Dynamic Modeling and Control of a Pressurized Steam Generator

Supplementary Research Note for PhD Applications

1. Project Overview

This project presents a first-principles dynamic model of a pressurized steam generator (SG) developed to study coupled thermal–hydraulic behavior and closed-loop control under transient operating conditions. The model integrates conservation-based mass and energy balances, two-phase thermodynamics, nonlinear drum level geometry, and feedback control within a modular Python framework.

The primary objective is to establish a transparent and extensible research platform suitable for doctoral-level investigation in nuclear engineering, thermal–fluids, and control systems.

2. Modeling Scope and Assumptions

The steam generator is represented as a lumped-parameter control volume with the following assumptions:

- Saturated liquid–vapor equilibrium in the drum
- Homogeneous two-phase mixture
- Negligible kinetic and potential energy effects
- Constant drum geometry
- Feedwater enters as subcooled liquid
- Steam exits as saturated vapor

Despite these simplifications, the formulation captures the dominant interactions between pressure, mass inventory, water level, and control actions.

3. Thermodynamic Property Framework

Thermophysical properties of water and steam are evaluated using the IAPWS-IF97 formulation. At each time step, the model computes:

- Saturated liquid and vapor density, $\rho_f(P)$ and $\rho_g(P)$
- Saturated liquid and vapor enthalpy, $h_f(P)$ and $h_g(P)$

- Saturation temperature, $T_{sat}(P)$

This ensures thermodynamic consistency across energy balance, steam generation, and pressure calculation.

4. Mass and Energy Balance Formulation

4.1 Total Mass Balance

The total working-fluid mass inside the steam generator drum evolves according to:

$$\frac{dm_{wf}}{dt} = \dot{m}_{fw} - \dot{m}_{steam,out} \tag{1}$$

The steam mass inventory satisfies:

$$\frac{dm_{steam}}{dt} = \dot{m}_{steam,gen} - \dot{m}_{steam,out} \tag{2}$$

4.2 Energy Balance

The total internal energy of the control volume is expressed as:

$$E = m_{wf} h_{wf} \tag{3}$$

The transient energy balance is given by:

$$\frac{d}{dt}(m_{wf} h_{wf}) = Q_{th} + \dot{m}_{fw} h_{fw} - \dot{m}_{steam,gen} h_g \tag{4}$$

In discrete time form:

$$h_{wf}^{n+1} = \frac{m_{wf}^{n} h_{wf}^{n} + Q_{th}\Delta t + \dot{m}_{fw} h_{fw}\Delta t - \dot{m}_{steam,gen} h_g \Delta t}{m_{wf}^{n+1}} \tag{5}$$

4.3 Steam Generation Model

Steam generation is governed by the available thermal energy and the latent heat of vaporization:

$$\dot{m}_{steam,gen} = \frac{Q_{th}}{h_{fg}}, \quad h_{fg} = h_g - h_f \tag{6}$$

This formulation ensures that phase change is strictly energy-limited and thermodynamically consistent.

5. Pressure Dynamics via Two-Phase Volume Constraint

Steam generator pressure is computed implicitly by enforcing the total volume constraint:

$$V_{total} = \frac{m_{steam}}{\rho_g(P)} + \frac{m_{liquid}}{\rho_f(P)} \tag{7}$$

where $m_{liquid} = m_{wf} - m_{steam}$.

At each time step, pressure is solved using a bisection method, ensuring numerical robustness without relying on empirical correlations.

6. Drum Water Level Modeling

The steam generator drum is modeled as a horizontal cylindrical vessel. The liquid cross-sectional area as a function of level h is given by:

$$A(h) = R^2 \cos^{-1}\left(\frac{R-h}{R}\right) - (R-h)\sqrt{2Rh - h^2} \tag{8}$$

The drum water level is obtained by numerically inverting:

$$A(h)L = \frac{m_{liquid}}{\rho_f} \tag{9}$$

This formulation captures the nonlinear geometric relationship between liquid volume and level.

7. Control System Formulation

7.1 Drum Level Control

The drum level controller regulates feedwater flow to maintain a target level. The volumetric level error is defined as:

$$e_L(t) = [A(h_{ref}) - A(h(t))]\, L \tag{10}$$

A PID controller generates the feedwater flow command:

$$\dot{m}_{fw}(t) = K_{p,L}e_L(t) + K_{i,L}\int_0^t e_L(\tau)d\tau + K_{d,L}\frac{de_L(t)}{dt} \tag{11}$$

7.2 Main Steam Pressure Control

Main steam pressure is regulated by adjusting steam discharge. The pressure control error is defined as:

$$e_P(t) = P(t) - P_{ref} \tag{12}$$

To account for steam compressibility, the scaled error is introduced:

$$e_P^*(t) = \frac{V_s}{RT_{sat}}e_P(t) \tag{13}$$

The steam outflow rate is computed using:

$$\dot{m}_{steam,out}(t) = K_{p,P}e_P^*(t) + K_{i,P}\int_0^t e_P^*(\tau)d\tau + K_{d,P}\frac{de_P^*(t)}{dt} \tag{14}$$

8. Numerical Implementation

The simulation advances in discrete time steps according to the following sequence:

1. Steam generation from thermal power input
2. Steam discharge via pressure control
3. Feedwater inflow via level control
4. Mass inventory update
5. Pressure solution from volume constraint
6. Drum level update from geometry

This ordering avoids algebraic loops and enhances numerical stability.

9. Research Significance and Future Work

The developed framework enables coupled simulation of pressure, level, mass, and control dynamics using physically interpretable state variables. Planned extensions include multi-region modeling, shrink–swell dynamics, coupling with reactor kinetics, advanced control strategies, and validation against plant data.

10. Relevance to Doctoral Research

This project demonstrates the ability to translate physical systems into mathematical models, apply thermodynamics and control theory coherently, and develop numerically stable simulation tools. The framework provides a strong foundation for PhD-level research in nuclear engineering and thermal–fluid systems.