Manuscript ID: 427037
Type: reply
Title: Reply to Comment on the memorandum "Large-area, high-numerical-aperture multi-level diffractive lens via inverse design"

Dear Editor,

We appreciate your efforts in procuring two thoughtful reviews to our submission, and furthermore are grateful for the opportunity to respond to the comments. Please find all our responses below in blue. The revised submission (where the revisions are highlighted in yellow) is appended to this letter. We have removed the numbered citations in the abstract in the revised reply.

Sincerely,

The Authors.

Response to reviewer comments.

Reviewer 1:

The authors, Meem et al., respond to the Comment submitted by Lalanne and Chavel about the authors' 2020 Optica Memorandum. The Comment provides useful context for Meem et al.'s work and calls attention to the Memorandam's comments on diffraction efficiency. Meem et al. acknowledge the prior work cited by Lalanne and Chavel and provide additional and useful clarification about diffraction efficiency.

The Response to Comment describes the difference between diffraction efficiency and what Meem et al. refer to as focusing efficiency. The latter is referenced to spot size, not simply the percentage of incident energy directed into a diffractive order. This distinction prompted the Memorandum's statement about "inconsistencies in reporting."

Unfortunately, the proximity of this verbiage to another statement that questions the diffraction efficiency claims by Harvard researchers leaves a reader like me to believe Meem et al. are accusing those researchers of poor science at best and, at worst, nefarious intent. Quoting, "This … is particularly troubling because the claimed focusing efficiency has been repeated in various review articles, but it is far higher than what is theoretically predicted to be possible in a recent article."

Lalanne and Chavel's Comment attempts to provide a more generous interpretation for Harvard reporting a diffraction efficiency that exceeds theoretical bounds published by Chung-Miller, namely, broad band operation. The Response to Comment counters that, although that publication addresses broad band operation, the bounds cited refer to the monochromatic analysis presented therein. I believe Meem et al. are correct on this point.

The Response to Comment generously removes the discussion further from veiled accusations and more toward an objective discussion about defining and measuring useful metrics, and the need to understand the metric Harvard used. Hopefully, the community will note the authors' definitions, the distinctions between them, and, for useful comparisons, provide the area over which energy is measured when citing element efficiencies.

Nonetheless, I hope that, in the future, Meem et al. will choose their words carefully. There is a vast difference between a questionable result and one that is "troubling."

Response R1.1:

We appreciate the reviewer's comments, and concur with the points mentioned. We acknowledge our poor choice of words in the original manuscript. We believe that this reply can indeed be useful for shedding light on these differences in definitions and the importance of reporting both the area in which energy is measured and how focus is defined (numerator and denominator when computing the ratio).

Reviewer 2:

In their reply to the comment on the above memorandum the repliers make two main points: The first is regarding the correctness of their method of measuring focusing efficiency, and the second is regarding a theoretical limit on focusing efficiency of unit-cell type metalenses. I strongly disagree with the first point and am skeptical about the second. Therefore, I do not recommend this reply for publication.

1. The repliers go to great length to explain how they define focusing efficiency, but in fact only add more errors and confusion.

   a. What the repliers call "focusing efficiency" in their reply is what theycalled "relative encircled power" in Fig. 2 of the Memorandum. In their reply they contradict the commentor's claim that encircled power is a measure of resolution since it is the product of focusing efficiency and incident power. Clearly when the commentors wrote that encircled power is a measure of resolution they meant the **relative** encircled power, so the replier's point is mute. Obviously, we are characterizing the lens and not the incident power! The commentor's point was that relative encircled power is a measure of resolution and not of efficiency!

Response R2.1:

The reviewer is correct that indeed the relative encircled power from Ref [1] is the general formulation of focusing efficiency. Firstly, we apologize that the reviewer was confused by our notation. Therefore, we have revised our reply to clarify that relative encircled power is, in fact, a generalization of what has been called "focusing efficiency". Using the reviewer's own statement below, we note that focusing efficiency was apparently first reported in ref [5] as the fraction of incident power within a spot of radius equal to 3 times its FWHM. We clearly showed in our reply that this number (the focusing efficiency) is simply the relative encircled power measured at a radius equal to 3 times its FWHM. Then, how can the relative encircled power <u>not</u> be a measure of focusing efficiency? Therefore, we respectfully disagree with this point.

We acknowledge that relative encircled power can be a measure of resolution (which we didn't deny in our reply, by the way), but resolution is typically determined using the Rayleigh criterion rather than relative encircled power. Therefore, we revised our reply to clarify that relative encircled power is a measure <u>both</u> of efficiency and of resolution.

   b. What the repliers call "focusing efficiency" or "relative encircled power" is in fact a measure of resolution, and has been know for over 50 years inthe optical community as encircled energy [1–4]. It is precisely a measure of resolution/spot size, as the commentor's wrote.

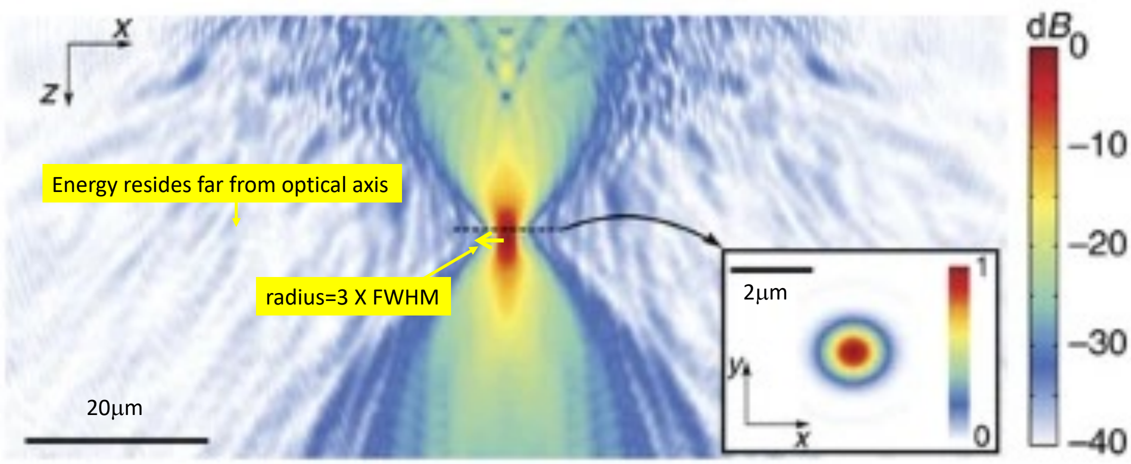
Response R2.2:

We agree with the reviewer that the relative encircled power can be used to estimate resolution or spot-size, but as we showed mathematically the focusing efficiency as defined by Refs [2, 5], and also agreed to by the reviewer, is simply one point on the relative encircled power plot. Our comment is simply that the relative encircled power is not only a measure of resolution, but also of focusing efficiency in a general manner. We further note that resolution is typically defined in terms of the Rayleigh criterion, which is, of course related to the relative encircled power through the point-spread function. In summary, our intention was not to say that relative encircled power is not useful to estimate resolution, but rather that to state that it is also the most appropriate measure of the "focusing efficiency" of a lens.

c. As far as I can tell, the first to coin the term "focusing efficiency" was Arbabi et al. in [5]. They defined it as the power in a circle of radius3FWHM relative to that of the light **incident on the lens aperture**. To my understanding the authors of [5] intended for their "focusing efficiency" tobe the same as the diffraction efficiency. They assumed their lens to be nearly diffraction limited, and therefore concluded that an aperture radiusof 3FWHM would include almost all the light directed to the first order of diffraction.

Response R2.3:

Let's look at ref [5] in some detail. Particularly Fig. 2c therein, where the simulated electric-field density is shown in detail. We have reproduced this below with a scale bar indicating the size of 3 X FWHM. A focusing efficiency of > 70% is claimed (note simulated d=25µm). From the figure below, we can easily see that outside the radius of 3 X FWHM (black dashed line in figure), the intensity values lie below ~ -15dB. Compare that to the Airy function, which has intensity below ~ -45dB. In fact, one can readily see that power lies in the tails of the PSF considerably far away from the optical axis. These tails have a disproportionate contribution to the relative encircled power. A simple radial integration will easily show that >70% of the integrated energy <u>cannot</u> lie within the 3 X FWHM. The reviewer is hypothesizing that the authors of Ref [5] "assumed their lens to be nearly diffraction-limited," when the presented data clearly shows that it is not.



Adapted from figure 2c of Ref[5].

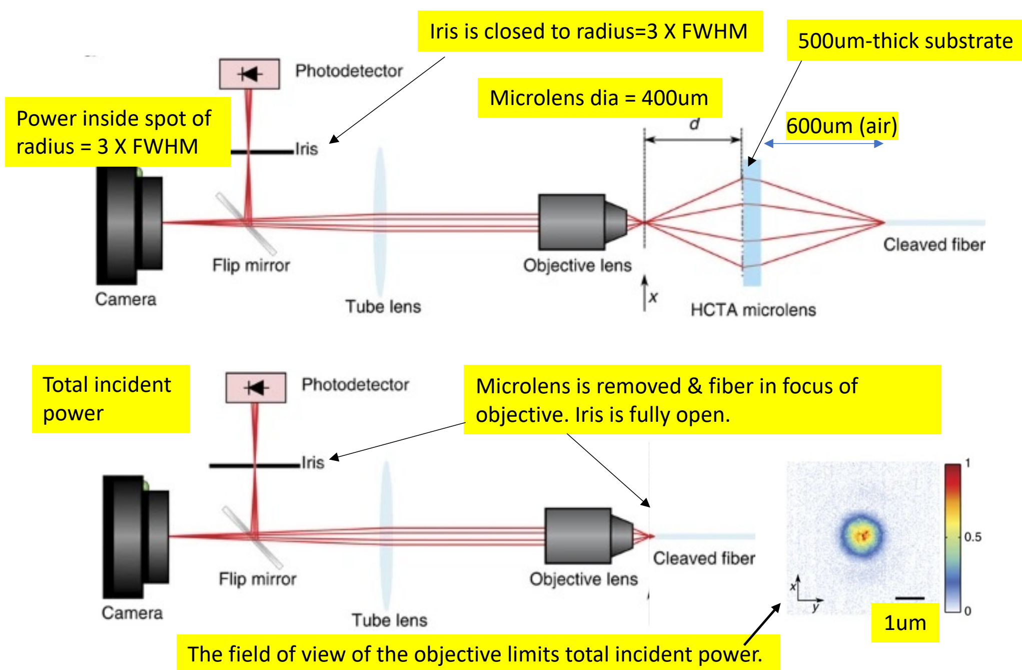
The reviewer also surmises that "the authors of [5] intended for their "focusing efficiency" tobe the same as the diffraction efficiency." As we explained in our reply, the focusing efficiency only considers power within a radius of 3 X FWHM as defined by the authors of ref [5], while diffraction efficiency assumes total power inside a converging spherical wave (which is equal to the total power in the Airy function integrated over the entire focal plane, and is not restricted to any particular radius). <u>Therefore, we emphasize that focusing efficiency and diffraction efficiency cannot be equal mathematically, except in the trivial case when the radius of the spot used to measure focusing efficiency is infinite.</u>

d. In the memorandum the repliers "approximate the incident power as the total power in the frame used to capture the PSF". This is a gross error, since even if we assume the reflection is small, there are other transmitted diffraction orders whose light is spread over an area on the order of the lens aperture – 4.13mm diameter (e.g., the zero and second order are spread over an area equal to the lens aperture [6]). The area of the frame used by the repliers to capture the PSF is over 200 times smaller that this in each dimension, since the camera size used is about the same as the lens diameter, but the PSF is imaged with 230X magnification. This error is what causes the replier's "focusing efficiency" to no longer represent diffraction efficiency but rather resolution.

Response R2.4:

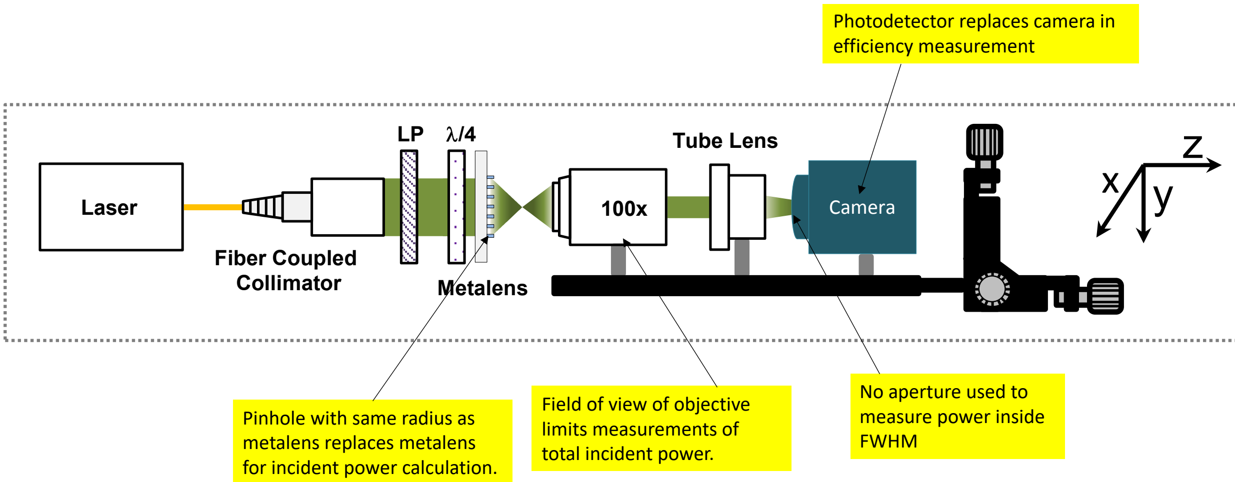
We agree with the reviewer and acknowledge this limitation of our experiment that leads to a gross over-estimation of relative encircled power and therefore, the focusing efficiency. We apologize that this was not clearly stated, and have revised the reply accordingly, including the point below.

We further emphasize that this problem is, unfortunately not limited to our paper, but to the vast majority of metalens papers including the ones cited by the review that we criticized in our memorandum [1, 4]. Let us consider the case of ref [5] as an example, whose measurement setup is illustrated in Fig. 4a (reproduced below with some edits). The total incident power is only collected from the field of view of the objective lens (Olympus UMPlan FL) with NA=0.95. The magnification of the system is not reported in the paper, but we can surmise the field of view from Fig. 4b of ref [5], reproduced below in bottom right inset. Also, with a typical field number of 26, and 100X magnification, the field of view ~3-6μm (also approximately consistent with Fig. 4b). Note that the diameter of a standard single mode fiber core is 9μm. From the reported information, we can hypothesize that only a small fraction of the total incident power is actually measured. Clearly, this experiment suffers from the same problem that the commenters pointed out (i.e., the total incident power is under-estimated, leading to a gross over-estimation of focusing efficiency) and that the reviewer is reiterating here.



Adapted from Fig. 4a of Ref[5]. Bottom right inset is Fig. 4b from Ref [5].

The same criticism can also be brought against the results in Ref [2], which were used to support the review written by the commentors. We reproduce below Fig. S1 from ref [2], which shows the experimental setup. The field of the view of the objective clearly limits the total incident power, thereby over-estimating the efficiency. In addition, in this case, as far as we can tell, no aperture was used to measure the power inside the focal spot, which over-estimates the efficiency even more.



Adapted from Fig. S1 of Ref[2].

Nevertheless, we point out that this is not a reason to reject our reply. In fact, this acknowledgement that the problem is general to almost all flat lens experiments corrects a mistake in the Comment [3], and also makes our reply even more urgent and pertinent for the community.

> In summary, the repliers are propagating an incorrect method ofmeasuring focusing/diffraction efficiency, by confusing it with a measureof resolution. Based on this incorrect method they give the false impression that the efficiency of their lens is 90%. The commentors have brought this to attention, and the repliers unfortunately are not willing to acknowledge their error.

Response R2.5:

We do acknowledge the incomplete measurement of the total incident power in our original experiment over-estimated the focusing efficiency, and have explicitly acknowledged this in our revised reply. But as pointed out above, this criticism is equally valid for papers that were cited by the commentators in their review paper [3].

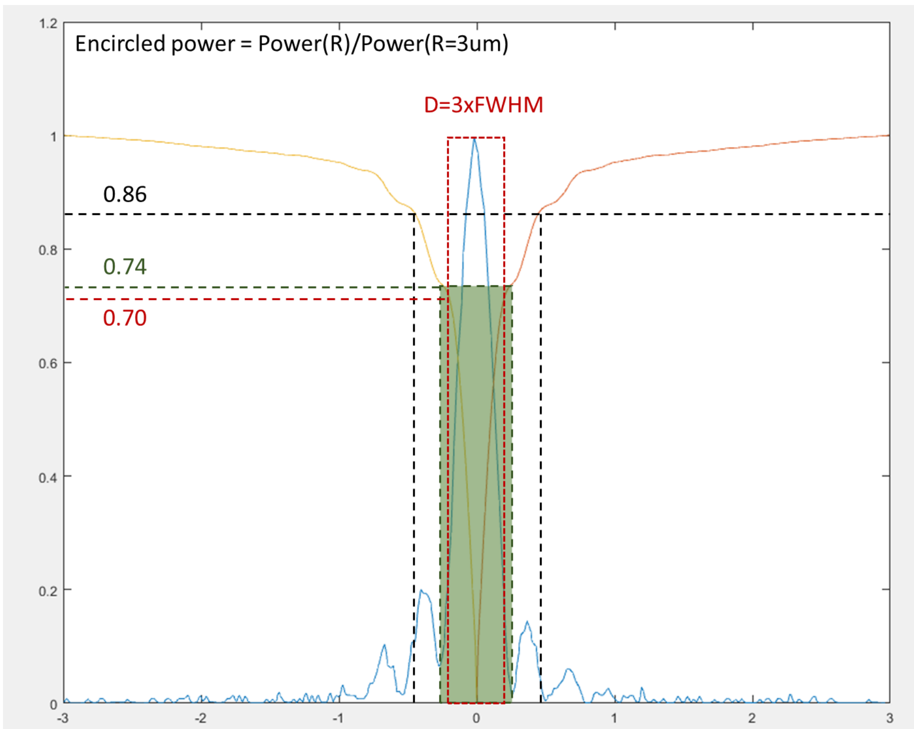
We reiterate that relative encircled power is mathematically the generalization of focusing efficiency as understood by the flat lens community (see our detailed responses above). Indeed it can be used to estimate resolution. These two statements are not mutually exclusive.

2. The second point made by the repliers is that the theoretical limit predicted for unit-cell design in [7] was indeed for monochromatic illumination, and not only for an achromatic design as claimed by the commentors. I agree with this. I also agree with the replier's note that in [8] the area in the focal plane over which light was collected for efficiency measurement is unfortunately not reported. However, I doubt this is the reason for the difference between the experimentand theory, since I assume the authors of [8] knew what they were doing.

Response R2.6:

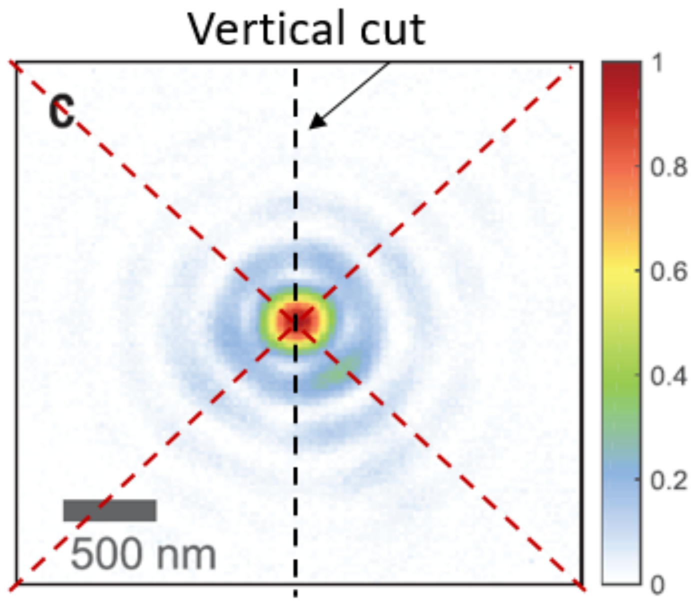
We appreciate the reviewer agreeing with our comments. We would like to take the opportunity to explain why we believe the ref [2] made mistakes in their efficiency calculation.

We have analyzed the data in [2] and have concerns about the extracted values and methods. Although no explicit discussion on how the focusing efficiency was computed in Ref. [2], the authors claim that "For the metalens designed at the wavelength of 405 nm, a measured focusing efficiency of 86% is achieved". Analysis of the measured focal spot intensity distribution, Figure 2(I), integrating and computing power relative to the power encircled in a 3μm radius region, thus neglecting the power outside the reported region, yields the encircled power vs radius plot depicted below:



It is noticed from this plot that the "measured focusing efficiency" reported in Ref. [2], that is 86%, is both larger than the "relative encircled power" in a diameter 3 x FWHM circle, which is ~70%, or the power in between the region defined by the first intensity minima, that is ~74%. Considering, as the reviewer points out that "relative encircled power" in a bounded frame (as computed in the Figure above) might be an upper bound for the actual "focusing efficiency" (because of the neglected power in this case outside the 3μm radius frame adding to the denominator), it is not clear how did the authors obtained a 86% focusing efficiency.

We also note that the data provided in Fig. 2(I) in [8] might actually represent already a best-case scenario. As exemplified in the sketch below, this data corresponds to a "vertical cut" of the PSF; however, cuts along most other planes, as depicted below, clearly show substantial larger intensities in the sidelobes (red dashed lines). In fact, sidelobes with intensities of ~0.2 relative to the peak can be seen as far away as ~5 X FWHM. Clearly, these tails make a disproportional contribution due to their large radial distance, and thereby, it is highly unlikely that the claimed focusing efficiency of 86% is correct. We believe that this is an honest mistake by the authors of ref [2].



Interestingly, in [7] this contradiction between theory and measurement is mentioned (text just before fig. 2), and is explained by the fact that the efficiency results given in [8] are polarization conversion efficiencies and not focusing efficiency. However, this explanation does not hold, since, in addition to polarization conversion efficiencies, focusing efficiencies are reported in [8], and are also higher that the theoretical limit of [7]. In addition, the efficiency resultsof [5] also exceed the theoretical limit. So, what is the explanation to this contradiction? I do not know. I think that bringing this contradiction to the attention of the community in hope that someone will come up with an explanation is important. However, rather than do this, the commentors indicate the explanation can be found in the different radii over which the focusing efficiency was measured. This seems to me like lookingfor the coin under the lamppost, when in fact it got lost a block away.

Response R2.7:

We first thank the reviewer for agreeing that the results of Ref [2] need to be explained. The commentors seem to indicate that there are no issues with the results of Ref [2]. Our reply is attempting to correct this mistake and we provide a hypothesis for where this mistake might come from (see our above responses). Therefore, we hope that the reviewer can accept our revised reply.

3. The other points made by the repliers are minor, so I do not think it is important to refer to them.

Response R2.8:

We agree with this comment.

References:

[1] M. Meem, S. Banerji, C. Pies, T. Oberbiermann, A. Majumder, B. Sensale-Rodriguez, and R. Menon, "Large-area, high-numerical-aperture multi-level diffractive lens via inverse design", Optica 7, 252–253 (2020).

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[3] P. Lalanne and P. Chavel, “Comment of the memorandum “Large-area, high-numerical-aperture multi-level diffractive lens via inverse design”, (to be published).

[4] P. Lalanne and P. Chavel, Laser Photon. Rev. **11**, 1600295 (2017).

[5] Arbabi, A., Horie, Y., Ball, A. *et al.* Subwavelength-thick lenses with high numerical apertures and large efficiency based on high-contrast transmitarrays. *Nat Commun* **6,** 7069 (2015).

Reply to Comment on the memorandum "Large-area, high-numerical-aperture multi-level diffractive lens via inverse design"

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Received XX Month XXXX; revised XX Month, XXXX; accepted XX Month XXXX; posted XX Month XXXX (Doc. ID XXXXX); published XX Month XXXX

We would like to thank the authors for their thoughtful comments, and first note that we agree with most of the points raised. However, there are a few comments that require clarifications, which we outline briefly below..

We proposed the use of relative encircled power as a measure of focusing efficiency [1]. A recent commentary [2] raised useful questions, which we address briefly here.

In the pioneering work of ref [3], the authors demonstrated an impressive 80% diffraction efficiency. With no intention of minimizing this achievement, we first note that diffraction efficiency and focusing efficiency are not the same, and focusing efficiency was unfortunately not reported in ref [3]. When a diffractive element (including a lens) is illuminated by a plane wave, the transmitted light may be decomposed into basis functions. In the case of lenses, these basis functions are usually spherical waves, where the diffraction orders represent the center of these spheres (positive and negative signs refer to converging and diverging spherical waves, respectively) [4]. The diffraction efficiency is the ratio of power carried by one of these spherical waves to the total incident power. When a lens is illuminated normally by a plane wave, the intensity in the focal plane can be written as the sum of contributions from the different diffraction orders: $I(\rho) = I_0 + I_{+1}(\rho) + I_{-1}(\rho) + \cdots$, where the subscript denotes the order of diffraction, and ρ is the radial co-ordinate in the focal plane. Let's take the example of a perfect lens, where the diffraction efficiency is 100% (into the +1 order), $I(\rho) = I_{+1}(\rho)$ and from Fourier optics, we know that this is the Airy function, $I_{A0}\left(\frac{2J_1(x)}{x}\right)^2$, where, I_{A0} is the incident intensity, $J_1(x)$ is the Bessel function of the 1st kind, and $x = \frac{\pi\rho}{\lambda f\#}$, where λ is the wavelength and $f\#$ is the f-number of the lens. Focusing efficiency was first introduced as the fraction of incident power within a focal spot of radius equal to 3 times its full-width at half-maximum (FWHM) [7]. We can generalize this by calculating the power within a spot of a general radius, R as

$$E_A(R) = \int_0^R I_A(\rho)\rho d\rho = E_{A0}\left(1 - J_0^2\left(\frac{\pi R}{\lambda f\#}\right) - J_1^2\left(\frac{\pi R}{\lambda f\#}\right)\right), \quad (1)$$

where, E_{A0} is the total incident power. Finally, the focusing efficiency is simply given by (this is also called the relative encircled power):

$$\eta_A(R) = \frac{E_A(R)}{E_{A0}} = \left(1 - J_0^2\left(\frac{\pi R}{\lambda f\#}\right) - J_1^2\left(\frac{\pi R}{\lambda f\#}\right)\right). \quad (2)$$

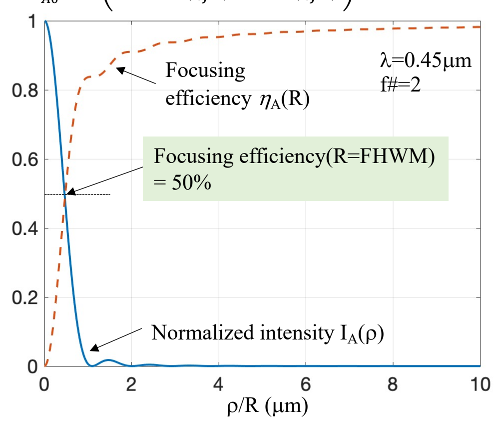

Fig. 1. PSF and Encircled power of a lens with 100% diffraction efficiency achieves a focusing efficiency of 50% for a spot-radius equal to FWHM. It is important to report the spot-radius along with focusing efficiency.

The subscript A refers to the ideal lens, whose PSF is the Airy function. From this expression, we can readily show that the focusing efficiency when R = FWHM of the PSF is 50% (Fig. 1). **In other words, a lens with 100% diffraction efficiency has focusing efficiency of 50% (measured with R = FWHM).** The focusing efficiency will be > ~90%, if R = 3 X FWHM. This example illustrates that reporting focusing efficiency without the value of R is not sufficient to give the full picture.

The commentors take issue with our critique regarding ref [5]. The gist of our comment was that the focusing efficiencies reported in ref [5] are higher than what is theoretically predicted possible with the concept of unit-cell design, as clearly explained in ref [6] both for monochromatic and broadband cases. This is a good example of a confusion arising from inconsistency in the size of the focal spot. Since the size of the focused spot used to compute focusing efficiency was not explicitly reported in ref [5], it is as well possible that the focusing efficiency reported there used a different focal spot than what was used in ref [6] for calculation of the upper bounds of focusing efficiency (where radius corresponding to the first zero in the PSF was used). This example simply proves our point in ref [1] that relative encircled power is a better metric for a lens than just a single value of focusing efficiency. In fact, we would like to correct the comment that relative encircled power is only a measure of resolution, which is clearly not correct as can be seen by the example of the Airy PSF above. In fact, the relative encircled power is a measure of both resolution and efficiency.

Next, we acknowledge the criticism from the commentors that the relative encircled power in ref [1] was over-estimated because the total incident power was under-estimated due to the limited frame size of our recorded image. We also note that this same mistake is likely present in the vast majority of the flat-lens publications, including the ones cited by the commentors in their review paper. For example, one can examine Fig. 4 of ref [7] or Fig. S1 of ref [2] to note that the total incident power is under-estimated due to the limited field of view of the microscope objectives used in both cases, which in turn leads to an over-estimation of the measured focusing efficiencies.

The commentors stated referring to our work that "*1-cm, 0.9-NA metalens (Ref. 2 in [1]) is called small.*" There are 3 reasons why we chose to ignore the metalens referred here. First, we note that this lens has NA=0.78 and not 0.9. Secondly, this metalens is a negative lens. Thirdly, no experimental validation of the PSF or MTF of this lens was provided that could validate the reported NA.

Finally, the commentors brought up an observation about a lack of scale in Fig. 2d. We apologize for this oversight as we naively assumed that the standard Air Force chart image is well known. Our results showed that the lines in Group 7, Element 6 (size ~2μm) were resolved, which is consistent with the reported PSF and MTF data.

Funding. Office of Naval Research grant N66001-10-1-4065, N000141512316, and DURIP grant: N000141912458. US National Science Foundation ECCS # 1936729.

Acknowledgments. Discussion with O. Miller is gratefully acknowledged

Disclosures. RM: Oblate Optics (I,E,P).

Data availability. Data underlying the results presented in this paper are available in Dataset 1, Ref. [3].

Supplemental document. See Supplement 1 for supporting content.

References

1. M. Meem, S. Banerji, C. Pies, T. Oberbiermann, A. Majumder, B. Sensale-Rodriguez, and R. Menon, "Large-area, high-numerical-aperture multi-level diffractive lens via inverse design", Optica 7, 252–253 (2020).
2. P. Lalanne and P. Chavel, "Comment of the memorandum "Large-area, high-numerical-aperture multi-level diffractive lens via inverse design", (to be published).
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7. Arbabi, A., Horie, Y., Ball, A. *et al.* Subwavelength-thick lenses with high numerical apertures and large efficiency based on high-contrast transmitarrays. *Nat Commun* **6,** 7069 (2015).