

**End Semester Examination of Semester-I, 2017**

**Subject : BCA**

**Paper : BCA-104 (Discrete Math)**

**Full Marks : 70**

**Time : 3 Hrs**

*The figures in the margin indicate the marks  
corresponding to the question*

*Candidates are requested to give their answers  
in their own word as far as practicable.*

*Illustrate the answers wherever necessary.*

**Group A**

1. Answer **any five** out of eight questions : 2x5=10
- i) Find the transitive closer of the relation  
 $R = \{(a, b), (b, a), (b, c), (d, a), (c, d)\}$  defined over  
the set  $A = \{a, b, c, d\}$ .
  - ii) Determine the coefficient of  $x^9y^3$  in  $(x+y)^{12}$ .
  - iii) Define recurrence relation and generating function.
  - iv) Solve the difference equation  $a_t - 3a_{t-1} + 2a_{t-2} = 0$
  - v) Define tautologies and contingency.
  - vi) Construct the truth table for the statement  
 $(p \wedge \sim q) \vee (q \vee (\sim p \wedge q))$ .
  - vii) Define Hamiltonian graph.

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- viii) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x^2$ ,  $x \in \mathbb{R}$ . Examine whether  $f$  is surjective or not. [Where  $\mathbb{R}$  denotes the set of all real numbers].

**Group B**

Answer **any five** out of seven questions :

5×4=20

2. Let  $X$  and  $Y$  be two non-empty sets and let  $f: X \rightarrow Y$  is an into mapping and also  $A \subseteq X$ ,  $B \subseteq X$  then prove that  $f(A \cap B) \subseteq f(A) \cap f(B)$ .
3. If  $R$  be a binary relation defined as :  $R = \{(a, b) \in \mathbb{R} : a - b \leq 3\}$ , determine whether  $R$  is reflexive, symmetric and transitive.
4. Prove by mathematical induction
$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}.$$
5. Prove that a connected graph  $G$  is a tree iff adding an edge between any two vertices in  $G$  creates exactly one circuit.
6. A shelf holds  $n$  books in a row. How many ways are there to choose  $r$  books so that no two adjacent books are chosen?
7. Given that  $A \cap B = A \cap C$  and  $A^c \cap B = A^c \cap C$ . Is it necessary that  $B = C$ ? Justify your answer.
8. Suppose 4 letters are to be placed in addressed envelope. Find the number of ways such that no letter is placed in the right envelope.

Group CAnswer **any four** out of six questions:

10x4=40

9. a) Establish the validity of the arguments

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$$p \Rightarrow r$$

$$\sim p \Rightarrow q$$

$$q \Rightarrow s$$

$$\therefore \sim r \Rightarrow s.$$

- b) Explain any one of traversal techniques in a graph faxing a suitable example. 5

10. a) Express Fibonacci sequence of numbers 1, 1, 2, 3, 5, 8, 13, 21, ... in term of

- i) general expression for the rth number  $a_r$  and
- ii) generating function.

- b) Define graph. Prove that the number of vertices of odd degree in a graph is always even. 5+5

11. a) Prove that if
- $f : X \rightarrow Y$
- and
- $g : Y \rightarrow Z$
- be two one-to-one onto function, then (gof) is also one-to-one onto function.

- b) Prove that
- $1^2 - 2^2 + 3^2 - \dots + (-1)^{n+1}n^2$

$$= \frac{(-1)^{n+1} n(n+1)}{2}$$

12. a) If function
- $f: \mathbb{R} \rightarrow \mathbb{R}$
- be defined by
- $f(x) = x^2 + 1$
- , then find
- $f^{-1}(-8)$
- and
- $f^{-1}(17)$
- . [
- $\mathbb{R}$
- denotes the set of Real numbers]

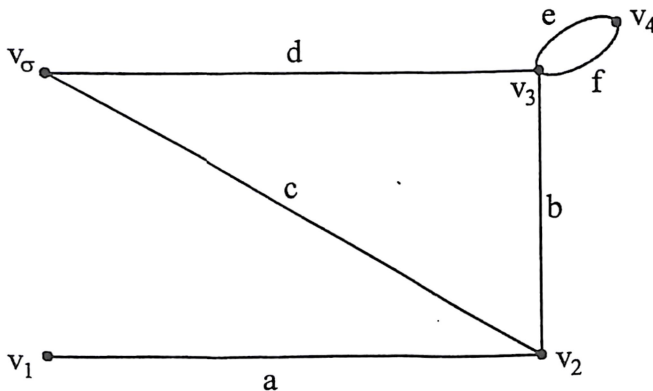
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- b) Show that simple graph with  $n$  vertices and  $k$  components can have at most  $\frac{(n-k)(n-k+1)}{2}$  edges. 5+5

13. a) A relation  $\rho$  is defined on the set  $z$  is "apb if and only if  $a - b$  is divisible by 6". Prove that  $\rho$  is an equivalence relation on  $z$ . [Where  $z$  denotes the set of integer numbers]

- b) Prove that any minimal set of edges containing at least one edge of every spanning tree of a connected graph  $G$  is a Cut-Set of  $G$ . 5+5

14. a) Find the incidence matrix of the following graph.



- b) Define Directed graph, Directed circuit, Binary Tree. 5+2+1+2
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