ASSIGNMENT-2
STOCHASTIC PROCESSES AND APPLICATIONS (MTH 371)

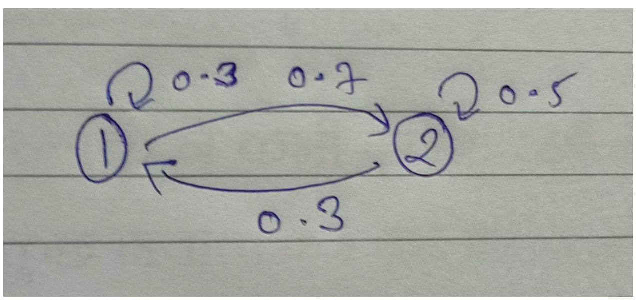
Ans-1

Given :-
We have discrete time markov chain with the state space S = {1,2}

Also, One Step Transition Probability :-

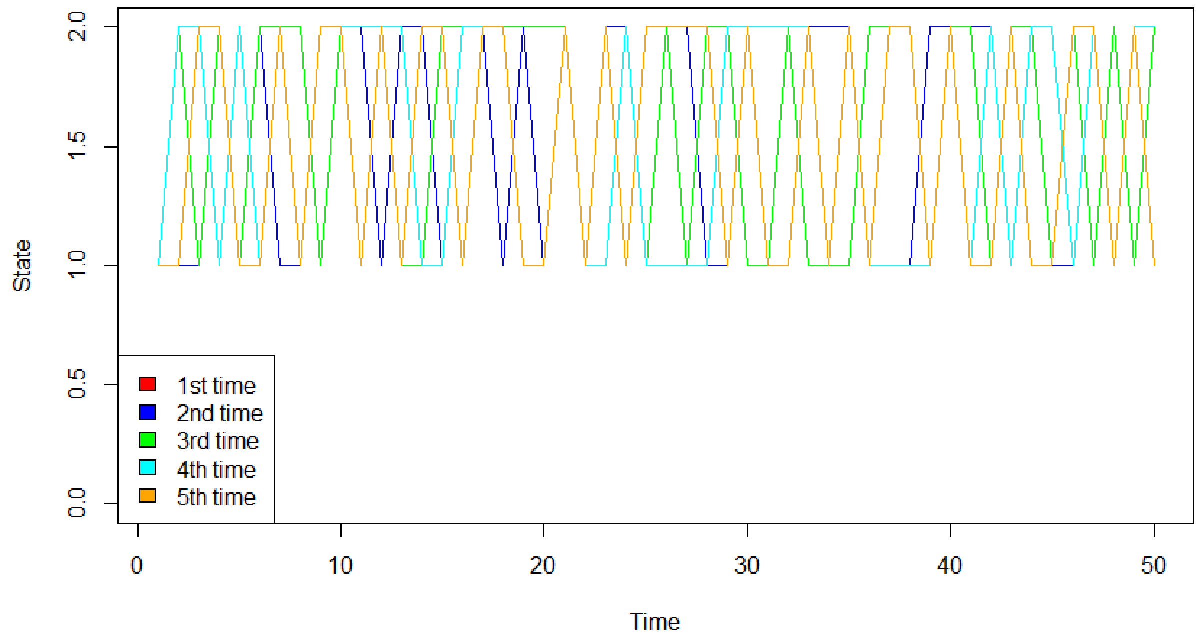
State	1	2
1	0.3	0.7
2	0.5	0.5

Corresponding State Diagram will be :-



Starting State is 1,

(a)
Plot for comparing time to the states of the process :-



(b)

P^{10} will be :-

State	1	2
1	0.4166667	0.4166667
2	0.5833333	0.5833333

P^{20} will be :-

State	1	2
1	0.4166667	0.4166667
2	0.5833333	0.5833333

P^{50} will be :-

State	1	2

1	0.4166667	0.4166667
2	0.5833333	0.5833333

Observation :-

(i) $P^{10} = P^{20} = P^{50}$, which means after P^{10}, our P^m where m >= 10 doesn't change which means it becomes static.
(ii) $\pi^{(n)} = [P(X_n = 1) \ \ P(X_n = 2)]$, so in the limiting distribution case our $P(X_n = 1) = P(X_n = 2)$, where n tends to infinity.

Proof for observation (ii) :-

Let's say we have initial distribution as $\pi^{(0)}$, therefore our limiting distribution will be $\pi^{(n)}$ where n tends to infinity,

$\Rightarrow \pi^{(10)} = \pi^{(0)} * P^{10}$
$\Rightarrow \pi^{(20)} = \pi^{(0)} * P^{20}$
$\Rightarrow \pi^{(50)} = \pi^{(0)} * P^{50}$

which means $\pi^{(10)} = \pi^{(20)} = \pi^{(50)}$,

Similarly for n tends to infinity,

$\pi^{(n)} = \pi^{(0)} * P^{(n)} = \pi^{(0)} * P^{(10)}$ [from observation (i)]

So, we have our limiting distribution as $\pi^{(0)} * P^{(10)}$,
Now, Let's say we have our initial distribution is [a 1-a]
So, $\pi^{(n)} =$ [a 1-a] [[0.4166667 0.4166667], [0.5833333 0.5833333]]
Clearly, at n tends to infinity :-
$P(X_n = 1) = P(X_n = 2)$, which means at infinity we have equal probability of state 1 and 2.

Ans-2

Given :-

We have 2 Gamblers, A and B,

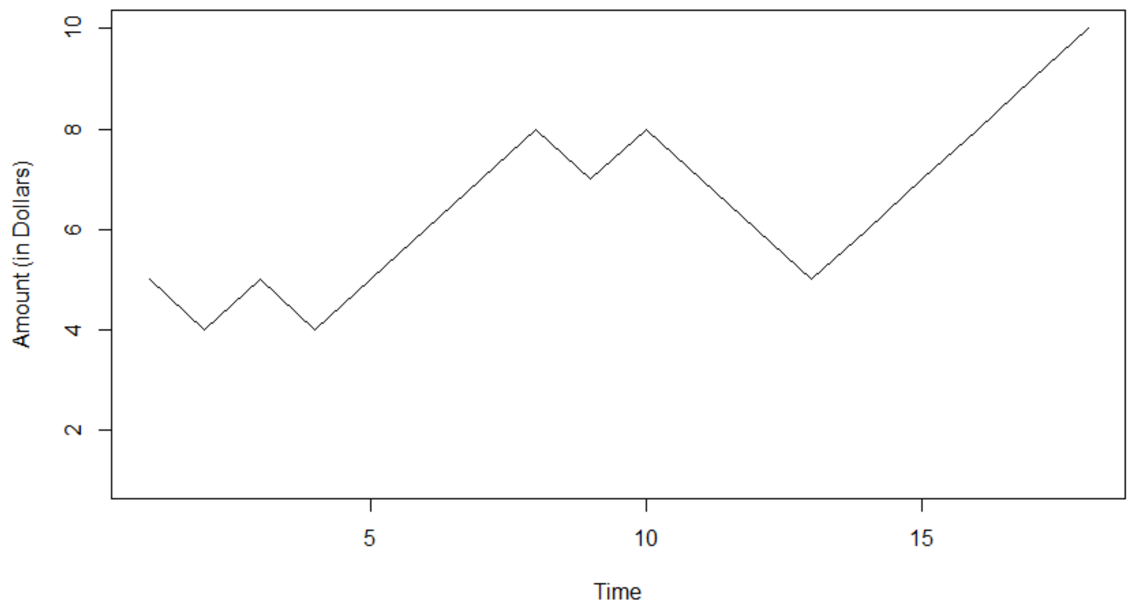
Total Amount (M) = 10 dollars and each gambler has 5 dollars which means start state of our process will be 5,

Probability of winning A = 0.8,

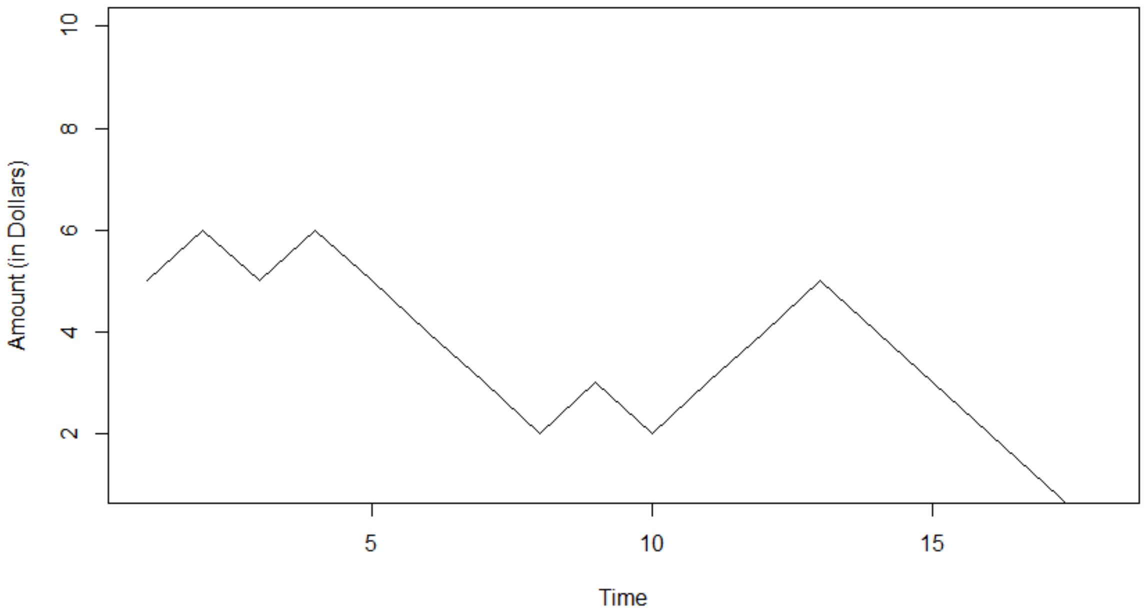
Probability of winning B = 0.2,

So after considering all the assumptions of 1-D random walk and Gambler's ruin problem, we model our process and plot the graphs for movement of gambler A and gambler B with respect to time and i.e.,

Plot for Movement of Gambler A w.r.t time :-



Plot for Movement of Gambler B w.r.t time :-


10
8
6
4
2
Amount (in Dollars)
5
10
15
Time