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Contest (1)

template.cpp

14 lines

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
  cin.tie(0)->sync_with_stdio(0);
  cin.exceptions(cin.failbit);
}
```

.bashrc

12 lines

```
{
    "shell_cmd": "g++ -std=c++14 \"$file\" -o \"$file_base_name\" &&
        timeout 4s ./$file_base_name < inputf.in > outputf.in",
    "selector": "source.c, source.c++",
    "working_dir": "$file_path",
    "file_regex": "^(.+\\.cpp):(\\d+):(\\d+): (.*)$",
    "variants": [
        {
            "name": "Run",
            "shell_cmd": "timeout 4s ./$file_base_name < inputf.in >
                outputf.in"
        }
    ]
}
```

.vimrc

6 lines

```
set cin aw ai is ts=4 sw=4 tm=50 nu noeb bg=dark ru cul
sy on   |   im jk <esc>   |   im kj <esc>   |   no ; :
" Select region and then type :Hash to hash your selection.
" Useful for verifying that there aren't mistypes.
ca Hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space:]' \
 \| md5sum \| cut -c-6
```

hash.sh

3 lines

```
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

troubleshoot.txt

52 lines

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
```

```
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed - bf}{ad - bc} \\ y &= \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Trigonometry

$$\begin{aligned} \sin(v + w) &= \sin v \cos w + \cos v \sin w \\ \cos(v + w) &= \cos v \cos w - \sin v \sin w \end{aligned}$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$

$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$\begin{aligned} a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi) \end{aligned}$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left[1 - \left(\frac{a}{b + c}\right)^2\right]}$

Law of sines: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

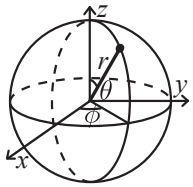
$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$. .

2.4.3 Spherical coordinates

.

.



$$\begin{array}{cc} x = r\sin\theta\cos\phi & r = \sqrt{x^2+y^2+z^2} \\ y = r\sin\theta\sin\phi & \theta = \text{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z = r\cos\theta & \phi = \text{atan2}(y,x) \end{array}$$

.

.

2.5 Derivatives/Integrals

$$\begin{array}{cc} \frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx}\tan x = 1 + \tan^2 x & \frac{d}{dx}\arctan x = \frac{1}{1+x^2} \\ \int \tan ax = -\frac{\ln|\cos ax|}{a} & \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2} \\ \int e^{-x^2} = \frac{\sqrt{\pi}}{2}\text{erf}(x) & \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1) \end{array}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$\begin{aligned} 1 + 2 + 3 + \cdots + n &= \frac{n(n+1)}{2} \\ 1^2 + 2^2 + 3^2 + \cdots + n^2 &= \frac{n(2n+1)(n+1)}{6} \\ 1^3 + 2^3 + 3^3 + \cdots + n^3 &= \frac{n^2(n+1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \cdots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \end{aligned}$$

2.7 Series

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty) \\ \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1) \\ \sqrt{1+x} &= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1) \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty) \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty) \end{aligned}$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n, p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n, p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii} = 1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j, is $a_{ij} = p_{ij} + \sum_{k\in\mathbf{G}} a_{ik}p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i = 1 + \sum_{k\in\mathbf{G}} p_{ki}t_k$.

Data structures (3)

oSet.h

Description: Ordered Set 3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

<bits/stdc++.h>, <ext/pb_ds/assoc_container.hpp>, <ext/pb_ds/tree_policy.hpp> d0c877, 18 lines

```
using namespace __gnu_pbds;
using namespace std;

template <typename T> using o_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  o_set<int> se;
  se.insert(4);
  se.insert(2);
  se.insert(5);
  // sorted set se = [2, 4, 5]
  cout << se.order_of_key(5) << '\n'; // number of elements < 5
  cout << se.order_of_key(6) << '\n'; // number of elements < 6
  cout << (*se.find_by_order(1)) << '\n'; // if you imagine this as a 0-
      indexed vector, what is se[1]?
  return 0;
}
```

DSU.h

Description: DSU to find parent. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

e22ea4, 16 lines

```
void make_set(int v) {
    parent[v] = v;
}

int find_set(int v) {
    if (v == parent[v])
        return v;
    return parent[v] = find_set(parent[v]);
}

void union_sets(int a, int b) {
    a = find_set(a);
    b = find_set(b);
    if (a != b)
        parent[b] = a;
}
```

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

d77092, 7 lines

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};
__gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{},{},{1<<16});
```

LazySegmentTree.h

Description: Segment tree. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

<bits/stdc++.h> c465a1, 62 lines

```
using namespace std;

const int N = 5e5 + 9;
int a[N];
struct ST {
  #define lc (n << 1)
  #define rc ((n << 1) | 1)
  long long t[4 * N], lazy[4 * N];
  ST() {
    memset(t, 0, sizeof t);
    memset(lazy, 0, sizeof lazy);
  }
  inline void push(int n, int b, int e) {
    if (lazy[n] == 0) return;
    t[n] = t[n] + lazy[n] * (e - b + 1);
    if (b != e) {
      lazy[lc] = lazy[lc] + lazy[n];
      lazy[rc] = lazy[rc] + lazy[n];
    }
    lazy[n] = 0;
  }
  inline long long combine(long long a,long long b) {
    return a + b;
  }
  inline void pull(int n) {
```

```
    t[n] = t[lc] + t[rc];
  }
  void build(int n, int b, int e) {
    lazy[n] = 0;
    if (b == e) {
      t[n] = a[b];
      return;
    }
    int mid = (b + e) >> 1;
    build(lc, b, mid);
    build(rc, mid + 1, e);
    pull(n);
  }
  void upd(int n, int b, int e, int i, int j, long long v) {
    push(n, b, e);
    if (j < b || e < i) return;
    if (i <= b && e <= j) {
      lazy[n] = v; //set lazy
      push(n, b, e);
      return;
    }
    int mid = (b + e) >> 1;
    upd(lc, b, mid, i, j, v);
    upd(rc, mid + 1, e, i, j, v);
    pull(n);
  }
  long long query(int n, int b, int e, int i, int j) {
    push(n, b, e);
    if (i > e || b > j) return 0; //return null
    if (i <= b && e <= j) return t[n];
    int mid = (b + e) >> 1;
    return combine(query(lc, b, mid, i, j), query(rc, mid + 1, e, i, j));
  }
};
int32_t main() {

}
```

LCA.h

Description: LEAST COMMON ANCESTOR Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

c5424a, 65 lines

```
#define mx 100002
int L[mx]; // Level of each node
int P[mx][22]; // Sparse table
int T[mx]; // Parent array
vector<int> g[mx];

void dfs(int from, int u, int dep) {
    T[u] = from;
    L[u] = dep;
    for (int i = 0; i < (int)g[u].size(); i++) {
        int v = g[u][i];
        if (v == from) continue;
        dfs(u, v, dep + 1);
    }
}

int lca_query(int N, int p, int q) { // N = number of nodes
    int tmp, log, i;

    if (L[p] < L[q])
        tmp = p, p = q, q = tmp;

    log = 1;
    while (1) {
        int next = log + 1;
        if ((1 << next) > L[p]) break;
        log++;
    }

    for (i = log; i >= 0; i--)
        if (L[p] - (1 << i) >= L[q])
            p = P[p][i];

    if (p == q)
        return p;

    for (i = log; i >= 0; i--)
        if (P[p][i] != -1 && P[p][i] != P[q][i])
            p = P[p][i], q = P[q][i];

    return T[p];
}

void lca_init(int N) {
    memset(P, -1, sizeof(P)); // Initialize all values to -1
    int i, j;
    for (i = 0; i < N; i++)
        P[i][0] = T[i];

    for (j = 1; (1 << j) < N; j++)
        for (i = 0; i < N; i++)
            if (P[i][j - 1] != -1)
                P[i][j] = P[P[i][j - 1]][j - 1];
}

int main(void) {
    g[0].push_back(1);
    g[0].push_back(2);
    g[2].push_back(3);
```

```
    g[2].push_back(4);
    dfs(0, 0, 0);
    lca_init(5);
    printf("%d\n", lca_query(5, 3, 4));
    return 0;
}
```

RMQ.h

Description: Find minimum range query Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

a5c3c2, 11 lines

```
int st[K + 1][MAXN];

std::copy(array.begin(), array.end(), st[0]);

for (int i = 1; i <= K; i++)
    for (int j = 0; j + (1 << i) <= N; j++)
        st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);

int i = log2[R - L + 1];
int minimum = min(st[i][L], st[i][R - (1 << i) + 1]);
```

SquareRoot.h

Description: SQRT decomposition Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

88b98e, 35 lines

```
int query(int input[], int segment_size, int l, int r) {
    int sum = 0;

    //loop the first segment
    //until we reach r or a starting index

    while (l < r && l % segment_size != 0) {
  sum += input[l];
  l++;
    }

    //Loop until we reach
    //segment that contains r
    while (l + segment_size <= r) {
        sum += segment[l / segment_size];
        l += segment_size;
    }

    //loop until r
    while (l<=r) {
        sum += input[l];
        l++;
    }

    return sum;
}

void update(int input[], int segment_size, int i, int val) {
    int segment_no = i / segment_size;

    segment[segment_no] -= input[i];
    segment[segment_no] += val;
    input[i] = val;
}
```

Trie.h

Description: TRIE Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

7b7a52, 66 lines

```
struct node {
    bool endmark;
    node* next[26 + 1];
    node()
    {
        endmark = false;
        for (int i = 0; i < 26; i++)
            next[i] = NULL;
    }
} * root;
void insert(char* str, int len)
{
    node* curr = root;
    for (int i = 0; i < len; i++) {
        int id = str[i] - 'a';
        if (curr->next[id] == NULL)
            curr->next[id] = new node();
        curr = curr->next[id];
    }
    curr->endmark = true;
}
bool search(char* str, int len)
{
    node* curr = root;
    for (int i = 0; i < len; i++) {
        int id = str[i] - 'a';
        if (curr->next[id] == NULL)
            return false;
        curr = curr->next[id];
    }
    return curr->endmark;
}
void del(node* cur)
{
    for (int i = 0; i < 26; i++)
        if (cur->next[i])
            del(cur->next[i]);
```

```
    delete (cur);
}
int main()
{

    puts("ENTER NUMBER OF WORDS");
    root = new node();
    int num_word;
    cin >> num_word;
    for (int i = 1; i <= num_word; i++) {
        char str[50];
        scanf("%s", str);
        insert(str, strlen(str));
    }
    puts("ENTER NUMBER OF QUERY";);
    int query;
    cin >> query;
    for (int i = 1; i <= query; i++) {
        char str[50];
        scanf("%s", str);
        if (search(str, strlen(str)))
            puts("FOUND");
        else
            puts("NOT FOUND");
    }
    del(root);
    return 0;
}
```

Numerical (4)

4.1 Polynomials and recurrences

Polynomial.h

c9b7b0, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h"

b00bfe, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0]*x^0+...+a[n-1]*x^{n-1}$. For numerical precision, pick $x[k] = c*\cos(k/(n-1)*\pi), k = 0 \ldots n-1$.
Time: $\mathcal{O}\left(n^2\right)$

08bf48, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  rep(k,0,n-1) rep(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
```

```
  return res;
}
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}\left(N^2\right)$

"../number-theory/ModPow.h" 96548b, 20 lines

```
vector<ll> berlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;

  ll b = 1;
  rep(i,0,n) { ++m;
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }

  C.resize(L + 1); C.erase(C.begin());
  for (ll& x : C) x = (mod - x) % mod;
  return C;
}
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \geq n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$

f4e444, 26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(tr);

  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
    res.resize(n + 1);
    return res;
  };

  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;

  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }

  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
}
```

4.2 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}\left(N^3\right)$

bd5cec, 15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}\left(N^3\right)$

3313dc, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
```

```
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

SolveLinear.h

Description: Solves $A*x=b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}\left(n^2m\right)$

44c9ab, 38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

"SolveLinear.h" 08e495, 7 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
```

SolveLinearBinary.h

Description: Solves $Ax=b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}\left(n^2m\right)$

fa2d7a, 34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
```

```
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1})$ (mod p^k) where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\}, \{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
Time: $\mathcal{O}(N)$

8f9fa8, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
}
```

Number theory (5)

5.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set `mod` to some number first and then you can use the structure.

"euclid.h" 35bfea, 18 lines

```
const ll mod = 17; // change to something else
struct Mod {
  ll x;
  Mod(ll xx) : x(xx) {}
  Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
  Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod); }
  Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
  Mod operator/(Mod b) { return *this * invert(b); }
  Mod invert(Mod a) {
    ll x, y, g = euclid(a.x, mod, x, y);
    assert(g == 1); return Mod((x + mod) % mod);
  }
  Mod operator^(ll e) {
    if (!e) return Mod(1);
    Mod r = *this ^ (e / 2); r = r * r;
    return e&1 ? *this * r : r;
  }
};
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

6f684f, 3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

b83e45, 8 lines

```
const ll mod = 1000000007; // faster if const

ll modpow(ll b, ll e) {
  ll ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;
  return ans;
}
```

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod m$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a.

Time: $\mathcal{O}(\sqrt{m})$

c040b8, 11 lines

```
ll modLog(ll a, ll b, ll m) {
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.

`modsum(to, c, k, m)` $= \sum_{i=0}^{to-1} (ki + c) \% m$. `divsum` is similar but for floored division.

Time: $\log(m)$, with a large constant.

5c5bc5, 16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.

Time: $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

bbbd8f, 11 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}\left(\log p\right)$ for most p

"ModPow.h" 19a793, 24 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

5.2 Primality

FastEratosthenes.h

Description: Prime sieve for generating all primes smaller than LIM.
Time: LIM=1e9 $\approx$ 1.5s

6b2912, 20 lines

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
  const int S = (int)round(sqrt(LIM)), R = LIM / 2;
  vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)*1.1));
  vector<pii> cp;
  for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
    cp.push_back({i, i * i / 2});
    for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
  }
  for (int L = 1; L <= R; L += S) {
    array<bool, S> block{};
    for (auto &[p, idx] : cp)
      for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
    rep(i,0,min(S, R - L))
      if (!block[i]) pr.push_back((L + i) * 2 + 1);
  }
  for (int i : pr) isPrime[i] = 1;
  return pr;
}
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" 60dcd1, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" d8d98d, 18 lines

```
ull pollard(ull n) {
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  auto f = [&](ull x) { return modmul(x, x, n) + i; };
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));
  return l;
}
```

5.3 Divisibility

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

33ba8f, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
}
```

CRT.h

Description: Chinese Remainder Theorem.

crt(a, m, b, n) computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.

Time: $\log(n)$

"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
```

5.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) :=$ # of positive integers $\le n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$

Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.

Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p$ $\forall a$.

cf7d6d, 8 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}
```

5.4 Fractions

ContinuedFractions.h

Description: Given N and a real number $x \ge 0$, finds the closest rational approximation p/q with $p, q \le N$. It will obey $|p/q - x| \le 1/qN$.

For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.

Time: $\mathcal{O}(\log N)$

dd6c5e, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
       a = (ll)floor(y), b = min(a, lim),
       NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives us a
      // better approximation; if b = a/2, we *may* have one.
      // Return {P, Q} here for a more canonical approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    }
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
}
```

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \le N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.

Usage: fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}

Time: $\mathcal{O}(\log(N))$

27ab3e, 25 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
  if (f(lo)) return lo;
  assert(f(hi));
  while (A || B) {
```

```
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \;\; b = k \cdot (2mn), \;\; c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

5.6 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.7 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

5.8 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \le m \le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \le m \le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11		12		13		14	15	16	17
$n!$	4.0e7		4.8e8		6.2e9		8.7e10	1.3e12	2.1e13	3.6e14
n	20		25		30		40	50	100	150	171
$n!$	2e18		2e25		3e32		8e47	3e64	9e157	6e262	>DBL_MAX

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.

Time: $\mathcal{O}(n)$

044568, 6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
    use |= 1 << x;                    // (note: minus, not ~!)
  return r;
}
```

6.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n} \right)$$

6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

6.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n-k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

6.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod p$.

6.2.3 Binomials

multinomial.h

Description: Computes $\binom{k_1+\cdots+k_n}{k_1,k_2,\ldots,k_n} = \frac{(\sum k_i)!}{k_1!k_2!\ldots k_n!}$.

<bits/stdc++.h> 15df88, 37 lines

```
using namespace std;

const int N = 1e6, mod = 1e9 + 7;

int power(long long n, long long k) {
  int ans = 1 % mod; n %= mod; if (n < 0) n += mod;
  while (k) {
    if (k & 1) ans = (long long) ans * n % mod;
    n = (long long) n * n % mod;
    k >>= 1;
  }
  return ans;
}
int f[N], invf[N];
int nCr(int n, int r) {
  if (n < r or n < 0) return 0;
  return 1LL * f[n] * invf[r] % mod * invf[n - r] % mod;
}
int nPr(int n, int r) {
  if (n < r or n < 0) return 0;
  return 1LL * f[n] * invf[n - r] % mod;
}
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  f[0] = 1;
  for (int i = 1; i < N; i++) {
    f[i] = 1LL * i * f[i - 1] % mod;
  }
  invf[N - 1] = power(f[N - 1], mod - 2);
  for (int i = N - 2; i >= 0; i--) {
    invf[i] = 1LL * invf[i + 1] * (i + 1) % mod;
  }
  cout << nCr(6, 2) << '\n';
  cout << nPr(6, 2) << '\n';
  return 0;
}
```

6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able).
$B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

6.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\dots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$

6.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

6.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2} C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max|w_i| < \sim 2^{63}$.

Time: $\mathcal{O}(VE)$

830a8f, 23 lines

```
const ll inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
struct Node { ll dist = inf; int prev = -1; };

void bellmanFord(vector<Node>& nodes, vector<Ed>& eds, int s) {
  nodes[s].dist = 0;
  sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled vertices
  rep(i,0,lim) for (Ed ed : eds) {
    Node cur = nodes[ed.a], &dest = nodes[ed.b];
    if (abs(cur.dist) == inf) continue;
    ll d = cur.dist + ed.w;
    if (d < dest.dist) {
      dest.prev = ed.a;
```

```
      dest.dist = (i < lim-1 ? d : -inf);
    }
  }
  rep(i,0,lim) for (Ed e : eds) {
    if (nodes[e.a].dist == -inf)
      nodes[e.b].dist = -inf;
  }
}
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j] = \texttt{inf}$ if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, inf if no path, or -inf if the path goes through a negative-weight cycle.
Time: $\mathcal{O}\left(N^3\right)$

531245, 12 lines

```
const ll inf = 1LL << 62;
void floydWarshall(vector<vector<ll>>& m) {
  int n = sz(m);
  rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
  rep(k,0,n) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) {
      auto newDist = max(m[i][k] + m[k][j], -inf);
      m[i][j] = min(m[i][j], newDist);
    }
  rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
}
```

TopoSort.h

Description: Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than n – nodes reachable from cycles will not be returned.
Time: $\mathcal{O}\left(|V| + |E|\right)$

<bits/stdc++.h> ef1438, 50 lines

```
using namespace std;

const int N = 1e5 + 9;
vector<int> g[N];
bool vis[N];
vector<int> ord;
void dfs(int u) {
  vis[u] = true;
  for (auto v: g[u]) {
    if (!vis[v]) {
      dfs(v);
    }
  }
  ord.push_back(u);
}
```

```
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  int n, m; cin >> n >> m;
  while (m--) {
    int u, v; cin >> u >> v;
    g[u].push_back(v);
  }
  for (int i = 1; i <= n; i++) {
    if (!vis[i]) {
      dfs(i);
    }
  }
  reverse(ord.begin(), ord.end());

  // check is feasible
  vector<int> pos(n + 1);
  for (int i = 0; i < (int) ord.size(); i++) {
    pos[ord[i]] = i;
  }
  for (int u = 1; u <= n; u++) {
    for (auto v: g[u]) {
      if (pos[u] > pos[v]) {
        cout << "IMPOSSIBLE\n";
        return 0;
      }
    }
  }

  // print the order
  for (auto u: ord) cout << u << ' ';
  cout << '\n';
  return 0;
}
// https://cses.fi/problemset/task/1679
```

7.2 DFS algorithms

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.
Usage: `scc(graph, [&](vi& v) { ... }) visits all components in reverse topological order. comp[i] holds the component index of a node (a component only has edges to components with lower index). ncomps will contain the number of components.`
Time: $\mathcal{O}\left(E + V\right)$

<bits/stdc++.h> ea07dd, 62 lines

```
using namespace std;
const int N = 3e5 + 9;
```

```cpp
// given a directed graph return the minimum number of edges to be added
    so that the whole graph  become an SCC
bool vis[N];
vector<int> g[N], r[N], G[N], vec; //G is the condensed graph
void dfs1(int u) {
  vis[u] = 1;
  for(auto v: g[u]) if(!vis[v]) dfs1(v);
  vec.push_back(u);
}

vector<int> comp;
void dfs2(int u) {
  comp.push_back(u);
  vis[u] = 1;
  for(auto v: r[u]) if(!vis[v]) dfs2(v);
}

int idx[N], in[N], out[N];
int main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);

  int n, m;
  cin >> n >> m;
  for(int i = 1; i <= m; i++) {
    int u, v;
    cin >> u >> v;
    g[u].push_back(v);
    r[v].push_back(u);
  }
  for(int i = 1; i <= n; i++) if(!vis[i]) dfs1(i);
  reverse(vec.begin(), vec.end());
  memset(vis, 0, sizeof vis);
  int scc = 0;
  for(auto u: vec) {
    if(!vis[u]) {
      comp.clear();
      dfs2(u);
      scc++;
      for(auto x: comp) idx[x]=scc;
    }
  }
  for(int u = 1; u <= n; u++) {
    for(auto v: g[u]) {
      if(idx[u] != idx[v]) {
        in[idx[v]]++, out[idx[u]]++;
        G[idx[u]].push_back(idx[v]);
      }
    }
  }
  int needed_in=0, needed_out=0;
  for(int i = 1; i <= scc; i++) {
    if(!in[i]) needed_in++;
    if(!out[i]) needed_out++;
  }
  int ans = max(needed_in, needed_out);
  if(scc == 1) ans = 0;
  cout << ans << '\n';
  return 0;
}
```

bridge.h

Description: Articulation Point + bridge representing the maximal clique.

Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

fe94e5, 55 lines

```cpp
// adj[u] = adjacent nodes of u
// ap = AP = articulation points
// p = parent
// disc[u] = discovery time of u
// low[u] = 'low' node of u

int dfsAP(int u, int p) {
  int children = 0;
  low[u] = disc[u] = ++Time;
  for (int& v : adj[u]) {
    if (v == p) continue; // we don't want to go back through the same
        path.
                          // if we go back is because we found another way
                              back
    if (!disc[v]) { // if V has not been discovered before
      children++;
      dfsAP(v, u); // recursive DFS call
      if (disc[u] <= low[v]) // condition #1
        ap[u] = 1;
      low[u] = min(low[u], low[v]); // low[v] might be an ancestor of u
    } else // if v was already discovered means that we found an ancestor
      low[u] = min(low[u], disc[v]); // finds the ancestor with the least
          discovery time
  }
  return children;
}

void AP() {
  ap = low = disc = vector<int>(adj.size());
  Time = 0;
  for (int u = 0; u < adj.size(); u++)
    if (!disc[u])
      ap[u] = dfsAP(u, u) > 1; // condition #2
}
vector<pair<int, int>> br;
```

```cpp
int dfsBR(int u, int p) {
  low[u] = disc[u] = ++Time;
  for (int& v : adj[u]) {
    if (v == p) continue; // we don't want to go back through the same
        path.
                          // if we go back is because we found another way
                              back
    if (!disc[v]) { // if V has not been discovered before
      dfsBR(v, u);  // recursive DFS call
      if (disc[u] < low[v]) // condition to find a bridge
        br.push_back({u, v});
      low[u] = min(low[u], low[v]); // low[v] might be an ancestor of u
    } else // if v was already discovered means that we found an ancestor
      low[u] = min(low[u], disc[v]); // finds the ancestor with the least
          discovery time
  }
}

void BR() {
  low = disc = vector<int>(adj.size());
  Time = 0;
  for (int u = 0; u < adj.size(); u++)
    if (!disc[u])
      dfsBR(u, u)
}
```

7.3 Trees

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.

Time: $\mathcal{O}(N \log N + Q)$

<bits/stdc++.h> ce5352, 56 lines

```cpp
using namespace std;

const int N = 3e5 + 9, LG = 18;

vector<int> g[N];
int par[N][LG + 1], dep[N], sz[N];
void dfs(int u, int p = 0) {
  par[u][0] = p;
  dep[u] = dep[p] + 1;
  sz[u] = 1;
  for (int i = 1; i <= LG; i++) par[u][i] = par[par[u][i - 1]][i - 1];
  for (auto v: g[u]) if (v != p) {
    dfs(v, u);
    sz[u] += sz[v];
  }
}
int lca(int u, int v) {
  if (dep[u] < dep[v]) swap(u, v);
  for (int k = LG; k >= 0; k--) if (dep[par[u][k]] >= dep[v]) u = par[u][k
      ];
  if (u == v) return u;
  for (int k = LG; k >= 0; k--) if (par[u][k] != par[v][k]) u = par[u][k],
       v = par[v][k];
  return par[u][0];
}
int kth(int u, int k) {
  assert(k >= 0);
  for (int i = 0; i <= LG; i++) if (k & (1 << i)) u = par[u][i];
  return u;
}
int dist(int u, int v) {
  int l = lca(u, v);
  return dep[u] + dep[v] - (dep[l] << 1);
}
//kth node from u to v, 0th node is u
int go(int u, int v, int k) {
  int l = lca(u, v);
  int d = dep[u] + dep[v] - (dep[l] << 1);
  assert(k <= d);
  if (dep[l] + k <= dep[u]) return kth(u, k);
  k -= dep[u] - dep[l];
  return kth(v, dep[v] - dep[l] - k);
}
int32_t main() {
  int n; cin >> n;
  for (int i = 1; i < n; i++) {
    int u, v; cin >> u >> v;
    g[u].push_back(v);
    g[v].push_back(u);
  }
  dfs(1);
  int q; cin >> q;
  while (q--) {
    int u, v; cin >> u >> v;
    cout << dist(u, v) << '\n';
  }
  return 0;
}
```

7.4 Math

7.4.1 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

7.4.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

Strings (8)

Zfunc.h

Description: z[i] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

ee09e2, 12 lines

```
vi Z(const string& S) {
  vi z(sz(S));
  int l = -1, r = -1;
  rep(i,1,sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  return z;
}
```

Hashing.h

Description: Self-explanatory methods for string hashing.

<bits/stdc++.h> bf11cb, 80 lines

```
using namespace std;

const int N = 1e6 + 9;
const int p1 = 137, mod1 = 127657753, p2 = 277, mod2 = 987654319;

int power(long long n, long long k, int mod) {
  int ans = 1 % mod; n %= mod; if (n < 0) n += mod;
  while (k) {
    if (k & 1) ans = (long long) ans * n % mod;
    n = (long long) n * n % mod;
    k >>= 1;
  }
  return ans;
}

int ip1, ip2;
pair<int, int> pw[N], ipw[N];
void prec() {
  pw[0] = {1, 1};
  for (int i = 1; i < N; i++) {
    pw[i].first = 1LL * pw[i - 1].first * p1 % mod1;
    pw[i].second = 1LL * pw[i - 1].second * p2 % mod2;
  }
  ip1 = power(p1, mod1 - 2, mod1);
  ip2 = power(p2, mod2 - 2, mod2);
  ipw[0] = {1, 1};
  for (int i = 1; i < N; i++) {
    ipw[i].first = 1LL * ipw[i - 1].first * ip1 % mod1;
    ipw[i].second = 1LL * ipw[i - 1].second * ip2 % mod2;
  }
}

pair<int, int> string_hash(string s) {
  int n = s.size();
  pair<int, int> hs({0, 0});
  for (int i = 0; i < n; i++) {
    hs.first += 1LL * s[i] * pw[i].first % mod1;
    hs.first %= mod1;
    hs.second += 1LL * s[i] * pw[i].second % mod2;
    hs.second %= mod2;
  }
  return hs;
}
pair<int, int> pref[N];
void build(string s) {
  int n = s.size();
  for (int i = 0; i < n; i++) {
    pref[i].first = 1LL * s[i] * pw[i].first % mod1;
    if (i) pref[i].first = (pref[i].first + pref[i - 1].first) % mod1;
    pref[i].second = 1LL * s[i] * pw[i].second % mod2;
    if (i) pref[i].second = (pref[i].second + pref[i - 1].second) % mod2;
  }
}
pair<int, int> get_hash(int i, int j) {
  assert(i <= j);
  pair<int, int> hs({0, 0});
  hs.first = pref[j].first;
  if (i) hs.first = (hs.first - pref[i - 1].first + mod1) % mod1;
  hs.first = 1LL * hs.first * ipw[i].first % mod1;
```

```
  hs.second = pref[j].second;
  if (i) hs.second = (hs.second - pref[i - 1].second + mod2) % mod2;
  hs.second = 1LL * hs.second * ipw[i].second % mod2;
  return hs;
}

int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  prec();
  string a, b; cin >> a >> b;
  build(a);
  int ans = 0, n = a.size(), m = b.size();
  auto hash_b = string_hash(b);
  for (int i = 0; i + m - 1 < n; i++) {
    ans += get_hash(i, i + m - 1) == hash_b;
  }
  cout << ans << '\n';
  return 0;
}
```

Various (9)

9.1 Misc. algorithms

TernarySearch.h

Description: Example structures and functions that doesn't really do anything. Latex commands are supported here, though! Like this: $2^2 = 3$

9155b4, 15 lines

```
template <class F>
int ternSearch(int a, int b, F f)
{
  assert(a <= b);
  while (b - a >= 5)
  {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid + 1))
      a = mid; // (A)
    else
      b = mid + 1;
  }
  rep(i, a + 1, b + 1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

LIS.h

Description: Example structures and functions that doesn't really do anything. Latex commands are supported here, though! Like this: $2^2 = 3$

2932a0, 17 lines

```
template<class I> vi lis(const vector<I>& S) {
  if (S.empty()) return {};
  vi prev(sz(S));
  typedef pair<I, int> p;
  vector<p> res;
  rep(i,0,sz(S)) {
    // change 0 -> i for longest non-decreasing subsequence
    auto it = lower_bound(all(res), p{S[i], 0});
    if (it == res.end()) res.emplace_back(), it = res.end()-1;
    *it = {S[i], i};
    prev[i] = it == res.begin() ? 0 : (it-1)->second;
  }
  int L = sz(res), cur = res.back().second;
  vi ans(L);
  while (L--) ans[L] = cur, cur = prev[cur];
  return ans;
}
```

boundedknapsack.h

Description: Example structures and functions that doesn't really do anything. Latex commands are supported here, though! Like this: $2^2 = 3$

ddd2b6, 44 lines

```
const int mod = 100000007;
int coin[50], amount[50], dp[51][1001];
int coinChange(int n, int sum)
{
    if (sum == 0)
        return 1;
    if (n == 0)
        return 0;
    if (dp[n][sum] != -1)
        return dp[n][sum];
    dp[n][sum] = 0;
    for (int i = 0; i <= amount[n - 1] && i * coin[n - 1] <= sum; i++)
        dp[n][sum] += (coinChange(n - 1, sum - i * coin[n - 1]) % mod);
    return (dp[n][sum] % mod);
}
void solve()
{
    int n, sum;
    cin >> n >> sum;
    for (int i = 0; i < n; ++i)
        cin >> coin[i];
    for (int i = 0; i < n; ++i)
        cin >> amount[i];
    int bottom_up_dp[n + 1][sum + 1];
    for (int i = 0; i <= sum; ++i)
        bottom_up_dp[0][i] = 0;
    for (int i = 0; i <= n; ++i)
        bottom_up_dp[i][0] = 1;
    for (int i = 1; i <= n; ++i)
    {
```

```
        for (int j = 1; j <= sum; ++j)
        {
            bottom_up_dp[i][j] = 0;
            for (int k = 0; (k <= amount[i - 1]) && ((k * coin[i - 1]) <=
                j); ++k)
            {
                bottom_up_dp[i][j] += (bottom_up_dp[i - 1][j - k * coin[i
                    - 1]]);
                bottom_up_dp[i][j] %= mod;
            }
        }
    }
    cout << bottom_up_dp[n][sum] << '\n';
    memset(dp, -1, sizeof(dp));
    cout << coinChange(n, sum) << '\n';
}
```

dfslargevalueofnodes.h

Description: Example structures and functions that doesn't really do anything. Latex commands are supported here, though! Like this: $2^2 = 3$

<bits/stdc++.h> 631c01, 24 lines

```
using namespace std;

map<int, vector<int>> adj;
set<int> visited;
void dfs(int n)
{
    if (visited.find(n) != visited.end())
        return;
    visited.insert(n);
    for (auto i : adj[n])
        dfs(i);
}
void solve(int testCaseNo)
{
    int n, u, v;
    cin >> n;
    for (int i = 0; i < n; ++i)
    {
        cin >> u >> v;
        adj[u].push_back(v);
    }
    dfs(n);
    cout << *visited.rbegin() << nl;
}
```

dsupathcompressionandorderbysize.h

Description: Example structures and functions that doesn't really do anything. Latex commands are supported here, though! Like this: $2^2 = 3$

<bits/stdc++.h> 28c2f8, 30 lines

```
using namespace std;

int parent[1000001], size[1000001];
void initialize(int n)
{
    for (int i = 1; i <= n; ++i)
    {
        parent[i] = i;
        size[i] = 1;
    }
}
int find_set(int n)
{
    if (parent[n] == n)
        return n;
    return parent[n] = find_set(parent[n]);
}

void union_sets(int a, int b)
{
    a = find_set(a);
    b = find_set(b);
    if (a != b)
    {
        if (size[a] < size[b])
            swap(a, b);
        parent[b] = a;
        size[a] += size[b];
    }
}
```

inversemodusingextendedeuclid.h

Description: Example structures and functions that doesn't really do anything. Latex commands are supported here, though! Like this: $2^2 = 3$

<bits/stdc++.h> 1c8e4a, 20 lines

```
using namespace std;

const int mod = 1000000007;

pair<int, int> Extended_Euclid(int a, int b) {
    if (b == 0)
        return {1, 0};
    pair<int, int> r = Extended_Euclid(b, a % b);
    return {r.second, r.first - r.second * (a / b)};
}

int inverseMod(int a) {
    pair<int, int> ans = Extended_Euclid(a, mod);
    return ans.first;
}
```

```
int main() {
    int a; cin >> a;
    cout << inverseMod(a);
}
```

lazysegmenttree.h

Description: Example structures and functions that doesn't really do anything. Latex commands are supported here, though! Like this: $2^2 = 3$

3136b0, 51 lines

```
int arr[100001];
struct info {
    int sum = 0, prp = 0;
} tree[200002];
void init(int node, int b, int e) {
    if (b == e) {
        tree[node].sum = arr[b];
        return;
    }
    int mid = (b + e) / 2;
    int l = node + 1;
    int r = node + 2 * (mid - b + 1);
    init (l, b, mid);
    init (r, mid + 1, e);
    tree[node].sum = tree[l].sum + tree[r].sum;
}
void update(int node, int b, int e, int i, int j, int x) {
    int mid = (b + e) / 2;
    int l = node + 1;
    int r = node + 2 * (mid - b + 1);
    if (tree[node].prp) {
        tree[node].sum += ((e - b + 1) * tree[node].prp);
        if (b != e)
            tree[l].prp += tree[node].prp, tree[r].prp += tree[node].prp;
        tree[node].prp = 0;
    }
    if (b > j || e < i) return;
    if (b >= i && e <= j) {
        tree[node].sum += ((e - b + 1) * x);
        if (b != e)
            tree[l].prp += x, tree[r].prp += x;
        return;
    }
    update(l, b, mid, i, j, x);
    update(r, mid+1, e, i, j, x);
    tree[node].sum = tree[l].sum + tree[r].sum;
}
int query(int node, int b, int e, int i, int j) {
    int mid = (b + e) / 2;
    int l = node + 1;
    int r = node + 2 * (mid - b + 1);
    if (tree[node].prp) {
        tree[node].sum += ((e - b + 1) * tree[node].prp);
        if (b != e)
            tree[l].prp += tree[node].prp, tree[r].prp += tree[node].prp;
        tree[node].prp = 0;
    }
    if (b > j || e < i) return 0;
    if (b >= i && e <= j) return tree[node].sum;
    return (query(l, b, mid, i, j) + query(r, mid+1, e, i, j));
}
```

MatrixExponentiation.h

Description: Example structures and functions that doesn't really do anything. Latex commands are supported here, though! Like this: $2^2 = 3$

<bits/stdc++.h> df9dec, 85 lines

```
using namespace std;

const int mod = 998244353;

struct Mat {
  int n, m;
  vector<vector<int>> a;
  Mat() { }
  Mat(int _n, int _m) {n = _n; m = _m; a.assign(n, vector<int>(m, 0)); }
  Mat(vector< vector<int> > v) { n = v.size(); m = n ? v[0].size() : 0; a
      = v; }
  inline void make_unit() {
    assert(n == m);
    for (int i = 0; i < n; i++)  {
      for (int j = 0; j < n; j++) a[i][j] = i == j;
    }
  }
  inline Mat operator + (const Mat &b) {
    assert(n == b.n && m == b.m);
    Mat ans = Mat(n, m);
    for(int i = 0; i < n; i++) {
      for(int j = 0; j < m; j++) {
        ans.a[i][j] = (a[i][j] + b.a[i][j]) % mod;
      }
    }
    return ans;
  }
  inline Mat operator - (const Mat &b) {
    assert(n == b.n && m == b.m);
    Mat ans = Mat(n, m);
    for(int i = 0; i < n; i++) {
      for(int j = 0; j < m; j++) {
        ans.a[i][j] = (a[i][j] - b.a[i][j] + mod) % mod;
      }
    }
```

```
    return ans;
  }
  inline Mat operator * (const Mat &b) {
    assert(m == b.n);
    Mat ans = Mat(n, b.m);
    for(int i = 0; i < n; i++) {
      for(int j = 0; j < b.m; j++) {
        for(int k = 0; k < m; k++) {
          ans.a[i][j] = (ans.a[i][j] + 1LL * a[i][k] * b.a[k][j] % mod) %
              mod;
        }
      }
    }
    return ans;
  }
  inline Mat pow(long long k) {
    assert(n == m);
    Mat ans(n, n), t = a; ans.make_unit();
    while (k) {
      if (k & 1) ans = ans * t;
      t = t * t;
      k >>= 1;
    }
    return ans;
  }
  inline Mat& operator += (const Mat& b) { return *this = (*this) + b; }
  inline Mat& operator -= (const Mat& b) { return *this = (*this) - b; }
  inline Mat& operator *= (const Mat& b) { return *this = (*this) * b; }
  inline bool operator == (const Mat& b) { return a == b.a; }
  inline bool operator != (const Mat& b) { return a != b.a; }
};

int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  int n; long long k; cin >> n >> k;
  Mat a(n, n);
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      cin >> a.a[i][j];
    }
  }
  Mat ans = a.pow(k);
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      cout << ans.a[i][j] << ' ';
    }
    cout << '\n';
  }
  return 0;
}
// https://judge.yosupo.jp/problem/pow_of_matrix
```

profitbasedknapsack.h

Description: Example structures and functions that doesn't really do anything. Latex commands are supported here, though! Like this: $2^2 = 3$

93135f, 24 lines

```
const int infinity = 1e9 + 1;
void solve() {
    int n, w, sum = 0; cin >> n >> w;
    int weight[n], value[n];
    for (int i = 0; i < n; ++i) {
        cin >> weight[i] >> value[i];
        sum += value[i];
    }
    vector<int> dp(sum + 1, infinity);
    dp[0] = 0;
    for (int i = 0; i < n; ++i) {
        for (int j = sum; j >= value[i]; --j) {
            int temp = dp[j - value[i]] + weight[i];
            if ((temp <= w) && (temp < dp[j]))
                dp[j] = temp;
        }
    }
    for (int i = sum; i >= 0; --i) {
        if (dp[i] != infinity) {
            cout << i;
            return;
        }
    }
}
```

segmentedsieve.h

Description: Example structures and functions that doesn't really do anything. Latex commands are supported here, though! Like this: $2^2 = 3$

<bits/stdc++.h> a994dc, 56 lines

```
using namespace std;
using namespace std::chrono;

auto start = high_resolution_clock::now();
#define int long long

bool marked[1000001];
vector<int> primes;

void sieve(int n) {
    memset(marked, false, n+1);
    marked[1] = true;
    for (int i = 4; i <= n; i += 2)
        marked[i] = true;
```

```
    for (int i = 3; i*i <= n; i += 2) {
        if (!marked[i]) {
            for (int j = i*i; j <= n; j += i*2)
                marked[j] = true;
        }
    }
    for (int i = 2; i <= n; ++i) {
        if (!marked[i])
            primes.push_back(i);
    }
}

int32_t main() {
    int a, b; cin >> a >> b;
    int n = sqrt(b);
    sieve(n);
    vector<int> rangePrimes;
    int segments = ceil(sqrt(b-a+1));
    int low = a, high = a + segments - 1;
    for (int i = 0; i < segments; ++i) {
        int range = high - low + 1;
        memset(marked, false, range);
        for (auto j : primes) {
            for (int k = max(j*j, (low+j-1)/j * j); k <= high; k += j)
                marked[k-low] = true;
        }

        if (low == 1)
            marked[0] = true;
        for (int j = 0; j < range; ++j) {
            if (!marked[j])
                rangePrimes.push_back(j+low);
        }
        low = high + 1;
        high = min(b, high + segments);
    }

    cout << "Total Primes: " << rangePrimes.size();
    auto stop = high_resolution_clock::now();
    auto duration = duration_cast<milliseconds> (stop - start);
    cout << "\nTime Taken: " << duration.count() << " milliseconds.";
}
```

FastKnapsack.h

Description: Example structures and functions that doesn't really do anything. Latex commands are supported here, though! Like this: $2^2 = 3$

b20ccc, 16 lines

```
int knapsack(vi w, int t) {
  int a = 0, b = 0, x;
  while (b < sz(w) && a + w[b] <= t) a += w[b++];
  if (b == sz(w)) return a;
  int m = *max_element(all(w));
  vi u, v(2*m, -1);
  v[a+m-t] = b;
  rep(i,b,sz(w)) {
    u = v;
    rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x])
      v[x-w[j]] = max(v[x-w[j]], j);
  }
  for (a = t; v[a+m-t] < 0; a--) ;
  return a;
}
```