First-order Calculation of Band Structure of One-Dimensional Photonic Crystal

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Abstract

First Brillouin zone of one-dimensional photonic crystal is computed by transfer matrix analysis. Semiconductor heterostructure is considered as the material for the periodic arrangement, and Adachi's model is taken account for better accuracy of the result by considering the dependence of refractive indices on operating wavelength, material composition and bandgap of the material. Dependence of extension of Brillouin zone on the structural parameters is analyzed. Result is important for understanding the band structure and photonic properties of the crystal.

Introduction

Propagation of electromagnetic wave through the structures with periodicities along the direction of it suffers dispersive effects. These can be exhibited in photonic crystal where permittivity of the dielectric material undergo periodic changes, result in photonic bandgap [1-2]. This unique property helps to block the propagation of some wavelength, and allows other spectra; thus can effectively be considered as optical bandpass filter [3]. This phenomenon can be explained by the principle of Bragg's reflection [4], where we assume that wavelength of light will be of the order of layer dimensions [5]. Materials exhibit photonic bandgap can be used in designing photonic crystal fiber [6], which may replace the conventional optical fiber due to its highly improved performance from communication point-of-view [7]. It is used to construct optical transmitter [8], switch [9], waveguide [10] etc.

Bandgap of 2D photonic crystal is studied by varying column roundness by Hillebrand [11] using plane-wave expansion method. Recently, finite-difference-time-domain method is used to analyze the forbidden region of photonic crystal with different geometries [12]. Zhao calculated the width of bandgap [13] using Bragg's principle of reflection. Men optimized the computational problem using semidefinite programming and subspace methods [14]. Evolutionary algorithm [15] and level-set method [16] are also used for design of large bandgap crystal. However, for all the cases, SiO_2-air composition is considered as the material system for calculation. In the present paper, band structure of one-dimensional photonic crystal is calculated by first-order method using transfer matrix technique, and $GaN/Al_xGa_{1-x}N$ material is considered for the system. Layer dimensions along with material compositions are varied to observe the effect on First Brillouin zone. The introduction of semiconductor heterostructure adds novelty of the work in the respect that the structure can be treated as photonic multiple quantum well for electronic applications [17], and also it provides better performance as optical transmitting/emitting device [18]. This structure can also be embedded in the active region of VCSEL, which adds tunability of its filter characteristics [19]. Result is important for basic understanding of band structure of 1D photonic crystal.

Mathematical Modeling

Consider the smallest unit of 1D photonic crystal structure comprising of $GaN/A_{lx}Ga_{1-x}N$ material composition where forward and backward propagating waves are given by-

$$a_2 = t_{21}a_1 + r_{12}b_2 \quad (1)$$

$$b_1 = t_{12}b_2 + r_{21}a_1 \quad (2)$$

where r_{ij} and t_{ij} are reflectivity and transmissivity in passing from layer i to layer j.

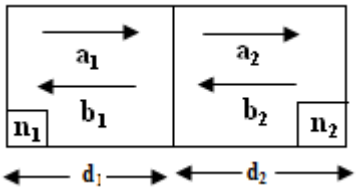

Fig 1: Schematic picture of forward and backward waves in smallest unit of 1D photonic crystal

From the wave equations, transfer matrix corresponding to the interface can be obtained as

$$M^{T}_{1,2} = \frac{1}{t}\begin{pmatrix} 1 & r_{21,12} \\ r_{21,12} & 1 \end{pmatrix} \tag{3}$$

Considering the phase factor of the field propagating through uniform medium, propagation matrix is given as

$$P_{1,2} = \begin{pmatrix} \exp[\,jk_{1,2}d_{1,2}\,] & 0 \\ 0 & -\exp[\,jk_{1,2}d_{1,2}\,] \end{pmatrix} \tag{4}$$

where d_i is the propagation length in i^{th} layer, and k_i is the wavevector in that layer. Thus, transfer matrix for the elementary cell is

$$M = M^{T}{}_{1}P_{1}M^{T}{}_{2}P_{2} \tag{5}$$

For a perfectly periodic medium composed of N such elementary cells, the total transfer matrix for such a structure is

$$M_{tot} = M_{N} \tag{6}$$

Since eigenvectors of the periodic structure are the Bloch modes, hence we can write

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = \exp(-jkz)\begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix} \tag{7}$$

But from transfer matrix analysis,

$$\begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix} = M\begin{bmatrix} a_n \\ b_n \end{bmatrix} \tag{8}$$

Hence we can obtain the eigenvalue equation as

$$M\begin{bmatrix} a_n \\ b_n \end{bmatrix} = \exp(\,jk\Lambda)\begin{bmatrix} a_n \\ b_n \end{bmatrix} \tag{9}$$

where Λ is the periodicity of the grating. Considering exponential term as λ, we may write

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0 \tag{10}$$

for which solution is given by

$$\lambda = \frac{1}{2}(a+d) \pm \sqrt{\left[\frac{1}{2}(a+d)\right]^{2} - 1} \tag{11}$$

Thus dispersion relation for the Bloch mode is

$$k = \frac{1}{\Lambda}\cos^{-1}\left[\frac{1}{2}(a+d)\right] \tag{12}$$

Eq. (12) states that once the transfer matrix for the single elementary cell (M) is determined, $\omega-k$ dispersion relation for the infinite periodic structure can easily be computed.

Result and Discussion

Using Eq. (12), band structure of the photonic crystal is computed and plotted. Fig 2 shows the wavenumber with normalized frequency for different material composition. From the plot, it is seen that with increase of AlN percentage, Brillouin zone is first quenched, but further increment makes its expansion. This is due to the fact that with increase of x, the mole percentage of AlN in AlGaN material increases, which causes decrease of refractive index following Adachi's model. In this context, we consider the refractive indices of the materials are function of incident wavelength, material composition and bandgap of the semiconductor material. This enhances the effective index difference between the constituent materials of the periodic structure. This affects the interface reflectivity, and the transfer matrices based on the reflection coefficients. Now the Bloch dispersion relation is a function of reflectivity matrices. Hence for a optimum difference of refractive indices, Brillouin zone takes the minimum size.

This is represented from the first two subplots of Fig 2. After that, it starts once again to increase with increase of x. Hence E-k diagram can be accurately controlled by changing the material composition of the periodic arrangement.

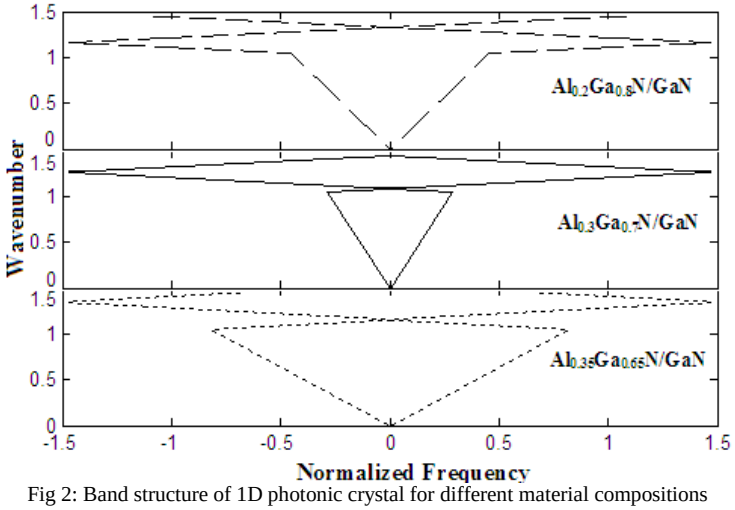

Fig 2: Band structure of 1D photonic crystal for different material compositions

In Fig 3, band structure is plotted with different layer dimensions. In Fig 3a, dimension of lower refractive index material (d_1) is varied keeping the other material dimension constant. It is observed form the graph that with increase of d_1, Brillouin zone first increases, then starts reducing. Thus E-k diagram becomes modified. Similar variation is observed when d_2 (dimension of higher refractive index material) is changed. This is plotted in Fig 3b. It may be noted from the comparative study of Fig 3a and Fig 3b that by changing d_2, size of the Brillouin zone can be made much wider, which may not be possible by varying d_1.

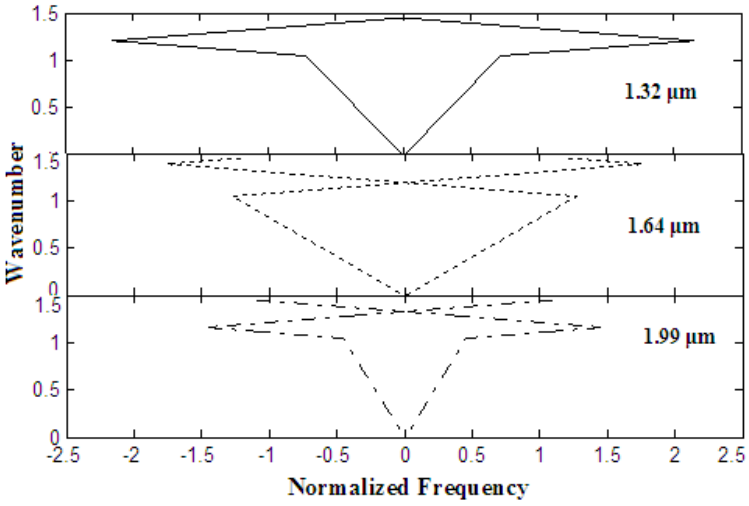

Fig 3a: Band structure of 1D photonic crystal for different values of d_1 keeping d_2 constant

Thus, tuning of layer dimension has a profound effect on Brillouin zone of the photonic crystal structure. However, the thickness of lower refractive index material has little tuning ability. This is because wavevector associated with the higher bandgap material is less in comparison with the wavevector associated with the lower bandgap material.

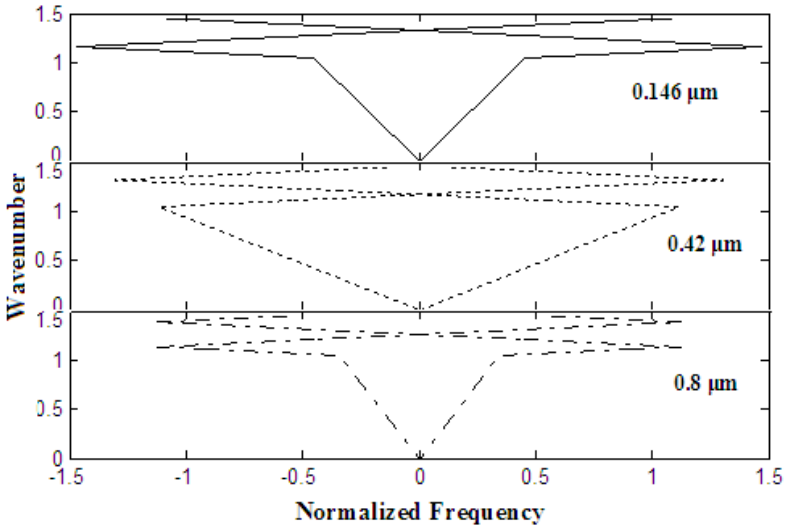

Fig 3b: Band structure of 1D photonic crystal for different values of d_2 keeping d_1 constant

Hence the propagation matrix computed for the former material has less effect on the Bloch dispersion function than the propagation matrix constructed for the other material. Results based on their independent tuning ability are plotted in Fig 3.

Conclusion

Computation of band structure using transfer matrix technique is an approximate first order method where one may have the idea of the behavior of the photonic crystal. Different structural parameters are varied to tune the first Brillouin zone. However, Green's function technique may be incorporated for accurate evaluation of that.

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