

Analysis of Reflectivity for Propagating Wave inside 1D Photonic Crystal with Different Material Systems

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Abstract— In this paper, reflective coefficient due to electromagnetic wave propagation in forward and backward directions inside one-dimensional photonic crystal is analytically computed using coupled mode theory for varying coupling coefficients and different material compositions. Grating length is tuned for near accurate study of the stratified periodic structure. The periodic dielectric array is essentially Bragg grating where increase in grating length enhances the reflection of electromagnetic wave, and strong coupling provides larger bandgap spectral width. For simulation purpose, GaN/Al_xGa_{1-x}N material composition is considered as unit block of the periodic organization, and refractive indices of materials are taken as function of bandgap and operating wavelength following Adachi's model. Results are compared with that obtained for SiO₂/air system for identical structural parameters and other propagating conditions. Simulation proves the dominance of usability of semiconductor heterostructure-based photonic crystal for optical communication in comparison to the conventional system.

Index Terms—One-dimensional photonic crystal, forward and backward waves, Bragg wavelength, reflectivity, strong and weak coupling

I. INTRODUCTION

Photonic crystal is a periodic arrangement of dielectric materials [1] where localization of e.m wave propagating inside the structure is controlled by tuning of structural parameters. This is possible due to the formation of photonic bandgap, a property which is extensively used to design photonic integrated circuits [2], optical transmitter [3], optical receiver [4], photonic crystal fibre [5], quantum information processing [6] etc. This novel microstructure has replaced conventional optical fibre for efficient communication.

Among the various types of confinements has been considered for photonic crystal analysis, it is suggested that only 1D and 2D structures are efficient to realize experimentally and for implementation purpose in different optoelectronic integrated circuits. 1D structure is very convenient to study because of ease of mathematical modeling. Propagating wave analysis in 1D structure is useful for designing four-wave mixing analysis in nonlinear photonic crystal [7]. Incorporation of semiconductor nanostructure makes it more interesting when filter characteristics is considered [8] including the polarization effect of incident light. Use of AlGaIn/GaN material composition is taken up because of predominant effect of carrier localization in undoped AlGaIn alloys [9], which enhances with the increase in Al contents. High Al-content also increases the overall figure of merit due to the combined advantages of enhanced band offset, lattice mismatch induced piezoelectric effect. The advantages of GaN can be summarized as ruggedness, power handling and low loss.

The present paper deals with computation of reflectivity in 1D photonic crystal for different grating lengths under strong and weak coupling conditions. Bragg wavelength is set at 1550 nm for both Al_xGa_{1-x}N/GaN and SiO₂/Air composition. Refractive indices of the materials are considered as function of bandgap

and operating wavelength. The spectral width of the structure i.e., photonic bandgap is qualitatively measured from the analysis.

II. MATHEMATICAL MODELLING

The field in a corrugated structure (grating) may be written as a sum of the forward and backward propagating modes

$$E_y = A(z)u(x) \exp[j(\omega t - \beta \cdot z)] + B(z)u(x) \exp[j(\omega t + \beta \cdot z)] \quad (1)$$

where A and B are the amplitudes of the forward and backward propagating waves, and u(x) is the mode profile. The perturbation in the corrugated region is

$$\vec{P}_{pert} = \Delta n(x, z)^2 \epsilon_0 \vec{E} \quad (2)$$

We now substitute the expression for the field into this expression to get

$$P_{pert} = \frac{1}{2} \Delta n(x)^2 \epsilon_0 \{A(z)u(x) \exp[j(\omega t - \beta \cdot z)] + B(z)u(x) \exp[j(\omega t + \beta \cdot z)]\} \quad (3)$$

The fundamental coupled mode equation is given by

$$-\frac{dA_i^+}{dz} \cdot \exp[j(\omega t - \beta \cdot z)] + \frac{dA_i^-}{dz} \cdot \exp[j(\omega t + \beta \cdot z)] + c.c = \frac{-j}{2\omega} \frac{\partial^2}{\partial t^2} \int_{-\infty}^{\infty} P_{pert}(x, z, t) u_i(x) dx \quad (4)$$

Assuming the grating is of square-wave nature, it can be expressed as a series in the following form

$$\Delta n^2(x, z) = \Delta n^2 \sum_m C_m e^{j \frac{2m\pi \cdot z}{\Lambda}} \quad (5)$$

By comparing this expression to the above coupled mode equations, we realize that only modes that are close to phase matched will experience significant coupling. In a range of wave vectors, the equations can be simplified to

$$\frac{dA}{dz} = \frac{j\omega\epsilon_0}{4} B e^{j2\beta \cdot z} C_m e^{-j \frac{2m\pi \cdot z}{\Lambda}} \int_{-\infty}^{\infty} \Delta n^2 u^2(x) dx \quad (6)$$

$$\frac{dB}{dz} = \frac{j\omega\epsilon_0}{4} A e^{-j2\beta \cdot z} C_m e^{j \frac{2m\pi \cdot z}{\Lambda}} \int_{-\infty}^{\infty} \Delta n^2 u^2(x) dx \quad (7)$$

Based on the energy conservation principle, the difference between the rates of change in the forward-propagating and backward-propagating energy is

$$\frac{d}{dz} |A|^2 - \frac{d}{dz} |B|^2 = A \cdot B^* \cdot K e^{-j2\Delta\beta \cdot z} + A^* \cdot B \cdot K^* e^{j2\Delta\beta \cdot z} - B^* \cdot A \cdot K e^{-j2\Delta\beta \cdot z} - B \cdot A^* \cdot K^* e^{j2\Delta\beta \cdot z} = 0 \quad (8)$$

After determining the expressions of **A** and **B**, reflectivity can be calculated as

$$r = \frac{B(0)}{A(0)} = \frac{A_0 \cdot j \cdot K \frac{\sinh[SL]}{-\Delta\beta \sinh[SL] + jS \cosh[SL]}}{A_0 \cdot \frac{-\Delta\beta \sinh[SL] + jS \cosh[SL]}{-\Delta\beta \sinh[SL] + jS \cosh[SL]}} \Rightarrow r = j \cdot K \frac{\sinh[SL]}{-\Delta\beta \sinh[SL] + jS \cosh[SL]} \quad (9)$$

III. RESULTS AND DISCUSSION

Fig 1 exhibits the reflectivity profile for different grating lengths under weak coupling condition ($\kappa=0.01$) for different material compositions ($\text{Al}_x\text{Ga}_{1-x}\text{N}/\text{GaN}$ and SiO_2/Air). For both the figures, it may be seen that reflectivity becomes higher for larger value of grating length.

Outside of this bandgap, reflectivity closes to zero, as one move away from Bragg wavelength. The width of the gap is extremely narrow in Fig 1b compared to Fig 1a at resonance. Thus, semiconductor system provides wider photonic bandpass filter than the conventional system, and hence preferred from application point of view. Moreover engineering the photonic bandgap is extremely difficult because as we move away from the Bragg wavelength the reflectivity falls off so sharply that it is very difficult to control the drastic fall.

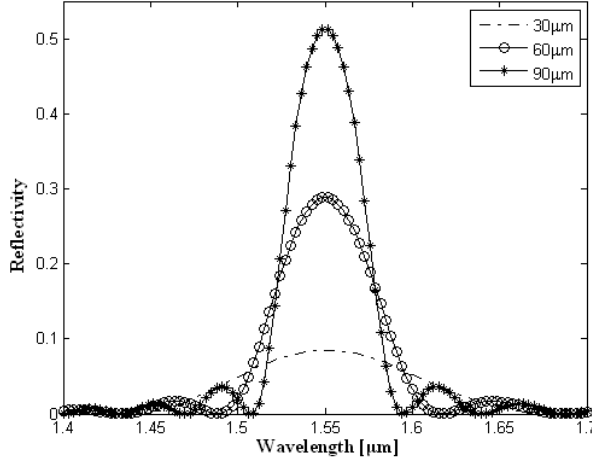


Fig 1a: Reflectivity vs. λ for different grating lengths for GaN/AlGaN system under weak coupling conditions

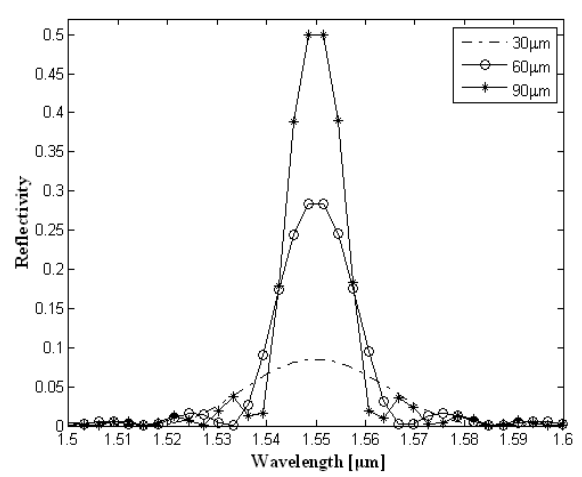


Fig 1b: Reflectivity vs. λ for different grating lengths for SiO_2 -air system under weak coupling conditions

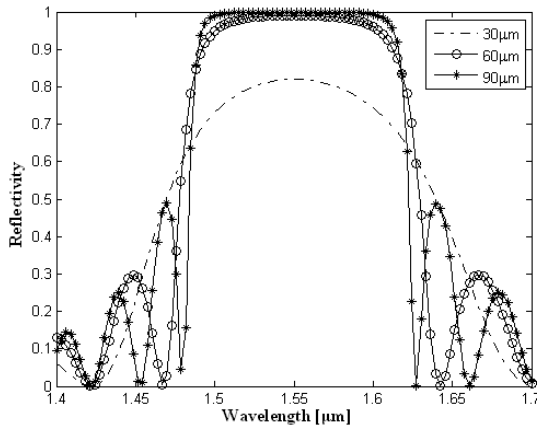


Fig 2a: Reflectivity vs. λ for different grating lengths for GaN/AlGaN system under strong coupling conditions

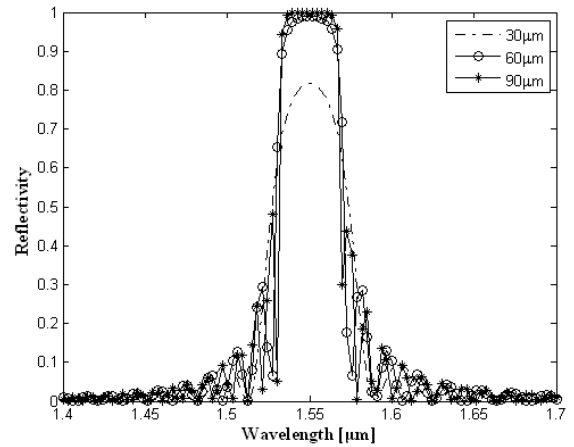


Fig 2b: Reflectivity vs. λ for different grating lengths for SiO_2 -air system under strong coupling conditions

Fig. 2 shows the reflectivity profile for the stratified periodic structures under strong coupling condition for varying grating length. For both the cases, it is observed that the size and width of the photonic bandgap is quite predominantly increased in comparison to the weak coupling condition. Moreover, magnitude of reflectivity goes up (touches unity) to nearly double the value for strong coupling. The fall off is gradual for semiconductor system whereas sharp for conventional one, depicted in Fig 2b. Also for higher values of the

grating length, reflectivity remains almost the same under the condition of strong coupling for the former composition. This is quite helpful in tailoring a perfect photonic bandgap. But for the later, width of the bandgap is too narrow for effective tailoring. A generalized comparative study indicates the fact that photonic bandgap is smaller for weak coupling condition, whereas it is wider for strong coupling condition.

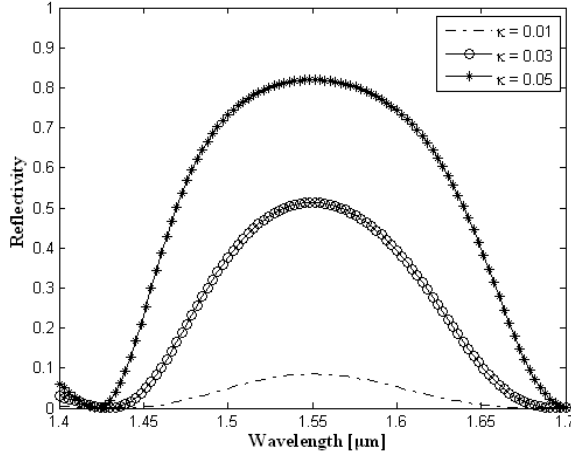


Fig 3a: Reflectivity with λ for different coupling coefficients with $30\mu\text{m}$ grating length for $\text{Al}_x\text{Ga}_{1-x}\text{N}/\text{GaN}$

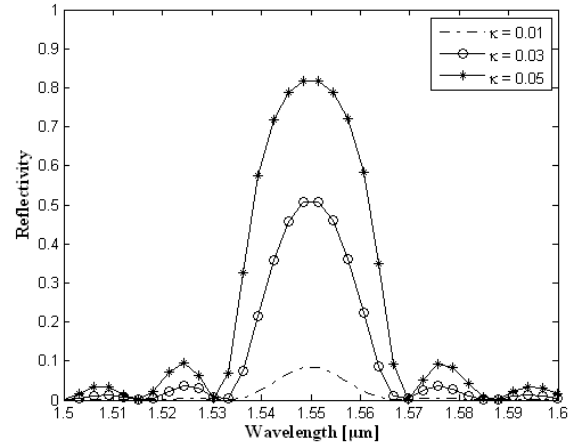


Fig 3b: Reflectivity with λ for different coupling coefficients with $30\mu\text{m}$ grating length for SiO_2/air

Fig. 3 shows the reflectivity profile for both the material compositions with smaller grating length ($30\mu\text{m}$). It is observed here that the complete reflection condition was less if coupling was relatively weak. For the present structure, 50% reflection can be obtained when coupling coefficient is $0.03\mu\text{m}^{-1}$, which increases to about 80% when the coupling coefficient is $0.05\mu\text{m}^{-1}$. In Fig. 3b, it is seen that the reflection was almost absent under weak coupling condition. As seen for the previous case, the extent of transmission in case of SiO_2/Air is much less i.e. opening of the bandgap is quite narrow.

CONCLUSIONS

Rigorous analysis is carried out to analyze the effect of the coupling and reflectivity on the propagation of e.m wave in 1D photonic crystal by varying the length of the grating, coupling coefficient and also for different material system. Observation revealed that $\text{Al}_x\text{Ga}_{1-x}\text{N}/\text{GaN}$ composition outperformed SiO_2/air in terms of wider spectral width and gradual roll-off at the edges, i.e., tailoring the photonic bandgap. Such characteristics remove problems which otherwise present in SiO_2/Air photonic crystal. The spectral width of photonic bandgap is a measure of bandpass filter characteristics suitable for optical communication, and hence our proposed system is superior to conventional configuration.

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