Total Pages : 4 MC(A)/UG/S–I/2019

End Semester Examination of Semester–I, 2019

Subject : BCA

Paper : BCA–104

Full Marks : 70

Time : 3 Hrs

The figures in the margin indicate the marks corresponding to the question

Candidates are requested to give their answers in their own word as far as practicable.

Illustrate the answers wherever necessary.

Group A

1. Answer **any five** out of eight questions : 2x5=10

   i) Show that the vectors (1, 5, 2), (1, 1, 0) and (0, 0, 1) are linearly dependent in the vector space R^3.

   ii) Apply Descart's rule of signs to find the nature of the roots of the equation

   $$3x^4 + 12x^2 + 6x - 10 = 0$$

   iii) Find the value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$, ω is an cube root of unity.

   iv) Evaluate $\int \frac{\sin 2x}{1+\sin^2 x} dx$.

v) State fundamental theorem of Classical Algebra.

vi) Show that for any event A, $P(\overline{A}) = 1 - P(A)$.

vii) Find Cov(x, y); if
$\Sigma x_i = 60,\ \Sigma y_i = 95,\ \Sigma x_i y_i = 574,\ n = 10.$

viii) Show that $A^2 - 2A + I_2 = 0$, where

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}.$$

Group B

Answer **any five** out of seven questions : 5×4=20

2. Show that the roots of the equation
$\frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3} + \frac{4}{x-4} + \frac{5}{x-5} = 6$ are all real.

3. Solve by crammer's rule :
$2x + 3y + z = 11, x + y + z = 6, 5x - y + 10z = 34.$

4. Prove that a square matrix can be expressed as sum of two matrices one is symmetric and another is skew symmetric.

5. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then use Cayley-Hamilton's theorem to show that $2A^5 - 3A^4 + A^2 - 4I = 138A - 403I_2$.

6. If the equation $x^4 + ax^3 + bx^2 + cx + d = 0$ has three equal roots, then show that each of them is equal to $\left(\frac{6c-ab}{3a^2-8b}\right)$.

7. Prove that $P(A + B) = P(A) + P(B) - P(AB)$. Where A and B are any events.

8. Determine the value of K such that defined by

$$f(x)=\begin{cases} Kx(1-x); & 0<x<1 \\ 0 & ; \text{ elsewhere} \end{cases}$$

is a probability density function.

Group C

Answer **any four** out of six questions: 10x4=40

9. a) Solve by Cardam's method :
$x^3 + 3x + 1 = 0.$

b) If $f(x) = x^4 - 3x^3 + 4x^2 - 5x - 3$ then show that
$f(x + 2) = x^4 + 5x^3 + 10x^2 + 7x - 11.$ 5+5

10. a) Determine the eigen value of $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ and eigen vectors corresponding to the positive value eigen of A.

b) Solve the equation $2x^3 + x^2 - 5x + 2 = 0$ if two of its roots α and β be connected by the relation $\alpha\beta + 1 = 0$. 6+4

11. a) Solve the differential equation (**any one**):

i) $(y + x)dx + x\,dy = 0.$

ii) $y\,dx + (x + e^y)dy = 0.$

b) Evaluate **any one** of the integral :

i) $\int_0^{n} \frac{dx}{(x^2+a^2)(x^2+b^2)}$

ii) $\int_0^{1} \frac{5x\,dx}{(x+2)(x^2+1)}$

12. a) State and proved Baye's theorem.

b) Define exhaustive set of events and conditional probability.

(2+5)+3

13. a) Solve : $(x + y)dy + (x - y)dx = 0$

b) Solve by Cardan's Method :

$x^3 - 9x + 28 = 0$. 5+5

14. a) State Leibnitz's theorem. If $y = \tan^{-1}x$, then prove that $(1 - x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$.

b) State Euler's Theorem of Homogeneous functions of two variable x & y. Verify Euler's theorem when $f(x, y) = ax^2 + 2hxy + by^2$. (1+4)+(1+4)