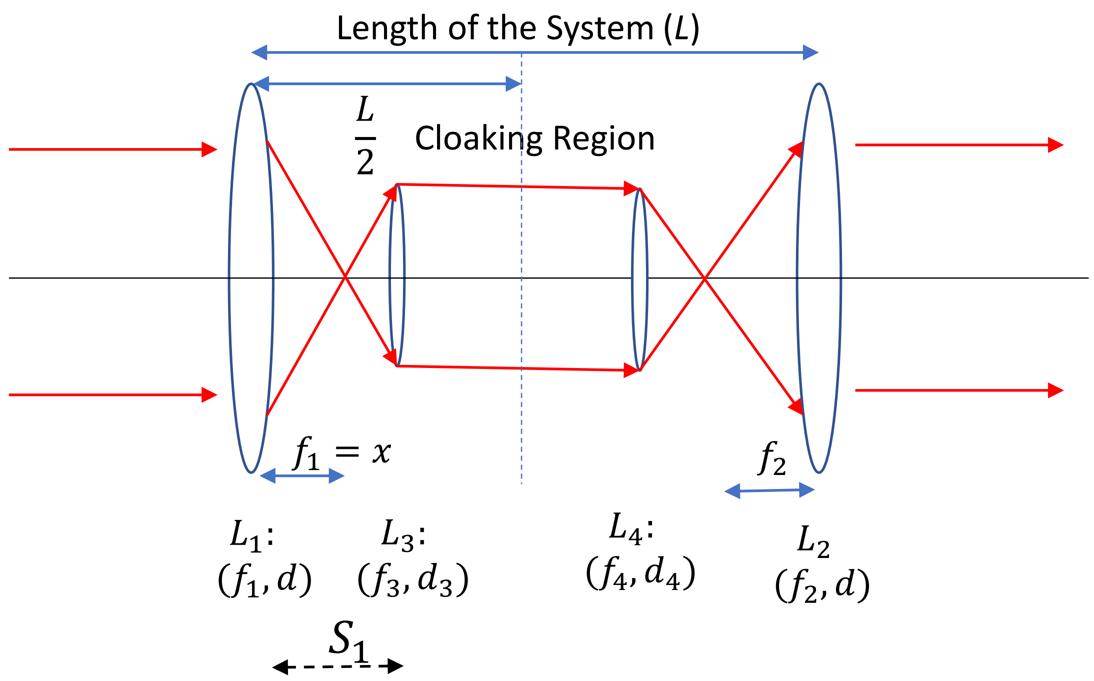
To simplify the construction: Assume symmetry in the design

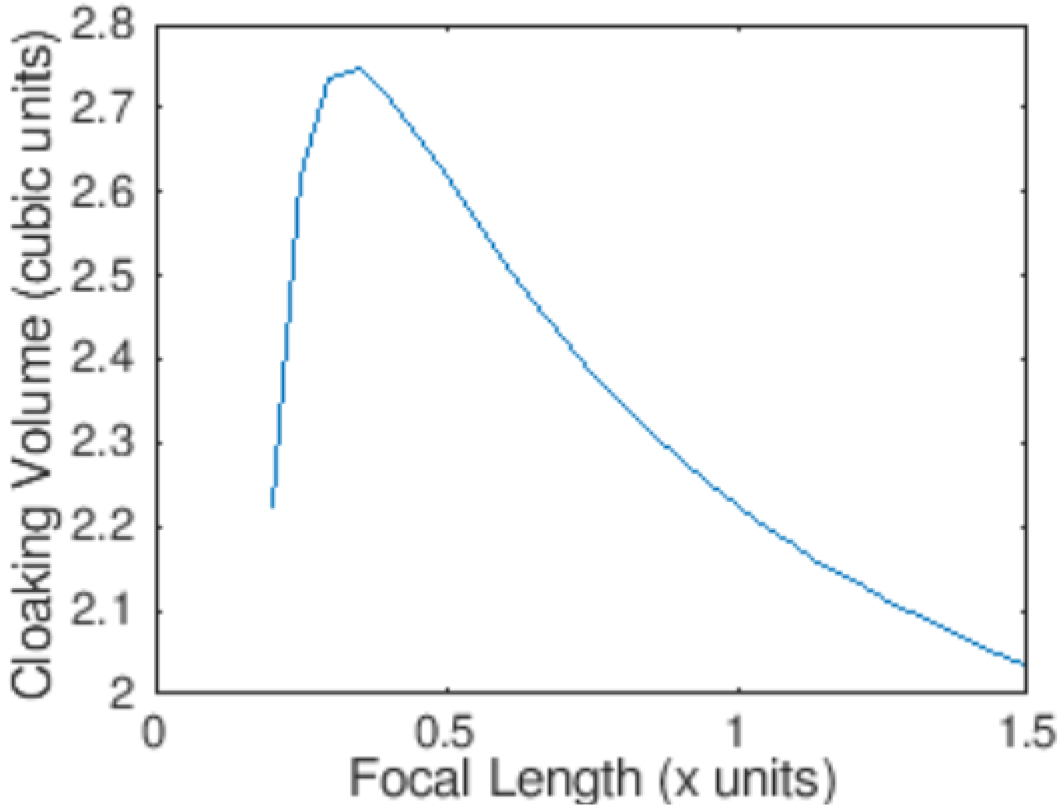
$$V_{cloaking} = \pi\left(\frac{d^2L}{4} - \frac{d^2 f_1}{12} - \left(d * \frac{S_1 - f_1}{2f_1}\right)^2 * \left(\frac{S_1 - f_1}{3} + \frac{L}{2} - S_1\right)\right)$$

$$S_1 - f_1 = \frac{L}{2} - S_1$$

$$V_{cloaking} = \pi\left(\frac{d^2L}{4} - \frac{d^2 f_1}{12} - \frac{1}{3}\left(d * \frac{S_1 - f_1}{2f}\right)^2 * \left(\frac{L}{2} - f_1\right)\right)$$

Four Lens Cloaking System : First System





$$V_{cloaking} = \pi\left(\frac{d^2 L}{4} - \frac{d^2 f_1}{12} - \left(d * \frac{S_1 - f_1}{2f_1}\right)^2 * \left(\frac{S_1 - f_1}{3} + \frac{L}{2} - S_1\right)\right)$$

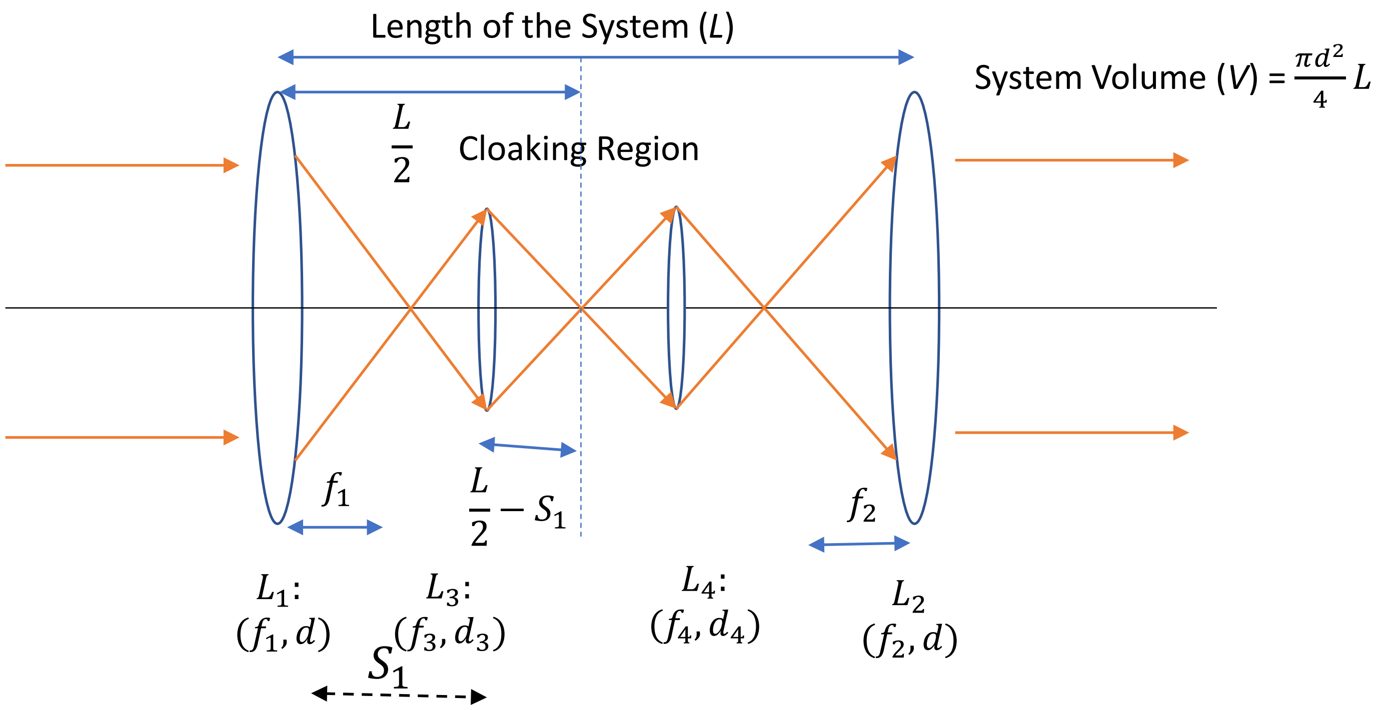
$Put\ f_1 = x$, $\quad \frac{L}{2} = 1$, $\quad S_1 = 0.5.$

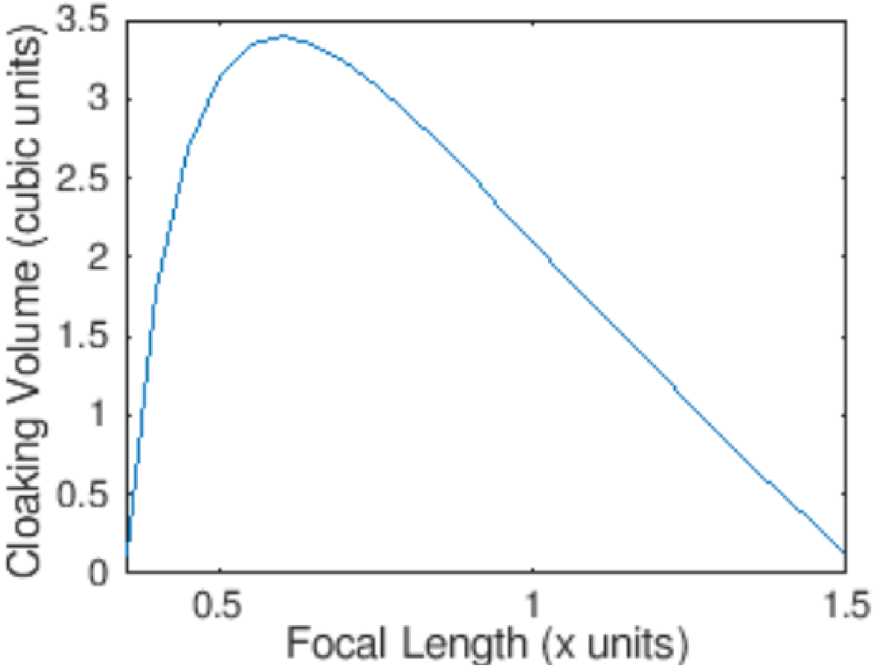
Solve cubic equation, find $x(f_1)$ to maximize the volume and then find $f_3 = S_1 - f$

Solution:

$L = 1\ units,$
$S = 0.5\ units,$
$f_1 = 0.38\ units$
$f_3 = 0.12\ units$

Four Lens Cloaking System : Second System





$$V_{cloaking} = \pi\left(\frac{d^2 L}{4} - \frac{d^2 f_1}{12} - \frac{1}{3}\left(d * \frac{S_1 - f_1}{2f}\right)^2 * \left(\frac{L}{2} - f_1\right)\right)$$

$$Put\ \ f_1 = x\,, \qquad \frac{L}{2} = 2S_1 - x, \qquad S_1 = 1$$

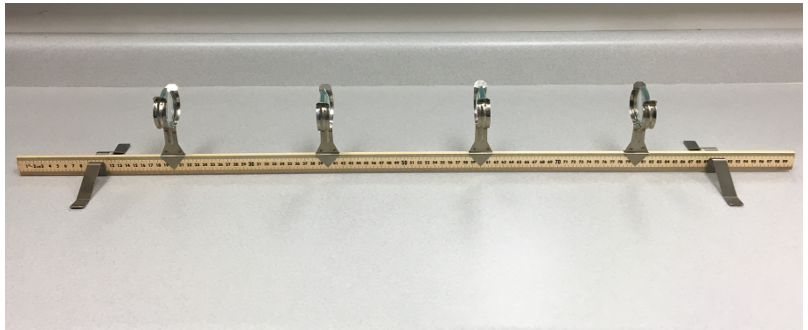
Solve cubic equation, find $x(f_1)$ to maximize the volume and then find $f_3 = S_1 - f$

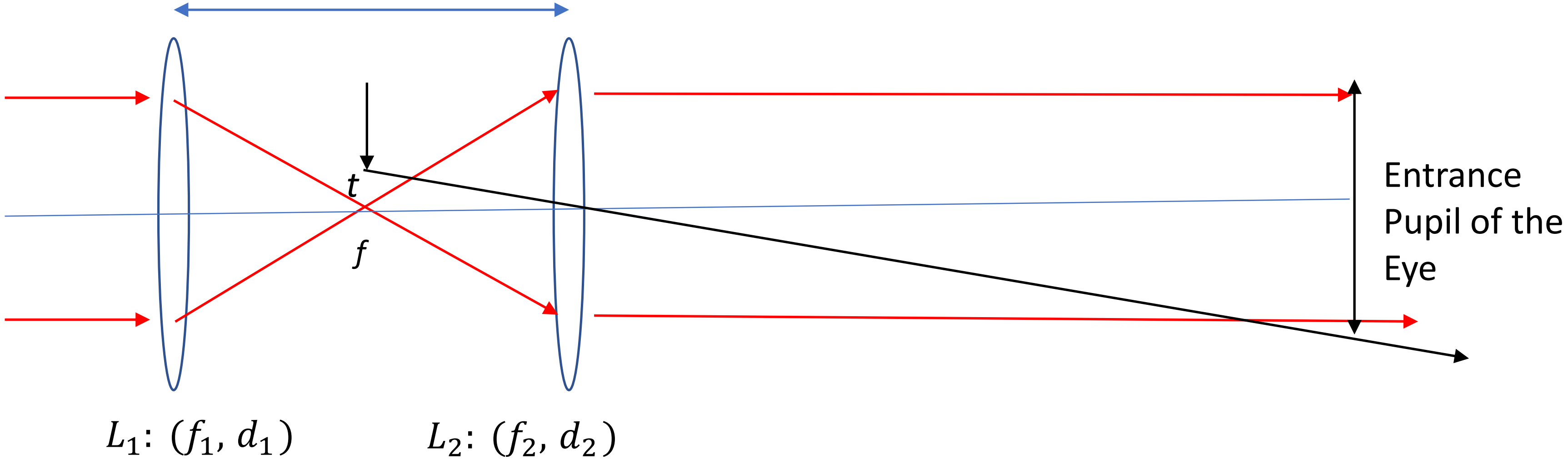
Solution:

$S_1 = 1\ units,$
$f_1 = 0.6\ units,$
$L = 3.8\ units$
$f_3 = 0.4\ units$

Conclusion

1. Cloaking
2. Requirements of the cloaking optical system
3. Literature survey and opportunities
4. Analysis of 2, 3, 4 lens cloaking system using paraxial assumptions.
5. System design to maximize the volume





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