Chain Rule

a) $f(z) = \ln(1+z)$

where

$$z = x^T x$$

we know. $\frac{df}{dx} = \frac{df}{dz} \times \frac{dz}{dx}$

$$\therefore \frac{d(f(z))}{dx} = \cancel{\frac{d(1+}{dz}} \frac{d}{dx}(\ln(1+z))$$

$$\frac{df}{dz} = \frac{1}{1+z} \quad \text{——— (i)}$$

$$\boxed{\cancel{\&) \ \frac{dz}{dx} = 2x \text{ — (ii)}}}$$

$$[x^T x \approx x^2]$$

$$\frac{dz}{dx} = \frac{d\, x^T x}{dx} = \frac{dx^2}{dx} = 2x \text{ ——— (ii)}$$

$$\therefore \frac{df}{dx} = \frac{1}{1+z} \times 2x = \frac{1}{1+x^T x} \times 2x$$

$$= \frac{2x}{1+x^T x}$$

(Solved)

b) $f(z) = e^{-z/2}$

$$z = g(y) = y^T S^{-1} y$$

$$y = h(x) = x - \mu$$

We know $\dfrac{df}{dx} = \dfrac{df}{dz} \times \dfrac{dz}{dy} \times \dfrac{dy}{dx}$

Here $\dfrac{df}{dz} = -\dfrac{1}{2} e^{-z/2}$

$$\frac{dz}{dy} = 2 S^{-1} y$$

$$\frac{dy}{dx} = I$$

$$\therefore \frac{df}{dz} = -\frac{1}{2} e^{-\frac{z}{2}} \cdot 2 S^{-1} y$$

$$= -\frac{1}{2} e^{-\frac{((x-\mu)^T S^{-1} (x-\mu))}{2}} \cdot 2 S^{-1} (x-\mu)$$