Advance Statistics

Project

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- *Batch DSBA June B Group B5 12'0*

A Business Report on Advance Statistics

Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected

	Striker	Forward	Attacking Midfielder	Winger	Total
Players Injured	45	56	24	20	145
Players Not Injured	32	38	11	9	90
Total	77	94	35	29	235

1.1 What is the probability that a randomly chosen player would suffer an injury?

1.2 What is the probability that a player is a forward or a winger?

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

1.4 What is the probability that a randomly chosen injured player is a striker?

1.5 What is the probability that a randomly chosen injured player is either a forward or an attacking midfielder?

Solution

1.1 From table:

Total Players Injured = 145

Total Players = 235

Probability of a random player would suffer an injury(p) $= \frac{Total\ Player\ Injured}{Total\ Players} = \frac{145}{235}$ $=0.617$

Hence, <u>There is a 61.7% Probability that a randomly selected player would suffer an injury.</u>

1.2 Forward Players = 94

Total Players = 235

Probability that a player is Forward $(p_F) = \frac{Forward\ Players}{Total\ Players} = \frac{94}{235} = 0.4$

Winger Players $=$ 29

Total Players = 235

Probability that a player is Winger $(p_W) = \frac{Winger\ Players}{Total\ Player} = \frac{29}{235} = 0.12$

Probability that a player is forward or winger $= p_F \cup p_W$

$$\Rightarrow p_F \cup p_W = p_F + p_w = 0.4 + 0.12 = 0.52$$

Hence, <u>There is a 52% Probability that a randomly selected player is a Forward or a winger.</u>

1.3 Players with Injury in Striker Position = 45

Total Players in Striker position = 77

Probability that a randomly chosen player in striker position has

foot injury $(p_{Striker\ with\ injury}) = \frac{Players\ with\ injury\ in\ Striker\ Position}{Total\ Player\ in\ striker\ position} = \frac{45}{77} = 0.584$

Hence, <u>There is 58.4% Probability that a randomly selected player in striker position has a foot injury.</u>

1.4 Injured Player in striker = 45

Total Players = 235

Probability that a randomly chosen injured player is a striker

$(p_{Injured\ Striker}) = \frac{Injured\ Player\ in\ Striker\ Position}{Total\ Players} = \frac{45}{235} = 0.1915$

Hence, <u>There is 19.15% Probability that a randomly selected injured player is a striker.</u>

1.5 Injured Player in Forward = 56

Injured Player in Attacking Midfielder = 24

Total Players = 235

Probability that a randomly chosen injured player is either a forward or attacking midfielder =
$probability\ of\ forward\ injured\ player\ +\ probability\ of\ attacking\ midfielder\ injured\ player$

$$\Rightarrow \frac{Injured\ player\ in\ Forward}{Total\ Players} + \frac{Injured\ Player\ in\ attacking\ Midfielder}{Total\ Players}$$

$$\Rightarrow \frac{56}{235} + \frac{24}{235} = \frac{80}{235} = 0.3404$$

Hence, There is 34.04% Probability that a randomly selected injured player is either a Forward or an attacking MidFielder

Problem 2

An independent research organization is trying to estimate the probability that an accident at a nuclear power plant will result in radiation leakage. The types of accidents possible at the plant are fire hazards, mechanical failure, or human error. The research organization also knows that two or more types of accidents cannot occur simultaneously.

According to the studies carried out by the organization, the probability of a radiation leak in case of a fire is 20%, the probability of a radiation leak in case of a mechanical 50%, and the probability of a radiation leak in case of a human error is 10%. The studies also showed the following;

- The probability of a radiation leak occurring simultaneously with a fire is 0.1%.
- The probability of a radiation leak occurring simultaneously with a mechanical failure is 0.15%.
- The probability of a radiation leak occurring simultaneously with a human error is 0.12%.

On the basis of the information available, answer the questions below:

2.1 What are the probabilities of a fire, a mechanical failure, and a human error respectively?

2.2 What is the probability of a radiation leak?

2.3 Suppose there has been a radiation leak in the reactor for which the definite cause is not known. What is the probability that it has been caused by:

- A Fire
- A Mechanical Failure
- A Human Error

Solution

2.1

- Fire :

  Given;

  Probability (Radiation Leak | Given In case of Fire) = $P(R|F)$ = 0.2

  Probability (Radiation leak occurring simultaneously with fire) = $P(R \cap F) = 0.001$

  Let Probability of Radiation leaked by Fire = $P(F)$

  So by conditional probability; $P(R|F) = \frac{P(R \cap F)}{P(F)}$

  $P(F) = \frac{P(R \cap F)}{P(R|F)} = \frac{0.001}{0.2} = 0.005$

  Hence, the Probability of Radiation Leak caused by Fire is 0.5%

- Mechanical Failure

  Given;

  Probability (Radiation Leak | Given In case of Mechanical Failure) = $P(R|M)$ = 0.5

  Probability (Radiation leak simultaneously with mechanical failure) = $P(R \cap M) = 0.0015$

  Let Probability of Radiation leaked by Mechanical Failure= $P(M)$

  So by conditional probability; $P(R|M) = \frac{P(R \cap M)}{P(M)}$

  $P(M) = \frac{P(R \cap M)}{P(R|M)} = \frac{0.0015}{0.5} = 0.003$

  Hence, the Probability of Radiation Leak caused by Mechanical Failure is 0.3%

- Human Error

  Given;

  Probability (Radiation Leak | Given In case of Human Error) = $P(R|H)$ = 0.1

Probability (Radiation leak simultaneously with a human error) = $P(R \cap H) = 0.0012$

Let Probability of Radiation leaked by Human Error= $P(H)$

So by conditional probability; $P(R|H) = \frac{P(R \cap H)}{P(H)}$

$$P(H) = \frac{P(R \cap H)}{P(R|H)} = \frac{0.0012}{0.1} = 0.012$$

Hence, the Probability of Radiation Leak caused by a Human Error is 1.2%

2.2 Probability of Radiation Leak $= P(R)$

$$P(R) = P(R \cap F) + P(R \cap M) + P(R \cup H)$$

$$P(R) = 0.001 + 0.0015 + 0.0012$$

$$P(R) = 0.0037$$

Hence, the Probability of Radiation Leak is 0.37%

2.3

- Fire :

  Probability (Fire | Radiation) = $P(F|R)$

  So,; $P(F|R) = \frac{P(R \cap F)}{P(R\cap F) + P(R\cap M) + P(R\cup H)}$

  $$P(F|R) = \frac{P(R \cap F)}{P(R\cap F) + P(R\cap M) + P(R\cup H)} = 0.27027$$

  Hence, the Probability of Radiation Leak caused by Fire is 27.027%

- Mechanical Failure

  Probability (Mechanical | Radiation) = $P(M|R)$

  So,; $P(M|R) = \frac{P(R \cap M)}{P(R\cap F) + P(R\cap M) + P(R\cup H)}$

$$P(M|R) = \frac{P(R \cap M)}{P(R \cap F) + P(R \cap M) + P(R \cup H)} = 0.40541$$

Hence, the Probability of Radiation Leak caused by Mechanical Failure is 40.541%

- Human Error

Probability (HumanError | Radiation) = $P(H|R)$

So,; $P(H|R) = \frac{P(R \cap H)}{P(R \cap F) + P(R \cap M) + P(R \cup H)}$

$$P(H|R) = \frac{P(R \cap H)}{P(R \cap F) + P(R \cap M) + P(R \cup H)} = 0.32432$$

Hence, the Probability of Radiation Leak caused by a Human Error is 32.432%

Problem 3

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information; (Provide an appropriate visual representation of your answers, without which marks will be deducted)

3.1 What proportion of the gunny bags have a breaking strength less than 3.17 kg per sq cm?

3.2 What proportion of the gunny bags have a breaking strength at least 3.6 kg per sq cm.?

3.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

3.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?

Solution

3.1 Mean (η) = $5 \frac{kg}{cm^2}$

Standard Deviation (σ) = $1.5 \frac{kg}{cm^2}$

As the breaking strength of the gunny bags are normally distributed, Fig 1 represents the graph of the Normal Distribution for Strength of Gunny Bags.

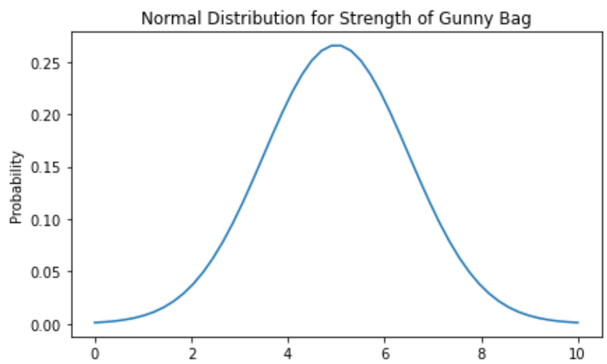


Fig 1: Strength of Gunny Bag (Kg. sq cm)

We have to find what proportion of Gunny Bag have a breaking strength less than 3.17 kg per sq. cm

$x = 3.17$

Fig 2, shows the same graph with shaded representing $x = 3.17$. Finding the area of the shaded region will give the proportion. $P(X \leq 3.17)$

<u>**Hence the probability of gunny bags having breaking strength less than 3.17 kg per sq cm. is 0.1112**</u>

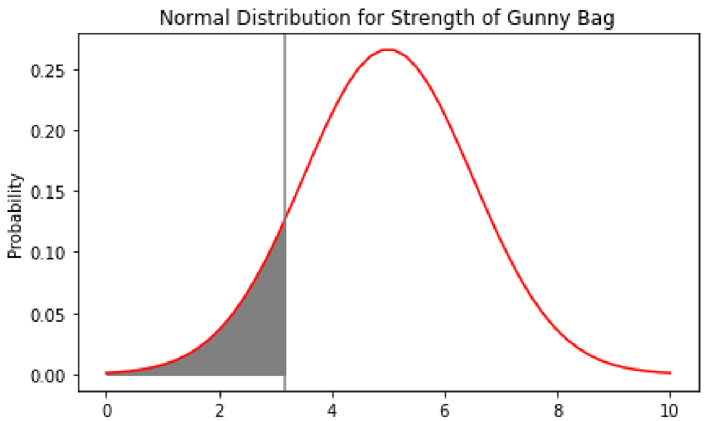


Fig 2: Strength of Gunny Bag (Kg. sq cm)

3.2 Fig 3. represents the same Normal distribution graph but the shaded region represents the breaking strength of at least 3.6 kg per sq.cm as per given. The area on the right side of this line will give the probability of the breaking strength at least starting from this point. $P(X \leq 3.6)$

<u>**Hence the probability of gunny bags having breaking strength at least 3.6 kg per sq cm. is 0.8247**</u>

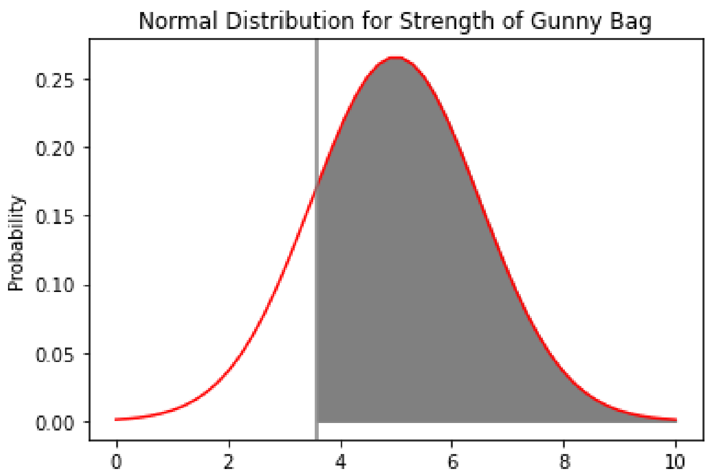


Fig 3: Strength of Gunny Bag (Kg. sq cm)

3.3 Fig 4. represents the same Normal distribution graph but the shaded region represents the breaking strength between 5 kg per sq.cm and 5.5 kg per sq.cm.

The area of the shaded region will give the proportion of the breaking strength between 5 and 5.5 kg per sq.cm. $P(5 \leq X \leq 5.5)$

Hence the probability of gunny bags having breaking strength between 5 and 5.5 kg per sq cm. is 0.1306

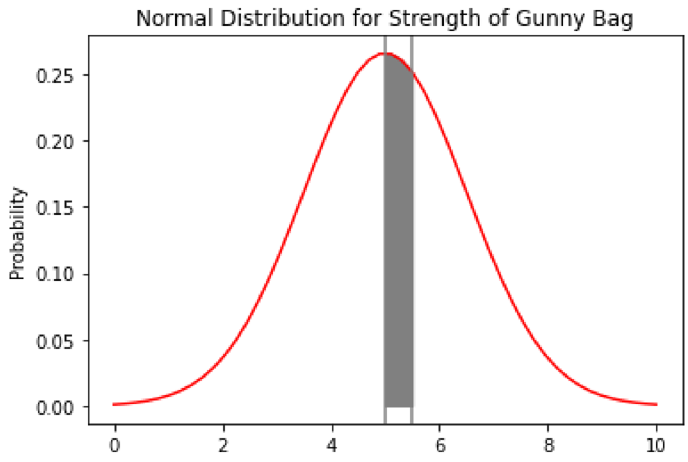


Fig 4: Strength of Gunny Bag (Kg. sq cm)

3.4 Fig 5. represents the same Normal distribution graph but here the two shaded regions represent the breaking strength. If we are excluding the area of the unshaded region from the total area, it will give the probability of the breaking strength NOT between 3 and 7.5 kg per sq.cm. $[1 - P(3 \leq X \leq 7.5)]$

Hence the probability of gunny bags having breaking strength NOT between 3 and 7.5 kg per sq cm. is 0.1390

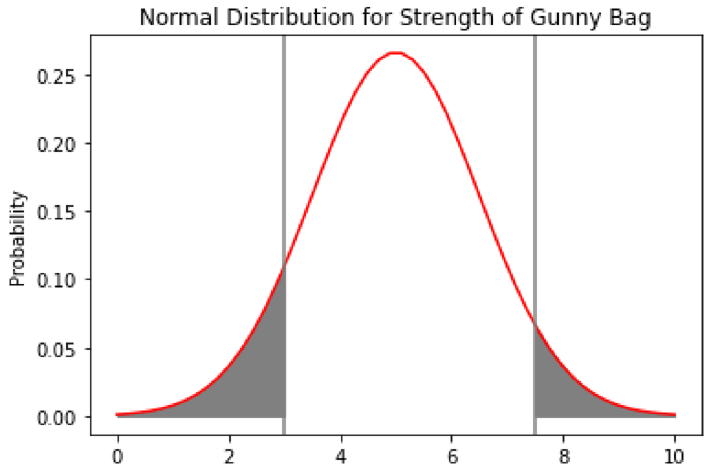


Fig 5: Strength of Gunny Bag (Kg. sq cm)

Problem 4

Grades of the final examination in a training course are found to be normally distributed, with a mean of 77 and a standard deviation of 8.5. Based on the given information, answer the questions below.

4.1 What is the probability that a randomly chosen student gets a grade below 85 on this exam?

4.2 What is the probability that a randomly selected student scores between 65 and 87?

4.3 What should be the passing cut-off so that 75% of the students clear the exam?

Solution

4.1 Mean (η) = 77, Standard Deviation (σ) = 8.5

$$P(X \leq 85)$$

From the graph Fig 6, we can clearly see from the shaded region shows that grades below 85 is obtained by students by 0.8267 portion

The area of the Cumulative distribution graph left to the X=85 represented by the orange dotted line will give the probability.

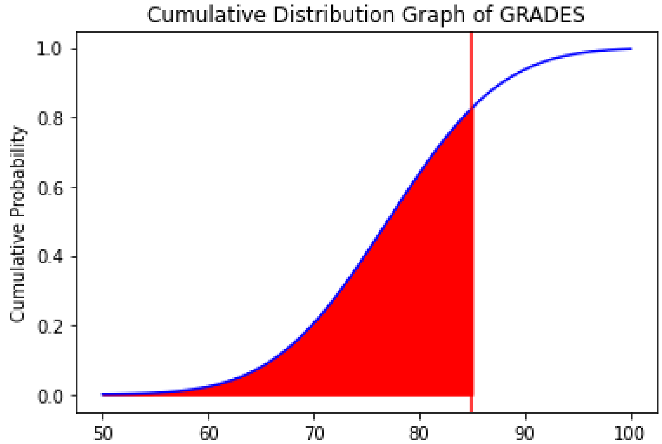


Fig 6: Grades of Final Exam

Hence the probability that a randomly chosen student gets a grade below 85 is 82.67%

4.2 $P(65 \leq X \leq 87)$

From the Fig 7, Cumulative distribution graph;

The area of the Cumulative distribution graph between 65 and 87 represented by green and red dotted lines respectively gives the probability.

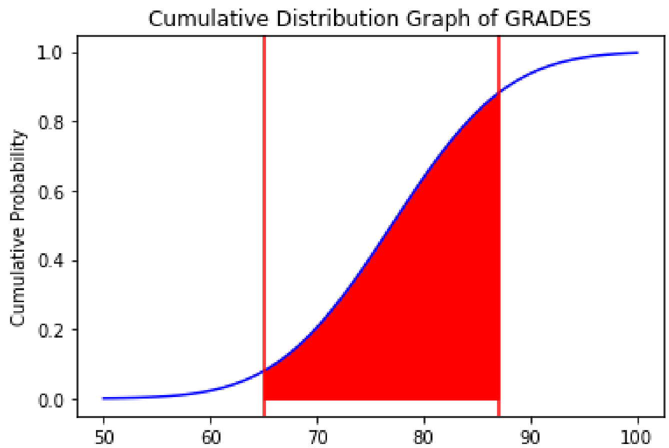


Fig 6: Grades of Final Exam

Hence the probability that a randomly chosen student gets a grade between 65 and 87 is 80.13%

4.3 Let the Passing cut-off marks be M.

$P(X \geq M) = 75\% \Rightarrow 1 - P(X < M) = 0.75 \Rightarrow P(X < M) = 0.25$

71.27 should be the passing Cut-off so that 75% of the students clear the exam

Problem 5

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

5.1 Earlier experience of Zingaro with this particular client is favorable as the stone surface was found to be of adequate hardness. However, Zingaro has reason to believe now that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

5.2 Is the mean hardness of the polished and unpolished stones the same?

Solution

5.1

Zingaro believes that unpolished stones may not be suitable for printing, which means Brinell's Hardness index is ! >= 150,

- STATE NULL & ALTERNATIVE HYPOTHESIS

$$H_0\ (Null\ Hypothesis):\ \mu \geq 150$$

$$H_1\ (Alternative\ Hypothesis):\ \mu\ ! \geq 150$$

- Decide Significance Level

$$\alpha\ (significance\ level)\ =\ 0.05$$

- Identification of the Test

We have to test for only Unpolished samples and we do not know the Population standard deviation, Hence we will be conducting the one sample test.

- Calculate the p-value and the test_Statistics

Test_stats: -4.164629601426758

p_value: 8.342573994839285e-05

- Conclusion / Check P-Value with Significance Level

    On the basis of calculated p_value: p-value <<< 0.05, so we have strong evidence to accept Alternative Hypothesis. Unpolished stones do not have a Brinell's Hardness index of at least 150.

So we can conclude by saying that Zingara's STATEMENT IS CORRECT

5.2

We have to conclude by finding evidence that Mean of Unpolished & Polished stones are different

- STATE NULL & ALTERNATIVE HYPOTHESIS

    $H_0\ (Null\ Hypothesis) : \mu_1 = \mu_2$

    $H_1\ (Alternative\ Hypothesis) :\ : \mu_1 \neq \mu_2$

- Decide Significance Level

    $\alpha\ (significance\ level)\ =\ 0.05$

- Identification of the Test

    We have two samples and we do not know the Population standard deviation

    Sample size for both the samples are same

    Both the sample are independent variables

    Two-Sample t-test to be conducted

- Calculate the p-value and the test_Statistics

    Test_stats: -3.242232050141406

    P_value: 0.001465515019462831

- Checking the mean value & comparing the confidence on our statement from the p-value and the test_Statistics

Mean of Zing Unpolished stones(μ_1): 134.11

Mean of Zing Polished stones(μ_2): 147.79

We have evidence (p_value(0.001)< 0.05) to Reject Null Hypothesis and conclude that :

Mean of Unpolished Stones is not equal to Mean of Polished Stones with 95% confidence level

Problem 6

Aquarius health club, one of the largest and most popular crossfit gyms in the country, has been advertising a rigorous program for body conditioning. The program is considered successful if the candidate is able to do more than 5 push-ups, as compared to when he/she enrolled in the program. Using the sample data provided can you conclude whether the program is successful? (Consider the level of Significance as 5%)

Note that this is a problem of the paired-t-test. Since the claim is that the training will make a difference of more than 5, the null and alternative hypotheses must be formed accordingly.

Solution

We have to statistically find out whether the program is considered successful or not.

- STATE NULL & ALTERNATIVE HYPOTHESIS

$$H_0\ (Null\ Hypothesis) : Pushup_2 - Pushup_1 \leq 5$$

$$H_1\ (Alternative\ Hypothesis) : Pushup_2 - Pushup_1 > 5$$

- Decide Significance Level

  $\alpha\ (significance\ level) = 0.05$

- Identification of the Test

  We have two samples and both the variables are dependent on each other, we will conduct PAIRED T-TEST

- Calculate the p-value and the test_Statistics

  Test_stats: -19.322619811082458

  P_value: 2.2920419252511966e-35

- Conclusion / Check P-Value with Significance Level

  On the basis of calculated p_value which is almost negligible:

  p(2.292e-35) << 0.05, so Null Hypothesis is rejected and Alternative Hypothesis is accepted here. Hence, Candidates are able to do more than 5 pushups as compared to when he/she enrolled in the program.

Thus we can conclude by stating that PROGRAM IS SUCCESSFUL

Problem 7

Dental implant data: The hardness of metal implant in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as on the dentists who may favor one method above another and may work better in his/her favorite method. The response is the variable of interest.

1. Test whether there is any difference among the dentists on the implant hardness. State the null and alternative hypotheses. Note that both types of alloys cannot be considered together. You must state the null and alternative hypotheses separately for the two types of alloys.?

2. Before the hypotheses may be tested, state the required assumptions. Are the assumptions fulfilled? Comment separately on both alloy types.?
3. Irrespective of your conclusion in 2, we will continue with the testing procedure. What do you conclude regarding whether implant hardness depends on dentists? Clearly state your conclusion. If the null hypothesis is rejected, is it possible to identify which pairs of dentists differ?
4. Now test whether there is any difference among the methods on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which pairs of methods differ?
5. Now test whether there is any difference among the temperature levels on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which levels of temperatures differ?
6. Consider the interaction effect of the dentist and method and comment on the interaction plot, separately for the two types of alloys?
7. Now consider the effect of both factors, dentist, and method, separately on each alloy. What do you conclude? Is it possible to identify which dentists are different, which methods are different, and which interaction levels are different?

Solution

7.1

As there are two types of Alloys. Alloy of Category 1 & Alloy of Category 2, We have to separate both the Alloy types.

- ALLOY TYPE 1:
  - <u>STATE NULL & ALTERNATIVE HYPOTHESIS</u>

$$H_0\ (Null):\ Implant\ Hardness\ for\ all\ dentist\ are\ same$$

$$H_1\ (Alternative):\ Implant\ Hardness\ for\ all\ Dentist\ are\ not\ same$$

- Decide Significance Level

  $\alpha\ (significance\ level)\ =\ 0.05$

- Identification of the Test

  We have to perform One Way Anova for Alloy 1

- Calculate the p-value and the test_Statistics

  From the Anova Table:

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	1.977112	0.116567
Residual	40.0	539593.555556	13489.838889	NaN	NaN

- Conclusion:

  Since the PR(>F) is 0.116567 which is greater than Significance Level(alpha) 0.05.

  We fail to Reject the Null Hypothesis for Alloy 1

- ALLOY TYPE 2:
  - STATE NULL & ALTERNATIVE HYPOTHESIS

    $H_0\ (Null):\ Implant\ Hardness\ for\ all\ dentist\ are\ same$

    $H_1\ (Alternative):Implant\ Hardness\ for\ all\ Dentist\ are\ not\ same$

  - Decide Significance Level

    $\alpha\ (significance\ level)\ =\ 0.05$

  - Identification of the Test

    We have to perform One Way Anova for Alloy 2

  - Calculate the p-value and the test_Statistics

    From the Anova Table:

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	5.679791e+04	14199.477778	0.524835	0.718031
Residual	40.0	1.082205e+06	27055.122222	NaN	NaN

○ Conclusion:

Since the PR(>F) is 0.718031 which is greater than Significance Level(alpha) 0.05.

We fail to Reject the Null Hypothesis for Alloy 2

7.2 Assumptions & Comments:

- ALLOY TYPE 1:

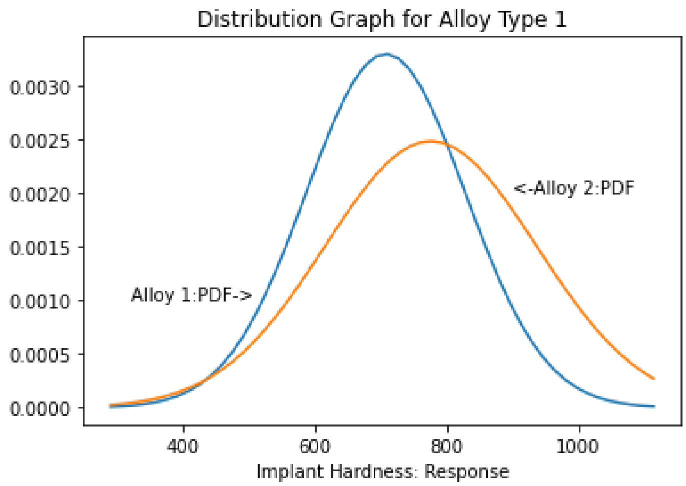
> Implant Hardness:Response is continuous & normally distributed for Alloy 1 can be clearly seen in the blue curve

> There are 5 dentist category hence, degrees of freedom in df is 4 (n-1) in Alloy 1

> Variance of Alloy 1 is not exact same to Alloy 2 but correspond to approximate

> There are 45 Samples, so residual ddof is 45-1 = 44, 44-(ddof accounted for Dentist), So it is 40

> The Variance between Dentist Category is about 1.97 times the variance within each Dentist Categories



- ALLOY TYPE 2:

> Implant Hardness: Response is continuous & normally distributed for Type 2 Alloy, can be seen in Orange curve

> There are 5 dentist category hence, degrees of freedom in df is 4 (n-1) in Alloy 2

> There are 45 Samples, so residual ddof is 45-1 = 44, 44-(ddof accounted for Dentist), So it is 40

> The Variance between Dentist Category is about 0.52 times the variance within each Dentist Categories

7.3

- Conclusion:

1. Both the anova test of Alloy(Type 1) & Alloy (Type2), the {p-value(f-critical)} PR(>F) is greater than Significance Level(alpha) 0.05.

2. For Alloy Type 1: the PR(>F) is 0.11 which is greater than Significance Level 0.05. Hence, We fail to Reject the Null Hypothesis. Hence, There is no difference among the Dentist on Implant Hardness for Alloy Type 1

3. For Alloy Type 2: the PR(>F) is 0.71 which is greater than Significance Level 0.05. Hence, We fail to Reject the Null Hypothesis. Hence, There is no difference among the Dentist on Implant Hardness for Alloy Type 2

4. For both the Alloy Types, Implant Hardness is not impacted by Dentist Category

5. Variability of Dentist for Alloy of Type 2 is small between variance between Dentists and variance within dentists

6. Dentist is a significant cause in Implant Hardness for Alloy 1

7.4

As there are two types of Alloys. Alloy of Category 1 & Alloy of Category 2, We have to separate both the Alloy types.

- ALLOY TYPE 1:
    - STATE NULL & ALTERNATIVE HYPOTHESIS

$$H_0\ (Null):\ Implant\ Hardness\ for\ different\ Method\ are\ same$$

$$H_1\ (Alternative):Implant\ Hardness\ for\ different\ Method\ are\ not\ same$$

    - Decide Significance Level

$$\alpha\ (significance\ level)\ =\ 0.05$$

    - Identification of the Test

We have to perform One Way Anova for Alloy 1

    - Calculate the p-value and the test_Statistics

From the Anova Table:

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	148472.177778	74236.088889	6.263327	0.004163
Residual	42.0	497805.066667	11852.501587	NaN	NaN

    - Conclusion:

Since the PR(>F) is 0.0041 which is less than Significance Level(alpha) 0.05

We will Reject the Null Hypothesis for Alloy 1

- ALLOY TYPE 2:
    - STATE NULL & ALTERNATIVE HYPOTHESIS

$$H_0\ (Null):\ Implant\ Hardness\ for\ different\ method\ are\ same$$

$$H_1\ (Alternative):Implant\ Hardness\ for\ different\ method\ are\ not\ same$$

    - Decide Significance Level

$$\alpha\ (significance\ level)\ =\ 0.05$$

    - Identification of the Test

We have to perform One Way Anova for Alloy 2

- Calculate the p-value and the test_Statistics

From the Anova Table:

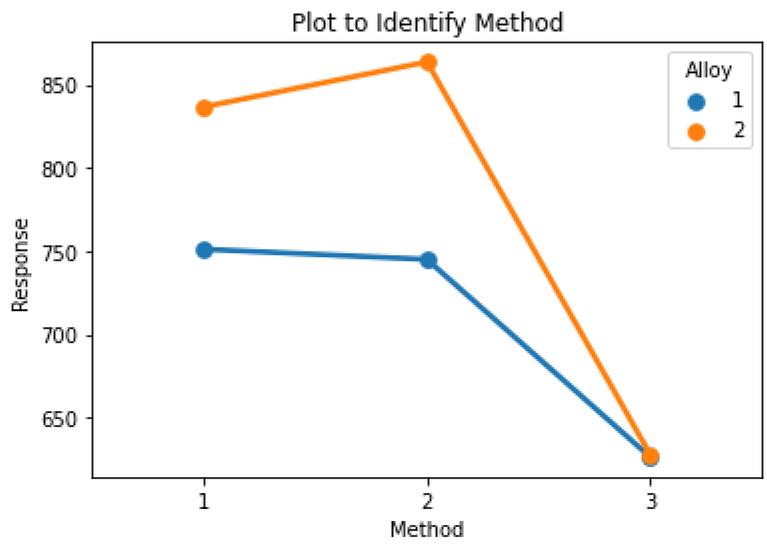
	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	499640.4	249820.200000	16.4108	0.000005
Residual	42.0	639362.4	15222.914286	NaN	NaN

- Interpretation of the Anova Test:

Since the PR(>F) is 0.000005 which is much less than Significance Level(alpha) 0.05,

We will Reject the Null Hypothesis for Alloy 2

From the graph of pointplot to identify Method difference:



1. The Method 3 of both Alloy types has significantly less mean of Response (Implant Hardness)

2. Method 2 of Alloy (type 2) has the best Response mean than all other method

3. Methods of Alloy (type 2) has greater mean Response than Alloy(type1)

Conclusion:

1. Both the anova test of Alloy(Type 1) & Alloy (Type2), the {p-value(f-critical)} PR(>F) is less than Significance Level(alpha) 0.05.

2. For Alloy Type 1: the PR(>F) is 0.0041 which is less than Significance Level 0.05.

3. Hence, We will Reject the Null Hypothesis. There is significance difference among the Methods on Implant Hardness for Alloy Type 1

4. For Alloy Type 2: the PR(>F) is 0.000005 which is much less than Significance Level 0.05. Hence, We will Reject the Null Hypothesis. Hence, There is significance difference among the Dentist on Implant Hardness for Alloy Type 2

5. For both the Alloy Types, Implant Hardness is impacted by Methods in different ways

6. The variance between methods in Alloy (type 2) is 16.41 times the variance within the methods

7.5

As there are two types of Alloys. Alloy of Category 1 & Alloy of Category 2, We have to separate both the Alloy types.

- ALLOY TYPE 1:
    - STATE NULL & ALTERNATIVE HYPOTHESIS

      H_0 $(Null):$ $Implant\ Hardness\ for\ different\ Temperature\ are\ same$

      H_1 $(Alternative): Implant\ Hardness\ for\ different\ Temperature\ are\ not\ same$

    - Decide Significance Level

      $\alpha\ (significance\ level)\ =\ 0.05$

    - Identification of the Test

      We have to perform One Way Anova for Alloy 1

    - Calculate the p-value and the test_Statistics

      From the Anova Table:

	df	sum_sq	mean_sq	F	PR(>F)
C(Temp)	2.0	10154.444444	5077.222222	0.335224	0.717074
Residual	42.0	636122.800000	15145.780952	NaN	NaN

- Conclusion:

  Since the PR(>F) is 0.71 which is greater than Significance Level(alpha) 0.05,

  We Fail to Reject the Null Hypothesis for Alloy 1

- ALLOY TYPE 2:
  - STATE NULL & ALTERNATIVE HYPOTHESIS

    $H_0\ (Null):\ Implant\ Hardness\ for\ different\ Temperatures\ are\ same$

    $H_1\ (Alternative): Implant\ Hardness\ for\ different\ Temperatures\ are\ not\ same$

  - Decide Significance Level

    $\alpha\ (significance\ level)\ =\ 0.05$

  - Identification of the Test

    We have to perform One Way Anova for Alloy 2

  - Calculate the p-value and the test_Statistics

    From the Anova Table:

	df	sum_sq	mean_sq	F	PR(>F)
C(Temp)	2.0	9.374893e+04	46874.466667	1.883492	0.164678
Residual	42.0	1.045254e+06	24886.996825	NaN	NaN

  - Interpretation of the Anova Test:

    Since the PR(>F) is 0.16 which is greater than Significance Level(alpha) 0.05,

    We Fail to Reject the Null Hypothesis for Alloy 2

Conclusion:

1. Both the anova test of Alloy(Type 1) & Alloy (Type2), the {p-value(f-critical)} PR(>F) is greater than Significance Level(alpha) 0.05.

2. For Alloy Type 1: the PR(>F) is 0.71 which is greater than Significance Level 0.05.

3. Hence, We fail to Reject the Null Hypothesis. There is no significance difference among the Temperatures on Implant Hardness for Alloy Type 1

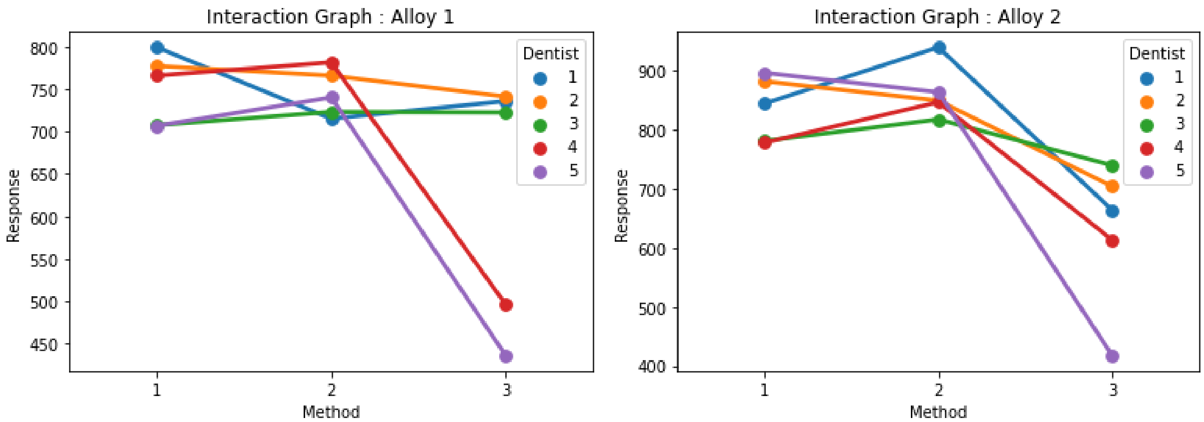
4. For Alloy Type 2: the PR(>F) is 0.16 which is greater than Significance Level 0.05. Hence, We fail Reject the Null Hypothesis. Hence, There is no significance difference among the Temperatures on Implant Hardness for Alloy Type 2

5. For both the Alloy Types, Implant Hardness is not impacted by Temperatures

6. The variance between Temperatures in Alloy (type 2) is 1.88 times the variance within the Temperatures

7.6

Although from the below graph we can illustrate that for both Alloy, different methods are behaving in similar patterns. But this is statistically not proven. Hence we need to conduct a two-way Anova test in order to find evidence on the interaction.



- STATE NULL & ALTERNATIVE HYPOTHESIS

$H_0\ (Null):\ Implant\ Hardness\ for\ Interaction\ Level\ are\ same$

$H_1\ (Alternative): Implant\ Hardness\ for\ Interaction\ Level\ are\ not\ same$

From the ANOVA table for ALLOY 1:

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist):C(Method)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

From the ANOVA table for ALLOY 2:

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist):C(Method)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

- Conclusions:

Alloy 1:

1. As p-value of Interaction between Dentist:Method < 0.05 the Null Hypothesis is Rejected.

2. There is an interacting effect of Dentist & Method in Alloy 1

Alloy 2:

1. As p-value of Interaction between Dentist:Method > 0.05 the Null Hypothesis is Failed to Reject.

2. Thus, there is no interacting effect of Dentist & Method in Alloy 2

Although from the interaction plot of Alloy 1 & 2, we could see that there is an interaction of Dentist:Method in both the types of Alloy to Response, but interaction of Dentist:Method in Alloy 2 it is not established by statistical method.

7.7

Alloy 1:

1. As p-value > 0.05 in Anova table of Dentist, we fail to reject the Null Hypothesis.The different Dentist Categories for Implant Hardness are same

2. Thus the Dentist Categories does not impacts Implant Hardness

3. In the Anova table of Method: p-value < 0.05, the Null Hypothesis is Rejected for Method Category.

4. Thus, the different Methods impact Implant Hardness

5. In Anova table of Dentist interacting with Method: p-value < 0.05, the Null Hypothesis is Rejected

6. There is an interacting effect of Dentist & Method in Alloy 1 as interatacting p-value < 0.05 which is statistically proved

Alloy 2:

1. As p-value > 0.05 in Anova table of Dentist, we fail to reject the Null Hypothesis.The different Dentist Categories for Implant Hardness are same

2. Thus the Dentist Categories does not impacts Implant Hardness

3. In the Anova table of Method: p-value < 0.05, the Null Hypothesis is Rejected for Method Category.

4. Thus, the different Methods impact Implant Hardness

5. In Anova table of Dentist interacting with Method: p-value > 0.05, the Null Hypothesis is Failed to Rejected

6. There is no interaction of Dentist & Method in Alloy 2 as interacting p-value > 0.05