

# Implementation of Elliptic Curve Diffie–Hellman Key Exchange in c

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## **Abstract**

In this report we will focus on the implementation ECDH (Elliptic Curve Diffie–Hellman Key Exchange) in C language. The ECDH is an anonymous key agreement scheme, which allows two parties, each having an elliptic-curve public–private key pair, to establish a shared secret over an insecure channel. ECDH is very similar to the classical DHKE (Diffie–Hellman Key Exchange) algorithm, but it uses ECC point multiplication instead of modular exponentiations. ECDH is based on the following property of EC points:

$$(a * G) * b = (b * G) * a$$

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# 1 Introduction

The Elliptic Curve Cryptography (ECC) is modern family of public-key cryptosystems, which is based on the algebraic structures of the elliptic curves over finite fields and on the difficulty of the Elliptic Curve Discrete Logarithm Problem (ECDLP). ECC implements all major capabilities of the asymmetric cryptosystems: encryption, signatures and key exchange. The ECC cryptography is considered a natural modern successor of the RSA cryptosystem, because ECC uses smaller keys and signatures than RSA for the same level of security and provides very fast key generation, fast key agreement and fast signatures.

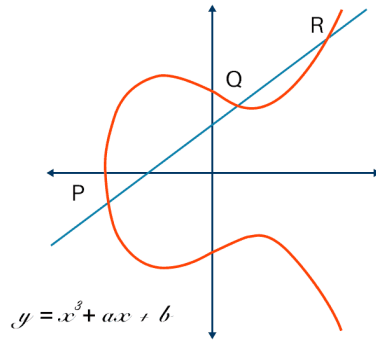


Figure 1: ECC CURVE

If we have two secret numbers  $a$  and  $b$  (two private keys, belonging to Alice and Bob) and an ECC elliptic curve with generator point  $G$ , we can exchange over an insecure channel the values  $(a * G)$  and  $(b * G)$  (the public keys of Alice and Bob) and then we can derive a shared secret:

$$\text{secret} = (a * G) * b = (b * G) * a.$$

For implementation purpose at first I build the elementary function individually then I merge them to build the whole algorithm. To do less number of elementary operations, I have taken numbers in 256 base then I converted them to  $2^{30}$  base using sagemath.

## 2 Change of Bases of digits

```
//base change to 2^30
void change_base(unsigned char *a,long long int *A){

    A[0] = (long long int)a[0] | ((long long int)a[1] << 8) | (long long int)a[2]<<16 | ((long long int)a[3] & 0X3F)<<24;
    A[1] = ((long long int)a[3] >> 6) | (long long int)a[4]<<2 | (long long int)a[5]<<10 | (long long int)a[6]<<18 | ((long long int)a[7] & 0XF)<<26;
    A[2] = ((long long int) a[7] >>4) | (long long int) a[8] <<4 | (long long int) a[9] << 12 | (long long int) a[10] <<20 | ((long long int)a[11] & 0x3 )<<28 ;
    A[3] = ( (long long int) a[11]>>2 | ((long long int) a[12] )<<6 | ((long long int) a[13] )<<14 | ((long long int) a[14] )<<22;

    A[4] = (long long int)a[15] | ((long long int)a[16] << 8) | (long long int)a[17]<<16 | ((long long int)a[18] & 0X3F)<<24;
    A[5] = ((long long int)a[18] >> 6) | (long long int)a[19]<<2 | (long long int)a[20]<<10 | (long long int)a[21]<<18 | ((long long int)a[22] & 0XF)<<26;
    A[6] = ((long long int) a[22] >>4) | (long long int) a[23] <<4 | (long long int) a[24] << 12 | (long long int) a[25] <<20 | ((long long int)a[26] & 0x3 )<<28 ;
    A[7] = ( (long long int) a[26]>>2 | ((long long int) a[27] )<<6 | ((long long int) a[28] )<<14 | ((long long int) a[29] )<<22;

    A[8] = (long long int) a[30] | (long long int) a[31] << 8 ;

}
```

Figure 2:  $2^8$  to  $2^{30}$  base change

Intuition is if I consider any binary representation of a number then if I keep taking 2 consecutive bits from lsb side that will give me conversion to base  $2^2$ . Similarly if I take 3 consecutive bits from lsb onwards that will give me conversion to base  $2^3$ . So here I have taken 30 consecutive bits from lsb to get the conversion to  $2^{30}$  base.

Mathematically we can easily see the conversion of any number to  $2^h$  base:

$$x = a_0 + 2 * a_1 + 2^2 * a_2 + \dots + a_n * 2^{n-1} = (a_0 + 2 * a_1 + 2^2 * a_2 + \dots + a_h * 2^{h-1}) + 2^h(a_h + 2 * a_{h+1} + 2^2 * a_{h+2} + \dots + a_{2*h-1}) + (2^h)^2(\dots) + \dots$$

## 2.1 Correctness Checking with an Example

I have taken a number in  $2^8$  base using sagemath. Then I verified my algorithm to check it is converting the number in  $2^{30}$  base correctly or not.

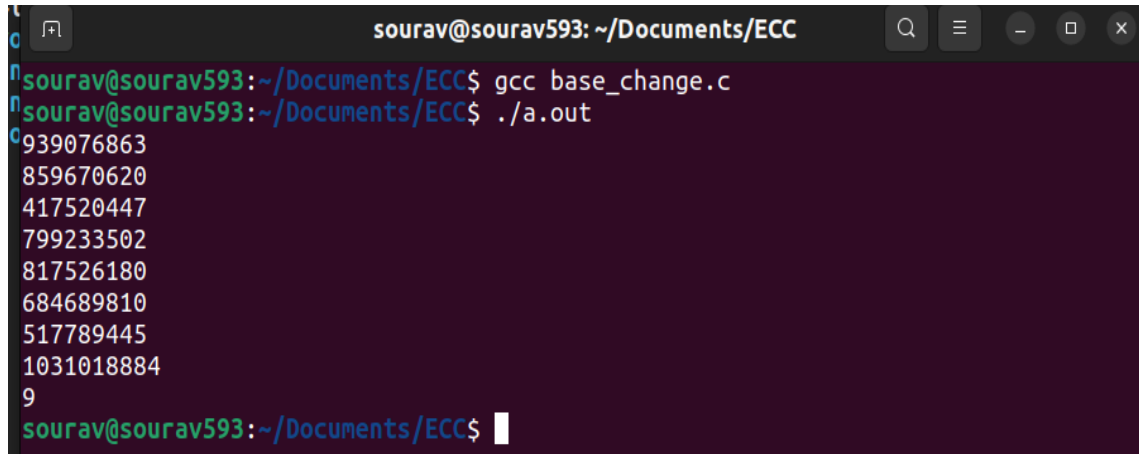
A terminal window with a dark background. The prompt is 'sourav@sourav593: ~/Documents/ECC'. The user enters 'gcc base\_change.c' and then './a.out'. The program outputs a list of numbers: 939076863, 859670620, 417520447, 799233502, 817526180, 684689810, 517789445, 1031018884, and 9. The prompt returns to 'sourav@sourav593: ~/Documents/ECC\$'.

Figure 3: output of C code of Base conversion

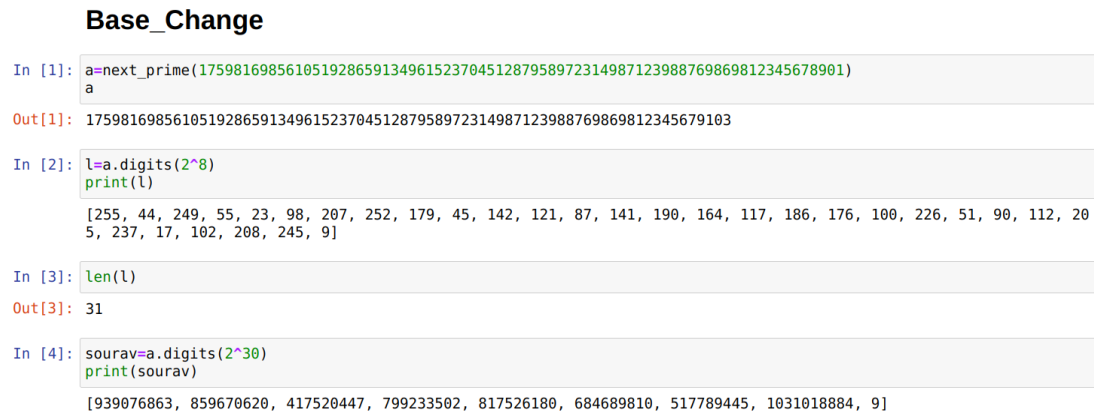
A SageMath notebook titled 'Base\_Change'. It contains four input-output pairs. The first input is 'a=next\_prime(17598169856105192865913496152370451287958972314987123988769869812345678901)' followed by 'a', and the output is the same large prime number. The second input is 'l=a.digits(2^8)' followed by 'print(l)', and the output is a list of 255 digits. The third input is 'len(l)' and the output is '31'. The fourth input is 'sourav=a.digits(2^30)' followed by 'print(sourav)', and the output is a list of 31 numbers, which matches the output of the C code in Figure 3.

Figure 4: Example taken and checked the answer in sage

## 3 Addition, Subtraction & Multiplication Of two $2^{30}$ base Numbers

### 3.1 Addition

Here I have taken long long int array as inputs. Since long long int has 64 bit size so I have enough space to compute my operations. I did the addition like we generally did in the case for decimal numbers. For example if We add 19 with 9 then We will add the unit places ( $9+9=18$ ) then we keep 8 in unit place and keep 1 in carry to add in the next step.

```
void addition( long long int *r, long long int *s,
long long int *t,int n){
    int i;
    long long int carry=0;
    for (i=0;i<n;i++){
        t[i]=r[i]+s[i]+carry;
        carry=(t[i]>>30);
        t[i]=t[i] & 0xffffffff;
    }
    t[9]=carry;
}
```

Here in the code also I add the numbers and to get the carry I have taken the bits after  $30^{th}$  bits. Then I add the carry to the next position.

### 3.2 Subtraction

Here like addition I used school book technique to do subtraction. Like in decimal subtraction  $21-9$ , we don't do  $1-9$ , we have to borrow 10 from the next digit so that we can subtract 9 from  $10+1$ . So the unit place becomes  $11-9=2$  & the answer becomes 12. So in decimal we borrow 10, here we have to borrow  $2^{30}$  since we have taken the number in  $2^{30}$  base.

```
void mod_subtract(long long int *r, long long int *s,
long long int *t){
    for(int i=0;i<9;i++){
        t[i]=r[i]-s[i];
        if((t[i]>>63) & 1){
            t[i]+= 1<<30;
            r[i+1]-=1;
        }
    }
}
```

But this can subtract a from b correctly if a is greater than b. So I defined a function to compare a & b.

Suppose I want to calculate x-y if x is greater than y then its fine But if y is greater than x then I have to think something. Finally I have to do subtraction in mod p for for some given large prime p. So what I did is for the case for y greater than x is calculate (y-x) then p-(y-x). That will equal to the number x-y in mod p.

### 3.3 Multiplication

I implemented schoolbook technique as well here. Just multiply to same position block and then if any carry exists I add them in the next block.

```
void multi(long long int *r, long long int *s, long long int *t,
int n){
    for(int i=0;i<n;i++){
        for(int j=0;j<n;j++){
            t[i+j]+=r[i]*s[j];
            t[i+j+1]+= (t[i+j]>>30);
            t[i+j]=t[i+j] & 0xffffffff;
        }
    }
}
```

#### 3.3.1 Karatsuba algorithm

It is used for multiplication. It is faster than school book method. Since addition is cheaper than multiplication here we do more elementary addition than multiplication.

Take

$$\begin{aligned} a &= x_1 * B^m + x_0 \\ b &= y_1 * B^m + y_0 \\ a * b &= x_1 * y_1 * B^{2m} + (z) * B^m + x_0 * y_0 \end{aligned}$$

$$z = x_0 * y_1 + y_0 * x_1 = (x_0 + y_0) * (y_0 + y_1) - x_1 * y_1 - x_0 * y_0$$

If we write z in this form then we only required 3 multiplications instead of 4. For implementation purpose it is easy to implement karatsuba using recursion. But in cryptology implementation we try to avoid recursion. So if someone want to implement karatsuba then they have to break the blocks according to their inputs.



### 3.4 Code

Combine the three function I verified the result of my functions.

```
int main(){
    unsigned char B[32]={243, 212, 5, 39, 33, 146, 119, 117, 245, 85, 53,
    unsigned char A[32]={109, 56, 115, 155, 240, 27, 194, 45, 201, 191, 24
    long long int R[9],S[9],T[10]={0},R_dash[9]={0},S_dash[9]={0};
    long long int w[10]={0},MULTI[18]={0};
    int i;
        //base change to 2^30
        change_base(A,R);
        change_base(B,S);
    for(int i=0;i<9;i++)
        R_dash[i]=R[i];
    for(int i=0;i<9;i++)
        S_dash[i]=S[i];
        addition(R_dash,S_dash,T);
        printf("\naddition\n");
    //after addition result
        for(i=0;i<9;i++){
            printf("%lld_\t", T[i]);
        }
        printf("\nsubtraction:::%d\n",Is_a_BiggerThan_b(R,S));
    //after subtraction result
    if(Is_a_BiggerThan_b(R,S)==1)
        mod_subtract(R_dash,S_dash,w);
    if(Is_a_BiggerThan_b(R,S)==-1)
        mod_subtract(S_dash,R_dash,w);
        //further we can add the prime factor while needed
    //after subtraction result
        for(i=0;i<9;i++){
            printf("%lld_\t", w[i]);
        }
        multi(R,S,MULTI);
        printf("\nmultiplication\n");
    //after multiplication result
        for(i=0;i<18;i++){
            printf("%lld_\t", MULTI[i]);
        }
}
```

### 3.4.1 Sage Output

```
In [2]: A=next_prime(451896591623598612387956182705646152248761239512307561238754612390)

In [3]: L=A.digits(2^8)
print(L)
[243, 212, 5, 39, 33, 146, 119, 117, 245, 85, 53, 42, 42, 143, 71, 204, 119, 202, 235, 101, 223, 162, 119, 122, 24
0, 127, 74, 4]

In [4]: B=next_prime(2383965981326587961327856238756123879568127365879123657123640982)

In [5]: S=B.digits(2^8)
print(S)
[109, 56, 115, 155, 240, 27, 194, 45, 201, 191, 243, 86, 84, 232, 251, 142, 52, 202, 249, 186, 3, 54, 47, 177, 139,
203, 5]

In [8]: print((A+B).digits(2^30))
[41487712, 216447047, 311516138, 282976160, 630500443, 459508867, 79151802, 276]

In [9]: print((A-B).digits(2^30))
[194157702, 517331138, 874078916, 317306228, 838878013, 565407403, 189156500, 273]

In [10]: print((A*B).digits(2^30))
[775541623, 904082450, 674284498, 698383547, 407923945, 227102914, 915866975, 271470896, 747622078, 249545757, 2455
33198, 814244225, 269765456, 933906280, 397]
```

Figure 5: output of Sage of Operation

### 3.4.2 C Code Output

```
sourav@sourav593:~/Documents/ECC$ gcc addition.c
sourav@sourav593:~/Documents/ECC$ ./a.out

addition
41487712      216447047      311516138      282976160      630500443      459508867      79151802      276      0
subtraction::-1
194157702     517331138      874078916      317306228      838878013      565407403      189156500      273      0
multiplication
775541623     904082450      674284498      698383547      407923945      227102914      915866975      271470896      747622078      249545757      245533198      8
14244225      269765456      933906280      397      0      0      0      sourav@sourav593:~/Documents/ECC$
```

Figure 6: output of C code of elementary function

Here I compared the result and they are matching. I tried for 2 or 3 more numbers also they are working fine. So good to go for the next steps.

## 4 Barrett Reduction

Barrett Reduction helped me to calculate values in mod p.

```

    void barret(long long int *r,long long int *R){
        int i;
        //p=0xffffffff00000001000000000000000000000000
0 ffffffff ;;; k=256//30 ;;; B=2^30
        long long int p[10]={1073741823, 1073741823,
1073741823, 63, 0, 0, 4096, 1073725440, 65535}; //calculated in sage
        long long int T[10]={805306368, 0, 0, 1073741820, 1073741807,
1073741759, 1073741567, 1073741823, 4095, 16384};
// T=B^2k/p calculated in sage
        long long int Q[10]={0},Q1[20]={0},Q2[10]={0},QP[20]={0};
        //Q=[x/B^k-1]=[r/2^240]
        for(i=0;i<10;i++)
            Q[i]=r[8+i];
        multi(Q,T,Q1,10);
        //Q2 <- Q1/B^k+1=Q/2^300
        for(i=0;i<10;i++)
            Q2[i]=Q1[10+i];
        multi(Q2,p,QP,10);
        long long int r_dash[10]={0},QP_dash[10]={0};
        for(i=0;i<10;i++){
            r_dash[i]=r[i]; //r mod 2^300
            QP_dash[i]=QP[i]; //QP mod 2^300
        }
        if(Is_a_BiggerThan_b(r_dash, QP_dash,10)==1)
            mod_subtract(r_dash, QP_dash,R,10);
        if(Is_a_BiggerThan_b(r_dash, QP_dash,10)==-1){
            long long int thikthik[10]={0};
            mod_subtract(QP_dash, r_dash, thikthik,10);
            mod_subtract(p, thikthik, R,10);
        }
        while(1==Is_a_BiggerThan_b(R,p,10)){
            long long int te[10]={0};
            mod_subtract(R,p,te,10);
            for(i=0;i<10;i++)
                R[i]=te[i];
        }
    }

```

#### 4.0.1 Sage Output

## Barrett Reduction

[illegible]

Figure 7: output of Sage for Barrett

### 4.0.2 C Code Output

```
sourav@sourav593:~/Documents/ECC$ gcc barret.c
sourav@sourav593:~/Documents/ECC$ ./a.out

Barret
151314446      876022499      88379576      774376329      357410509      150572782      857764354      422283974      40802  0      sourav@sourav593:~/Documents/
```

Figure 8: output of C code for Barrett

Here I have taken the prime from 256 bit prime field Weierstrass curve. I proceed according to Barrett algorithm: Here  $B=2^{30}$ ,  $k=\lceil 256/30 \rceil=9$ ,  $T=\lceil B^{2k}/p \rceil = 2^{540}/p$ , Input= $x$  (512 bit),  $Q=\lceil x/B^{k-1} \rceil = x/2^{240}$ ; Then  $Q1=Q*T$ ;  $Q2=Q1/2^{300}$ ;  $R=x-Q2*p \pmod{B^{k+1}}=x-Q2*p \pmod{2^{300}}$ ; Next is run a while loop to decrement  $R$  by  $p$  until  $R < p$ .

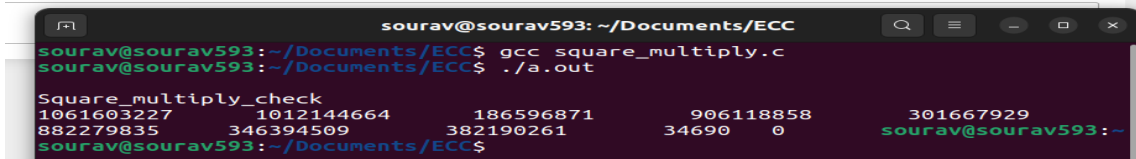
## 5 Square & Multiply

Briefly square and multiply method for calculating  $x^a \pmod{p}$  is start with  $z=1$ , then according to their bit representation of  $a$  from the side of MSB square in every iteration and multiply with  $x$  if  $\text{bit}=1$ . One can also check the correctness of the algorithm by checking for any number  $a$ ,  $a^{p-1}=1$  or not.

```
void square_multiply(long long int *x, long long int *a,
long long int *result){
    long long int zid[9]={1},temp[18]={0},temp1[18]={0};
    for(int i=8;i>=0;i--){
        for(int j=29;j>=0;j--){
            for(int k=0;k<18;k++){
                temp[k]=0;
            }
            multi(zid,zid,temp,9);
            for(int k=0;k<9;k++){
                zid[k]=0;
            }
            barret(temp,zid);
            if((a[i]>>j)&1){
                for(int k=0;k<18;k++){
                    temp1[k]=0;
                }
                multi(zid,x,temp1,9);
                for(int k=0;k<9;k++){
                    zid[k]=0;
                }
                barret(temp1,zid);
            }
        }
    }
    for(int i=0;i<9;i++){
        result[i]=zid[i];
    }
}
```

### Square & Multiply

```
In [24]: print(power_mod(A,B,Prime).digits(2^30))
[1061603227, 1012144664, 186596871, 906118858, 301667929, 882
279835, 346394509, 382190261, 34690]
```



```
sourav@sourav593:~/Documents/ECC$ gcc square_multiply.c
sourav@sourav593:~/Documents/ECC$ ./a.out
Square_multiply_check
1061603227 1012144664 186596871 906118858 301667929
882279835 346394509 382190261 34690 0
sourav@sourav593:~/Documents/ECC$
```

Figure 9: output of Square and Multiply

## 6 ECC ADDITION

### *Elliptic Curve Addition Algorithm*

$$E := Y^2 = X^3 + AX + B$$

be an elliptic curve and let  $P_1$  &  $P_2$  be two point on  $E$ .

- Input  $P_1 = (x_1, y_1), P_2 = (x_2, y_2)$
- Output  $P_1 + P_2 = P_3 = (x_3, y_3)$
- if  $P_1 = 0$  then  $P_1 + P_2 = P_2$
- if  $P_2 = 0$  then  $P_1 + P_2 = P_1$
- If  $x_1 = x_2, y_1 = -y_2$  then  $P_1 + P_2 = 0$
- Define  $\lambda = (y_2 - y_1)/(x_2 - x_1)$  if  $P_1 \neq P_2$
- if  $P_1 = P_2, \lambda = (3x^2 + p - 3)/2y_1$

$$x_3 = \lambda^2 - x_1 - x_2$$

$$y_3 = \lambda(x_1 - x_3) - y_1$$

Here I just implemented the above algorithm line by line and have taken help from sagemath to calculate powers and verify the result. I am showing the important part of calculating  $\lambda$ .

```

if ((Is_a_BiggerThan_b(a_x, b_x, 9) == 0) &&
(Is_a_BiggerThan_b(a_y, b_y, 9) == 0)) {
    multi(a_x, a_x, e1, 9);
    barret(e1, x_sq);
    multi(x_sq, three, three_x, 9);
    barret(three_x, three_x_barret);
    addition(three_x_barret, p_3, num, 9);

    multi(two, a_y, den, 9);
    barret(den, den_bar);
    square_multiply(den_bar, p_2, sol);
    multi(sol, num, lamda, 9);
}
else {
    if (Is_a_BiggerThan_b(b_y, a_y, 9) == 1)
        mod_subtract(b_y, a_y, y, 9);
    if (Is_a_BiggerThan_b(b_y, a_y, 9) == -1) {
        long long int thik[9] = {0};

```

```

        mod_subtract(a_y, b_y, thik, 9);
        mod_subtract(p, thik, y, 9);
    }
    if(Is_a_BiggerThan_b(b_x, a_x, 9)==1)
        mod_subtract(b_x, a_x, x, 9);
    if(Is_a_BiggerThan_b(b_x, a_x, 9)==-1){
        long long int thik1[9]={0};
        mod_subtract(a_x, b_x, thik1, 9);
        mod_subtract(p, thik1, x, 9);
    }
    square_multiply(x, p_2, x_inv);
    multi(x_inv, y, lamda, 9);

```

## ECC ADDITION

```

In [29]: ##### NIST P-256
p256 = 2^256-2^224+2^192+2^96-1
a256 = p256 - 3
b256 = 41058363725152142129326129780047268409114441015993725554835256314039467401291
## Base point
gx = 48439561293906451759052585252797914202762949526041747995844080717082404635286
gy = 36134250956749795798585127919587881956611106672985015071877198253568414405109
## Curve order
qq = 11579208921035624876269744694940757352999695522413576034242259061068512044369
FF = GF(p256)
EC = EllipticCurve([FF(a256), FF(b256)])
EC.set_order(qq)
# Base point
G = EC(FF(gx), FF(gy))
## Alice's private key
a = 545456567897987
## Alice's public key
A = a*G
print(A)

(85843274658334699305043628116802730568687465077193738908167644400996701448582 : 1127185829199726846088203346886573
93631572712981800368689363891854254539376747 : 1)

In [30]: print(85843274658334699305043628116802730568687465077193738908167644400996701448582.digits(2^30))

[822618502, 839769931, 1026660504, 406381500, 813785078, 904312073, 299183961, 613803269, 48585]

In [31]: print(112718582919972684608820334688657393631572712981800368689363891854254539376747.digits(2^30))

[413763691, 1072626329, 790597169, 524569649, 465465364, 885174302, 230441977, 490776745, 63796]

```

Figure 10: ECC

```

In [26]: b=54545656789798986
         b*G
Out[26]: (24488783781291031289044693414742267011758631797059251508710471768703341781691 : 4022666962906534760084419746947849
5199919812062201795325281959960505552766687 : 1)

In [33]: print(24488783781291031289044693414742267011758631797059251508710471768703341781691.digits(2^30))
[850223803, 164210561, 32244799, 704277377, 1017435543, 478851327, 879500052, 172265376, 13860]

In [34]: print(40226669629065347600844197469478495199919812062201795325281959960505552766687.digits(2^30))
[616859359, 1031785013, 520361210, 738402539, 162460059, 22895108, 692511935, 524158533, 22767]

In [27]: c=a+b

In [28]: c*G
Out[28]: (424276640822084966573446071852479557359529511786210742483291313821172333436 : 269053212446202711627450290027833383
78226335762420493407339252786313472686587 : 1)

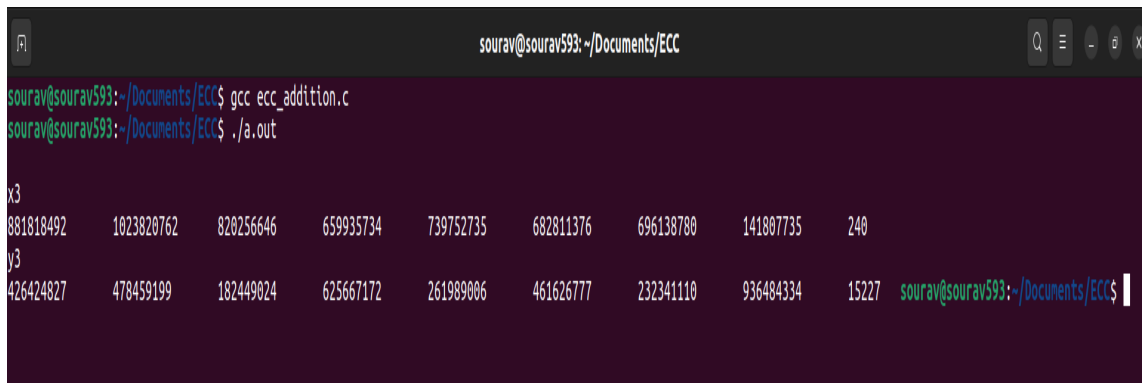
In [35]: print(424276640822084966573446071852479557359529511786210742483291313821172333436.digits(2^30))
[881818492, 1023820762, 820256646, 659935734, 739752735, 682811376, 696138780, 141807735, 240]

In [36]: print(26905321244620271162745029002783338378226335762420493407339252786313472686587.digits(2^30))
[426424827, 478459199, 182449024, 625667172, 261989006, 461626777, 232341110, 936484334, 15227]

```

Figure 11: Ecc points

I have taken this points from sagemath to do the calculations further and in c i got the correct result. I have already implemented addition function



```

sourav@sourav593: ~/Documents/ECC
sourav@sourav593:~/Documents/ECC$ gcc ecc_addition.c
sourav@sourav593:~/Documents/ECC$ ./a.out

x3
881818492    1023820762    820256646    659935734    739752735    682811376    696138780    141807735    240
y3
426424827    478459199    182449024    625667172    261989006    461626777    232341110    936484334    15227

```

Figure 12: ecc addition c output

for general case. So I can use this thing to implement doubling. That is for  $2*x = x + x$ .



## 7 Ecc Scalar Multiplication

Just like square and multiply I implemented this ecc scalar multiplication also. To calculate  $a \cdot G$  the difference from square and multiply is here we have to start with 0 and then for every bit we have to double the number and if the bit is 1 then we have to add  $G$ . We have to be carefull regarding the starting step I have set a flag to start the process only after i got a bit=1.

```

void ecc_scalar_multiply(long long int *x, long long int *y,
long long int *c, long long int *a, long long int *b){
    long long int z1[9]={0}, z2[9]={0}, temp[9]={0},
temp1[9]={0}, sou1[9]={0}, sou2[9]={0};
    int flag=0, i, j, B, k;
    for(i=0; i<9; i++){
        z1[i]=x[i];
        z2[i]=y[i];
    }
    for(i=8; i>=0; i--){
        for(j=29; j>=0; j--){
            B=(c[i]>>j) & 1;
            printf("\nbit %d\n", B);
            if(flag==1){
                ecc_addition(z1, z1, z2, z2, temp, temp1);
                for(k=0; k<9; k++){
                    z1[k]=temp[k];
                    z2[k]=temp1[k];

                    temp[k]=0;
                    temp1[k]=0;
                }
            }
            if(B==1){
                ecc_addition(z1, x, z2, y, sou1, sou2);
                for(k=0; k<9; k++){
                    z1[k]=sou1[k];
                    z2[k]=sou2[k];

                    sou1[k]=0;
                    sou2[k]=0;
                }
            }
        }
        if(B==1)
            flag=1;
    }
}

```

```

    }
}
for ( i=0; i < 9; i++){
    a [ i ]=z1 [ i ];
    b [ i ]=z2 [ i ];
}
}

```

Like the previous cases I can verify this using sagemath.

## Scalar Multiplication

```

In [38]: print(gx.digits(2^30))
[412664470, 310699287, 515062287, 14639179, 608236151, 865834382, 69500811, 880588875, 27415]

In [39]: print(gy.digits(2^30))
[935285237, 785973664, 857074924, 864867802, 262018603, 531442160, 670677230, 280543110, 20451]

In [40]: 250*G
Out[40]: (42816713642517519830642598718239551603454468247529013466436607916954616566201 : 1303912648148581117724883854580081
1647913026559060138776014452474689200219750 : 1)

In [41]: print(42816713642517519830642598718239551603454468247529013466436607916954616566201.digits(2^30))
[1064014265, 521290241, 992775702, 43075731, 28569876, 883937578, 715184055, 430701780, 24233]

In [42]: print(13039126481485811177248838545800811647913026559060138776014452474689200219750.digits(2^30))
[102393446, 222751136, 1012228591, 173923507, 691539925, 177923810, 712638746, 949247134, 7379]

```

Figure 13: Ecc scalar multiply sage output

```

sourav@sourav593: ~/Documents/ECC$ gcc ecc_final.c
sourav@sourav593: ~/Documents/ECC$ ./a.out
ecc_scalar multiplication
cx1
1064014265      521290241      992775702      43075731      28569876      883937578      715184055      430701780      24233
cy1
102393446      222751136      1012228591     173923507     691539925     177923810     712638746     949247134     7379
sourav@sourav593: ~/Documents/ECC$

```

Figure 14: Ecc scalar multiply C output

## 8 Elliptic Curve Diffie–Hellman Key Exchange

Now I have all the function to do key exchange.

$$(a * G) * b = (b * G) * a$$

In the program I first calculated  $a*G$  then  $b*(a*G)$ . Similarly for the other party. Then they have the common key.

```
int main(){
    unsigned char r[32]={243, 212, 5, 39, 33, 146, 119, 117, 245,
71, 204, 119, 202, 235, 101, 223, 162, 119, 122, 240, 127, 74, 4},s[32]={
84, 232, 251, 142, 52, 202, 249, 186, 3, 54, 47, 177, 139, 203, 5};
    long long int a[9]={0},b[9]={0};
    //base change to 2^30
    change_base(r,a);
    change_base(s,b);
    int i;
    long long int gx[9]={412664470, 310699287, 515062287, 14639179,
14639179, 14639179, 14639179, 14639179, 14639179};
    long long int aG_x[9]={0},aG_y[9]={0};
    //calculating aG
    ecc_scalar_multiply(gx,gy,a,aG_x,aG_y);
    printf("\naGx1\n");
    print_array(aG_x,9);
    printf("\naGy1\n");
    print_array(aG_y,9);
    printf("\n");
    //calculating bG
    long long int bG_x[9]={0},bG_y[9]={0};
    ecc_scalar_multiply(gx,gy,b,bG_x,bG_y);
    printf("\nbGx1\n");
    print_array(bG_x,9);
    printf("\nbGy1\n");
    print_array(bG_y,9);
    printf("\n");
    //calculate a*(bG)
    printf("\na*(bG)\n");
    long long int aBG_x[9]={0},aBG_y[9]={0};
    ecc_scalar_multiply(bG_x,bG_y,a,aBG_x,aBG_y);
    printf("\na*(bG)x1\n");
    print_array(aBG_x,9);
    printf("\na*(bG)y1\n");
    print_array(aBG_y,9);
```

```

printf("\n");
//calculate b*(aG)
printf("\nb*(aG)");
long long int bAG_x[9]={0},bAG_y[9]={0};
ecc_scalar_multiply(aG_x,aG_y,b,bAG_x,bAG_y);
printf("\nb*(aG)x1\n");
print_array(bAG_x,9);
printf("\nb*(aG)y1\n");
print_array(bAG_y,9);
printf("\n");
if(Is_a_BiggerThan_b(bAG_x,aBG_x,9)==0 && Is_a_BiggerThan_b(bA
    printf("\nNow they have common key ..... \n");
else
    printf("\nError in key exchange ..... \n");
}

```

```

sourav@sourav593:~/Documents/ECC$ gcc ecc_final.c
sourav@sourav593:~/Documents/ECC$ ./a.out

aGx1
812812144      413351939      175593752      36621247      348616328      229844039      330928752      374507044      53634

aGy1
1048372997     837446331      735075336     938204959     522143284     304556449     77464032      563213970     23864

bGx1
300973972      97194001      299744040     264020083     639732053     284336951     88827051      451183582     23732

bGy1
680982865      883972688     735672104     325441668     807874354     408655276     533915784     933361001     13184

a*(bG)
a*(bG)x1
110989031      982503481     701980360     1001413779     655806820     1029467079     171723775     103899102     11446

a*(bG)y1
274667958      314429414     239905683     455132376     859819234     955703339     804258419     435316589     1503

b*(aG)
b*(aG)x1
110989031      982503481     701980360     1001413779     655806820     1029467079     171723775     103899102     11446

b*(aG)y1
274667958      314429414     239905683     455132376     859819234     955703339     804258419     435316589     1503

Now they have common key.....
sourav@sourav593:~/Documents/ECC$

```

Figure 15: ecc addition c output