 MC(A)/UG/S–I/2018

End Semester Examination of Semester–I, 2018

Subject : BCA

Paper : BCA–104 (Discrete Math)

Full Marks : 70

Time : 3 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary.

Group A

1. Answer **any five** out of eight questions : 2x5=10

   i) Define equivalence relation on a non-empty sets.

   ii) Prove that if in a graph G there is one and only one path between every pair of vertices the G is a tree.

   iii) Define Isomorphic graph with example.

   iv) What are the difference between walks and circuits?

   v) Define recurrence relation and generating function.

   vi) If $a_0 = 3$, $a_1 = 4$ and $a_n = a_{n-1} + a_{n-2}$, then find the value of a_4, a_5.

   vii) By means of truth table, show that,

   $\sim(p \Leftrightarrow q) = \sim p \Leftrightarrow q = p \Leftrightarrow \sim q$

viii) Let U be the set of all integers.

$A = \{x \in U : x^2 - 5x + 6 = 0\}$ and

$B = \{x \in U : x^2 - 1 = 0\}$

Find i) $A \cap B$, ii) $A \cup B$, iii) A^c, iv) $A \times B$

Group B

Answer **any five** out of seven questions : 5X4=20

2. What is composite mapping? Let $f : \mathbb{R} \to \mathbb{R}$ be define by f(x) $= 3x + 1$, $x \in \mathbb{R}$, Prove that f is invertible. Then find f^{-1}. 1+2+1

3. What is anti-symmetric relation?
Verify the relation $R : |a| \geq |b|$ where a, b $\in$ real number is equivalence or not. 1+3

4. Show that $(P \Rightarrow Q) \wedge (R \Rightarrow Q) \equiv (P \vee R) \Rightarrow Q$ using logical identities.

5. If the function $f : R \to R$ be defined by $f(x) = x^2 + 1$ then find $f^{-1}(-8)$ and $f^{-1}(17)$.

6. Solve $a_{r+2} + a_{r+1} + a_r = r.2^r$

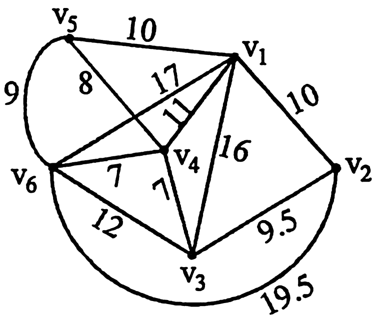
7. If a connected planner graph G has n vertices, e edges and r region then $n - e + r = 2$.

8. A relation ρ is defined on the set Z is "aρb if and only if ab>0 for examine if a, b$\in$Z". Examine if ρ is (i) reflexive (ii) symmetric (iii) transitive.

Group C

Answer **any four** out of six questions: 10x4=40

9. Described Prim's and Kruskal's algorithm to finding the shortest spanning tree. Using those algorithms find the shortest spanning tree of (3+3)+(2+2)



10. a) What is disjoint set? Describe with an example. Prove that (A – B) and (B – A) are disjoint sets. 1+1+3

b) Prove that

i) $A \cap (B - C) = (A \cap B) - (A \cap C)$

ii) $(A - B) \times C = (A \times C) - (B \times C)$. 2+3

11. a) Prove that for three non-empty sets A, X, Y if $A \cup X = A \cup Y$ and $A \cap X = A \cap Y$ then $X = Y$.

b) Using set theory find the H.C.F. & L.C.M. of 36 & 48.

c) Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

3+2+5

12. a) How many 2-digits numbers greater than 40 can be formed by using the digits 1, 2, 3, 4, 6 7

    i) When repetition is allowed

    ii) When repetition is not allowed. 5

b) i) Draw the multi-graph G whose adjacency matrix M_A is shown in Fig-1

$$M_A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

ii) Define a Hamiltonian graph and Eularian graph. Give an example of a graph which is Hamiltonian but not Eularian and vice-verse. 2+3

13. a) A mapping $f : R \to R$ is defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$

is one to one and onto. Find f^{-1} if exists. 5

b) Prove by induction that the sum of the cubes of three consecutive integers is divisible by 9. 5

14. a) Construct the binary expression tree for the expression $(a + b) * (d / c)$. 2

b) Verify that proposition $PV \sim (p \wedge q)$ is tautology. 3

c) Prove $\frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots + \frac{1}{2^n} > \frac{13}{24}$, for $n \geq 2$. 5