SECTION II

FAULT CALCULATIONS

19

Introduction to Fault Calculations

Introduction — Procedure of Fault Calculation — Representation of Power Systems — Per Unit Method — Advantages of Per Unit System — Selection of Bases — Single Phase Circuit — Determination of Per Unit Resistance and Reactance — Summary.

19.1. INTRODUCTION

Section II of this book deals with steady state fault calculations. This section covers symmetrical faults, unsymmetrical faults, method of symmetrical components and use of digital computer and network analyzer in fault calculations. Some simple problems have been solved for understanding of the procedure of fault calculations.

The circuit-breakers should be capable of breaking and making the currents as per their ratings and should also have rated short-time capacity. Hence, for proper selection of circuit-breakers and other switching devices/switchgear components, knowledge of current during normal and abnormal conditions (at various respective locations) is necessary.

The design of machines, bus-bars, isolators, circuit-breakers, etc. is based on considerations of normal and short-circuit currents.

The protective relaying schemes can be selected only after ascertaining the fault levels and normal currents at various locations.

Fault studies are also necessary for system design, stability considerations, selection of layout, etc.

The faults are classified as

1. Three phase faults
2. Single phase to earth fault
3. Phase to phase fault
4. Double phase to earth fault
5. Simultaneous faults.

For steady state fault calculations, the steady state reactances are considered. The current and voltage are in r.m.s. value.

In fault calculations, many assumptions are made for simplifying the calculations *e.g.* resistances are neglected when their value is negligible as compared with the reactance. Capacitance is neglected. Machine reactances are assumed to be constant. Saturation effects are neglected. Generated voltages are assumed to be constant. Contribution of shunt capacitor banks is usually neglected.

Two machine-models are assumed in some problems for understanding the procedure. The fault current and fault levels are calculated for steady state.

Consider a point in the power system. Suppose the normal current flowing through the conductor at the point is I_n amperes and the phase to phase voltage at the system is V_n. The normal MVA supplied through the part is given by :

$$\text{Normal MVA}_n = \sqrt{3}\, V_n I_n \qquad ...(19.1)$$

where MVA_n = Normal MVA

V_n = normal phase to phase voltage, kV rms.

I_n = normal current, kA rms.

Now, consider a *three phase* fault occurring at the point under considerations, the fault current being I_f. The MVA in case of three phase fault would be equal to

$$\text{Fault} \quad MVA_f = \sqrt{3}\, VI_f \qquad ...(19.2)$$

where MVA_f = Fault MVA

V = Phase to phase voltage, kV, rms

I_f = Fault current, kA. rms, (90° lag)

The fault impedance being low, the power flowing into the fault depends on the system reactance upto the fault point. Fault MVA is generally several times the normal MVA, and with lagging p.f.

Table 19.1
Typical Values of Fault Currents in Distribution and Transmission Systems in India

Nominal Voltage (line to line) kV	*Steady State Fault Levels kA, r.m.s*
11	10 to 40
33	10 to 25
132	10 to 30
220	15 to 40
400	20 to 40

With increase in generation and new interconnection, the fault levels at all points go up.

While determining the rating of circuit-breakers, bus-bars, CT's etc. the fault-level at the point under consideration should be known. Fault MVA is of lagging p.f. with current lagging behind voltage by 90°. Large capacitor banks provide leading MVA and *reduce* the fault level. (Ref. Sec. 20.14)

Effective Fault MVA = [Calculate Fault MVA – MVA rating of capacitor Bank]

The Fault-level at the various points in the power system can be calculated by the well established procedures of Fault-calculations. These are for steady state. (Sec. 3.5)

Faults cause drop in voltage, unbalance and loss of stability. Hence another aim of fault calculations is to provide data required for system studies under various fault conditions.

19.2. PROCEDURE OF FAULT CALCULATIONS

Fault calculations deal with determination of current and voltages for various fault conditions at different locations of the power system. Such calculations provide the necessary data for selections of circuit-breakers and design of protective scheme. Fault calculations normally begin with drawing single line or one line diagram of the given system. Next, suitable kV and kVA bases are chosen for each voltage level. From these, the base quantities for current and impedance are calculated for each voltage level. Thereafter reactance diagram or positive sequence network of the system is drawn. These are the preliminary steps in fault calculations.

The faults are classified as symmetrical faults and unsymmetrical faults. Symmetrical faults include three phase faults. Such faults can be solved on per phase basis. The system is represented by a single phase system considering phase and neutral. The unsymmetrical faults are solved by using the method of symmetrical components.

For simple systems, calculations can be performed directly be means of calculator. But for modern complex systems, a.c. network analyzers or digital computers are used for faults calculations.

Per unit system is adopted for fault calculations as it simplifies the analysis. Steady state rms values are calculated.

19.3. REPRESENTATION OF POWER SYSTEMS

A balanced three phase system can be conveniently solved on single-phase basis. It can be represented by a single-phase system having one phase and a neutral. The first step in fault calculation is normally drawing a *Single Line Diagram*. In this diagram the components of the system are usually drawn in the form of their symbol. The neutral earthing is indicated. Fig. 19.1 shows single line diagram of a simple system.

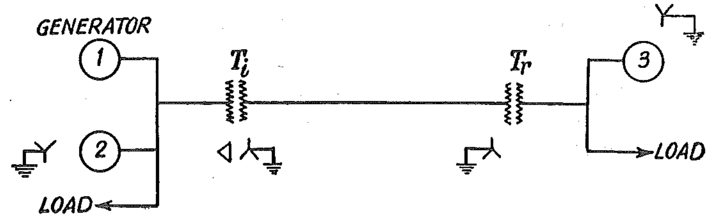


Fig. 19.1. Single line diagram of system.

The next step is to draw :

'Impedance or Reactance Diagram' or 'Positive Sequence Network'.

In impedance diagram each component is represented by its equivalent circuit. Fig. 19.2 represents impedance diagram of systems of Fig. 19.1. In majority of problems of fault calculations, resistance is neglected. Further, some more approximations can be made such as neglecting the capacitance and magnetizing current, etc. the rotating machines are represented by e.m.f. source in series with a reactance. Static loads are omitted. The induction motors are omitted for steady state analysis. Thereby the impedance diagram reduces to simplified reactance diagram. Contribution of large capacitor banks is considered. *Effective Fault MVA* is obtained by subtracting capacitor bank MVA from calculated Fault MVA.

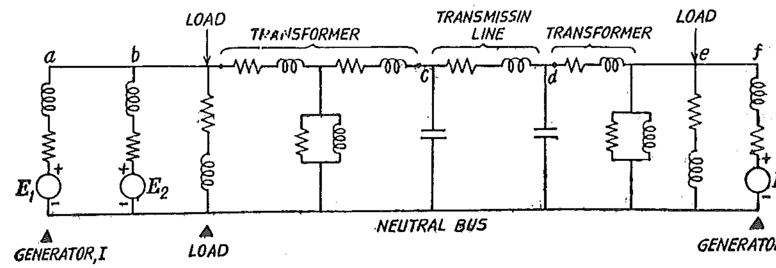


Fig. 19.2. Impedance diagram of Fig. 19.1.

Fig. 19.3 shows the reactance diagram of the system of Fig. 19.1. Reactance diagram is also called 'Positive Sequence Network'.

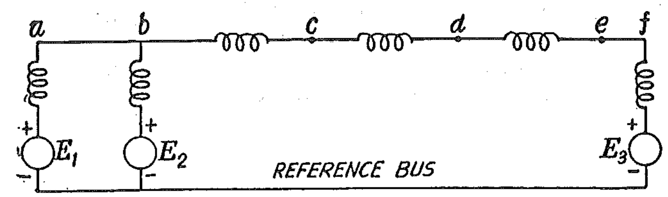


Fig. 19.3. 'Reactance diagram' or 'Positive Sequence Network of the system in Fig. 19.1.

19.4. PER UNIT METHOD

The quantities voltage (V), current (I), kVA, impedance Z are often expressed as percentage or per unit of their selected bases. Such a method simplifies the calculations.

For example let us call, 200 volts equal to 1 per unit or 100 per cent. Then 200 volts is the base voltage. Now, other voltages are expressed as multiple the base voltage as follow :

$$\text{Per unit voltage} = \frac{\text{Actual voltage}}{\text{Base voltage}}$$

e.g.,
$$20\ \text{V} = \frac{20}{200} = 0.1 \text{ p.u. or 10 per cent}$$
$$100\ \text{V} = \frac{100}{200} = 0.5 \text{ p.u. or 50 per cent}$$

and likewise.

Similarly, other quantities, I, Z etc. are expressed as per unit of their selected bases, *e.g.*, if base current is 10 amperes, 100 amperes will be 10 p.u., 10 amperes will be 1.p.u. etc.

After completing the calculations, the actual values of I, V, Z etc., are obtained by the reverse method, *i.e.*,

$$\text{Actual value} = \text{p.u. Value} \times \text{Base value.}$$

Example 19.1. Per Unit Method. *For a single phase system, selected bases are as follows :*
Base current 10 amperes.
Base voltage 200 volts.
Calculate base impedance.
Express the following quantities in per unit form
20 A, 0.2 A, 50 V, 1000 V, 2 Ω.

Solution. Base current = 10 A
Base voltage = 200 V

$$\text{Base impedance} = \frac{\text{Base Voltage}}{\text{Base current}} = \frac{200}{10} = 20 \text{ ohms.}$$

$$20\ \text{A} : \frac{20}{10} = 2 \text{ p.u.} \qquad \text{[Current]}$$
$$0.2\,A = 0.02 \text{ p.u.}$$
$$50\ \text{V} : \frac{50}{200} = 0.25 \text{ p.u.} \qquad \text{[Voltage]}$$
$$2\,\Omega : \frac{2}{20} = 0.1 \text{ p.u.} \qquad \text{[Resistance]}$$

19.5. ADVANTAGES OF PER UNIT SYSTEM

1. Calculations are simplified.
2. For circuits connected by transformers, per unit system is particularly suitable. By choosing suitable base kV's for the circuits the per unit reactance remains the same, referred to either sides of the transformer. Therefore, the various circuits can be connected in the reactance diagram.
3. Machine reactances given in per unit, give a basis for comparison. The micro-machines are built to represent the actual machines for the purpose of research. They have nearly the same per unit reactance as their parent machine. Thus per unit system gives a method of comparison.

19.6. SELECTION OF BASES

(*I*) As a rule only two bases should be selected first and from these two, the remaining bases should be calculated. This is so, because kV, kVA, I and Z are interrelated. They must obey Ohm's law. If we choose Base kV and kVA, the other bases, *i.e.*, Base I and Base Z are calculated from Base kV and Base kVA. As we will see later, it is convenient to select Base kV and Base kVA.

(*II*) For circuits connected by transformer, choose same kVA base for both the circuits. Choose base kV's such that the ratio of Base kV's is same as the ratio of transformer. Such a selection gives same p.u. reactance of transformer referred to both the circuits [Refer example 19.4]

19.7. SINGLE PHASE CIRCUITS : DETERMINATIONS OF BASE-IMPEDANCE (or Resistance or Reactance)

Select Base kV and Base kVA
Actual kV given in kV

$$\text{P.u. kV} = \frac{\text{Actual kV}}{\text{Base kV}}$$

$$\text{Base Current } I \text{ (Amperes)} = \frac{\text{Base kVA}}{\text{Base kV}} \qquad \ldots(1)$$

$$\text{Base Impedance } Z \text{ (Ohms)} = \frac{\text{Base kV} \times 1000}{\text{Base Current } I} \qquad \ldots(2)$$

$$= \text{Base kV} \times \frac{\text{Base kV}}{\text{Base kVA}} \times 1000 \qquad \ldots(3)$$

$$= \frac{[\text{Base kV}]^2 \times 1000}{\text{Base kVA}} \qquad \ldots(4)$$

Base power kW = Base kVA

$$\text{P.u. Impedance} \quad Z = \frac{\text{Actual Z}}{\text{Base Z}} \qquad \ldots(5)$$

$$= \text{Actual } Z \times \frac{\text{Base kVA}}{(\text{Base kV})^2 \times 1000} \qquad \ldots(6)$$

19.8. CHANGE OF BASE

From Eq. (6) we get important conversion :
P.u. Z referred to new base

$$= \text{P.u. } Z \text{ referred to old base} \times \left(\frac{\text{Base kV old}}{\text{Base kV new}}\right)^2 \times \left(\frac{\text{Base kVA new}}{\text{Base kVA old}}\right) \qquad \ldots(7)$$

Example 19.2. *Convert 2 ohms into per unit. Base kV 11, Base kVA 1000.*

Solution.
$$\text{Base } Z = \frac{\text{Base } kV^2 \times 1000}{\text{Base kVA}} = \frac{11^2}{1000} \times 1000 = 121$$

Hence $121\ \Omega = 1$ p.u.

Then $2\ \Omega = \dfrac{2}{121} = 0.0165$ p.u.

Example 19.3. *A 11 kV, 15,000 kVA generator has reactance of 0.15 p.u. referred to its ratings as bases. The new bases chosen for calculations are 110 kV and 30,000 kVA.*

Calculate the new p.u. reactance.

Solution. Eq. (6) gives

$$\text{New p.u. } Z = \text{Old p.u. } Z \times \left(\frac{\text{Old kV Base}}{\text{New kV Base}}\right)^2 \times \left(\frac{\text{New kVA Base}}{\text{Old kVA Base}}\right)$$

In this problem,

$$\text{P.u. } X \text{ new} = 0.15 \times \left(\frac{11}{110}\right)^2 \times \left(\frac{30{,}000}{15{,}000}\right)$$
$$= 0.15 \times (0.1)^2\,(2) = 0.0003 \text{ p.u.}$$

Three phase Systems

(*i*) Three-phase systems are solved on single phase basis, the base voltage represents phase to neutral voltage, and currents represents the phase currents. On this basis, the equations for base impedance are as follows :

Base kV phase to neutral kV

$$\text{Base kVA} = \frac{\text{3-phase kVA}}{3}$$

$$\text{Base current} = \frac{\text{Base kVA}}{\text{Base kV}}$$

$$\text{Base Impedance (ohms)} = \frac{\text{Base kV}^2}{\text{Base kVA}} \times 1000.$$

(*ii*) Suppose we take a three phase base kVA and base kV as phase to phase kV, we obtain the following expressions :

$$\text{Base Current} = \frac{\text{Base kVA}}{\sqrt{3} \times \text{Base kV}}$$

$$\text{Base impedance} = \frac{\left(\frac{\text{Base kV}}{\sqrt{3}}\right)^2 \times 1000}{\text{Base kVA}/3} = \frac{\text{Base kV}^2 \times 1000}{\text{Base kVA}}$$

We note that, the same expression of p.u. impedance is obtained for a single phase and three phase systems. In power system problems the data are given in terms of 3-phase kVA, phase to phase kV. The machine reactances are given in percentage of p.u. quantities based on machine kVA and machine phase to phase kV rating.

Further the direct axis synchronous reactance is also known as positive sequence reactance of the machine.

19.9. CIRCUITS CONNECTED BY TRANSFORMER

Consider two circuits *A* and *B* connected by means of a transformer. While selecting the bases for these two circuits choose same kVA base for both the circuits, but different kV bases. The kV bases for the two circuits should have the same ratio of transformation. With such a selection of bases the p.u. reactance referred to either side with bases of that side remains same (Refer Example 19.4).

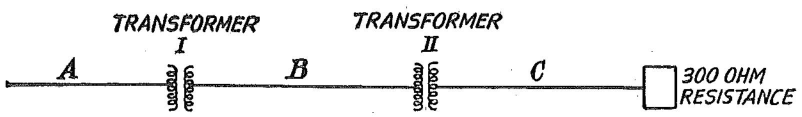


Fig. 22·4. Single line diagram.

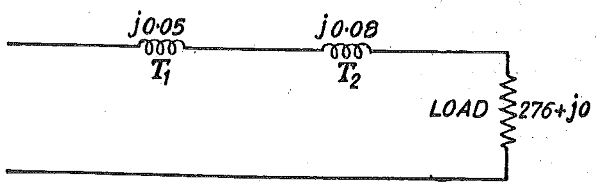


Fig. 19.5. Reactance diagram of the system. (Ans.)

Example 19.4. *Three-single-phase circuit A, B, C are connected as shown in Fig. 19.4. Draw reactance diagram of the system. Transformer I is rated 10,000 kVA and has a ratio 11/22 kV, leakage reactance of 5%. Transformer II has rating of 10,000 kVA, ratio 22/3.3 kV and leakage reactance 8%. Load is 300 ohm resistance. Draw reactance diagram for the system.*

Solution. Select same base kVA for *A*, *B*, *C*. Say 10,000 kVA.

Select base kV's of the same ratio as the transformation ratio. If we take base kV for circuit *A* as 11 kV. Base kV for circuit *B* is 22 kV Base, kV for circuit *C* is 3.3 kV.

$$\text{Base Impedance} = \frac{\text{Base kV}^2 \times 1000}{\text{Base kVA}}$$

$$\text{For circuit } A: \quad \frac{(11)^2 \times 1000}{10,000} = 12.1 \text{ ohm}$$

$$\text{Circuit } B: \quad \frac{(22)^2 \times 1000}{10,000} = 48.4 \text{ ohm}$$

$$\text{Circuit } C: \quad \frac{(3.3)^2 \times 1000}{10,000} = 1.09 \text{ ohm}$$

Load impedance = 300 Ω (resistive)

$$\text{In circuit } C: \quad \frac{300}{1.09} = 276 \text{ p.u.} \qquad ...(i)$$

Load impedance referred to circuit *B*

$$= 300 \times \left(\frac{22}{3.3}\right)^2 = 13,350 \text{ ohms}$$

In p.u. this is equal to

$$\frac{13,350}{48.4} = 276 \text{ p.u.} \qquad ...(ii)$$

Load impedance referred to circuit *A*

$$= 133,50 \left(\frac{11}{22}\right)^2 = 3337.5 \text{ ohms.}$$

In p.u. from which is equal to

$$\frac{3337.5}{12.1} = 276 \text{ p.u. (ohms)} \qquad ...(iii)$$

From (*i*), (*ii*), (*iii*), we note that the p.u. Impedance of load has same value in all the three circuits, with the proper selection of bases. The reactance diagram shows the two transformers and load represented as reactances the reactance diagram is given by Fig. 19.5.

The principle of selecting base kV and base kVA, mentioned above can also be applied to 3-phase transformers, irrespective of their connections of various star-delta combinations.

19.10. REACTANCES OF CIRCUIT ELEMENTS

The manufacturer mentions the percentage or per unit reactance of the machine. It is understood that the base quantities are machine ratings. Tables are available giving approximate values of per unit impedances of synchronous machines, transformers, induction motors.

19.11. INDUCTION MOTORS

Induction motors of large size have their own contribution to fault current which cannot be neglected. The kVA rating of the motor is taken from rating plate.

19.12. SYNCHRONOUS MOTOR

kVA rating is taken from rating plate.

19.13. THEVENIN'S THEOREM

The current in a branch of a network having one or more voltage sources can be conveniently determined by Thevenin's theorem. The current in the branch I_b is given by the expression

$$I_b = \frac{V_{oc}}{Z_{th} + Z_b}$$

where V_{oe} is the voltage across the terminal, with the branch disconnected.

Z_{th} = Thevenin's equivalent impedance, *i.e.* The impedance of the network between the terminals of the branch with the branch disconnected and the voltage sources replaced by short circuits.

Z_b = Impedance of the branch whose current is to be determined.

Thevenin's theorem will be used in the fault calculations.

Table 19.1
Reference Values for Reactance of Two Winding Power Transformers

kV of hv, side / *MVA rating*	*3.3 kV*	*11kV*	*33kV*	*66kV*	*132kV*	*275kV*	*400kV*
Less than 1 MVA	4.75	4.75—6	4.75—6	6			
5 MVA		6—7	6—7	7.5			
10 MVA		12—15	12—15	11	10		
15 MVA		12—15	13	11	10		
30 MVA			12.5	11	10		
60 MVA				11	12.5		
120 MVA						15	
210 MVA						5	
600 MVA							15

Note. The percentage reactance depends on kV rating of h.v. side

Table 19.2
Reference Values for Percentage Reactances of Alternators*

Rating and type	X''	X'	X_s	X_2	X_o	*SCR*
11 kV Salient Pole, Without Damper	25	35	112	20	6	
11.6 kV, 600 MW, Turbo	11	17	200	13	6	0.55
11.6 kV, 60 MW, Gas-Turbo	11	14	175	13	5.2	0.7
13.5 kV, 100 MW, Turbo	20	29	205	22	10	0.58
18.5 kV, 300 MW, Turbo	20	26	260	19	11	0.4
22.2 kV, 500 MW, Turbo	20	28	250	20	9	0.4

*Where X'' : Sub transient reactance
X' : Transient reactance
X_s : Steady state reactance
} Positive Sequence Reactance
X_2 : Negative sequence reactance
X_0 : Zero sequence reactance

SCR : Short circuit ratio = $\frac{\text{Field current to get rated o.c. voltage}}{\text{Field current to get rated s.c. current}}$

For steady state calculations, use the steady state reactance value X_s..

Example 19.5. *Thevenin's Theorem. Determine the fault current in the circuit [Fig. 19.6 (a)]. Impedance of fault is negligible.*

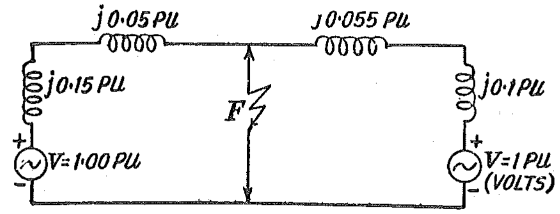


Fig. 19.6 (*a*). Ex. 19.5.

Solution. Since load current is zero the o.c. voltage across F is 1 p.u. Hence $V_{oc} = 1$. The Thevenin's impedance is obtained between terminals of F by shunting the voltage sources.

$$Z_{th} = \frac{j0.2 \times j0.155}{j0.2 + j0.155}$$

$$= j\frac{0.0310}{0.355} = j\ 0.087 \text{ p.u.}$$

$$I_f = \frac{V_{oc}}{Z_{th} + Z_f}$$

$$Z_f = 0$$

$$I_f = \frac{1 + j0}{j0.087} = -j\ 11.5 \text{ p.u.}$$

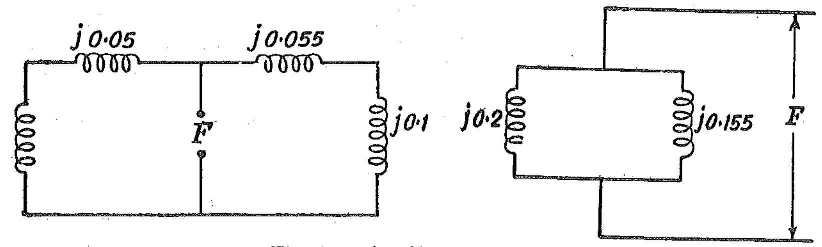


Fig. 19.6 (*b*) of Ex. 19.5.

Example 19.6. *Draw reactance diagram of the system shown in Fig. 19.7. The generator is 11 kV, 30,000 kVA with sub-transient reactances of 15%. The generator supplies power to two motors through a transmission line having transformers at both the ends. Motors have rated input 10,000 kVA, at 11 kV, 11/110 kV with leakage reactance of 10 per cent. Series reactance of transmission line 80 ohms.*

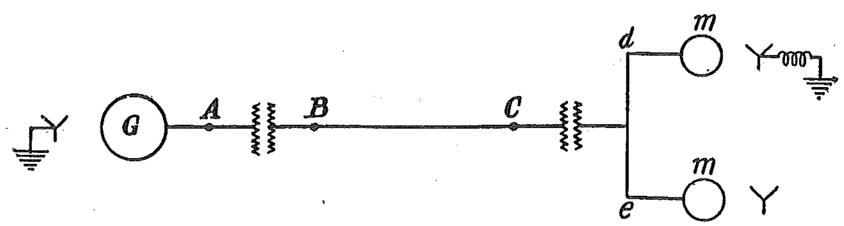


Fig. 19.7. Single line diagram of Ex. 19.6.

Solution.

Base kVA for the complete system	30,000 kVA
Base kV's generator circuit	11 kV
Transmission line circuit	110 kV
Motor circuit	11 kV

Base impedance $= \frac{\text{Base kV}^2}{\text{Base kVA}} \times 1000$

Base impedance of transmission line circuit

$$= \frac{(110)^2}{30{,}000} \times 1000 = 403.3 \text{ ohms.}$$

P.u. reactance of transmission line

$$\frac{80}{403.3} = 0.198 \text{ p.u.}$$

P.u. reactance of transformer to new base kVA

$$= j0.1 \times \frac{30{,}000}{35{,}000} = j\ 0.0857 \text{ p.u.}$$

P.u. reactance of motor to new base kVA

$$= j0.2 \times \frac{30{,}000}{10{,}000} = j\ 0.6 \text{ p.u.}$$

The reactance diagram or positive sequence network is drawn from these p.u. values (Refer Fig. 19.8) **(Ans.)**

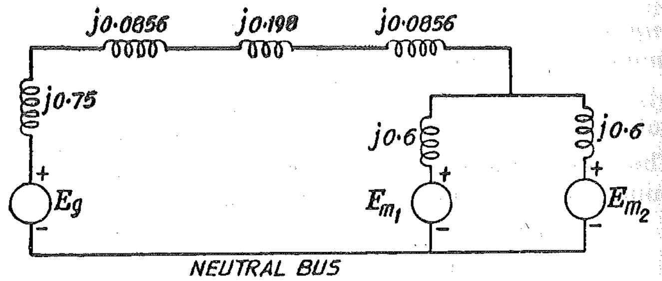


Fig. 19.8. Reactance diagram of Example 19.6. Also called positive sequence network. (Ans.)

Per Unit Impedance of Three Winding Transformers. The three windings of a three winding transformer may have different kVA ratings. Impedance of each winding may be given in per unit based on the rating of that winding. However, the per unit impedances in impedance diagram must be expressed on the same base kVA. The impedance measured from short circuit test may be denoted as follows :

Z_{ps} = leakage impedance measured in primary with secondary short circuited and tertiary open.

Z_{pt} = leakage impedance in primary with tertiary short-circuited and secondary open.

Z_{st} = leakage impedance measured in secondary with tertiary short-circuited and primary open.

If the three impedances measured in ohms are referred to voltage of one of the windings, the impedances of each separate winding referred to that winding are as follows :

$$Z_{ps} = Z_p + Z_s$$
$$Z_{pt} = Z_p + Z_t$$
$$Z_{st} = Z_s + Z_t$$

Where Z_p, Z_s and Z_t are the impedances of primary, secondary and tertiary winding referred to primary circuits, solving the equations simultaneously, we get

$$Z_p = \frac{1}{2}\,[Z_{ps} + Z_{pt} - Z_{st}]$$
$$Z_s = \frac{1}{2}\,[Z_{ps} + Z_{st} - Z_{pt}]$$
$$Z_t = \frac{1}{2}\,[Z_{pt} + Z_{st} - Z_{ps}]$$

The three impedances are star connected to represent single-phase equivalent circuit. The three outer points of star connection are connected to the parts of the impedance diagram which represent in connections of primary, secondary and tertiary.

Example 19.7. *A three phase rating of a three winding transformers are :*

Primary — Star connected 66 kV, 10 MVA.
Secondary — Star connected 11 kV, 7.5 MVA.
Tertiary — Delta connected 3.3 kV, 5 MVA.

The leakage impedances defined as mentioned earlier are :

$$Z_{ps} = 7\% \text{ based on 10 MVA, 66 kV base}$$
$$Z_{pt} = 9\% \text{ based on 10 MVA, 66 kV base}$$
$$Z_{st} = 6\% \text{ based on 7.5 MVA, 11 kV base.}$$

Find p.u. impedances of equivalent circuit for 10 MVA, 66kV base on primary circuit.

Solution. Bases primary 10 MVA, 66 kV.

Secondary 10 MVA, 11 kV.

Tertiary 10 MVA, 3.3 kV.

$$Z_{st} = 0.06 \times \frac{10}{7.5} = 0.08 \text{ p.u.}$$

$$Z_p\ \frac{1}{2}\,[Z_{ps} + Z_{pt} - Z_{st}]$$
$$Z_p = \frac{1}{2}\,[j\,0.07 + j\,0.09 - j\,0.08] = j\,0.04 \text{ p.u.}$$
$$Z_s = \frac{1}{2}\,[Z_{ps} + Z_{st} - Z_{pt}]$$
$$= [j\,0.07 + j\,0.08 - j\,0.09] = j\,0.03 \text{ p.u.}$$
$$Z_t = \frac{1}{2}\,[Z_{pt} + Z_{st} - Z_{ps}]$$
$$= \frac{1}{2}\,[j\,0.09 + j\,0.08 - j\,0.07] = j\,0.05 \text{ p.u.}$$

Equivalent Circuit

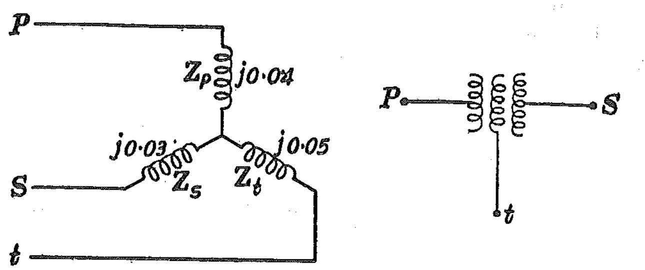


Fig. 19.8 (*a*). Solution of Ex. 19.7.

Example 19.8. *Define percentage reactance of an electrical machine and show that the short-circuit current is inversely proportional to percentage reactance.*

Solution. Let E be the rated voltage and I be the rated current. If the electrical machine is the only impedance in the circuit the short-circuit current is given by E/X which is

$$I_{sh} = \frac{E}{X}$$

The percentage reactance of the machine is defined as

$$\%X\ \frac{IX}{E} \times 100$$

Therefore, $$I_{sh} = \frac{I \times 100}{\%X} \quad \textbf{Ans.}$$

Example 19.9. *Fault level at a 110 kV bus in a receiving substation neglecting new capacitor bank was 5000 MVA. A new 200 MVA, 110 kV capacitor bank was commissioned. Calculate new fault level.*

Solution. New fault level = Effective fault level
= Fault MVA – Capacitor Bank MVA
= 5000 – 200 = 4800 MVA.

19.14. SOME TERMS

1. **Fault Power.** Also called Fault level or Short-circuit level-(Symbol : *Sa*). It is the product :

$\sqrt{3}$ × Fault current × System voltage.

Fault power in MVA = $\sqrt{3}$ kV × kA

where kV = line to line voltage
kA = fault current

3-phase fault is assumed. Contribution of Capacitor Banks is neglected.

2. **Initial Symmetrical Fault Power**

(Symbol *Sa''*)

$\sqrt{2}$ × initial symmetrical fault current × Service voltage.

3. **Peak Short Circuit Current, I_S.** The highest instantaneous value of current after appearance of short-circuit.

With full asymmetry.

Peak value $= 1.8\sqrt{2} \times$ a.c. component.

4. **Initial Symmetrical Short Circuit Current, I_k''.** The r.m.s. value of symmetrical short circuit current at the instant of short circuit, determined from sub-transient reactances X_d''.

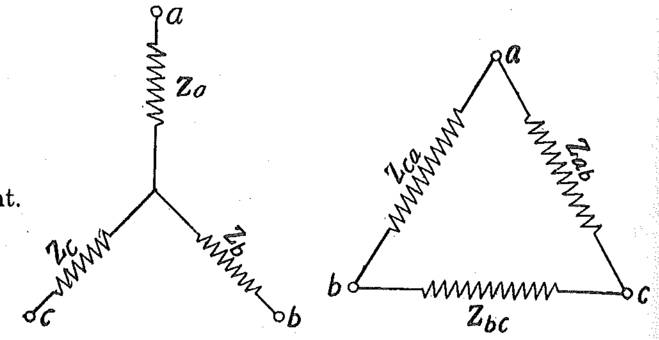


Fig. 19.9. Star-delta transformation.

5. **Transient Short Circuit Current, I_k'.** RMS value of short-circuit current determined from transient reactance X_d'.

6. **Sustained Short Circuit Current, I_k.** Short-circuit current determined from steady-state reactance.

[Refer Sec. 3.5]

19.15. STAR-DELTA TRANSFORMATION

Case I. *Star to Delta*

$$Z_{ab} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}$$

$$Z_{bc} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a}$$

$$Z_{ca} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b}$$

Case II. *Delta to Star*

$$Z_a = \frac{Z_{ab}\, Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

$$Z_b = \frac{Z_{bc} Z_{ab}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

$$Z_c = \frac{Z_{ca} Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

19.16. NOTATION j

A vector Z can be written as

$$\overline{Z} = x + jy$$

where x = component of $\overline{Z}$ along real axis

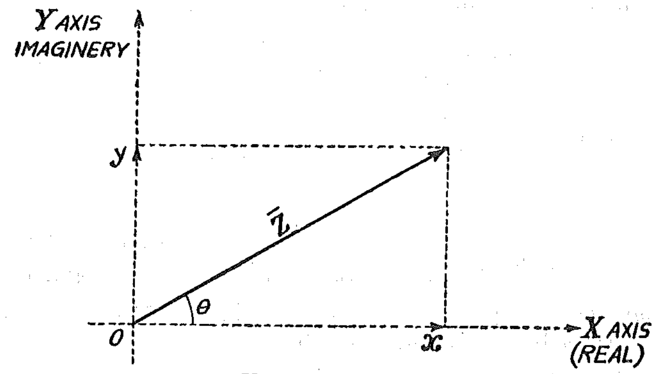
y = component of $\overline{Z}$ along imaginary axis

j = vector operator $= \sqrt{-1}$

$|Z|$ = magnitude of $\overline{Z}$

$= \sqrt{x^2 + y^2}$

$\theta = \tan^{-1} \frac{y}{x}$



Vector $Z = x + iy$

Fig. 19.10.

Addition of vectors : $\overline{Z}_1 + \overline{Z}_2 = \overline{Z}$

$$= (x_1 + jy_1) + (x_2 + jy_2)$$

$$= (x_1 + x_2) + j(y_1 + y_2)$$

$$|Z| = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}$$

Subtraction of $\overline{Z}_1$ and $\overline{Z}_2$

$$\overline{Z} = \overline{Z}_1 - \overline{Z}_2 = (x_1 - x_2) + (y_1 - y_2)$$

Multiplication of $\overline{Z}_1$ and $\overline{Z}_2$

$$\overline{Z}_1\,\overline{Z}_2 = (x_1 + jy_1)(x_2 + jy_2)$$

$$= x_1 x_2 + jx_1 y_2 + jx_2 y_1 - y_1 y_2$$

$$= (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$$

$$\overline{Z}_1\,\overline{Z}_2 = |Z_1|\,|Z_2| \,\underline{/\theta_1 + \theta_2}$$

Division of $\overline{Z}_2$ and $\overline{Z}_1$

$$\frac{Z_1}{Z_2} = \left|\frac{Z_1}{Z_2}\right| \underline{/\theta_1 - \theta_2}$$

19.17. SUMMARY

(*a*) Procedure of fault calculation is as follows :

1. Draw single line diagram of the system in which the machines are represented by their symbols.

2. Draw reactance diagram or positive sequence network.

3. For symmetrical faults, only positive sequence network is enough.

4. For unsymmetrical faults, method of symmetrical components is applied in which three sequence networks are drawn.

(*b*) The per unit system is used for fault calculations. The per unit reactances of machines are given referred to their ratings as the bases.

The kV, I, Z, kVA are expressed as per unit or per cent of their selected bases ; only two bases, (usually kV and kVA) are selected first then the remaining two bases are calculated.

(*c*) The p.u. Impedance is given by following expressions.

Base $$Z = \frac{\text{Base kV}^2}{\text{Base kVA}} \times 1000$$

$$\text{p.u. } Z = \frac{\text{Ohmic Z}}{\text{Base Z}} = \frac{Z \times \text{Base kVA}}{\text{Base kV}^2 \times 1000}$$

Change of Base

$$\text{p.u. } Z_{new} = \text{p.u. } Z_{old} \times \left(\frac{\text{kVA Base New}}{\text{kVA Base Old}}\right) \times \left(\frac{\text{kV base Old}}{\text{kV base New}}\right)^2$$

(*d*) For circuits connected by transformer choose same kVA base on both sides and different kV bases on either sides. kV bases should have the ratio same as transformation ratio. The p.u. reactance remains same on either sides.

(*e*) Thevenin's theorem and network reduction methods are useful in fault calculations.

QUESTIONS

1. Explain the Per Unit System. What are its advantages ?
   Derive expression for p.u. reactance from the chosen base kVA and base kV.
2. The p.u. reactance of a 11 kV, 20,000 kVA, 3 ph. 50 Hz alternator is 0.20 p.u. What will be its p.u. reactance referred to the base kV 110 and base kVA 10,000?
3. A generator is connected to a transmission line through a transformer. The ratings are as follows :
   Generator : 20,000 kVA, 11 kV, positive sequence reactance 15 p.u.
   Transformer 10,000 kVA, 11/110 kV, leakage reactance 6 per cent. Transmission line total reactance 10 ohms. A fault occurs at the other end of the line. Draw reactance diagram taking suitable base kV and 200,000 base kVA.
   Transmission line total reactance 10 ohms. A fault occurs at the other end of the line. Draw reactance diagram taking suitable base kV and 200,000 base kVA.
4. A 20,000 kVA, 11 kA generator 3 ph. has a direct axis synchronous reactance of 20 per cent. A 3-phase short circuit near the terminals. Calculate the steady short circuit current. Generator is at rated voltage and on load. **[Ans.** 5250 A]
5. Two generators are operating in parallel and have sub-transient reactance $X'' = 10\%$. Generator I is rated 2500 kVA, 3.3 kV, generator 2 is rated 500 kVA, 32 kV. Find p.u. reactances of each generator on 15,000 kVA, 3 kV base. What is equivalent p.u. reactance of the two generators on 1500 kVA, 3 kV base ?
6. Three reactance 0.5, 0.2 and 0.3 are connected in delta. Find the equivalent star reactances.
7. Three motors are connected to a common bus. The motors are rated 5000 h.p., 3.3 kV, 0.8 p.f., $X'' = 17\%$. They are supplied by a generator 20,000 kVA, 11 kV and of $X'' = 10\%$ through a transformer 11/3.3 kV, 18,000 kVA and having 5% leakage reactance. Draw the reactance diagram of the system. Take kVA = 1.10 × H.P. Take 20,000 kVA base.
8. Distinguish clearly between per unit method and percentage reactance method. Show that the per unit reactance referred to the circuits connected by transformer is same if same base kVA is taken for both circuits and the base kVs have ratio equal to transformation ratio.
9. State whether correct or wrong. Write corrected statements if necessary.
   (*i*) Circuit-breakers open during steady state fault condition.
   (*ii*) The positive sequence reactance of generators is less than their negative sequence reactance.
   (*iii*) During short-circuits, the current increases with time during first 10 cycles.
   (*iv*) Percentage reactance of power transformers is always less than one per cent.
   (*v*) Short-circuit ratio of generators is less than 1.
   (*vi*) Power system stability depends upon fault clearing time.

20

Symmetrical Faults and Current Limiting Reactors

Fault MVA and Fault Current – Summary. **Solved Examples 20.1 to 20.22** of various types by different methods — Current Limiting Reactors.

In this chapter some examples on symmetrical faults have been solved. Per unit system and procedure mentioned in chapter 19 has been followed.

Significance of Fault MVA.

High fault MVA at a point signifies high strength of power system at that point, low equivalent reactance upto that point. Therefore, large loads can be connected at that point. Low fault level signifies weak system. Fault level indicates the strength of the power system.

20.1. FAULT MVA AND FAULT CURRENT (STEADY STATE)

Calculate total p.u. reactance upto fault point, by network reduction of positive sequence network or reactance diagram. If same kVA base is taken for complete system

$$\text{P.u. fault current} = \frac{\text{P.u. voltage at fault point}}{\text{P.u. } X_{equivalent}}$$

P.u. fault level = $\sqrt{3}$ p.u. fault current × p.u. service voltage. Or in other way

$$\text{Fault MVA} = \frac{\text{Base MVA}}{\text{P.u. } X_{equivalent}} \text{ (MVA)} \quad \text{... reactive, lagging.}$$

$$\text{Fault current} = \frac{\text{Fault MVA} \times 10^3}{\sqrt{3} \times \text{Base kV}} \text{ (Amp.) } \angle -90°$$

Hence for three phase (symmetrical) fault, we get the expressions, neglecting load current

$$\text{Fault MVA} = \frac{\text{Base MVA}}{X_{p.u.\ the\ venine's\ equivalent}} \quad \text{... reactive, lagging.}$$

$$\text{Fault current} = \frac{\text{Fault MVA} \times 1000}{\sqrt{3} \times \text{Base kA}} \text{ (Amp.) } \angle -90°$$

(Base kV at the fault point, assuming the fault occurs at normal voltage. Fault current is of lagging power factor).

20.2. SOLVED EXAMPLES 20.1 TO 20.21

Note. In this chapter some problems on symmetrical faults have been solved. Further, the term sub-transient, transient and steady state are described in Ch. 3 (Notation *j* is omitted).

Example 20.1. *A 3-phase, 5000 kVA, 6.6 kV generator having 12% sub-transient reactance. A 3-phase short circuit occurs at its terminals, calculate fault MVA and current.*

Solution. (Method I)

$$\text{Fault MVA} = \frac{\text{Base MVA}}{\text{\%Reactance}} \times 100$$

Generator % reactance is based on its own voltage and kVA ratings. Hence choose 5000 kVA as base.

Fault MVA $= \frac{5}{12} \times 100$ ...(subtransient) $=$ **41.60 MVA. Ans.**

Fault current $= \frac{VA}{\sqrt{3}V}$ ...(subtransient)

$$= \frac{\text{Fault MVA} \times 1000}{\sqrt{3}\text{kV}} = \frac{41.66 \times 100}{\sqrt{3} \times 6.6} = \mathbf{3644\ A\ \angle{-90}}. \quad \textbf{Ans.}$$

Method II. From base voltage and p.u. reactance, calculate **p.u. fault** current. Then calculate fault current in amperes. Then you can calculate fault MVA.

Generator reactance (0.12 p.u.) is base on its kV and kVA ratings.

Base kV (phase) $= \frac{6.6}{\sqrt{3}} = 3.8$ kV

Hence 3.8 kV = 1 p.u. (voltage)

Fault current p.u. $= \frac{\text{p.u. voltage}}{\text{p.u. reactance}} = \frac{1}{j0.12} = 8.32$ p.u. $\angle -90°$ (current)

Base current $= \frac{\text{Rating kVA}}{\sqrt{3} \times \text{Rated kVA}} = \frac{5000}{\sqrt{3} \times 6.6} = 437$ Amp.

Fault Current = p.u. current × Base current

$= 8.32 \times 437 =$ **3644 Amp. Ans.**

Fault power $= \sqrt{3} \times$ Fault current × Service voltage

$= \sqrt{3} \times 3.64 \times 6.6$ MVA

$=$ **41.6 MVA** $\angle -90°$. **Ans.**

Change of Voltage

Suppose in same problem, the generator voltage is 6.4 kV, when fault occurs and not rated voltage, Method II should be adopted. The service voltage is then

$$\frac{6.4}{6.6} = 0.97 \text{ p.u.}$$

p.u. Fault current $= \frac{0.97}{0.12} = 8.09$ p.u.

Fault current in ampere = p.u. Current × Base current

$= 8.09 \times 437 = 3538$ Amp.

Check. Current reduces proportionately.

$$\frac{3538}{3640} = 0.97 = \frac{6.4}{6.6} \text{ (checked)}$$

Fault power at service voltage 6.4 kV

$= \sqrt{3} \times$ Fault current × Service voltage

$= \sqrt{3} \times 5.53 \text{ kA} \times 6.4 \text{ kV} =$ **39.2 MVA**

Note. For service voltage other than rated voltage adopt method II, to calculate p.u. fault current from p.u. Service voltage/p.u. reactance. If given, load current is added to fault current.

Example 20.2. *A 3-phase, 11 kV 5000 kVA, generator has a steady state reactance X_d of 20%. It is connected to a 3000 kVA transformer having 5.0% leakage reactance and ratio of 11/33 kV. The 33 kV side is connected to a transmission line. A three-phase fault occurs at the other end of the transmission line. The series reactance between the faulted point and the transformer is 30 ohms. Calculate the steady state fault current assuming no load prior to the fault.*

Solution. Let,

Base kVA for complete system = 5000 kVA

Base kV = 11 kV for generator side

Base kV = 33 kV for transmission line side.

P.u. reactance of generator = 0.2 p.u. (given)

New p.u. reactance of transformer

$$= 0.05 \times \frac{5000}{3000} = 0.083 \text{ p.u.}$$

P.u. reactance of transmission line

$$= \frac{\text{Ohms} \times \text{Base kVA}}{\text{Base kV}^2 \times 1000} = 30 \times \frac{5000}{(33)^2 \times 1000} = 0.139 \text{ p.u.}$$

Total p.u. reactance up to fault

$= j0.2 + j\,0.083 + j0.139 = j0.422$ p.u.

Fault MVA $= \frac{\text{Base MVA}}{X_{p.u.\ eq.}} = \frac{5000 \times 10^{-3}}{0.422} =$ **11.9 Ans.**

Fault current $= \frac{\text{Fault MVA} \times 10^2}{\sqrt{3}\text{ Base kV}} = \frac{11.9 \times 10^3}{3 \times 33} =$ **208 Amp.** $\angle -90°$ **Ans.**

Example 20.3. *Four 11 kV, 25 MVA alternators having a subtransient reactance of 16% are operating in parallel when a 3-phase fault occurs on the generator bus. Find the 3 phase fault MVA fed into the fault.*

Solution. Let,

Base MVA = 25

Base kV = 11

Fault MVA $= \frac{\text{Base MVA}}{X_{p.u.\ eq.}}$

The equivalent p.u. reactance of four generators operating in parallel is :

$$\frac{16}{4} = 4\% = j\,0.04 \text{ p.u.}$$

Fault MVA $= \frac{25}{0.04} = 625$ MVA

Other Method

Fault current $= \frac{\text{p.u. voltage}}{\text{p.u. } X_{eq.}}$

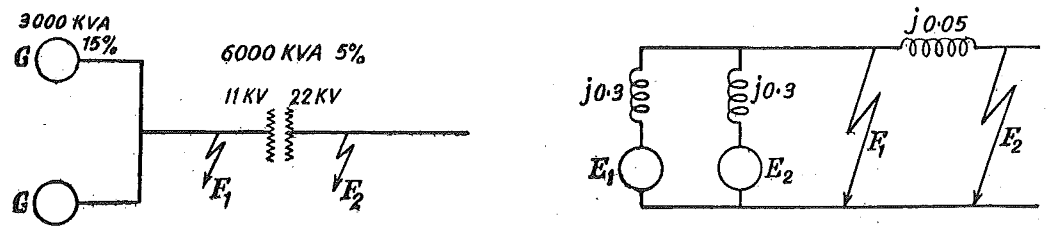
$$\frac{1}{j\,0.04} = 25 \text{ p.u. } \angle -90°$$

Base current $= \frac{25 \times 10^3}{\sqrt{3} \times 11} = 1310$ A

Short circuit current $= 25 \times 1310 = 32{,}750$ A

Short circuit MVA $= \frac{\sqrt{3} \times V \times I}{10^6} = \frac{\sqrt{3} \times 32{,}750 \times 11}{10^3} =$ **625 MVA Ans.**

Example 20.4. *Two 11 kV, 3-phase, 3000 kVA generators having sub-transient reactance of 15% operate in parallel. The generators supply power to a transmission line through a 6000 kVA transformer of ratio 11/22 kV and having a leakage reactance of 5%. Calculate fault current and fault MVA for three phase fault on (a) H.T. side (b) L.T. side of a transformer.*



(a) One line diagram. (b) Reactance diagram.

Fig. 20.1. Diagrams of example 20.4.

Solution.

Draw single line diagram [Fig. 20.1 (*a*)].

Let Base kVA = 6000

Base kV = 11 kV for generator side.

Base kV = 22 kV for transmission side.

Now, p.u. reactance of generator, (subtransient)

$$= j0.15 \times \frac{6000}{3000} = j0.3 \text{ p.u.}$$

Draw reactance diagram [Fig. 20.1 (*b*)].

Calculate Thevenin's equivalent reactance for the faults.

(*a*) *Fault on generator side :* F_1

Equivalent of two reactance (j 0.3 each) in parallel

$$= \frac{j0.3}{2} = j0.15 \text{ p.u.}$$

Fault MVA $= \dfrac{\text{Base MVA}}{X_{equivalent}} = \dfrac{6000 \times 10^{-3}}{0.15} = \mathbf{40\ MVA.}$ **Ans.**

Fault current $= \dfrac{\text{Fault MVA} \times 10^3}{\sqrt{3} \times \text{Base kV}} = \dfrac{40 \times 10^3}{\sqrt{3} \times 11} = \mathbf{2100\ A\underline{/-90^\circ}.}$ **Ans.**

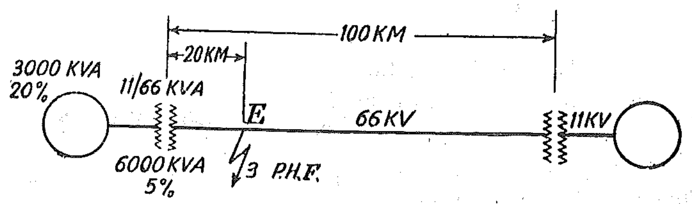
(*b*) *Fault on transmission side :* F_2

Equivalent reactance $= j0.15 + j0.05 = j\,0.20$ p.u.

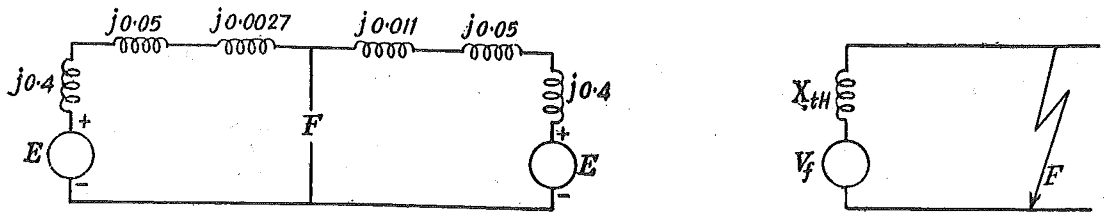
Fault MVA $= \dfrac{6000 \times 10^{-3}}{0.20} = \mathbf{30\ MVA.}$ **Ans.**

Fault current $= \dfrac{30 \times 10^3}{\sqrt{3} \times 22} = \mathbf{786\ A\underline{/-90^\circ}\ (lag).}$ **Ans.**

Example 20.5. (*a*) Two generators rated 11 kV, 3000 kVA, having 20% reactance are interconnected by a 100 km long transmission line. The reactance of line is 0.10 ohms per km. The transformers near the generators are rated 6000 kVA, 11 kV/66 kV and have 5% reactance. A 3 phase fault occurs at a distance of 20 km from one end of the line when the system is on no load but at rated voltage. Calculate fault MVA and fault current.



(*a*) Single line diagram of the system of Ex. 20.5.



(*b*) Reactance diagram or positive sequence network.

(c) Thevenin's equivalent circuit.

Fig. 20.2. Figures of Example 20.5.

Solution. Base kVA for the complete system = 6000 kVA

Base kV = 11 for generator sides

Base kV = 66 for transmission circuits.

Per unit reactance to these bases :

(1) Generators : $0.2 \times \dfrac{6000}{3000} = 0.4$ p.u.

(2) Transformers : $= 0.05$ p.u.

Base reactance of transmission line circuit

$$\frac{(66)^2 \times 100}{6000} = \frac{4356}{6} = 726\ \Omega$$

Per unit reactance of 20 km line

$$= \frac{20 \times 0.1}{726} = 0.00276 \text{ p.u.}$$

Per unit reactance of 80 km line = 0.01104 p.u.

From these values, the reactance diagram [Fig. 20.2 (*b*)] is drawn.

Thevenin's equivalent reactance from *F*.

Note that on short-circuiting the voltage sources, there are two parallel branches, the reactances being.

$$j0.4 + j0.05 + j0.0027 = j0.4527 \text{ p.u.}$$

and

$$j0.4 + j0.05 + j0.011 = j0.461 \text{ p.u.}$$

The equivalent is given by

$$\frac{j0.4527 \times j0.461}{j0.4527 + j0.461} = \frac{j0.209}{0.9137} = j0.229 \text{ p.u.}$$

Fault current $= \dfrac{V_f}{X_{p.u.}}$

$$\frac{1 + j0}{j0.229} = -j4.36 \text{ p.u.}$$

V_f is the p.u. voltage at fault point.

Base current in transmission line circuit

$$\frac{\text{Base kVA}}{\sqrt{3}\ \text{Base kV}} = \frac{6000}{\sqrt{3} \times 66} = 52.5 \text{ amperes.}$$

Fault current $= 4.36 \times 52.5 = 229$ amperes.

Fault MVA $= \sqrt{3} \times kV \times I_f \times 10^{-3}$

$= \sqrt{3} \times 66 \times 229 \times 10^{-3} = 26.2$ MVA.

Another method :

Fault MVA $= \dfrac{\text{Base MVA}}{X_{p.u.\ eq.}} = \dfrac{6000 \times 10^{-3}}{0.299} = \mathbf{26.2\ MVA.}$ **Ans.**

Fault current $= \dfrac{26.2 \times 10^3}{\sqrt{3} \times 66} = \mathbf{229\ A, \underline{/-90^\circ}}$ **Ans.***

Example 20.5. (*b*) *Further to example 20.5 (a) calculate the fault current supplied by each transformer and each generator.*

Neglect load current.

* **Note.** As the fault current flows through system reactance, and resistance being negligible, the fault current lags behind corresponding voltage by 90°.

Solution. The total fault current is supplied by two generators. Their contribution depends upon reactances in their branches.

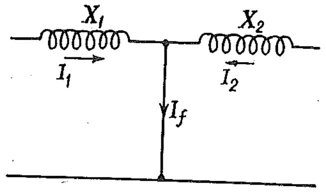


Fig. Ex. 20.5 (*a*)

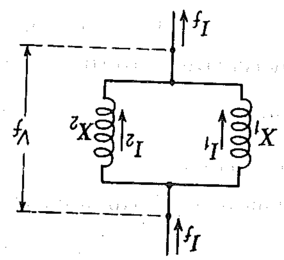


Fig. Ex. 20.5 (*b*)

Division of current through parallel branches.

$$I_f \times X_{eq} = V_f = I_1 \times X_1 = I_2 \times X_2$$

$$X_{eq} = \frac{X_1 \times X_2}{X_1 + X_2}$$

$$I_f \times \frac{X_1 \times X_2}{X_1 + X_2} = I_1 \times X_1 = I_2 \times X_2$$

Therefore,

$$I_1 = I_f \times \frac{X_2}{X_1 + X_2} \quad \text{...(1)}$$

$$I_2 = I_f \times \frac{X_1}{X_1 + X_2} \quad \text{...(2)}$$

$$I_f = I_1 + I_2 \quad \text{...(3)}$$

These are general equations which give the division of total current I_f in two parallel branches having reactance X_1 and X_2.

In this example, $I_f = 229$ A

$$I_1 = 229 \times \frac{0.461}{0.4527 + 0.461}$$

$$= 229 \times \frac{0.461}{0.913} = \mathbf{115.5\ A}$$

$$I_2 = 229 \times \frac{0.4527}{0.913} = \mathbf{113.5\ A} \quad \textbf{Ans.}$$

Check $\quad I_1 + I_2 = 115.5 + 113.5 = \mathbf{229\ A.}$ **Ans.**

Current supplied by generator = Current supplied by transformer × Transformation Ratio

$$I_{G1} = I_1 \times \frac{66}{11}$$

$= 115.5 \times 6 =$ **693 A. Ans.**

$$I_{G2} = I_2 \times \frac{66}{11}$$

$= 113 \times 6 =$ **678 A. Ans.**

Note. Load current neglected.

$$I_1 = \frac{I_f \times X_2}{X_1 + X_2} = \frac{I_f \times X_1}{X_1 + X_2}$$

$$I_1 + I_2 = I$$

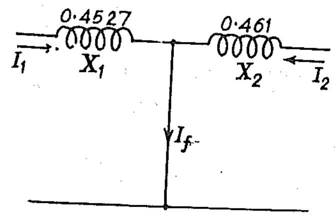


Fig. Ex. 20.5 (*c*)

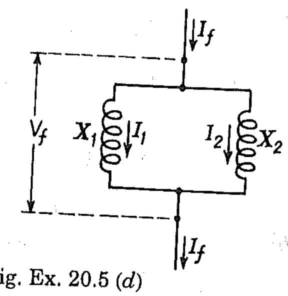


Fig. Ex. 20.5 (*d*)

Example 20.6. *A 3-phase 6000 kVA, 11 kVA alternator has 10% direct axis sub-transient reactance. It is connected to a 6000 kVA, 11/66 kV transformer having 9% leakage reactance. A symmetrical delta connected fault of impedance 12 + j3 ohms occurs between the lines near the H.T. terminals of the transformer when the system is on no load. Calculate the current supplied by alternator.*

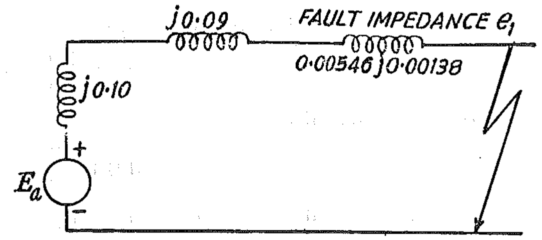


Fig. 20.3. Reactance diagram of Ex. 20.6.

Solution. Base kVA = 6000 for complete system, base kV = 11 and 66 kV for the two circuits connected by transformer.

P.u. reactance of generator $j0.10$ p.u.

P.u. reactance of transformer $j0.09$ p.u.

Converting delta connected impedance to equivalent star connected impedance, we get

$$Z_{eq.} = \frac{12 + j3}{3} = 4 + j1 \text{ ohms/phase}$$

Base *Z* on H.T. side $$= \frac{\text{Base kV}^2 \times 1000}{\text{Base kVA}} = \frac{(66)^2 \times 1000}{6000} = 66 \times 11 = 726 \text{ ohm}$$

$$\text{P.u. } Z_{eq.} = \frac{4 + j1}{726} = 0.00546 + j0.00138$$

$$E_a = 1 \text{ p.u.} = 1 + j0$$

$$I_f = \frac{1 + j0}{0.00546 + j0.19138}$$

$$= \frac{0.00546 - j0.19138}{0.03663} = 0.149 - j5.225 \text{ p.u.} = 5.23 \text{ p.u.}$$

Base *I*, HT side $$= \frac{\text{Base of kVA}}{\sqrt{3} \times \text{Base kV}} = \frac{6000}{\sqrt{3} \times 66} = 52.5 \text{ Amp.}$$

$$I_f = 5.23 \times 52.5 = \mathbf{274\ Amp.}$$

$$\text{Current from generator} = 5.23 \times \frac{6000}{\sqrt{3} \times 11} = \mathbf{1644\ Amp.} \quad \textbf{Ans.}$$

Note. The respective base voltages are taken to calculate base currents on the two sides of transformer. We can also calculate as :

Current from Generator $= I_f \times 66/11$.

Example 20.7. *Two generators are connected to their unit transformer as shown in the figure. Generators and transformers are rated as follows :*

Generator I	:	*20 MVA, 11 kV, 0.2 p.u.*
Transformer 1	:	*20 MVA, 11/110 kV, 0.08 p.u.*
Generator 2	:	*30 MVA, 0.2 p.u., 11 kV*
Transformer 2	:	*30 MVA, 11/110 kV*

0.1 p.u.

Reactance of transmission lines 0.516 p.u. (based on 110 kV, 30 MVA bases).

A 3-phase short-circuit occurs at the receiving end 110 kV bus-bar.

Determine the current supplied by the generators.

Solution.

Base MVA = 30

Base kV = 11, 110 ; respectively on generator, transmission sides

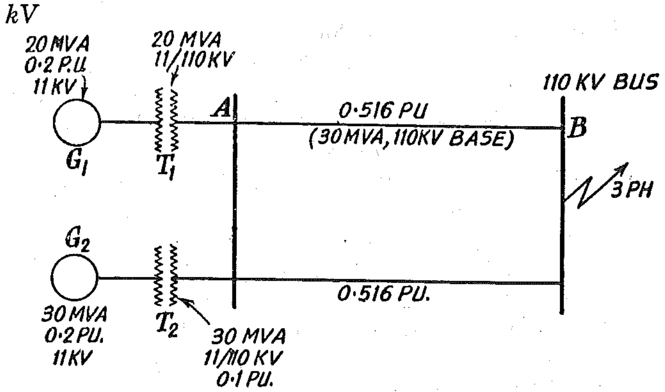


Fig. 20.4. Circuit diagram of Ex. 20.7.

P.u. reactance of Generator 1 to new base

$$= 0.2 \times \frac{30}{20} = 0.3 \text{ p.u.}$$

P.u. reactance of transmission 1 to new base

$$= 0.08 \times \frac{30}{20} = 0.12 \text{ p.u.}$$

P.u. reactance of $G_2 = 0.2$ p.u.

P.u. Reactance of $T_2 = 0.1$ p.u.

P.u. Reactance of transmission line = 0.516 p.u.

From these values the reactance diagram is drawn.

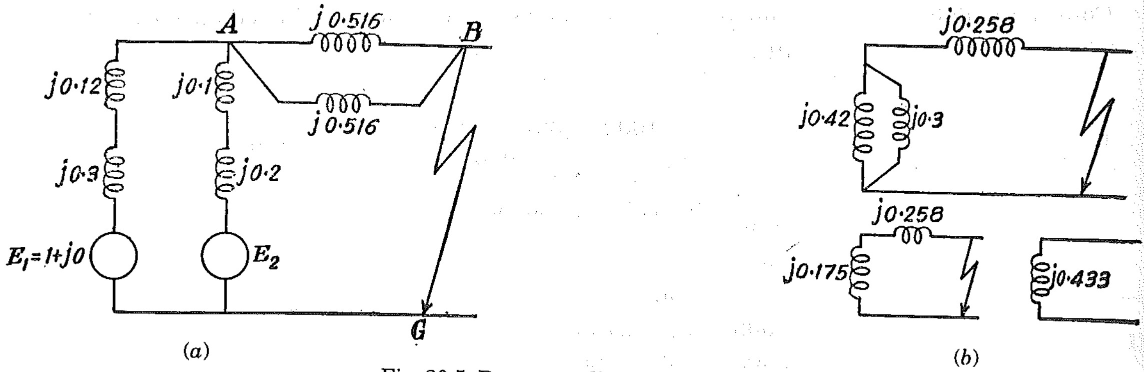


Fig. 20.5. Reactance diagram of Ex. 20.7.

Thevenin's equivalent reactance between B, G is obtained by network reduction and is equal to $j0.433$ p.u.

Short-circuit current $= \dfrac{E_a}{X_{eq.\ p.u.}} = \dfrac{1+j0}{j0.433} = 2.3$ p.u.

*Current from G_1 $= 2.3 \times \dfrac{0.3}{0.3+0.42} = 2.3 \times \dfrac{0.3}{0.72} = 0.96$ p.u.

Current from G_2 $= 2.3 \times \dfrac{0.42}{0.72} = 1.34$ p.u.

$$I_{G1} + I_{G2} = 0.96 + 1.34 = 2.30 \text{ p.u. (check)}$$

Current taken from G_1 in Amp.

$$= 0.96 \times \frac{30{,}000}{\sqrt{3} \times 11} = 1510 \text{ A (lagging)}$$

Current from G_2 in Amp. $= 1.34 \times \dfrac{30{,}000}{\sqrt{3} \times 11} = 2105$ A (lagging)

(Since Base kVA = 30,000 and Base kV is 11.)

Example 20.8. *A 25000 kV, 11 kV generator with 15% subtransient reactance is connected through a transformer to a bus that supplies 4 identical motors as shown in Fig. 20.6. Each motor has $X_d'' = 30\%$, $X_d' = 30\%$ based on 5000 kVA, 6.6. kV. Three-phase rating of the transformer is 25000 kVA, 11/6.6 kV and leakage reactance is 10%. Bus voltage is 6.6. kV when a three-phase fault occurs the terminals of one motor. Calculate the following :*

(1) Sub-transient fault current.

(2) Sub-transient current in breaker A.

*(3) Momentary current rating** of circuit breaker.*

* Current gets distributed amongst parallel generators according to proportion of reactances. See Ex. 20.5 (b).

** Momentary current rating is defined as r.m.s. value of the short-circuit current at the instant of first current peak. The American Standards on circuit breakers specify the momentary current rating of breaker.

(4) Current to be interrupted by breaker A; breaker time is 5 cycles. Use multiplying factor 1.1 for d.c. component (Asymmetry).

Solution.*

Base kVA = 25,000 for complete-system, p.u. reactance of motor

$$X_d' = j0.2 \times \frac{25{,}000}{5{,}000} = j1.0 \text{ p.u. ... subtransient}$$

$$X_d' = j0.3 \times \frac{25{,}000}{5000} = j1.5 \text{ p.u. ...transient}$$

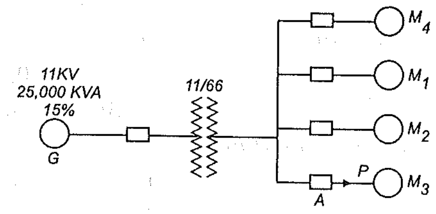
Correspondingly the reactance diagram, one for sub-transient and other for transient state.

Sub-transient state

$$Z_{th} = j0.125$$

$$V_f = 1.0 \text{ p.u.}$$

$$I_f'' = \frac{V_f}{Z_{th}} = \frac{1.0 + j0}{j0.125} = -j8.0 \text{ p.u.}$$

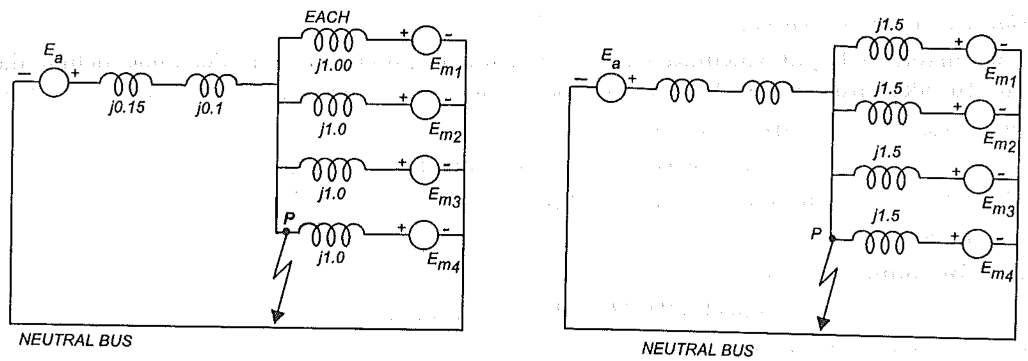


(a) Circuit diagram.

Base current is 6.6. kV circuit is

$$\frac{25{,}000}{\sqrt{3} \times 6.6} = 2182 \text{ Amp.}$$

I_f'' in Amp $= 2182 \times 8 = 17{,}456$ Amp. $\underline{/-90^\circ}$



(b) Reactance diagram with sub-transient reactances. (c) Reactance diagram with transient reactances

Fig. 20.6. Diagrams of Ex. 20.8.

Generator Contribution

$$-j8.0 \times \frac{0.25}{0.5} = -j4.0 \text{ p.u.}$$

Each motor contributes 25% of the remaining fault current.

Total motor contribution

$$= -j8 - (-4j) = -j4 \text{ p.u.}$$

$\therefore$ Contribution of each motor $-$ $j1.0$ p.u.

Short-circuit current passing through circuit-breaker A

$$= \text{Contribution of 3 motors} + \text{Contribution of generator}$$

$$= -j4.0 + 3\,(-j1.0) = -7.0$$

$$I_A'' = -j7.0 \times 2182 = 15{,}274 \text{ Amp. (r.m.s.)}$$

* For solution of Subtransient current, take Subtransient reactances X'' For solution of Transient Current, take Transient Reactances X' Refer Ch. 3.

Momentary current

$$= 1.6 \times I_A'' = 1.6 \times 15274 = 24{,}440 \text{ A} \quad \text{[Factor 1.6 for Doubling Effect*]}$$

Now come to the Transient State

Assume transient reactance of generator $X' = 0.15$ p.u.

$$Z_{th}' = j\frac{0.375 \times 0.25}{0.375 + 0.25} = j0.15 \text{ p.u.}$$

S.C. Current (Transient)

$$I' = \frac{E_a}{Z_{th}'} = \frac{1}{j0.15} = -j6.66 \text{ p.u.}$$

Generator Contribution

$$\frac{1}{j0.15} \times \frac{0.375}{j0.625} = k4.0 \text{ p.u.}$$

Motor Contribution (each)

$$\frac{1}{4} \times \frac{1}{j0.15} \times \frac{0.25}{0.625} = -j0.67 \text{ p.u.}$$

Current to be interrupted

Generator contribution + Contribution of 3 motors

i.e. $[4.0 + 3 \times 0.67] = 6.01$ p.u.

Current to be interrupted

The current 6.01 p.u. calculated above is symmetrical current and does not include d.c. component. To take into account d.c. component multiply by 1.1 because breaker time is 5 cycles.

Breaking current (Asymmetrical)

$$= 1.1 \times 6.01 = 6.611 \text{ p.u. (r.m.s.)}$$

and *Breaking current (Symmetrical) = 6.01 p.u. (r.m.s.)*

In amperes :

Asy. Breaking current

$$= 6.611 \times 21.82 = 14{,}500 \text{ A (r.m.s.)}$$

Symm. Breaking current

$$= 6.01 \times 21.82 = 13{,}000$$

Breaking capacity in kVA of breaker A (Required Minimum)

$$= \sqrt{3} \times \text{kV} \times I_{sh}$$

$$= \sqrt{3} \times 6.6 \times 14{,}500 = 166{,}000 \text{ kVA.}$$

Example 20.9. *A 3 ph, 50 Hz, 1 MVA, 66/3.3 kV power transformer having percentage reactance of 6 per cent is connected singly to a 66 kV bus having fault level of 100 MVA. Calculate fault level (short circuit level) on 3.3. kV side of the transformer.*

Solution. Fault level on 66 kV side = 100 MVA (given)

This means, for a 3 phase fault on h.v. side, the fault MVA would be 100. Applying the rule

$$\text{Fault MVA} = \frac{\text{Base MVA}}{\text{p.u. } X_{eq.}} \qquad \ldots(1)$$

We can determine the p.u. $X_{eq.}$ of the source up to H.V. side. Any base MVA may be selected.

Let Base MVA = 1 MVA for both sides of transformer

Fault MVA on H.V. Side = 100 MVA (given)

* Ref. Sec. 3.4.

p.u. $X_{eq.}$ of source up to H.V. side, from Eq. (1),

$$= \frac{\text{Base MVA}}{\text{Fault MVA}} = \frac{1}{100} = 0.01 \text{ p.u. (on selected base)}$$

P.u. Reactance of Transformer for same base as its rating

$$= \frac{\text{Per cent reactance}}{100} = \frac{6}{100} = 0.06 \text{ p.u.}$$

Total equivalent reactance

= Eqt. reactance of source + Eqt. reactance of transformer

= 0.01 + 0.6 = 0.61 p.u.

Fault level on 3.3. kV side

$$= \frac{\text{Base MVA}}{\text{Eqt. p.u. reactance}} = \frac{1}{0.61}$$

$$= \mathbf{1.64 \text{ MVA.}} \quad \textbf{Ans.} \qquad \ldots(a)$$

$$\text{Fault current} = \frac{1.64 \times 10^3}{\sqrt{3} \times 3.3} = \mathbf{288 \text{ A.}} \quad \textbf{Ans.}$$

Note. If we assume source of zero reactance, fault MVA on L.V. side would be, from Eqn. (1)

$$= \frac{1}{0.6} = 1.67 \text{ MVA} \qquad \ldots(b)$$

which is on higher and safer side, as regards selection of circuit breaker, compared with value (*a*) above.

Hence fault level on L.T. side of a transformer can be approximately calculated by

$$\text{Fault level on L.V. Side} = \frac{\text{Transformer kVA}}{\text{p.u. reactance of transformer}}$$

The value thus obtained is on higher and safer side.

Example 20.10. *Calculate maximum possible fault level on Low-tension side of the transformer 500 kVA, 4.75 p.c. Reactance. State the assumption made.*

Solution. Assuming the source of zero equivalent reactance.

Maximum fault level on L.T. side of Transformer

$$= \frac{\text{Transformer kVA}}{\text{p.u. Reactance of transformer}}$$

$$\therefore \text{ Fault level on L.T. side } = \frac{500}{4.75} \times 100 = 10{,}500 \text{ kVA.}$$

Example 20.11. *Two incoming lines with fault levels at their terminals equal to 75 MVA and 150 MVA, terminate on a common bus in a sub-station. A 1 MVA step-down transformer having 5 per cent reactance is connected to this bus. No other lines need be considered. Calculate fault level on L.T. side of the transformer.*

Solution. Total fault level on the bus on H.T. side of transformer,

= 75 + 150 = 225 MVA

Let Base MVA = 225.

∴ p.u. reactance of transformer w.r.t. new base

$$= 0.05 \times \frac{225}{1} = 11.25 \text{ p.u.}$$

Equivalent reactance of source

$$= \frac{\textbf{Base MVA}}{\textbf{Fault MVA}} = \frac{225}{225} = 1 \text{ p.u.}$$

Total p.u. reactance upto L.T. Side

= Eqt. p.u. reactance of source + p.u. reactance of transformer

= 1 + 11.25 = 12.25 p.u.

$$\text{Fault level} = \frac{\text{Base MVA}}{\text{P.u. reactance (total)}} = \frac{225}{11.25} = \textbf{18.38 MVA.} \quad \textbf{Ans.}$$

Note. Had we neglected source reactance the answer would have been

$$= \frac{225}{11.25} = 20 \text{ MVA}$$

$$\text{or simply, } \frac{\text{Transformer MVA}}{\text{p.u. reactance}} = \frac{1}{0.05} = 20 \text{ MVA.}$$

Part-B. A 1 MVA capacitor bank was installed on the L.t. side of power transformer. Calculate effective fault MVA on LT side.

Answer : 18.38 – 1 = 17.38 MVA

Example 20.12. *Two buses having fault level of 50 MVA and 100 MVA respectively are inter-connected by a line of negligible impedance. Calculate fault level at any point on the line.*

Solution. Let base MVA = 100

Equivalent reactance of source behind the bus I

$$= \frac{\text{Base MVA}}{\text{Fault level of bus I}} = \frac{100}{50} = 2 \text{ p.u.}$$

Similarly, the equivalent reactance of source behind bus II

$$= \frac{100}{100} = 1 \text{ p.u.}$$

These two sources are in parallel as regards Thevenin's equivalent reactance.

Hence total equivalent reactance

$$\frac{1}{1.5} = 0.667 \text{ p.u.}$$

$$\text{Hence fault level on the line} = \frac{\text{Base MVA}}{\text{Equivalent reactance}} = \frac{100}{0.667} = 150 \text{ MVA.}$$

Note. Total fault level here is the sum of two fault levels as the reactance of line is neglected (*i.e.* 100 + 50 = 150 MVA.)

20.3. PROCEDURE RECOMMENDED BY STANDARDS FOR SHORT-CIRCUIT CALCULATIONS IN DISTRIBUTION SYSTEMS.

Single source, symmetrical short-circuit

(1) Initial symmetrical short-circuit current, I_k''

$$I_k'' = \frac{1.1\,V}{\sqrt{3}\,\sqrt{R^2 + X^2}} \qquad \text{...(20.1)}$$

where I_k'' = initial symmetrical short circuit current

i.e. the r.m.s. value of symmetrical short-circuit current.

V = rated voltage line to line

R = resistance per phase, ohms

X = reactance per phase, ohms (subtransient for this case)

1.1 = factor to take into account the rise in generator voltage. [Refer example 20.14]

(2) peak short circuit current, I_s

$$I_s = x\sqrt{2}I_k'' \qquad \text{...(20.2)}$$

where I_s = Peak short-circuit current, *i.e.* highest instantaneous value of current after the short-circuit.

x = factor to take into account the asymmetry, depends on R/X ratio of generator

$\sqrt{s}$ = To convert r.m.s. to peak value. [Refer Example 20.13]

Example 20.13.

Generator : *3750 kVA*

6600 V

23% X″

0.866 Ω per phase R.

Calculate, (a) Initial symmetrical short-circuit current.

(*b*) Peak short-circuit current for a 3 phase terminal fault.

Solution. (*a*) **Initial sym. short-circuit current**

$$I_k'' = \frac{1.1\,V}{\sqrt{3}\,\sqrt{R^2 + X^2}}$$

$$X'' = 23\%$$

$$= \frac{23}{100} \times \frac{\text{kV}^2}{\text{kVA}} \times 1000 \text{ ohms}$$

$$= \frac{23 \times (6.6)^2 \times 10}{3750} = 2.67 \text{ ohms}$$

$$R = 0.866\ \Omega \text{ per phase}$$

$$\sqrt{R^2 + X^2} = Z = \sqrt{(2.67)^2 + (0.866)^2} = 2.8\ \Omega$$

$$I_k'' = \frac{1.1 \times 6.6 \times 1000}{\sqrt{3} \times 2.8}$$

Initial sym. short-circuit current

$$I_k'' = \textbf{1500 Amp.} \quad \textbf{Ans.}$$

(*b*) **Peak short-circuit current I_x**

$$I_s = \sqrt{2}xI_k''$$

$$R/X = \frac{0.866}{2.67} = 0.32$$

*Ratio	R/X	0	0.1	0.2	0.3	0.4	0.5
*Factor	x	2	1.75	1.55	1.4	1.3	1.22

x = 1.38 (from table by interpolation)

$$I_s = \sqrt{2} \times 1.38 \times 1500 = \textbf{2930 Amp.}$$

Example 20.14. *Two alternators A and B are connected in parallel. The details of the alternators are as follows :*

Alternator A : 50,000 kVA, sub-transient reactance 25%

Alternator B : 25,000 kVA, sub-transient reactance 25%.

These alternators are connected to delta star transformer T of rating 75,000 kVA, 11 kV Delta ; 66 kV Star of 10% reactance.

A three-phase fault occurs on HT side of the transformer. Find the sub-transient currents in each generator and in the HT side of the transformer. The system is on no load before fault, with voltage on HT side equal to 66 kV.

Solution. Let us adopt per unit system.

Select Base kVA 75,000

Base kV = 11 kV on L.T. side

Base kV = 66 kV on H.T. side.

Machine reactances will be converted to per unit reactances with new bases.

Percentage reactance with new base

$$= \left[\begin{matrix}\text{\%Reactance with} \\ \text{old base}\end{matrix}\right] \times \frac{\text{Base kVA new}}{\text{Base kVA old}} \times \left(\frac{\text{Base kV old}}{\text{Base kV new}}\right)^2$$

Alternator A :

Per unit reactance

$$= 0.25 \times \frac{75{,}000}{50{,}000} \times \left(\frac{11}{11}\right)^2 = 0.375 \text{ per unit.}$$

Alternator B :

Per unit reactance

$$= 0.25 \times \frac{75{,}000}{25{,}000} = 0.75 \text{ per unit.}$$

Transformer : per unit reactance = 0.10 p.u. (unchanged).

The currents can be easily calculated from the solution of network of Fig. 20.7.

$E_a = 1 + j0$ p.u. (voltage per phase)

Total reactance consists of a parallel branch in series with a reactance, *i.e.*,

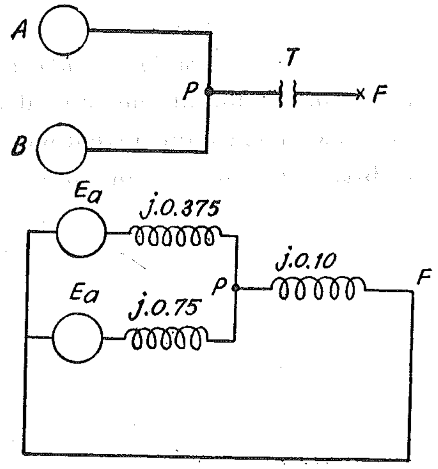


Fig. 20.7. Diagram of Ex. 20.14.

$$\frac{j0.375 \times j0.75}{j0.375 + j0.75} + j0.10 = j\,0.25 + j0.10 = j0.35$$

Note. To calculate sub-transient currents, take sub-transient reactance. To calculate transient currents, take transient reactance.

The total sub-transient current

$$= \frac{E_a}{j0.35} = \frac{1}{j0.35} = -j2.860 \text{ p.u.}$$

This current gets divided into two parallel branches in inverse proportion of their reactance. Thus

$$I_A'' = -j2.86 \times \frac{j0.75}{j0.75 + j0.375}$$

$$= -j2.86 \times \frac{0.75}{1.125} = -j1.91 \text{ p.u.}$$

$$I_B'' = -j2.86 \times \frac{0.375}{1.125} = -j0.955 \text{ p.u.}$$

The total sub-transient current on H.T. side

$$= -j\,2.86 \text{ p.u.}$$

$$= -j2.86 \times \frac{\text{Base kVA}}{\sqrt{3}\text{ Base kVA}} \text{ amp.}$$

$$= -j2.86 \times \frac{75{,}000}{\sqrt{3} \times 66} = 1876 \text{ amp.}$$

Base kV is 66 on H.T. side.

Current in Alternator A $= -j1.91 \times \dfrac{75{,}000}{\sqrt{3} \times 11} = 7519$ amp.

Base kV is 11 on H.T. side.

Current in Alternator B $= -j\,0.955 \times \dfrac{75{,}000}{\sqrt{3} \times 11} = 3{,}760$ Amp.

[**Ans.** *Sub-transient current :* Generator A 7519 A
Generator B 3,760 A
H.T. Side 1,875 A]

Example 20.15. *A 7,500 kVA, 6.6 kV generator connected through a 5 cycle breaker has reactances*

$$X_d'' = 9\%, X_d' = 15\%, X_d = 100\%.$$

It is operating no-load at rated terminal voltage when a three phase short-circuit occurs beyond the breaker. Find :

(1) Sustained short-circuit current.

(2) Initial symmetrical r.m.s. current.

(3) Maximum possible d.c. component of short-circuit current.

(4) Making capacity required for circuit-breaker.

(5) Breaking capacity required for circuit-breaker.

(6) Interrupting MVA of circuit-breaker.

Solution. (1) Sustained short-circuit current is the steady state current.

Adopting per unit method :

Sustained short-circuited current = I

$$I = \frac{E_a}{X_d} = \frac{1.0}{1.0} = 1 \text{ p.u.}$$

where E_a = voltage per phase

I = sustained short circuit current, Amp.

X_d = Synchronous reactance.

Rated current $= \dfrac{\text{Generator rated kVA}}{\sqrt{3} \times \text{Rated kV}} = \dfrac{7500}{\sqrt{3} \times 6.6} = 656$ A.

(1) Since the per unit reactances of generator are based on its ratings as bases, the rated current refers to its per unit current.

∴ Per unit current 1.p.u. = 656 A

(1) $X_d = 100\%$, I_s = Sustained short-circuit current = 1 p.u. = 656 A.

(2) Initial symmetrical r.m.s. current = I''

$$= \frac{E_a}{X_d''} = \frac{1.00}{0.09} \text{ p.u.} = \frac{1}{0.09} \text{ p.u.}$$

$$= 656 \times \frac{1}{0.09} = 7289 \text{ A} = 7.28 \text{ kA r.m.s.}$$

(3) Maximum D.C. component

$$= \frac{E_m}{X_d''} = \frac{\sqrt{2}E_a}{X_d''} = \sqrt{2} \times 7.28 = 10.3 \text{ kA.}$$

(4) Making capacity required

$$= 2 \times \sqrt{2} \times \text{Initial Symmetrical Current}$$

Where, factor 2 is for doubling factor and $\sqrt{2}$ is for converting r.m.s. to peak

$$= 2\sqrt{2} \times 7.28 = 20.6 \text{ kA peak.}$$

(5) Consider transient reactance for calculating the breaking current. Since breaker operates in transient state,

$$I' = \frac{E_a}{X_d'} = \frac{1}{0.15} \text{ p.u.}$$

$$= 656 \times \frac{1}{0.15} = 4373 \text{ Amp.}$$

Transient Current = 4.37 kA.

Breaking Current = $1.4 \times I'$

Where 1.4 is a factor for Asymmetry of the wave for a 5 cycle breaker (Assumed)

$= 1.4 \times 4.37 = 6.12$ kA

(6) MVA capacity $= \sqrt{3} \times \text{kV} \times \text{kA} = \sqrt{3} \times 6.6 \times 6.12 =$ **69.96 kA.** Asy.

[**Answers** :

(1) Sustained short circuit current $= I = 656$ A.

(2) Initial Sym. S.C. Current $I'' = 7.28$ kA.

(3) Maximum d.c. comp. = 10.3. kA.

(4) Making capacity required = 20.6 kA peak.

(5) Transient short-circuit current $= I' = 4.37$ kA.

(6) Interrupting capacity required = 69.96 kA, Asy.

Note. (Refer Ch. 3).

$$I'' > I > I$$

SUMMARY

(1) For three-phase symmetrical fault

$$\text{Fault MVA} = \frac{\text{Base MVA}}{\text{Equivalent } X_{p.u.}}$$

$$\text{Fault current} = \frac{\text{Fault MVA} \times 10^3}{\sqrt{3} \times \text{kV at fault point}} = \frac{V_{p.u.}}{X_{eq.\ p.u.}}$$

(2) For symmetrical fault calculations, the Thevenin's equivalent reactance upto fault is calculated and then fault MVA is calculated by applying the expressions given above.

(3) The problems are either for steady state, transient or sub-transient state. Correspondingly the steady state or transient or sub-transient reactances are used for calculations.

(4) Fault current is of lagging power factor.

(5) Capacitor banks provide leading fault MVAr.

SERIES REACTORS

20.4. REACTORS IN POWER SYSTEMS

There are several types of reactors used in Power Systems.

These include :

— Current limiting reactors : Saturated. (Series reactors).

— Reactors in neutral to earth connection called arc suppression coils/Peterson coil/ground fault neutralizers.

— Shunt reactors (Compensation Reactors). (Ref. Sec. 18.25)

— Reactors in harmonic filters.

— Smoothing reactors in HVDC systems.

Current limiting reactors are inserted in series with the line, to limit the current flow in the event of a short-circuit and thereby bring down the fault level. The current limiting reactors are also called Series Reactors.

The reactors between neutral and ground help in eliminating or suppressing arcing grounds. They are covered in the chapter 'Neutral Earthing'. (Ch. 18)

Shunt reactors are connected with transmission lines, for absorbing reactive power.

20.5. PRINCIPLE OF CURRENT LIMITING REACTORS

A current limiting reactor is an inductive coil having a large inductive reactance (ωL) and is used for limiting short-circuit currents to be interrupted by circuit-breakers. If X is the reactance

of a circuit, E is the voltage, neglecting the resistance, the short circuit current I_{sc} is given by E/X. Therefore by increasing series reactance, X of the system, the short circuit currents can be decreased. The short circuit currents depend upon the generating capacity, voltage at the fault point and the total reactance between the generators and the fault point. The circuit breakers should have enough breaking current capacity such that the fault currents are less than the breaking current capacity. If fault currents are beyond the capacity of the circuit breaker, the circuit breaker, may not interrupt the fault current. In a system where several generating stations are interconnected by short feeders, the fault currents can be high, the circuit breakers of suitable breaking capacity may not be available. The fault current, then, should be limited by some means so that available or existing circuit-breakers can be used safely. Further, when the system is extended by adding more generating stations or more generator units, the fault current to be interrupted by the same circuit breaker will be greater than before. In such a case the circuit breaker should be replaced by another of higher breaking current capacity or the fault current can be limited by means of reactors. By including a reactor or a few reactors at strategic locations, the short circuit current at several points can be reduced. Hence current limiting reactors are useful in limiting short-circuit current so that the circuit breakers can interrupt them. However the voltage drops and losses caused by reactors should be small.

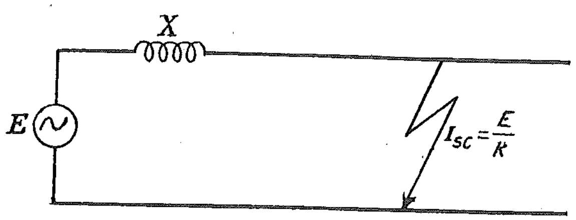


Fig. 20.8. By increasing reactance X, short circuit current I_{sc} can be reduced.

Summarising (1) Reactors limit the short-circuit currents.

(2) They are used in systems when extensions are made and the circuit-breaker breaking current capacities become inadequate.

(3) They are employed in large systems, so as to limit the short circuit MVA, to match with the breaking current capacity of circuit breaker.

Reactors are also used in short-circuit test plants. Are furnace installations. In furnace-plant, reactors are used to limit the arc current. These reactors are connected in the primary circuit of furnace transformer.

It is reported that in France, the power system is designed such that the fault levels at various points of the system are below certain limit. This is achieved by inserting series reactors at strategic locations in the system. Since 1980's suitable SF_6 and vacuum circuit breakers of high ratings are available for each voltage level and the need of series reactors is practically eliminated.

20.6. DESIGN FEATURES OF CURRENT LIMITING REACTORS

The essential requirement of current limiting reactor is that the reactance should not reduce due to saturation under short-circuit conditions. If fault current is more than about three times rated full load current, an iron core reactor having essentially constant permeability would necessitate a very large cross-section of core. Hence air core coils are sometimes used for current limiting reactors. Air core reactors are of two type (1) Dry-type Air Core Reactors and (2) Oil immersed Air Core Reactor, magnetically shielded or without shielding.

Dry type reactors are generally cooled by natural or forced air cooling. These are used only upto 33 kV. For higher voltage oil immersed designs prevail. The air insulated (dry type) reactors occupy relatively larger space. They need a large clearance from adjacent constructional work. Because of the absence of iron, the reactance of air-corded reactors is almost constant. With oil filled design, laminations of iron shields are provided around the outside conductors so as to avoid the entering of magnetic flux in the surrounding iron parts. Due to iron shield a drop of about 10% occurs in the reactance, during short-circuits. Oil- immersed reactors can be used upto any voltage level, for outdoor or indoor constructions. The advantages of oil-immersed reactors are :

1. High factor of safety against flashover.
2. Smaller size.
3. High thermal capacity.

The oil-immersed type of reactor employs insulation and cooling arrangements similar to those used in power transformers. If air core is used, laminated iron-shield is provided outside the coils. If iron core is used, air gaps are introduced in the core to prevent saturation and to get desired value magnetising current.

Special care should be taken about the foundation, that it should withstand the electrodynamic forces during the short-circuit. In case of dry-reactors, there should be clearance between the reactor and surrounding metal structures, reinforcement, fabrication work etc.

20.7. DRY, AIR CORED SERIES REACTOR

In this type of reactor, the core is of air and the entire construction of the reactor, the core is free from ferromagnetic materials (iron, steel). There is absence of dielectric oil and cooling is provided by air. The reactor consists of concrete supports of glass-reinforced synthetic resins on which the winding is rigidly placed. The whole construction is rugged. Due to absence of iron, the reactance remains fairly constant during high short-circuit currents. Such reactors are not shielded, hence require special room free from reinforcement, closed metal circuits. The magnetic fluxes surrounding the reactor cause heating of structural works, metal bodies etc. the vicinity of the reactors.

20.8. OIL IMMERSED NON-MAGNETICALLY SHIELDED REACTOR

Principle. Imagine a current carrying inductance coil without iron core. The magnetic flux of such a coil will surround the coil. Now, suppose the coil is placed axially in a cylindrical aluminium tube, and paths are provided for the current induced in the aluminium enclosures. The induced currents in the enclosures will flow longitudinally in the enclosures and will provide magnetic flux. By proper design of the enclosures, the fluxes due to enclosure currents can be made almost equal to the fluxes due to the magnetic flux due to coil current. The magnetic flux due to the enclosure current opposes the magnetic flux due to coil current : outside the enclosure thereby provides magnetic shield.

The non-magnetically shielded oil immersed reactor looks like a power transformer. It has a coil without iron core. The coil assembly is placed in tank filled with transformer oil. Features are similar to power transformer. The aluminium enclosure of tube shape are placed in between the tank and the inductance coil. Paths are provided for circulating the induced currents. This method of shielding is simpler in construction.

20.9. OIL IMMERSED SHIELDED REACTORS

Such a reactor is similar to power transformers in several aspects, but has no continuous iron core. There is either no core or gapped core. Single disc-winding is placed on the central core of the magnetic circuit. The core is with air-gaps. Strong ceramic discs are placed between adjucent disc coils of the reactor. The entire winding is held under pressure to prevent vibrations and noise. The coil assembly is oil immersed and is enclosed in a tank. Cooling is similar to that in power transformers.

Sub-assemblies of laminated silicon-steel sheets are fixed rigidly at strategic locations between the inductive coil and the tank. The magnetic fields surrounding the coil are thereby get a path and the fluxes outside the shields are minimised.

20.10. TERMS AND DEFINITIONS

1. **Series Reactor**. It is an inductance coil connected in series with the system for one of the following purposes :

— Limiting current during fault condition.

— Limiting current during synchronising, load sharing, fluctuating loads, etc.

2. **Continuous Rated Current.** The r.m.s. value of current which the reactor can carry continuously, with temperature rise of current carrying parts and other parts, within specified limits. [*e.g.* 800 A].

3. **Rated Short-time Current.** The symmetrical r.m.s. value of fault current which the reactor can carry for specified short time [*e.g.* 40 kA for 1 sec.]

4. **Rated Impedance.** Impedance expressed in ohms per phase or in per unit specified for the reactor.

5. **Rated Over Current Factor.** The ratio of rated short time current to continuous current, e.g. 25.

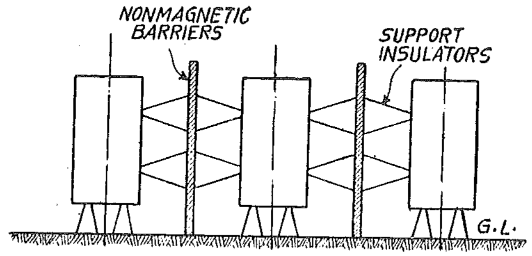
6. **Rated Through-put kVA.** $\sqrt{3} \times$ Rated Voltage $\times$ Rated Current (for 3 phase reactors).

7. **Rated Voltage**. The line to line service voltage for which the reactor is designed.

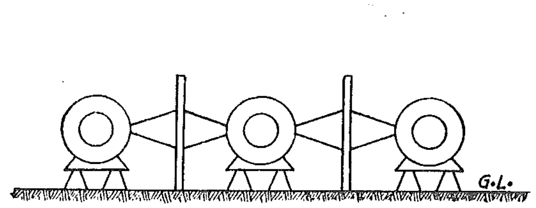
8. **Short Circuit Rating**. The reactors should be capable of withstanding the mechanical and thermal stresses during short circuit at its terminals for a specified period of time.

20.11. PHYSICAL ARRANGEMENT OF SERIES REACTORS

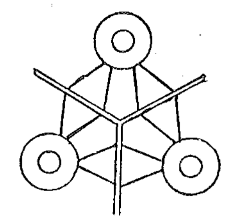
Fig. 20.9 illustrates various alternative methods of the physical arrangement of series reactors for 3-phase circuits.



ELEVATION
(*a*) With axis vertical and in a single row.



ELEVATION
(*b*) With axis horizontal, side by side.



PLAN
(c) With axis vertical, Triangular Formation.

Fig. 20.9. Physical Arrangement of Reactor for 3-phase system.

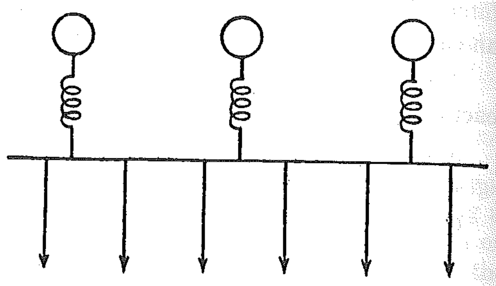
20.12. SELECTION OF REACTORS

While selecting a current limiting reactor, following aspects should be noted :

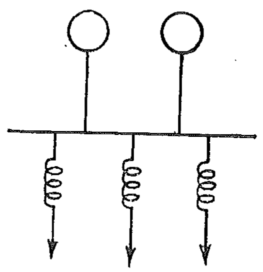
(1) **Type.** Dry or oil-immersed, iron-cored or Air-cooled, etc.
(2) **Phases**. Single-phase or three-phase.
(3) Indoor or outdoor.
(4) Reactance in ohms.
(5) Normal current rating, short-time current rating.
(6) Reactor through-put kVA.
(7) Rated voltage.
(8) Circuit characteristics-Frequency, voltage etc.

20.13. LOCATION OF SERIES REACTORS

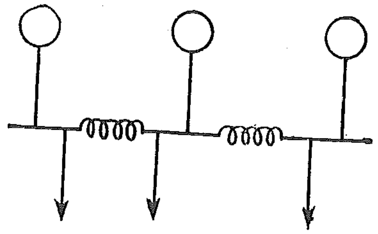
(a) *Generator Reactors.* In this scheme reactors are inserted between the generator and the generator bus [Fig. 20.10 (a)]. Modern turbo-generators have large reactance obtained by means of deep slots and other design features. The high reactance is provided to safeguard the generators in case of dead three-phase short-circuit as its terminals. Therefore, generator reactors are normally unnecessary in the modern installations. When new generators are installed in an old power station, generator reactors may be added for the older generators. The magnitude or such reactors is very approximately about 0.05 per unit. In this method when fault occurs on one feeder or busbar, the voltage of the common busbar drops down to a low value and stability is likely to be lost.



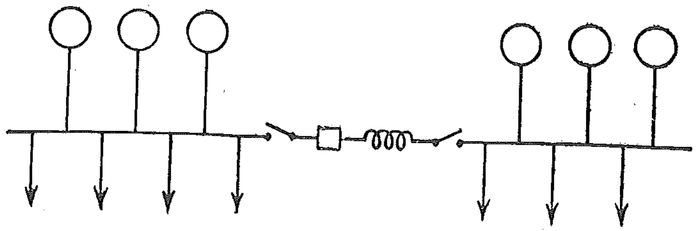
(a) Generator reactor.



(b) Feeder reactor.



(c) Busbar reactor.



(d) Busbar reactors between sections of bus.
Fig. 20.10. Locations of Series Reactors.

(b) *Feeder Reactors.* The reactors in this case are connected in series with the feeders. The advantage is that the voltage of the bus does not drop substantially in the event of a fault on the feeder. Hence the other feeder are affected less. The disadvantage is that, there are so many feeders and, therefore, many reactors are necessary. Further there is a constant loss in the reactor as full feeder current flows through it.

(c) *The Busbar Reactor.* The constant loss in reactor is avoided by inserting the reactors is busbars [Fig. 20.10 (c)]. In this system only a small powers-flows through the reactors during normal condition. During short-circuit on the feeder, only one generator feeds the fault directly, by-passing the reactors. While the other generators, feed the fault through the reactor.

Sometimes busbars are sectionalized and the reactors are included only between sections of the bus [Fig. 20.10 (d)].

With increasing size of today's power system, there is a constant need of checking the fault levels at all the important power stations and sub-stations almost every year. The capacities of circuit breakers and other switchgear should be verified. If necessary, new circuit breakers of higher rating should be installed or current limiting reactors should be introduced to limit the fault level

within the capacity of existing switchgear. Otherwise, due to inadequate capacity, the circuit-breakers may not be able to clear short-circuits, resulting in disasters. Such incidents occurred during 1950s in our system. Now (1980's) circuit-breakers of adequate ratings are available for every voltage level and series reactors are rarely used.

(d) **Smoothing reactors.** These are installed in series with rectifier sets on DC side to reduce ripple in DC current and minimise requirement of harmonic filters.

Example 20.16. (a) *Find short-circuit current in a single-phase system shown below. The reactance between the transformer and the fault point F is 2 ohms. The voltage at F is 6.6 kV.*

Solution.

Note. It is a single phase system.

(i) Take 2000 kVA as base kVA.

(ii) % Reactance of transformer to the new base kVA

$$= 7 \times \frac{2000}{1200} = 11.7$$

(iii) Base $I = \dfrac{\text{Base kVA}}{\text{Base kV}}$ for single phase circuit

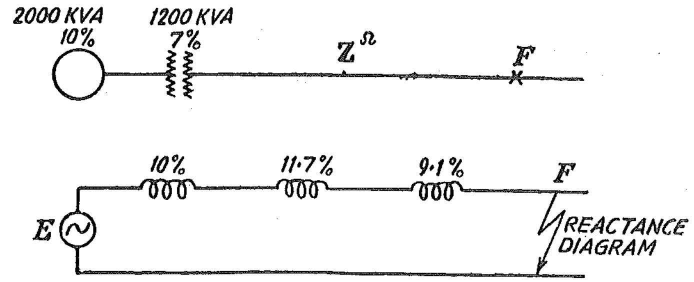


Fig. Ex. 20.16 (a)

$$= \frac{2000}{6.6} = 303 \text{ A}$$

(iv) Base impedance $= \dfrac{\text{Base Voltage}}{\text{Base Current}} = \dfrac{6.6 \times 10^3}{303} = 21.8\ \Omega$

Example 20.16. (b) *Solved by direct ohmic method.*

Solution. Ohmic reactance of generator, from Eqn. (20.1)

$$= \frac{F \times \%X}{I \times 100} = \frac{6600 \times 10}{303 \times 100} = 2.18\ \Omega.$$

Similarly ohmic reactance of transformer

$$\frac{6600 \times 7 \times \dfrac{2000}{1200}}{303 \times 100} = 2.56\ \Omega$$

Total reactance $= 2.18 + 2.57 + 2 = 6.75$ ohms

$$I_{sh} = \frac{6600}{6.75} = 980 \text{ A.}$$

OR by % reactance method.

% Reactance of generator 10%

% Reactance of transformer 11.7%, from (ii) above

% Reactance of line upto F

$$= \frac{\text{Ohmic Reactance}}{\text{Base Reactance}} \times 100 = \frac{2}{21.8} \times 100 = 9.2\%$$

Total percentage reactance

$$= 10 + 11.7 + 9.2 = 30.9\%$$

Short-circuit current

$$I_{sh} = I \times \frac{100}{\%X} = \frac{303 \times 100}{30.9} = \mathbf{980\ Amp.} \quad \textbf{Ans.}$$

Example 20.16. (*c*) *Further to problem 20.1, reactance of 5% is connected to the generator, in between the generator and transformer. Calculate the new short-circuit current. The 5% reactance is based on generator ratings.*

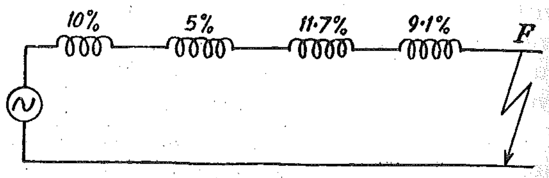


Fig. Ex. 20.16 (*c*)

Solution. Add 5% reactance in series with the generator.

Total % reactance $= 10 + 5 + 11.7 + 9.1 = 35.8\%$

$$I_{sh} = \frac{I \times 100}{\%X} = 303 \times \frac{100}{35.8} = \mathbf{845\ Amp.} \quad \textbf{Ans.}$$

We note that by including a reactor, I_{sh} has reduced.

Example 20.17. (*a*) *Two-three-phase generators of ratings 1000 kVA and 1500 kVA and voltage 3.3 kV have percentage reactances of 10 and 20 respectively per cent with respect to their ratings. These are connected to bus-bar. A three-phase short-circuit occurs on the bus. Find the short circuit current.*

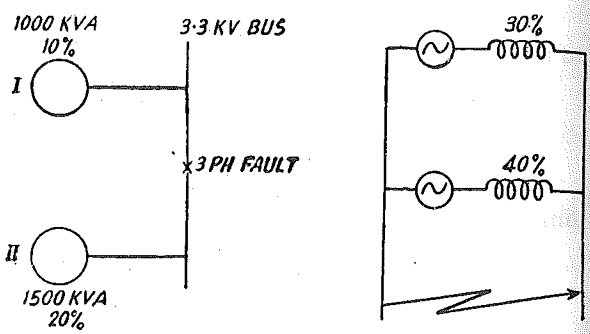


Fig. Ex. 20.17 (*a*)

Solution. Assume base kVA 3000

%Reactance of generator I to this base

$$= 10 \times \frac{3000}{1000} = 30\%$$

%Reactance of generator II to new base

$$= 20 \times \frac{3000}{1500} = 40\%$$

These two reactance are in parallel, total %X

$$= \frac{1}{\frac{1}{30} + \frac{1}{40}} = 17.14$$

Short circuit kVA $= \dfrac{\text{Base kVA} \times 100}{\%X}$

$$= \frac{3000}{17.14} \times 100 = 17.500 \text{ kVA}$$

Short-circuit current $= \dfrac{\text{Short circuit kVA}}{\sqrt{3} \times \text{kV}}$

$$I_{sh} = \frac{17{,}500}{\sqrt{3} \times 3.3} = \mathbf{3050\ A.} \quad \textbf{Ans.}$$

Example 20.17. (*b*) *In the above problem find the reactance of a reactor to be connected in series with generator of 1000 kVA to limit the short circuit kVA of bus-bar to 10,000.*

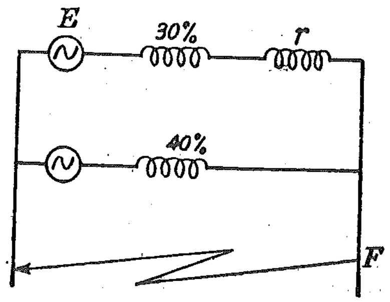


Fig. Ex. 20.17 (*b*)

Solution. [Continued from problem 20.17 (*a*)]

Short circuit kVA = 10,000

$$= \frac{\text{Base kVA}}{\%\text{X}} \times 100$$

$$\therefore \quad \%X = \frac{3000 \times 100}{10{,}000} = 30\%$$

This resultant percentage reactance is obtained by adding the reactor in series with the 1000 kVA generators. Suppose the newly added reactor is of % reactance r, then

$$\frac{1}{30} = \frac{1}{30 + r} + \frac{1}{40}$$

$$\frac{1}{120} = \frac{1}{30 + r}$$

$$30 + r = 120$$

$$r = 90\%$$

The reactance of 90%, based on the base 3000 kVA, should be connected in series with the generator to limit the short-circuit kVA to 10,000.

Example 20.18. (*a*) *Two generators of 3000 kVA and 10% reactance and one grid supply are connected to a generator bus as shown in the figure. The rating of the circuit-breakers on the feeder is 150 MVA. The capacities and reactances of the generators and the transformer are as shown in the figure. Calculate the reactance of the reactor X to limit the short circuit MVA on feeders to 150.*

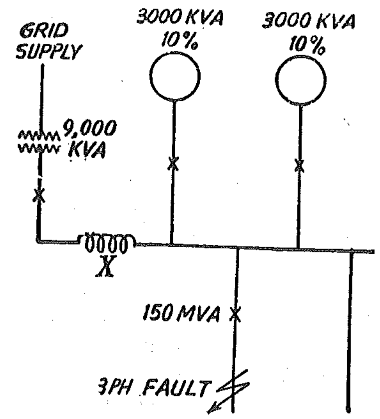


Fig. Ex. 20.18 (*a*)
(Note. Transformer reactance is 5%)

Neglect the equivalent reactance of the grid and assume the grid supply to be of infinite fault level.

Solution. Step 1. Convert the % reactance to the common new base.

Step 2. Calculate total reactance upto fault by series parallel combination.

Step 3. Short Circuit MVA is limited to 150 MVA

$$= \frac{\text{Base MVA}}{\%X_T} \times 100$$

From which the unknown X can be determined. Let the base kVA = 9000.

% Reactance of generators to the new base kVA

$$= \frac{9000}{3000} \times 10 = 30\%$$

The two generators are in parallel the combined reactance is given by

$$\frac{1}{30} + \frac{1}{30} = \frac{1}{X_c} = \frac{1}{15}$$

$$X_c = 15\%.$$

This is parallel with the grid-transformer and the reactors X. The combined reactance of the set up is given by :

$$\frac{1}{X_T} = \frac{1}{5 + X} + \frac{1}{15}$$

$$= \frac{15 + (5 + X)}{15\,(5 + X)} = \frac{20 + X}{75 + 15X}$$

$$X_T = \frac{75 + 15X}{20 + X} \ \% \text{ reactance.}$$

Short-circuit MVA $= \dfrac{\text{Base MVA}}{X_T} \times 100$

Short-circuit MVA is limited to 150

$$150 = \frac{9}{\frac{75 + 15X}{20 + X}} \times 100$$

$$\therefore \quad 3 = \frac{9 \times 2}{\frac{75 + 15X}{20 + X}}$$

$$\therefore \qquad 3\,(75 + 15X) = 18\,(20 + X)$$
$$225 + 45X = 360 + 18X$$
$$135 = 27X$$
$$X = \frac{135}{27} = 5.$$

Reactance of 5% (based on base kVA 9000) should be added between the transformer and generator bus.

Example 20.18. (*b*) *The incoming grid supply in example 20.17 has a fault level of 4000 MVA instead of infinity. Calculate fault level at generator bus neglecting reactor X.*

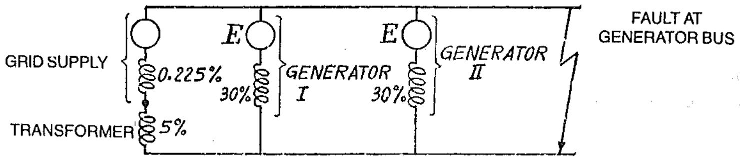


Fig. Ex. 20.18 (*a*) Equivalent Diagram.

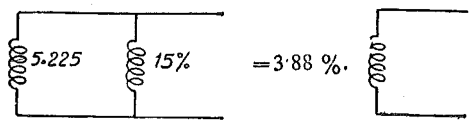


Fig. Ex. 20.18 (*b*) Equivalent Reactance.

Solution. (Consider from Ex 20.18 (*a*), with above changes)

Fault MVA

$$= \frac{\text{Base MVA}}{\%\text{Reactance}} \times 100 \qquad \ldots(1)$$

The fault level of the incoming supply is given. Hence, the supply can be considered as a generator having equivalent reactance. The equivalent reactance can be calculated from the expression (1) above.

Equivalent % reactance of the grid supply

$$= \frac{\text{Base MVA}}{\text{Fault MVA}} \times 100$$

Choosing same bases as in Ex. 20.17 (*a*) (*i.e.* 9 MVA).

Equivalent % reactance of the grid supply

$$= \frac{9}{4000} \times 100 = \frac{900}{4000} = 0.225\%.$$

Equivalent diagrams :

Equivalent reactance upto generator bus = 3.88%.

Fault level $= \dfrac{\text{Base kVA}}{\%\text{Reactance}} \times 100$

$$= \frac{9000}{3.88} \times 100 = 231{,}000 \text{ kVA} = 231 \text{ MVA}$$

New fault level at generator bus = **231 MVA. Ans.**

Example 20.19. *Explain briefly the advantages gained by insertion of reactors in the busbars of a large generating station. A generating station has four identical three phase alternators A, B, C, D, each of 20,000 kVA, 11 kV having 20% reactance. They are connected to a bus-bar which has a bus-bar reactor of 25% reactance on the basis of 20,000 kVA base, inserted between B and C. A 66 kV feeder is taken off from the bus-bars through a 10,000 kVA transformer having 5% reactance. A short circuit occurs across all phases at the high voltage terminals of the transformer, calculate the current fed into the fault.*

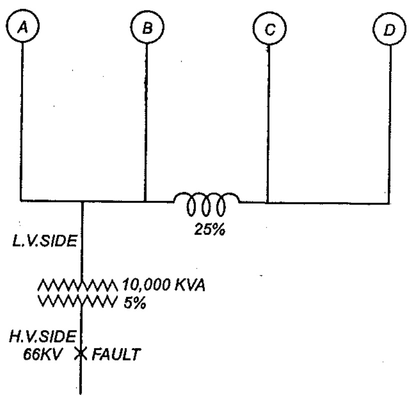


Fig. Ex. 20.19 (*a*)

Solution. Select base kVA and base kV.

Note. Choose base kVA, base kV : choose same base kVA on both sides of transformer. Choose different kV bases on either sides of transformer. The kV bases on both sides should

have the ratio same as the ratio of transformer. With this choice of bases the percent reactances referred to either sides remains same.

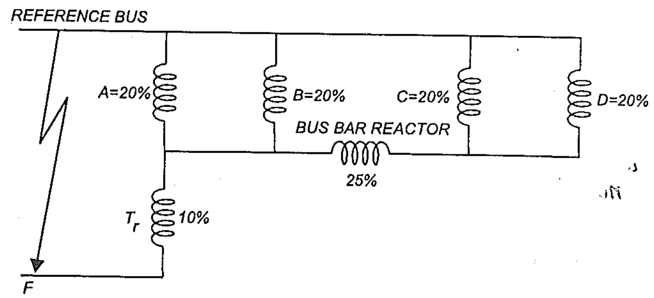
Base kVA = 20,000 on either sides of transformer

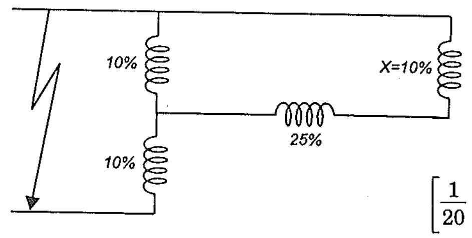
Base kV = 11 kV on generator side, 66 kV on feeder side.

Per cent reactance of transformer referred to new base kVA.

$$= \%\text{ reactance on old base} \times \frac{\text{New base kVA}}{\text{Old base kVA}}$$

$$= 5 \times \frac{20{,}000}{10{,}000} = 10\%$$





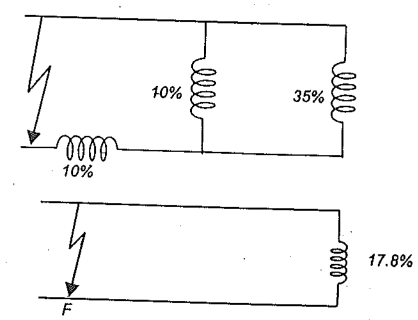
$$\left[\frac{1}{20} + \frac{1}{20} = \frac{1}{10}\right]$$

Fig. Ex. 20.19 (*b*).

Other % reactances remain unchanged. Thevenin's equivalent of the circuit contains reactances of generators *A*, *B* in parallel, generators *C*, *D* in parallel as shown in Fig. 20.19 (*b*).

Thevenin's equivalent reactance between *F* and reference bus. It is obtained by reduction of the network by series-parallel simplification as follows :

Thevenin's equivalent reactance is 17.8%.



Short circuit MVA $= \dfrac{\text{Base MVA}}{\text{Thev. \% reactance}} \times 100$

$$= \frac{20}{17.8} \times 100 = 112.5 \text{ MVA}$$

Short-circuit current $= \dfrac{\text{S.C. MVA} \times 1000}{\sqrt{3} \times \text{kV at fault point}}$

$$= \frac{112.5 \times 100}{\sqrt{3} \times 66} = 977 \text{ A}$$

Ans. Short circuit MVA 112.5.

Short circuit current = 977 A.

Example 20.20. (*a*) *Determine the ratio of the percentage reactance of the reactors to that of generators in a tie bar system if short circuit current is not to exceed two times the short circuit current of single section.*

Solution. Let the percentage reactance of a generator be g and that of the reactor be R. When there is a short-circuit on a feeder, except one R, remaining reactors and generators are in parallel. Suppose there are n number of sections. The percentage reactance of reactors in parallel is

$$\frac{g+R}{n-1}$$

This is in series with the reactor R connected to feeder on which fault occurs,

$$\frac{g+R}{n-1}+R=\frac{g+nR}{n-1}$$

This is in parallel with generator on whose feeder the fault occurs. The resulting reactance X is given by

$$\frac{1}{X}=\frac{1}{g}+\frac{1}{\dfrac{g+nR}{n-1}}$$

Giving $\quad X=g\dfrac{g+nR}{ng+nR}$

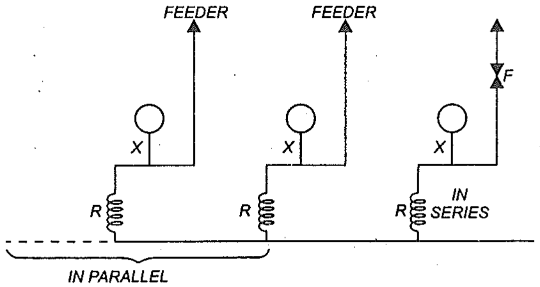


Fig. 20.20. Tie bar system.

Short circuit current

$$=\frac{\text{Full load current}}{\%\text{ reactance}}\times 100$$

Let $\quad I_{sh}$ = Short circuit current

I = Normal full load current

$$I_{sh}=I\times 100\,\frac{(ng+nR)}{g\,(g+nR)} \qquad \text{...(1)}$$

Short circuit current of one section, $n=1$

$$I_1=\frac{I}{g}\times 100 \qquad \text{...(2)}$$

According to the example the short circuit current in (1) should be twice that given by (2),

Assuming n is α,

$$I_{sh}=I\times 100\,\frac{\frac{1}{n}(ng+nR)}{g\left(\frac{g}{n}+\frac{nR}{n}\right)}$$

Taking limit as $n\to\alpha$ in this example

$$I_{sh}=I\times 100\,\frac{g+R}{gR}$$

$$I_{sh}=I_1\left(\frac{g+R}{R}\right)$$

$$\frac{I_{sh}}{I_1}+\frac{g+R}{R}$$

which is equal to 2.

$\therefore \quad \dfrac{g+R}{R}=2$

$\therefore \quad$ **g = R. Ans.**

Example 20.20 (*b*). **Tie Bar Reactors.**

A generating station has four generators, each rated 11 kV, 20 MVA, 50 Hz with transient reactance of 20%. The main busbars are divided into four sections, each section is connected to a tie-bar via a reactor R (Ref. Fig. 20.20).

If a three phase short-circuit takes place on of the sectional busbars, the voltage of remaining three busbar sections falls to 60% of normal value.

Calculate the % reactance of reactors R. Also calculate ohmic value of reactor R.

Solution. Draw single line diagram.

Ref. Fig. 20.20. Draw similar diagram with one more generator and reactor.

Consider fault F as indicated in the Figure.

Let $\quad$ Base kV = 11

Base kVA = 20,000

Let reactance per R per cent

Let percentage reactance of generator $=g$

For fault F,

The total equivalent reactance upto fault includes the following combination :

One of the generators feed the fault directly. Other three generators feed the fault *via* their respective reactors R's and then together through one reactor R.

Ref. Example 20.20 (*a*),

Thus in this case $n=4$

Percentage Reactance of three parallel branches :

$$\frac{g+R}{n-1}=\frac{20+R}{3}\%=(6.67+0.33\,R)\%$$

This is in series with one R.

Total Equivalent Reactance

$$=6.67+0.33+R+R=(6.67+1.33\,R)\%$$

Voltage of three sections dropped to 60% of nominal value. Hence 100 – 60 = 40%. Voltage drop takes place in the reactances X of generators, (Ref. Fig. 20.20) and remaining 60% voltage drop takes place in reactors R.

$\dfrac{g+R}{n-1}$ is total reactance in parallel branches

$$=\frac{g}{n-1}+\frac{R}{n-1}.$$

The first term on right hand side gives equivalent reactance of three generators in parallel. The second term gives equivalent reactance of three reactors in parallel

Thus for circuit containing three parallel branches and one series R,

$$\left(\frac{g+R}{n-1}+R\right)\text{ becomes}$$

$$\frac{g+nR}{n-1}\text{ as seen in Ex. 20.20 }(a)$$

This has two terms *i.e.*

$$\frac{g}{n-1}\text{ for generator reactance}$$

$$\frac{nR}{n-1}\text{ for series reactors}$$

Coming back to the example,

Voltage drop in generator reactance g is 40% of total voltage drop upto fault.

Hence 40% voltage drops in reactance

$$\frac{g}{n-1}\ i.e.\ \frac{20}{4-1}=\frac{20}{3}=6.67\% \qquad \text{...(I)}$$

100% voltage drop is in total reactance, *i.e.* in

$$= \frac{g + nR}{n - R}$$

$$= (6.67 + 1.33\,R)\% \qquad \text{...(II)}$$

But (I) is 40% of (II)

Hence,
$$6.67 = \frac{40}{100}(6.67 + 1.33\,R)$$

$$6.67 = 2.668 + 0.532\,R$$

Solving this for R, we get

$$R = \frac{6.67 - 2.67}{0.532} = \frac{4.00}{0.532} = 7.5\%$$

Hence Reactance of Reactors is 7.5% based on 11 kV and 20,000 kVA Base **(Ans.)**
Ref. Eqn. 6 in sec. 19.6 for conversion.

$$\text{Actual Reactance Ohms} = \text{p.u. Reactance} \times \frac{\text{Base kv}^2}{\text{Base kVA}} \times 1000$$

$$= \frac{7.5}{100} \times \frac{11^2}{20{,}000} \times 1000$$

$$= \frac{7.5 \times 121}{2000} = \mathbf{0.455\ ohms.} \quad \textbf{(Ans.)}$$

Example 20.21. *Fig. 20.21 illustrates a typical unit in thermal power station. The reactances of Generators and Transformal are as follows.*

Generator : *Subtransient reactance* $X'' = 20\%$
Transient reactance $X' = 28\%$
Synchronous Direct Axis reactance $X_s = 250\%$
Transformer leakage reactance = 15%
The generator is rated 25 kV, 500 MW, 0.8 pf
The transformer is rated 220/25 kV, 600 MVA.

Assume infinite fault level at 200 kV Bus and neglect fault level of Auxiliary Bus.

For a three phase symmetrical fault on the T-off, calculate the following of sub-transient, transient and steady state condition.

(1) Fault MVA
(2) Fault current in T-off
(3) Fault current contributed by generator side.
(4) Fault current contributed from transformer side.

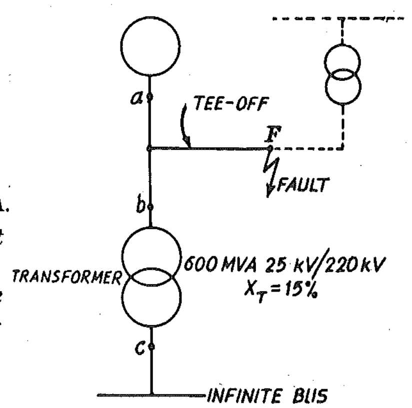


Fig. Ex. 20.21 (*a*) Line Diagram.

Solution. Select base kVA and base kV.

Let base kVA $= 600 \times 10^3$
base kV $= 25$ kV

(The rating of Transformer).

Calculate new p.u. reactances.

The p.u. Reactance of transformer given in the example refers to its own bases.

Hence p.u. Reactance of transformer referred to selected Bases in the same *i.e.*

$$X_t = 0.15$$

X_t remains unchanged for sub-transient, transient and steady state.

The generator rating is 500 MVA, 0.8 p.f.

$$\text{MVA} = \frac{\text{MW}}{\text{p.f.}} = \frac{500}{0.8} = 625$$

Generator reactances referred to new base MVA are calculated as follows :

$$\text{p.u. } X_q \text{ New} = \text{p.u. } X_g \text{ Old} \times \frac{\text{kVA Base New}}{\text{kVA Base Old}}$$

$$= \text{p.u. } X_g \text{ Old} \times \frac{600}{625}$$

$$= \text{p.u. } X_g \text{ Old} \times 0.96$$

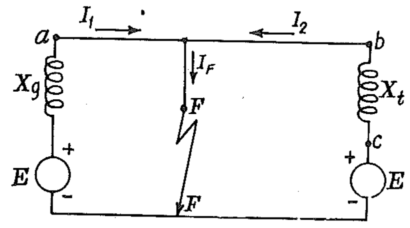


Fig. Ex. 20.21 (*b*) Reactance Diagram.

New p.u. Reactances of Generator

$$X_g'' = 0.2 \times 0.96 = 0.192 \text{ p.u.}$$

$$X_g' = 0.28 \times 0.96 = 0.269 \text{ p.u.}$$

$$X_s = 2.5 \times 0.96 = 2.4 \text{ p.u.}$$

For calculating sub-transient current, use sub-transient reactance X_g''.

For calculating transient current, use transient reactance X_g'.

For calculating steady state current, use steady state direct axis-synchronous reactance X_s.

$$X_t = 0.15 \text{ p.u. in all cases.}$$

Draw Reactance Diagram (Fig. Ex. 20.21 *b*).

Derive Equivalent Reactance as seen from fault point by removing fault branch and short-circuiting the e.m.f. sources.

Representation of Infinite Source

In this example the 220 kV bus has infinite fault level. It means, if a fault occurs on this bus, there is no internal reactance to limit the fault power. Hence infinite bus can be represented by an e.m.f. source with zero internal reactance as shown in Fig. Ex. 20.21 (*b*).

Equivalent Reactances

Refer Fig. Ex. 20.21 (*c*).

$$X_{eq} = \frac{1}{\dfrac{1}{X_g} + \dfrac{1}{X_t}} = \frac{X_g \times X_t}{X_g + X_t}$$

Sub-transient Reactance

$$X_{eq}'' = \frac{X_g\, X_t}{X_g'' + X_t} = \frac{0.192 \times 0.15}{0.192 + 0.15} = \frac{0.0298}{0.342} = \mathbf{0.083\ p.u.}$$

Transient Reactance

$$X_{eq}' = \frac{X_g'\, X_t}{X_g' + X_t} = \frac{0.269 \times 0.15}{0.269 + 0.15} = \mathbf{0.0965\ p.u.}$$

Steady State Reactance

$$X_{eg} = \frac{2.5 \times 0.15}{2.5 + 0.15} = \frac{0.375}{2.65} = \mathbf{0.143\ p.u.}$$

Total fault current I_F flows through T-off branch,

$$I_F'' = \frac{E}{X_{eq}''} = \frac{1}{0.083} = 12.05 \text{ p.u.}$$

$$I_F' = \frac{E}{X_{eq}'} = \frac{1}{0.0065} = 10.35 \text{ p.u.}$$

$$I_F = \frac{E}{X_{eq}} = \frac{1}{j\,0.14} = 7 \text{ p.u.}$$

The fault current I_F is composed of two components I_1 and I_2 [Refer Fig. Ex. 20.21 (*e*)].

$$I_1 = I_F \times \frac{X_t}{X_g + X_t}$$

$$I_2 = I_F \times \frac{X_g}{X_g + X_t}$$

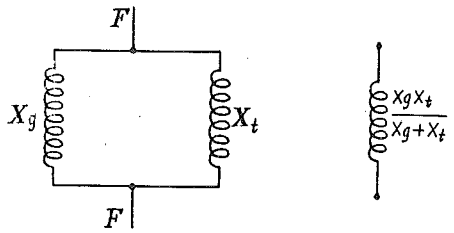


Fig. Ex. 20.21 (*c*) Thevenin's Equivalent reactance seen from Fault-branch.

[Refer Example 20.5 (*b*)]

$$I_1'' = I_F'' \times \frac{X_t}{X_g'' + X_t} = 12.05 \times \frac{0.15}{0.342} = 5.3$$

$$I_2' = I_F'' \times \frac{X_g}{X_g'' + X_t} = 12.05 \times \frac{0.192}{0.342} = 6.75.$$

Check $I_1'' + I_2'' = I_F''$

Base current $= \dfrac{\text{Base MVA} \times 10^3}{\sqrt{3} \times \text{Base kV}} = \dfrac{600 \times 10^3}{\sqrt{3} \times 22.2} = 15.6$ kA

Sub-transient currents are as follows :

Total sub-transient fault current in T-off.

$= 12.05 \times 15.6 \times 10^3 =$ **188 k Amp. Ans.**

Sub-transient current from Generator side

$= I_1'' = 5.3 \times 15.6 \times 10^3$ A = **82.9 k Amp. Ans.**

Sub-transient current from Transformer side

$= I_2'' = 6.75 \times 15.6 \times 10^3$ = **105.2 k Amp. Ans.**

Check $I_1'' + I_2'' = 82.9 + 105.2 = 188.1$ kA.

Calculate transient current and steady state current by following similar procedure.

20.14. EFFECTIVE SHORT CIRCUIT LEVEL (ESCL) BY CONSIDERING kVAr CONTRIBUTION OF SHUNT CAPACITOR BANKS

In earlier Sections of Ch. 20, the Normal Short Circuit Level (Normal Fault Levels) have been calculated by *neglecting* the contribution of large capacitor banks. The Normal Fault Levels so calculated are of lagging power factor currents with phase angle of 90° lag behind the voltage phasors. It is a universal practice to install *High voltage shunt capacitor banks* in receiving substations for power factor improvement and voltage regulation. Very large AC Shunt Capacitor Banks are installed in HVDC Substations for harmonic filters and shunt compensation of convertor. Shunt Capacitors *reduce* fault level by supplying current at leading power factor.

During a three phase symmetrical busbar fault at a particular voltage level, the capacitor banks connected to that busbar contribute fault MVA at leading power factor at phase angle 90° lead *i.e.* opposite to the normal fault level supplied by the generators.

The *Effective Short Circuit Level MVAe* (Effective Fault Level) is calculated by taking into account the contribution of shunt capacitor banks. Refer Fig. 20.21.

The Effective Fault Level MVAe at point *P* in a 3 Phase AC System is :

$$\begin{bmatrix}\text{Effective Fault} \\ \text{Level MVAe}\end{bmatrix} = \begin{bmatrix}\text{Normal Fault} \\ \text{Level MVAn}\end{bmatrix} - \begin{bmatrix}\text{Fault Level Contributed} \\ \text{by Capacitor Bank MVAc}\end{bmatrix}$$

$$\text{MVAe} = \text{MVAn} - \text{MVAc}$$

Note : *While calculating Effective Fault Level. Calculate the normal fault level as per procedure of symmetrical fault calculations. Then deduct MVAr contribution of the capacitor bank to obtain the Effective Fault Level.*

20.15. EFFECTIVE SHORT CIRCUIT RATIO (ESCR)

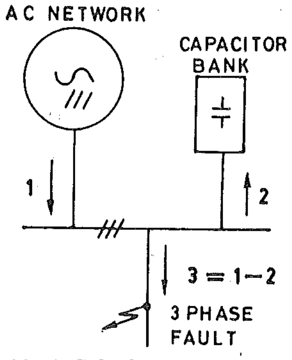


Fig. Ex. 20.21. Calculation of Effective Fault Level
MVAe (3) = MVAn (1) − MVAc (2)
S = Effective fault MVA 1 = Normal fault MVA, 2 = MVAr of Capacitor Bank

The concept of Effective Fault level (Effective Short Circuit Level) and Effective Short Circuit Ratio (ESCR) is useful in evaluating the strengthy of AC Power System to incorporate HVDC System.

The AC MVAr supplied by capacitor banks in HVDC Substations is of the order of 60% of Convertor MVA load. These capacitor banks contribute very significantly to the fault level on AC busses behind the convertor transformers. In this regard SCR ESCR are considered at the p lanning state of HVDC Project for detemining suitability of SC System to accomodate the HVDC System.

Normal Short Circuit Ratio

$$= \frac{\text{Normal Fault Level of AC Bus MVAn}}{\text{Rated Power of HVDC System MW}}$$

Effective Short Circuit Ratio

$$= \frac{\text{Effective Fault Level of AC Bus}}{\text{Rated Power of HVDC System}}$$

The effective Short Circuit Ratio of AC System at the AC substation busses should be more than 5 for planning the HVDC system connection, (Ref. Ch. 11).

Example 20.22. *Normal Short Circuit Level (Normal Fault level) at the 132 kV bus in a receiving station (before connecting the 132 kV, 200 MVAr shunt capacitor bank), was 6800 MVA. During 1995 a new 132 kV, 200 MVAr Shunt Capacitor Bank was connected to the 132 kV bus without any transformer between the bus and the capacitor bank. (a) Calculate the new Effective Short Circuit Level at the 132 kV Bus.*

(*b*) Calculate Normal Fort Current and Effective Fault Current.

Solution.

Normal Short Circuit Level at 132 kV Bus :

MVAn = 6800 MVA ($\angle - 90°$ Lag)

Contribution of New Capacitor Bank :

MVAc = 200 MVA ($\angle + 90°$ Lead)

Effective Short Circuit level at 132 kV Bus :

$$\text{MVAe} = \text{MVAn} - \text{MVAc}$$
$$\text{MVAe} = \text{MVAn} - \text{MVAc}$$
$$= 6800 - 200 = 6600 \angle - 90°$$

Normal Fault Current

$$If \cdot n = \frac{\text{MVAn}}{\sqrt{3}\,\text{KV}} = \frac{6800 \text{ MVA}}{\sqrt{3} \times 132 \text{ kV}} = 29.74 \text{ kA} \angle - 90°$$

Effective Fault Current

$$If \cdot e = \frac{\text{MVAe}}{\sqrt{3}\text{ KV}} = \frac{6600 \text{ MVA}}{\sqrt{3} \times 132 \text{ kV}} = 29.74 \text{ kA} \angle - 90°.$$

Example 20.23. *The Normal Short Circuit Level at the 400 kV bus of Rihand Power Station of Rihand Delhi HVDC System was 30,000 MVA. A 600 MVAr Shunt Capacitor Bank has been connected to the 400 kV Bus for AC harmonic filter and shunt compensation for HVDC power level of 1500 MW. Calculate the Effective (Equivalent) Short Circuit level by considering the contribution of the capacitor bank. Calculate the (a) Short Circuit Ratio (b) Effective short Circuit Ratio of the HVDC DC System. Give your comment regarding acceptability of the ESCR.*

Solution. Normal Fault Level at 400 kV Bus MVAn = 30000 MVA (given)

Normal Short Circuit Level of HVDC System

$$= \frac{\text{MVAn}}{\text{MW rating of HVDC System}} = \frac{30000}{1500} = 20 \quad \textbf{(Ans.)}$$

Effective fault level at 400 kV Bus

$$\text{MVAe} = \text{MVAn} - \text{MVAc}$$

$$= 30000 - 600 = 24000 \text{ MVA.} \quad \textbf{Ans.}$$

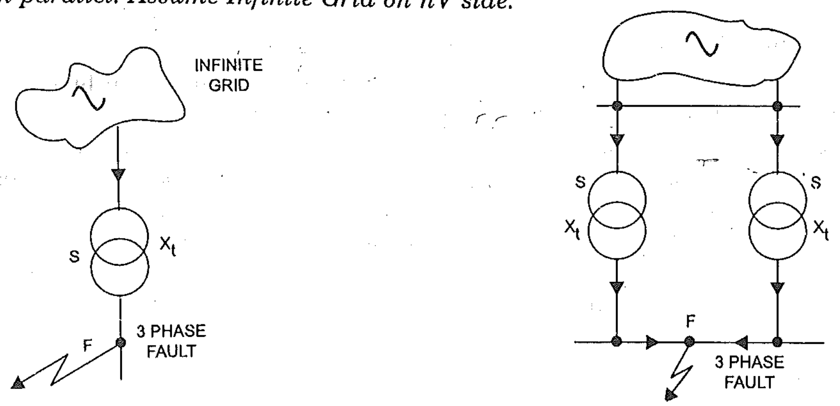
Effective Circuit Level of HVDC System

$$= \frac{\text{Effective Fault level MVAe}}{\text{MW rating of HVDC System}} = \frac{24000}{1500} = 16 \quad \textbf{Ans.}$$

Comment : ESCR is 16 and is acceptable. (Minimum ESCR = 5 required for locating HVDC Substation) **Ans.**

Example : 20.24. *Fault levels on secondary sides of power transformers.*

Ref. Fig. 20.22 (a) and (b). Rating of Transformer is S = 1 MVA ; Reactance X_t = 5 per cent. Calculate Fault Levels for a 3-phase fault F on secondary side for (a) Single transformer, (b) Two transformers in parallel. Assume Infinite Grid on hV side.



(*a*) (*b*)

Fig. Ex. 20.22

Solution. (*a*) Fault Level on LT side

$$= \frac{S}{X_t} \times 100 = \frac{1}{5} \times 100 = \mathbf{20\ MVA} \quad \textbf{Ans.}$$

(*b*) Fault Level on LT side $= \dfrac{S}{X_{t/2}} \times 100 = \mathbf{40\ MVA}$ **Ans.**

[Reactance of X_t of two transformers in parallel is $X_{t/2}$. Ref. Ex. 20.10 also.]

QUESTIONS

1. Describe the use of Series Reactors in power system.

Fault level at the incoming bus in a sub-station fed from a single incoming line is 400 MVA. Calculate the % reactance to be inserted in the incoming line to limit the fault level at the bus to 350 MVA. Select Base MVA equal to 500 MVA. [**Ans.** 18% p.u.]

2. Describe the following :

1. Magnetically shielded oil immersed reactors,
2. Air-cooled dry type reactors.
3. Non-magnetically shielded reactors.

3. Discuss the need of current limiting reactors in power systems.

4. Describe the principle of current limiting reactors.

5. Write short notes on the following :

1. Ratings of series reactors.
2. Physical arrangement of three phase series reactors.
3. Difference between applications of series reactor and shunt reactor.

6. Two sections *A* and *B* linked with a bus-bar reactor of 10% reactance at 5000 kVA base. The bus-bar *A* is connected to two generators, 11kV, 1000 kVA and 10% reactance each. Bus-bar *B* is connected to two 8000 kVA generators of 12% reactance each. A 3-phase dead short circuit occurs on bus bars of section *B*. Calculate fault MVA and steady state fault current. [**Ans.** 174 MVA, 9.1 kV]

7. Two generators are connected in parallel to low voltage delta side of three-phase transformer, generator I is rated 50,000 kVA, 13.8 kV. Generator II is rated 25,000 kVA, 13.8 kV. Each generator has subtransient reactance of 25%. Transformer is rated 75,000 kVA, 13.8 kV delta (generator side), 69 kV star and has a reactance of 10%. Before fault occurred, the voltage on H.T. side is 66 kV. Transformer is without load and there are no circulating, currents. A 3-phase short circuit occurs on H.T. side of the transformer. Calculate the sub-transient current in each generator.

[**Ans.** 5720 A, 2860 A]

8. Explain briefly the advantages of inserting series reactances in bus-bars. There are four identical 3-phase generators, *A*, *B*, *C*, *D* in a generating station each rated 20,000 kV and having 20% reactance. There is a reactor of 25% reactance based on 11 kV, 20,000 kVA between *B* and *C* bus sections. A 66 kV feeder is taken from bus *A* and is connected via transformer rated 10,000 kVA, 5% reactance. A three-phase short circuit occurs on 66 kV side of transformer. Calculate fault current.

[**Ans.** 975 amperes]

9. A 3-phase 6000 kVA, 6.6 kV alternator has a reactance of 10% and is connected through a 6000 kVA, 6.6 kV/33 kV transformer of 9% leakage reactance to a transmission line having resistance 0.09 ohm and reactance of 0.36 ohm per km respectively. A 3-phase symmetrical delta connected fault occurs between the three phases at a distance of 10 km from transformer. Alternator voltage is 7.2 kV. Find alternator current. Neglect the load current.

10. Show that a generating plant having *N* section bus bars each rated *Q* kVA and have *x*% reactance, connected on the tie bar system through bus bar reactances of *p*% has a total short circuit kVA on one section given by

$$\frac{Q}{x} + Q\,\frac{(N-1)}{(pN+x)} \times 100.$$

If the section rating is 50,000 kVA, *x*=20% and *p*=10%, find the short circuit kVA with (*a*) 3 sections (*b*) 9 section (*c*) show that with infinite sections the maximum fault kVA=750,000.

[**Hint.** Solved problem 20.20]

11. The estimated short circuit MVA at the bus bars of a generating station is 10,000 MVA, and at another station of 570 MVA. Generator voltage at each station is 11 kV. These stations are now linked by an inter-connector having reactance of 0.4 (ohm) per phase. Estimate the fault MVA at each station.

12. A generator connected through a 5 cycle circuit breaker (multiplying factor to obtain breaking capacity=1.1) is connected to a transformer with a breaker in-between.

The generator has reactances

$$X_d'' = 9\%,\ X_d' = 15\%,\ X_d = 100\%,$$

and rating 7500 kVA, 6.9 kV. A 3-phase, short circuit occurs between the breaker and the transformer.

Find (*a*) Sustained short circuit current in the breaker

(*b*) Initial symmetrical r.m.s. current.

(*c*) Maximum possible d.c. component at the short-circuit current.

(*d*) Making current of the circuit-breaker.

(*e*) Current to be interrupted by the circuit-breaker (r.m.s.).

(*f*) Breaker interrupting MVA.

[Hint. Refer Chapter 3]

13. A 150 MVA, 6.6 kV, 3-phase generator has 15% reactance and current limiting reactor of 8%. Find the ratio of electrodynamic forces of short circuit current to forces on full load currents (*a*) without reactor (*b*) with reactor.

[**Hint**. Electrodynamic force is proportional to I^2]. [**Ans.** 44:1, 19:1]

14. Two generators operate in parallel and are connected to a 33 kV transmission line through a transformer. Calculate fault MVA if a 3-phase fault occurs.

(*a*) at the beginning of transmission line.

(*b*) far end of the transmission line.

The data given are as follows:

Generator I, 10,000 kVA, 11 kV, 10% reactance

Generator II, 50,000 kVA, 11 kV, 7.5% reactance

Transformer 15,000 kVA, 11 kV/33, 6% reactance

Impedance of transmission line : (5 + j20) ohm.

15. A delta connected fault, each branch of the delta having reactance of 1.2 ohm is applied to an alternator on no load, and operating at rated terminal voltage. The generator has p.u. reactance of 0.2, and is rated 10 kV, 10,000 kVA. Calculate the line currents during the fault and fault MVA.

16. Three 6.6 kV alternators of rating 2000, 5000 and 8000 kVA and per unit reactances (positive sequence) of 0.08, 0.12 and 0.16 respectively are reactance of 0.125 ohms, 3 phase fault occurs at the other end of the feeder. Calculate fault power. [**Ans.** 87.2 MVA]

17. Three star connected 11 kV alternators are operating in parallel. Each of them is connected the common bus through a reactor. The alternators are rated 11 kV, 10 MVA and have subtransient reactance of 0.06 p.u.

Two 10 MVA, 0.02 p.u. 11 kV/33 kV transformers are connected in parallel to this bus bar and supply a transmission line of impedance 0.2 + j 0.7 ohm per km.

At another substation 10 km away from the generating station is 25 MVA 33 kV/11 kV transformer of 0.06 p.u. reactance. Calculate the reactance of current limiting reactors if each alternator is not to carry $2\frac{1}{2}$ times full load current, when a symmetrical 3 phase short circuit occurs on 11 kV Bus bar in the sub-station.

18. Three 11 kV alternators rated 10,000 kVA with resistance of 0.02 p.u. and reactance of 0.15 p.u. have their bus-bars connected in 3-phase delta connected (mesh connection) reactors of each (0.015 + j0.39) p.u. impedance per phase on 10,000 kVA rating. A 20,000 kVA transformer having (0.01 + j0.10) p.u. impedance is connected to the bus bar of one machine and feeds 132 kv transmission line having resistance of 9 ohms and reactance of 50 ohms per phase. A 3-phase short-circuit occurs at the load end of the line. What must be the minimum rating of the circuit- breaker located at the point of fault ? Find currents which flow from each of the machines. [**Ans.** 73.7 MVA, 1725/1075 A]

19. A sub-station bus receives power from two incoming lines. The fault levels of the lines at the sub-station end are 50 MVA and 150 MVA respectively when they are not connected to the sub-station. Calculate the fault level at the sub-station bus when lines are connected to it. [**Ans.** 200 MVA]

20. Fault level at incoming-bus of a sub-station is 150 MVA. A single 50 MVA transformer of p.u. reactance 8% is connected between the incoming bus and outgoing bus.

Calculate the fault level at out-going bus by

(*a*) Neglecting the equivalent reactance from source size.

(*b*) Without neglecting the above. [**Ans.** 625; 122 MVA]

21. Calculate the maximum possible fault level (short circuit level) at the secondary side of a single connected power transformer ; 500 kVA, 5% reactance. [**Ans.** 10 MVA]

22. A generator is rated 500 kVA, has a reactance of 132%. Calculate 3-ph. fault current when generator was on no load with terminal voltage of 6.5kV. (Note change in voltage).

A generator is rated : 500 kVA, 6.6 kV

X_d = 132% (steady state direct axis reactance)

A sustained 3-phase fault occurred when the generator was on no-load and at 6.5 kV voltage. Calculate fault current. (Note change in voltage)

23. A generator is rated 6.6 kV, 3600 kVA and has X_d = 132%, X_d'' = 23%, X_d' = 31%.

Calculate (1) Sustained S.C. current I ; (2) Transient S.C. current I and (3) Sub-transient S.C. current. When generator was at rated voltage and on :

(A) Full load, (B) No load. [**Ans.** B: 240; 1020 A; 1370 A]

[**Hint.** Add Full load current and fault current to get total current].

A generator has following ratings :

3600 kVA, 6.6 kV, X_d =132%, X_d''= 23%, X_d' = 31%

Calculate (1) Sustained short-circuit current.

(2) Transient short-circuit current.

(3) Initial symmetrical short-circuit current.

When generator is at rated voltage and (A) Full Load, (B) No Load.

Ans. $\begin{bmatrix} [B] & (1) = 240\,A \\ & (2) = 1020\,A \\ & (3) = 1370\,A \end{bmatrix}$

[**Hint.** (A) Add full load current and fault current to get total current.

21

Symmetrical Components

Introduction — Symmetrical Components of 3 Phase System — Operator 'a' — Some Trigonometrical Relations — Zero Sequence Currents-Unbalanced Supply Voltage. **Example 21.1 to 21.10.**

21.1. INTRODUCTION

For unsymmetrical faults such a single phase to ground fault, phase to phase fault, double phase to ground fault, simple single phase representation is not valid. The method of *'Symmetrical Components'* is generally used. The method of symmetrical components is a very powerful approach and has simplified the procedure of fault calculation in a miraculous way. Dr. C.L. Fortesque introduced the method of symmetrical components to the solution of polyphase networks in his paper presented in the year 1918. The principle of symmetrical components is as follows. Suppose we have to solve an unbalanced systems of n vectors. It is then resolved into n balanced systems, each of which consists of n vectors. These balanced vectors are called symmetrical components of the original components. Let us concentrate on unbalanced three-phase systems.

21.2. SYMMETRICAL COMPONENTS OF 3-PHASE SYSTEMS

In unbalanced systems of three vectors [I_a, I_b, I_c, or V_a, V_b, V_c] can be resolved into three balanced systems of vectors, the vectors of the balanced system are called symmetrical components of the original system, which are :

1. **Positive Sequence Components** V_{a1}, V_{b1}, V_{c1} or I_{a1}, I_{b1}, I_{c1} comprising three balanced systems, of vectors, displaced mutually by 120° and having the same phase sequence as that of the original system.

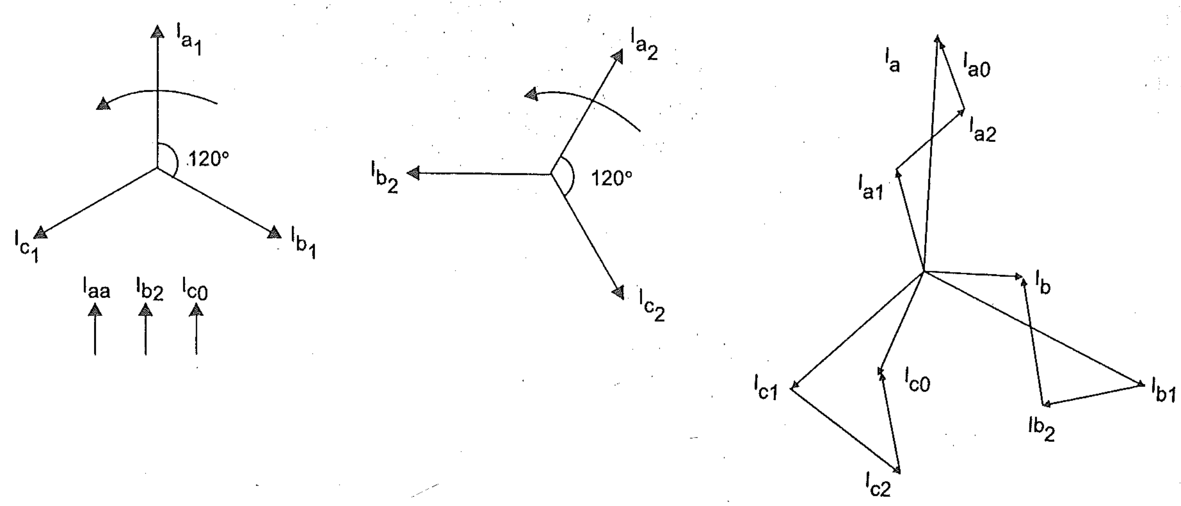


Fig. 21.1. Symmetrical components.

2. **Negative Sequence Components.** [V_{a2}, V_{b2}, V_{c2} or I_{a2}, I_{b2}, I_{c2}] comprising three balanced vectors of equal magnitude displaced mutually by 120° and having phase sequence opposite that of the original system of vectors. (If the original system of vectors have a phase sequence $a - b - c$; then positive sequence components too have phase sequence $a_1 - b_1 - c_1$ but negative phase sequence components have phase ($a_2 - c_2 - b_2$).

3. **Zero Phase Sequence Components.** [V_{a0}, V_{b0}, V_{c0} or I_{a0}, I_{b0}, I_{c0}] comprising three equal vectors having zero phase displacement, *i.e.* having same phase.

Symbolically,

Subscript 1 for positive sequence entities.

Subscript 2 is for negative sequence entities.

and subscript 0 for zero sequence entities.

$V_a, V_b, V_c, I_a, I_b, I_c$ } Original System of Unbalanced Vectors [Meaning : They may not be equal in magnitude or/and do not have same phase displacement.]

V_{a1}, V_{b1}, V_{c1}, I_{a1}, I_{b1}, I_{c1} } Positive Sequence Components

V_{a2}, V_{b2}, V_{c2}, I_{a2}, I_{b2}, I_{c2} } Negative Sequence Components.

V_{a0}, V_{b0}, V_{c0}, I_{a0}, I_{b0}, I_{c0} } Zero Sequence Components.

The original unbalanced system of vectors can be resolved into their symmetrical components or the respective symmetrical components can be added to get the original system of vectors.

Thus
$$\Leftrightarrow$$
$$V_a = V_{a0} + V_{a1} + V_{a2} \quad \text{...(I)}$$
$$V_b = V_{b0} + V_{b1} + V_{b2}$$
$$V_c = V_{c0} + V_{c1} + V_{c2}$$

and
$$\Leftrightarrow$$
$$I_a = I_{a0} + I_{a1} + I_{a2} \quad \text{...(II)}$$
$$I_b = I_{b0} + I_{b1} + I_{b2}$$
$$I_c = I_{c0} + I_{c1} + I_{c2}$$

Fig. 21.1, illustrates the Eqn. (II).

21.3. OPERATOR 'a'

Letter 'a' is commonly used to designate the operator that causes a counter-clockwise rotation of 120°.

It has unit magnitude and angle 120°. The vector operator 'a' is defined as :

$$a = 1e^{+j\frac{2\pi}{3}}$$
$$= \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3} = -0.5 + j0.866$$
$$a = 1\angle 120° = -0.5 + j0.866$$
$$a^2 = 1e^{+j\frac{4\pi}{3}}$$
$$= \cos\frac{4\pi}{3} + j\sin\frac{4\pi}{3}$$
$$= -0.5 - j0.866 = 1\angle 240°$$
$$a^3 = 1e^{+j2\pi} = 1\angle 360° = 1 + j0$$

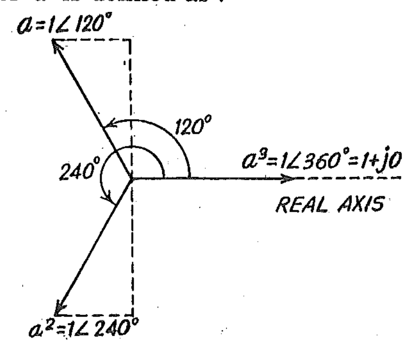


Fig. 21.2. Operator 'a'.

21.4. SOME TRIGONOMETRIC RELATIONS

$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{\text{Opposite side}}{\text{Base}}$$

$$\sin 30° = \frac{1}{2} = 0.5$$

$$\sin 60° = \frac{\sqrt{3}}{2} = 0.866$$

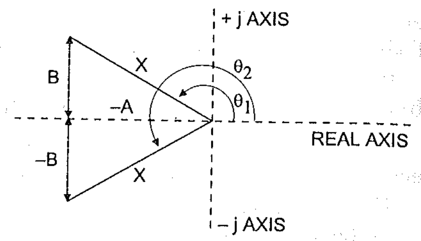


Fig. 21.3. Trigonometric relations.

$$\sin 120° = \sin(180° - 60°) = \sin 60° = 0.866$$
$$\sin 240° = \sin(180° + 60°) = -\sin 60° = -0.866$$
$$\cos 30° = \frac{\sqrt{3}}{2} = 0.866$$
$$\cos 60° = 0.5$$
$$\cos\theta_1 = -\frac{A}{X}$$
$$\cos\theta_2 = -\frac{A}{X}$$
$$\sin\theta_1 = +\frac{B}{X}$$
$$\sin\theta_2 = -\frac{B}{X}$$
$$\cos 120° = \cos(180° - 60°) = -0.5$$
$$\cos 240° = \cos(180° + 60°) = -0.5$$

Remember :

$$a = 1\angle 120° = \cos 120° + j\sin 120°$$
$$= -0.5 + j0.866$$
$$a^2 = 1\angle 240° = \cos 240° + j\sin 240°$$
$$= -0.5 - 0.866$$
$$a^3 = 1 - j0$$

Table of Operator 'a'

$$a = 1\angle 120° = -0.5 + j0.866$$
$$a^2 = 1\angle 240° = -0.5 - j0.866$$
$$a^3 = 1\angle 360° = 1 + j0$$
$$1 + a = 1\angle 60° = 0.5 + j0.866$$
$$1 - a = \sqrt{3}\angle -30° = 1.5 - j0.866$$
$$1 + a = 1\angle -60° = 0.5 + j0.866$$
$$a^3 - a^2 = \sqrt{3}\angle 30° = 1.5 + j0.866$$
$$a + a^2 = 1\angle 180° = -1 - j0$$
$$1 + a + a^2 = 0 \qquad = 0 + j0.$$

From Balanced Vector to Symmetrical Components

Positive Sequence	*Negative Sequence*	*Zero Sequence*
V_{a1}	V_{a2}	V_{a0}
$V_{b1} = a^2 Va_1$	$V_{b2} = aVa_2$	$V_{b0} = Va_0$
$V_{c1} = aVa_1$	$V_{c2} = a^2 V_{a2}$	$V_{c0} = V_{a0}$

From set equation I

$$V_a = V_{a0} + V_{a1} + V_{a2} \qquad \text{...(III)}$$
$$V_b = V_{a0} + a^2 V_{a1} + aVa_2$$
$$V_c = V_{a0} + aVa_1 + a^2 Va_2$$

Written in matrix form,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \qquad \text{...(IV)}$$

Let
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

Then
$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

Multiplying both sides of Eq. (IV) by A^{-1}, we get

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad \text{...(V)}$$

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c) \qquad \text{...(VI)}$$
$$V_{a1} = \frac{1}{3}(V_a + aV_b + a^2 V_c)$$
$$V_{a2} = \frac{1}{3}(V_a + a^2 V_b + aV_c)$$

From these equations, we can get symmetrical components of unbalanced system of vectors.
Summarizing

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$V_a = V_{a0} + V_{a1} + V_{a2}$ $V_b = V_{a0} + a^2 V_{a1} + aV_{a2}$ $V_c = V_{a0} + aV_{a1} + a^2 V_{a2}$	$I_a = I_{a0} + I_{a1} + I_{a2}$ $I_b = I_{a0} + a^2 I_{a1} + aI_{a2}$ $I_c = I_{a0} + aI_{a1} + a^2 I_{a2}$	
$V_{a0} = \frac{1}{3}[V_a + V_b + V_c]$ $V_{a1} = \frac{1}{3}[V_a + aV_b + a^2 V_c]$ $V_{a2} = \frac{1}{3}[V_a + a^2 V_b + aV_c]$	$I_{a0} = \frac{1}{3}[I_a + I_b + I_c]$ $I_{a1} = \frac{1}{3}[I_a + aI_b + a^2 I_c]$ $I_{a2} = \frac{1}{3}[I_a + a^2 I_b + aI_c]$	
V_{a1},	$V_{b1} = a^2 V_{a1}$,	$V_{c1} = aV_{a1}$
V_{a2},	$V_{b2} = aVa_2$,	$V_{c2} = a^2 V_{a2}$
$V_{a0} = V_{b0} = V_{c0}$		
$I_n = I_a + I_b + I_c = 3I_{a0}$		$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$

In the following text, the word phase currents implies currents in R, Y, B phases of a 3 phase supply line.

21.5. ZERO SEQUENCE CURRENTS

In three-phase systems, when there is a neutral return path for currents,

$$I_n = I_a + I_b + I_c$$

We get

$$I_a + I_b + I_c = 3\ [Ia_0]$$

$$\therefore \quad I_{a0} = I_n/3$$

In delta connected load, the line currents do not find return neutral path. Hence line currents do not have zero sequence components.

In star connected system without neutral path or neutral grounding, I_n is zero. Hence zero sequence currents are zero.

Example 21.1. *Calculate the symmetrical components of the following unbalanced line to neutral voltages.*

$$V_a = 100\angle 90° ;\ V_b = 116\angle 0° : V_c = 71\angle 224.8°.$$

Solution.

$$V_{a0} = \frac{1}{3}[V_a + V_b + V_c]$$

$$V_{a1} = \frac{1}{3}[V_a + aV_b + a^2V_c]$$

$$V_{a2} = \frac{1}{3}[V_a + a^2V_b + aV_c]$$

$$V_{a0} = \frac{1}{3}[100\ \angle 90° + 116\ \angle 0° + 71\ \angle 224.8°]$$

$$= \frac{1}{3}[0 + j100 + 116 + j0 + (-50 - j50)]$$

$$= \frac{1}{3}[66 + j50] = \frac{83}{3}\angle 37°$$

$$V_{a0} = 22 + j16.66 = 27.77\angle 37°$$

$$V_{a1} = \frac{1}{3}[V_a + aV_b + a^2V_c]$$

$$V_{a1} = \frac{1}{3}[100\angle 90° + 116\angle 0° + 120° + 71\ \angle 224.8° + 240°]$$

$$= \frac{1}{3}[100\ \angle 90° + 116\ \angle 120° + 71\ \angle 104.8°]$$

$$= \frac{1}{3}[(0 + j100) + (-58 + j100) + (-18 + j68)]$$

$$= \frac{1}{3}(-76 + j268) = -25.33 + j89.33 = 98\angle 106°$$

$$V_{a2} = \frac{1}{3}(V_a + a^2V_b + aV_c)$$

$$= \frac{1}{3}[100\ \angle 90° + 116\ \angle 0 + 240 + 71\angle 224.8° + 120]$$

$$= \frac{1}{3}[100\angle 90° + 116\angle 240° + 71\angle 344.8°]$$

$$= \frac{1}{3}[0 + j100 + (-58 - j100 + 68 - j18)]$$

$$= \frac{1}{3}[10 - j18] = 3.33 - j6 = 6.85\angle 299.$$

Check :

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$= (22 + j6.66) - (25.33 + j89.33) + (3.33 - j6.0)$$

$$= 0 + j99.99 = \text{app.}\ j100 = V_a = 100\ \angle 90°$$

Example 21.2. *The given symmetrical components are $V_{a0} = 22 + j16.66$, $V_{a1} = -25.33 + j89.34$ and $V_{a2} = 3.33 - j6.00$; calculate V_a, V_b, and V_c.*

Solution.

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{c0} + a^2V_{a1} + aV_{a2}$$

$$V_c = V_{ag} + aV_{c1} + a^2V_{c2}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$= (22 + j16.66) + (-25.33 + j89.34) + (3.33 - j6.00)$$

$$= 0 + j100$$

$$V_b = V_{a0} + a^2V_{a1} + aV_{a2}$$

$$= (22 + j16.16) + (-0.5 - j0.866)(-25.33 + j89.34) + (-0.5 + j0.866)(3.33 - j6.00)$$

$$= 115.6 + j0 = 115.6\angle 0°$$

$$V_c = V_{a0} + aV_{a1} + a^2V_{a2}$$

$$= (22 + j16.16) + (-0.5 + j0.866)(-25.33 + j89.34) + (-0.5 - j0.866)(3.33 - j6.0)$$

$$= -50 - j50.$$

Example 21.3. *Determine I_a, I_b, I_c, from the symmetrical components :*

$$I_{a1} = 50\angle 0°,\ I_{a2} = 10\angle 90°,\ I_{a3} = 10\angle 180°.$$

Solution.

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{a0} + a^2I_{a1} + aI_{a2}$$

$$I_c = I_{a0} + aI_{a1} + a^2I_{a2}$$

$$I_a = 10\angle 180° + 50\angle 0° + 10\angle 90°$$

$$= 10\ [-1 + j0) + 50[1 + j0] + 10[0 + j1]$$

$$= -10 + j\,0 + 50 + j0 + 0 + j10 = 40 + j10$$

$$I_b = 10\angle 180° + 50\angle 0 + 240° + 10\angle 90 + 120°$$

$$= -10 + 50\ (-0.5 - j0.866) + 10\ (-0.866 - j0.5)$$

$$= -10 - 25 - 8.66 - j43 - j5$$

$$= -43.66 - j48$$

$$I_c = I_{a0} + aI_{a1} + a^2I_{a2} = -30 + j34.84.$$

Example 21.4. *A delta connected load is connected to three-phase supply. One line of supply is open. The current in other two lines is 20 A. Find the symmetrical components of the line currents.*

Solution. Let a, b, c be the supply lines and c is open. Therefore, currents in the two liens are equal in magnitude.

$$I_a = 20\angle 0°; \quad I_b = 20\angle 180° ; \quad I_c = 0$$

$$I_{ao} = \frac{1}{3}[I_a + I_b + I_c] = \frac{1}{3}[20\angle 0° + 20\angle(180°) + 0] = 0$$

$$I_{a1} = \frac{1}{3}[20\angle 0° + 20\angle(180° + 120) + 0]$$

$$= \frac{1}{3}[20 + j0 + 20\ (0.5 + j0.866) + 0] = \frac{1}{3}[30 - j17.32]$$

$$= 10 - j5.77 = 11.56\ \angle -30°\ \text{Amp.}$$

$$I_{a2} = \frac{1}{3}[20\angle 0° + 20\angle(180° + 240°) + 0]$$

$$= 10 + j5.77 = 11.56\angle 30°\ \text{Amp.}$$

$$I_{b1} = a^2 I_{a1} = 11.56\angle -30 + 240° = 11.56\angle -210°$$
$$I_{b2} = aI_{a2} = 11.56\angle 30° + 120° = 11.56\angle 150°$$
$$I_{b0} = I_{c0} = 0.$$
$$I_{c1} = aI_{a1} = 11.56\angle -30° + 120° = 11.56\angle 90°$$
$$I_{c2} = a^2 I_{a2} = 11.56\angle 30° + 240° = 11.56\angle -90°$$
$$I_{c0} = I_{a0} = 0. \quad \text{(Ref. Sec. 21.5)}$$

Check :

$$I_c = I_{c0} + I_{c1} + I_{c2}$$
$$= 0 + 11.56\angle -90° + 11.56\angle +90° = 0.$$

Example 21.5. *In a three-phase system, the voltage of phase of neutral had the following sequence components during phase to phase fault condition. Calculate the voltages of the phases with respect to neutral, voltages between phases. Given :*

$$V_{a1} = 0.584 + j0 \text{ p.u.}, V_{a2} = 0.584 + j.0 \text{ p.u.}, V_{a0} = 0$$

Solution.
$$V_a = V_{a1} + V_{a2} + V_{a0}$$
$$= 0.584 + 0.584 + 0 = 1.68\angle 0° \text{ p.u.}$$
$$V_b = a^2 V_{a1} + aV_{a2} + V_{a0}$$
$$= (-0.5 - j0.866)\ V_{a1} + (-0.5j + 0.866)\ V_{a2} + 0 = -0.584 \text{ p.u.}$$
$$V_c = V_b = -0.584 \text{ p.u.}$$
$$V_{ab} = V_a - V_b = 1.168 + 0.584 = 1.752\angle 0° \text{ p.u.}$$
$$V_{bc} = V_b + V_c = -0.584 + 0.584 = 0$$
$$V_{ca} = -0.584 - 1.168 = -1.752\angle 180° \text{ p.u.}$$

Fault is between phases b and c. Hence V_{bc} is zero. $V_{ab} = -V_{ca}$.

Example 21.6. *In a single phase equivalent circuit the positive sequence components of current is given by*

$$I_{a0} = \frac{E_a}{X_1 + X_2 + X_0}$$

Further I_{a0} is equal to I_{a1} and I_{a2}. Calculate I_a. Given $E_a = 1 + j0$ p.u., $X_1 = j0.25$, $X_2 = j0.35$, $X_0 = j0.10$.

Solution. Given $I_{a0} = \dfrac{E}{X_1 + X_2 + X_0}$

Substituting the given values

$$I_{a0} = \frac{1 + j0}{j0.25 + j0.35 + j0.10} = \frac{1}{j0.70} = -j1.43 \text{ p.u.}$$

It is given, $\quad I_{a0} = I_{a1} = I_{a2}$

$\therefore$
$$I_a = I_{a0} + I_{a1} + I_{a2} = 3\ [I_{a0}]$$
$$= 3 \times [-j1.43] = -j4.29 \text{ p.u.}$$

Example 21.7. *In a problem on fault calculations, following expressions were obtained.*

$$I_a = \frac{E_a}{Z_1 + \dfrac{1}{\left(\dfrac{1}{Z_2} + \dfrac{1}{Z_0}\right)}}$$
$$V_{a1} = V_{a2} = V_{a0} = E_a - I_{a1} Z_1$$
$$I_2 = \frac{-V_{a2}}{Z_2},\ I_0 = \frac{-V_{a0}}{Z_0}$$

Determine I_a and V_a

Given : $Z_1 = j0.25$, $Z_2 = j0.35$, $Z_o = j0.1$ p.u., $E_a = 1 + j0$ p.u.

Solution.

$$I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_2 Z_0}{Z_2 + Z_0}} = \frac{1 + j\,0}{j\,0.25 + \dfrac{j0.35 \times j0.10}{j0.35 + j0.10}}$$
$$= \frac{1.0}{j0.25 + j0.0778} = \frac{1.0}{j0.3278} = -j3.05 \text{ p.u.}$$

Given : $\quad V_{a1} = V_{a2} = V_{a0} = E_a - I_{a1} Z_1$

$$= 1 + j0 - (-j3.05)\ (j0.25) = 1 - 0.763 = 0.237 \text{ p.u.}$$
$$I_{a0} = \frac{-V_{a0}}{Z_0} = \frac{-0.237}{j0.1} = j2.37 \text{ p.u.}$$
$$I_{a2} = \frac{-V_{a2}}{Z_2} = \frac{-0.237}{j0.35} = j0.68 \text{ p.u.}$$
$$I_a = I_{a1} + I_{a2} + I_{a0} = -j3.05 + j0.68 + j2.37 = 0$$
$$V_a = V_{a1} + V_{a2} + V_{a0} = 3V_{a1} = 3 \times 0.237 = 0.711 \text{ p.u.}$$

Example 21.8. *The positive sequence network of a system is shown in the figure given below. Draw Thevenin's equivalent network and determine the positive sequence component of fault current, assuming zero fault impedance and voltage at fault point 1.0 p.u.*

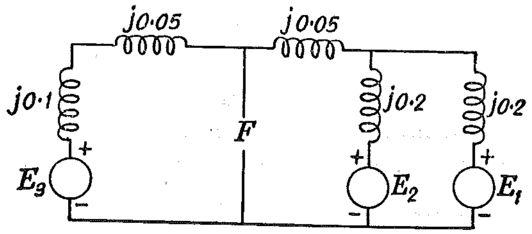
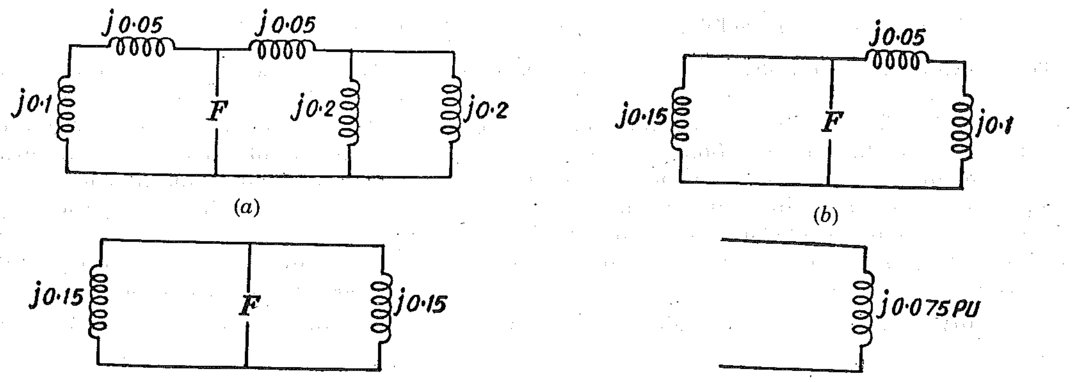


Fig. of Ex. 21.8.

Solution. Thevenin's equivalent impedance

V_f = Thevenin's O.C. voltage 1 p.u.



(a) (b) (c) (d)

Positive sequence current

$$= \frac{V_t}{X_{h\,(eq)}} = \frac{1 + j0}{j0.075} = 13.35 \text{ p.u.}$$

Example 21.9. *In a three-phase 4-wire system the phase currents in R, Y, B phases are $I_R = 100\angle 30°$, $I_Y = 60\angle 300$ A, $I_B = 30\angle 180°$. Calculate positive, negative and zero sequence components of current in the phase R and current in the return neutral current.*

Solution.
$$I_R = 100\angle 30° = 100\ [\cos 30° + j \sin 30°]$$
$$= 100\ [0.866 + j0.5] = 86.6 + j50$$

$$I_Y = 50\angle 300°$$
$$= 50[\cos 300° + j\sin 300] = 50\,[0.5 - j0.866] = 25 - j43.3$$
$$I_B = 30\angle 180° = 30[-1 + j0] = -30$$

I_n = Current in return neutral path

$$= I_R + I_Y + I_B = 86.6 + j50 + 25 - j43.3 - 30 = 81.6 + j6.7 = 82 \text{ A}$$
$$I_{R0} = \frac{1}{3}\,[I_R + I_Y + I_B] = \frac{1}{3}\,[81.6 - j6.7] = 27.2 + j2.25$$
$$I_{R1} = \frac{1}{3}\,[I_R + aI_Y + a^2 I_B]$$
$$= \frac{2}{3}[(86.5 + j50.0)] + (-0.5 + j0.866) \times (25 - j43.3) + (-0.5 - j0.866)\,(-30)]$$
$$= 42.2 + j39.8$$
$$I_{R2} = \frac{1}{3}[I_R + a^2 I_Y + aI_B] = 17.2 + j8$$

Check : $I_R = I_{R0} + I_{R1} + I_{R2}$

$$= 27.2 + 42.2 + 17.2 + j2.25 + j39.8 + j8.0 = 86.6 + j50.05$$

which agrees closely with I_R.

Ans. $I_n = 82$ A   Check : $I_n = 3I_0$

$I_{R0} = 27.2 + j2.25$ A

$I_{R1} = 42.2 + j39.8$ A

$I_{R2} = 17.2 + j8$ A.

Example 21.10. *Given current in neutral to ground connection 1.9 Amperes. Calculate zero sequence component of current in phases.*

Solution. $I_{a0} = I_{bo} = I_{c0} = \frac{I_n}{3} = \frac{9}{3} = 3$ Amperes. **Ans.**

21.6. PHASE DISPLACEMENT IN STAR-DELTA TRANSFORMERS

The angular difference between vectors representing the voltages induced between h.v. and l.v. terminals having same marking letter and the corresponding neutral points (real or imaginary), expressed with reference to h.v. side is termed as phase displacement of transformer. Even under normal condition, the phase to phase voltages and phase to neutral voltages of h.v. side are displaced from corresponding voltages of l.v. side, in case of star-delta transformers. The phase displacement of + 30° comes in Group 4 and that of – 30° comes in Group 3 (IS : 2026, 1962 reprint 1972—Specifications for power transformers). Similarly the currents on the two sides are also displaced. While applying the method of symmetrical components, the inherent phase shirt should be considered. A phase shift can be expected in p.s., n.s., z.s. components on either sides.

Generally in short-circuits calculations the phase displacement is neglected. The procedure is as follows :

Consider a star-delta transformer. The analysis of the positive sequence currents and positive sequence voltages can be corrected if necessary for phase displacement by taking into account the inherent phase displacement. Similarly, the same reasoning applies to negative sequence currents and negative sequence voltages. Magnitude of phase shift is same for positive sequence components and negative sequence components. However, the direction of phase shift in case of negative phase sequence components is reverse of that applicable to the positive sequence components (Due to reverse phase sequence). The phase shifts pf p.s. components and n.s. components are equal in magnitude but opposite in direction. The magnitude and direction of phase displacement depends on transformer group and allocation of phase references. Phase displacement of zero-sequence quantities need not be considered in star-delta transformer. Since the zero sequence currents do not flow in lines on delta-connected side.

QUESTIONS

1. Given $V_a = 100 + j0$, $V_b = -2.7 + j32.3$, $V_c = -37.3 + j2.3$. Find the symmetrical components V_{a0}, V_{a1}, V_{a2}. [**Ans.** $20 - j18$, $50 + j15$, $30 - j5$]
2. Given $V_a = 100\angle 0$, $V_b = 50\angle 225$, $V_c = 111\angle 2.6$. Find the sequence components. [**Ans.** $66.66 + j16.66$, $60.1 - j8.17$, $-26.5 - j8.3$]
3. The current in three line conductors, a, b, c are $40 + j60$, -90, $-80 + j10$ Amp.
   If the reactance per phase for positive, negative and zero sequence current is respectively 20, 20, 50 ohms, find the voltage drop in conductor 'a'. [**Ans.** $-900 + j400$ V]
4. (*a*) Show that
   $$a + a^2 + 1 = 0$$
   $$a - a^2 = j\sqrt{3}$$
   $$a - 1 = 1.5 - j0.866,$$
   (*b*) Evaluate the following : $\frac{1+a^2}{1-a}, \frac{1-a}{1+a}, \frac{1+a}{1+a^2}$
5. Given $E_a, E_b, E_c = 60 + j0$, $45 - j75$, $-51 + j120$ respectively.
   Determine E_{a0}, E_{a1}, E_{a2}. [**Ans.** $28 + j15$, $72.2 + j11.5$, $-40.2 - j26.5$]
6. Given $I_a = 0 + j100$, $I_b = 20 + j0$, $I_c = 0$. Find I_{a1}, I_{a2}, I_{a0}.
7. Assume $I_a = 100$ Amp., $I_b = I_c = -50$ Amp. What are the sequence currents ?
8. The phase to phase voltage of a 3-phase system are 100, 150, 200 volts. Find the magnitudes of positive and negative sequence components.
9. A grounded neutral system has positive sequence voltage of E_{a1}, show that if neutral ground be removed and one phase wire grounded the sequence voltage remains unchanged.
10. Three resistors of 5, 10, 20 ohms are connected in delta across the bases A, B, C respectively to a balanced supply to 100 volts. What are the sequence components of current in the resistors and the supply line ?
11. Given $I_{a1} = j1.56$ p.u.
    $I_{a2} = j1.56$ p.u.
    $I_{a0} = 0$
    Calculate I_a, I_b, I_c. [**Ans.** $I_a = 0$, $I_b = -2.70 + j0$ p.u. $I_c = 2.70 + j0$ p.u.]
12. Given $I_{a1} = I_{a2} = I_{a0} = 0.255 - j0.32$ p.u.
    Calculate I_a, I_b, I_c [**Ans.** $I_a = 0.765 - j0.156$; $I_b = 0$; $I_c = 0$]
13. In a problem on fault calculations.
    $$I_{a1} = I_{a2} = I_{a0} = \frac{E}{Z_2 + Z_2 + Z_0} \text{ p.u.}$$
    $$E = 1 + j0$$
    Calculate I_a, I_b, I_c
    Given : $Z_1 = j0.5$, $Z_2 = j0.5$, $Z_0 = 0$.
14. Given $I_a = 0$ p.u., $I_b = -2.70 + j0$. p.u., $I_c = -0.70 + j0$. p.u.
    Calculate I_{a1}, I_{a2}, I_{a0}. [**Ans.** $-j1.560$ p.u., $j\,1.560$ p.u.]
15. Explain the principle of symmetrical components. What is the difference between positive sequence and negative sequence components ? Derive the relation between V_a, V_b, V_c and $V_{a0}\, V_{a1}\, V_{a2}$.
16. Given V_{a0}, V_{a1}, V_{a2} derive an expression to obtain V_a, V_b, V_c.
17. Show that the current is neutral to ground connection is three times zero sequence component of current, *i.e.* $I_n = 3I_{a0}$.
18. A star connected three phase winding is with earthed neutral. During a fault, the current in neutral to ground path was 9 Amp. Calculate the zero sequence component of current in the winding.

22

Unsymmetrical Faults on an Unloaded Generator

Sequence Impedances — Sequence Network of Alternator — L-G Faults on Alternator — L-L Fault on Alternator — 2 L-G Fault on Alternator — Solved Examples.

22.1. SEQUENCE IMPEDANCES

The impedances offered by a rotating machine to positive sequence component of current, differ from those offered to negative sequence components of currents. Consider a circuit component, the voltage drop across it, for given sequence component of current will be equal to magnitude of that sequence current into impedance offered to it. Thus we come across positive sequence impedance or reactance, negative sequence impedance or reactance and zero sequence impedance or reactance.

The impedance offered by a circuit to positive sequence component current is called Positive Sequence Impedance of that circuit. Likewise the negative sequence impedance and zero sequence are defined.

22.2. SEQUENCE NETWORKS OF ALTERNATOR

(I) The positive sequence network of 3 phase alternator (shown in Fig. 22.1) consists of an e.m.f. source E_a in series with positive sequence impedance Z_1. E_a is the induced e.m.f. of one phase. Z_1 is replaced by jX_1 if the resistance is neglected. X_1 is positive sequence reactance of generator.

E_a = Voltage behind the reactance (induced e.m.f.) per phase

X_1 = Positive sequence reactance Direct axis reactance

$V_1 = E_a - I_{a1} X_1$

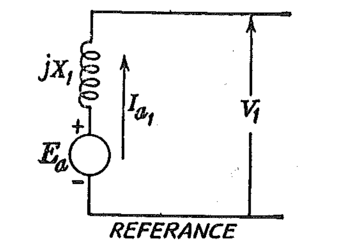


Fig. 22.1. Positive sequence network of an alternator.

It is same as direct axis reactance. It may be sub-transient or transient or steady state reactance depending upon the problem to be solved. Typical values of reactance are given in Table 19.2.

The generator e.m.f's are balanced voltages and, therefore, considered to be positive sequence e.m.f's. Generator does not induce negative or zero sequence e.m.f. The phase sequence of positive sequence voltages is the same as the phase sequence of induced e.m.f.

X_1 = positive sequence reactance of generator

E_a = e.m.f. induced in phase a.

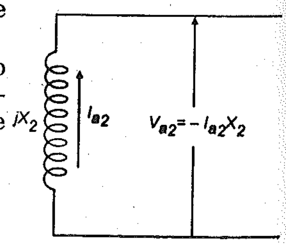


Fig. 22.2. Negative sequence network of a generator.

(II) Negative Sequence Network of Generator. The negative sequence network of a generator consists simply of negative sequence reactance, jX_2 (Fig. 22.2) as there are no negative sequence e.m.f's induced by the alternator. Only

negative sequence current flows through negative sequence impedance (or reactance) and the voltages drop in the negative sequence network is given by

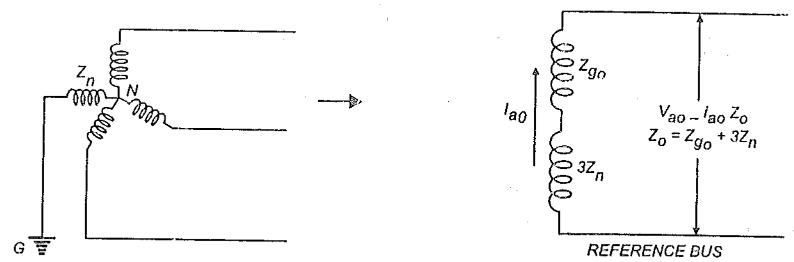
$V_{a2} = - I_{a2} Z_2$ or $-jI_{a2} X_2$

jX_2 = Negative sequence of reactance generator.

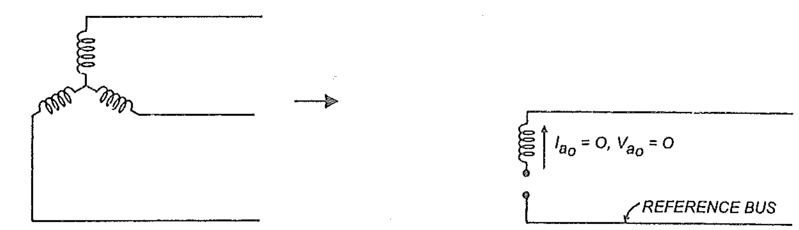
(III) Zero Sequence Network. Zero sequence network of an alternator consists of the zero sequence impedance of alternator per phase, plus three times the impedance in neutral to ground circuit, *i.e.*,

$$Z_0 = Z_{g0} + 3Z_n \text{ ; voltage drop} = I_n Z_0$$

It may be recalled* that the current in the neutral circuit is $I_n = 3I_0$. Hence the voltage drop is equal to $3I_{a0}Z_n$, where Z_n, is the reactance in neutral to ground circuit. We consider that only current I_{a0} flows through the neutral circuit. Hence Z_n is multiplied by 3 to get the voltage drop $3I_{a0} Z_n$. The zero sequence network of an alternator is shown in Fig. 22.3. If neutral is not grounded there is a gap in the zero sequence network and zero sequence component of current I_a is zero. Hence I_n is also zero.



[A] (*a*) Neutral grounded. (*b*) Zero sequence network.



[B] (*a*) Neutral ungrounded. (*b*) Zero sequence network.

Fig. 22.3. Zero sequence network of alternator.
[A] With neutral grounded. [B] Without neutral grounding.

22.3. VOLTAGE EQUATIONS

We observe that the currents of a particular sequence produce voltage drop of like sequence. Referring to Figs. 22.1, 22.2, 22.3 we write the following equations :

$$V_{a1} = E_a - I_{a1}Z_1 \quad \text{or} \quad E_a - jI_{a1} X_1$$
$$V_{a2} = - I_{a2}Z_2 \quad \text{or} \quad - jI_{a2}X_2$$
$$V_{a0} = - I_{a0}Z_0 \quad \text{or} \quad - jI_{a0} X_0$$
$$Z_0 = Z_{g0} + 3Z_n \quad \text{or} \quad X_0 = X_{g0} + 3X_n$$

* Refer Sec. 21.5 $I_{ao} = I_{bo} = I_{co} = I_{n3}$

The word line refers to one conductor of 3 phase system.

22.4. SINGLE LINE* TO GROUND FAULT ON AN UNLOADED THREE-PHASE ALTERNATOR AT RATED TERMINAL VOLTAGE

Solution. Let a, b, c be the terminals of the unloaded generator whose star point N is grounded through impedance Z_n. A single line to ground fault occurs on terminal a. We have to determine fault current and voltage of the lines.

V_a = Voltage of terminal a with respect to N.

$I_b = 0$ and $I_c = 0$. Since generator is on no load.

$V_a = 0$. Neglecting drop in Z_n.

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{3} I_a.$$

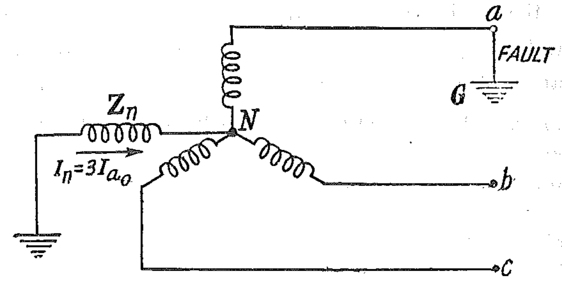


Fig. 22.4. Circuit condition of L-G fault.

Hence for single line to ground fault on terminal a, we get

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a \qquad ...(a)$$

Coming to voltage equations,

$$V_{a1} = E_a - I_{a1} Z_1$$

$$V_{a2} = -I_{a2} Z_2$$

$$V_{a0} = -I_{a0} Z_0$$

Since $\quad I_{a1} = I_{a2} = I_{a0}$

$$V_a = V_{a1} + V_{a2} + V_{a0} = E_a - I_{a1}(Z_1 + Z_2 + Z_0) = 0$$

As $\quad V_a = 0,$

Hence
$$I_{a1} = \frac{E_a}{Z_0 + Z_1 + Z_2} \qquad ...(b)$$

Taking a look at equations (a) and (b), we feel that there should be some easy way to remember these expressions. And there is ! This is a wonderful part of the method of symmetrical components. The apparently dull and complicated complexities can be brought to a simple systematic form which makes the analysis interesting and easy.

Connect the three sequence networks of the generator in series. Equal current flows through the three networks and the above equations are satisfied. Fig. 22.5 shows the connection of sequence network to represent Single Line to Ground fault.

The sequence currents can be easily calculated from this simple series circuit.

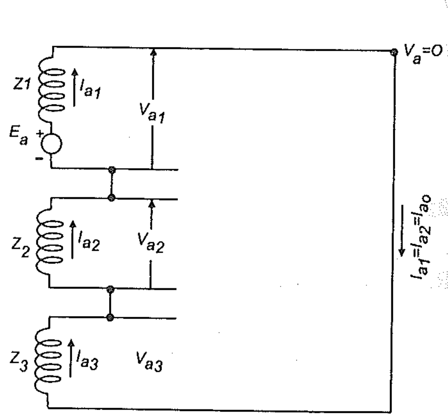


Fig. 22.5. Connection of sequence networks to represent single line to ground fault on phase a.

Example 22.1. *A 25,000 kV, 11 kV, 3 phase alternator has direct axis sub-transient reactance of 0.25 per unit, negative sequence reactance and zero sequences are respectively 0.35 and 0.1 p.u.*

Neutral of the generator is solidly grounded. Determine subtransient line currents and line-to-line voltages, for

(*a*) *Single line to ground fault.*

(*b*) *Double line to ground fault.*

(*c*) *Line to line fault.*

Generator is on no load and rated terminal voltage. Resistance is negligible.

Solution. (a) Let a, b, c be three terminals of the generator. Let fault occur between terminal a and ground. Let the induced voltage of phase a line to neutral E_a be 1 p.u.

$$E_a = 1 + j0 \text{ p.u.} = \frac{11}{\sqrt{3}} \text{ kV, actual}$$

$$X_n = 0, X_0 = X_{g0} + 0$$

Fig. 22.5 represents the fault condition. For L-G fault :

$$I_{a0} = I_{a1} = I_{a2} = \frac{E_2}{X_{g0} + X_1 + X_2}$$

$$= \frac{1 + j0}{j0.25 + j0.35 + j0.1} = \frac{1 + j0}{j0.70} = -j\,1.43 \text{ p.u.}$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = 3I_{a0} = -j\,4.29 \text{ p.u.}$$

Base current $\quad = \dfrac{\text{Base kVA}}{\sqrt{3}\text{ Base kV}} = \dfrac{25{,}000}{\sqrt{3} \times 11} = 1310$ Amp.

Fault current $\quad I_a = -j4.29 \times 1310 = 5630 \text{ A } \underline{/-90^\circ}$

$$V_{a1} = E_a - I_{a1} Z_1$$

$$= 1 - (-j1.43)(j0.25) = 1 - 0.357 = 0.643 \text{ p.u.}$$

$$V_{a2} = -I_{a2} Z_2 = (-j1.43)(j0.35) = -0.50 \text{ p.u.}$$

$$V_{a0} = -I_{a0} Z_0 = -(-j\,1.43)(j0.1) = -0.143 \text{ p.u.}$$

Line to ground voltages

$$V_a = V_{a1} + V_{a0} = 0.643 - 0.50 - 0.143 = 0 \text{ [check]}$$

$$V_b = a^2 V_{a1} + aV_{a2} + V_{a0}$$

$$= 0.643(-0.5 - j0.866) + (-0.5 + j0.866)(-0.5) - 0.143 = 0.215 - j0.989 \text{ p.u.}$$

$$V_c = aV_{a1} + a^{2V_{a2}} + V_{a0}$$

$$= 0.643(-0.5 + j0.866) - 0.5(-0.5 - j0.866) - 0.143 = 0.215 + j0.989$$

Line to line voltages

$$V_{ab} = V_a - V_b = 0.215 + j0.989 = 1.01\angle 77^\circ \text{p.u.}$$

$$V_{bc} = V_b - V_c = 0 - j1.978 = 1.978\angle 270^\circ \text{ p.u.}$$

$$V_{ac} = V_c - V_a = +0.215 + j989 = 1.01\angle 102.3^\circ \text{ p.u.}$$

Line to line voltages in kV

Since 1 p.u. voltage $= \dfrac{11}{\sqrt{3}}$ kV

$$V_{ab} = 1.01 \times \frac{11}{\sqrt{3}} = 6.42\angle 77^\circ \text{ kV}$$

$$V_{bc} = 1.978 \times \frac{11}{\sqrt{3}} = 12.55\ \angle 270^\circ \text{ kV}$$

$$V_{ca} = 1.01 \times \frac{11}{\sqrt{3}} = 6.42\ \underline{/102.3} \text{ kV}$$

Fault current I_a = 3630 $\underline{/90}$ A

Parts (B) and (C) will be solved later.

Example 22.2. A 3-phase 11 kV. 10,000 kVA alternator has $X_0 = 0.05$ p.u., $X'' = 0.15$ p.u., $X_2 = 0.15$ p.u. It is on no load and rated terminal voltage. Find the ratio of the line currents for a Single Line to Ground Fault, to 3 phase fault. Neutral is solidly grounded.

Solution. Let $E_a = 1$ p.u.

(1) **L-G Fault**

$$I_{a1} = \frac{E_a}{X_1 + X_2 + X_0} = \frac{1}{j0.05 + j0.15 + j0.15} = \frac{1}{j0.35}$$

$$I_a = 3I_{a1} = \frac{3}{j0.35} = -j8.57 \text{ p.u.}$$

(2) **Three-Phase Fault**

$$I_a = \frac{E_a}{X_1} = \frac{E_a}{X'} \qquad \{X_1 = X' \text{ or } X'' \text{ or } X_s\}$$

$$= \frac{1}{j0.15} = -j6.66 \text{ p.u.}$$

Ratio of line currents $= \dfrac{8.57}{6.66} = 1.285$

Single line to ground fault current is 1.285 times three phase fault current. **Ans.**

Example 22.3. *A 3-phase, 11 kV, 25,000 kVA alternator with $X_{g0} = 0.05$ p.u., $X_1 = 0.15$ p.u. and $X_2 = 0.15$ p.u. is grounded through a reactance of 0.3 ohms. Calculate the line current for a single line to ground fault.*

Solution. Base Z $= \dfrac{\text{Base kV}^2 \times 1000}{\text{Base kVA}}$

Let Base kV = 11

and Base kVA = 25,000

Base Z $= \dfrac{121 \times 1000}{25,000} = 4.84$ ohms

P.u. X_d of neutral connection $= \dfrac{0.3}{4.84} = 0.062$ p.u.

$$X_0 = X_{g0} + 3X_n$$

$$= j0.05 + 3\,[0.062] = j0.05 + j0.15 + j0.186 = j0.236 \text{ p.u.}$$

$$X_1 + X_2 + X_0 = j0.15 + j0.236 = j0.536 \text{ p.u.}$$

For single to ground fault, refer Fig. 22.5.

$$I_{a0} = \frac{E_a}{X_1 + X_2 + X_0} = \frac{1 + j0}{j0.536} = -j1.86 \text{ p.u.}$$

$$I_a = 3 \times I_{a0} = 3 \times 1.86 = 5.58 \text{ p.u.}$$

I_a in amperes $= 5.58 \times \dfrac{25,000}{\sqrt{3} \times 11} = 5.58 \times 1310 =$ **7312 amperes.**

22.5. DOUBLE LINE TO GROUND FAULT ON AN UNLOADED GENERATOR

Let fault involve terminals b, c and ground (Fig. 22.6)

Observing the fault condition, $V_b = 0$, $V_c = 0$ and $I_a = 0$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

Hence $V_{a1} - V_{a2} = V_{a0} = \dfrac{V_a}{3}$

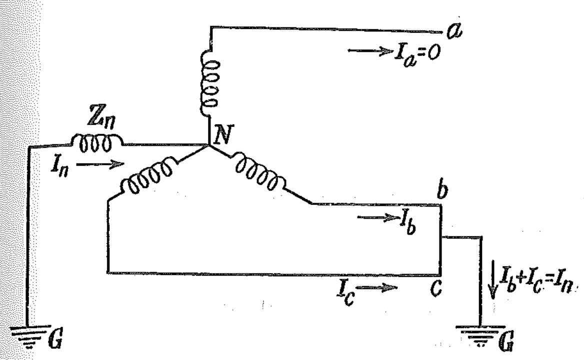


Fig. 22.6. 2 *L-G* fault.

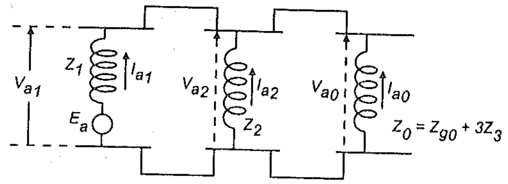


Fig. 22.7. Connection of sequence networks of 2 *L-G* fault.

Further $I_a = I_{a1} + I_{a2} + I_{a0} = 0$

This suggests that the three sequence networks of the generator may be connected in parallel as shown in Fig. 22.7.

From the figure, for double line to ground fault, we get

$$I_{a1} = \frac{E_a}{Z_1 + 1\Big/\left(\dfrac{1}{Z_2} + \dfrac{1}{Z_0}\right)} = \frac{E_a}{Z_1 + \dfrac{Z_2 Z_0}{Z_2 + Z_0}}$$

$$V_{a1} = E_a - I_{a1} Z_1$$

$$V_{a1} = V_{a/3}$$

$$V_{a0} = V_{a1} = V_{a2}$$

$$I_{c2} = \frac{-V_{a1}}{Z_2} \qquad \text{...[Note the – ve sign]}$$

$$I_{a0} = \frac{-V_{a1}}{Z_0}$$

From these symmetrical components the currents and voltages can be determined.

Example 22.4. *(A) Part (b)* of example 22.1.

Given : Generator 11 kV, 25,000 kVA

$$X_1 = j0.25,\ X_2 = j0.35 \text{ p.u.},\ X_{g0} = j0.1,\ X_n = 0$$

Fault : Double line to ground, between terminals b, c and ground.

Solution. Let $E_a = 1 + j0$ p.u. $= \dfrac{11}{\sqrt{3}}$ kV

Refer Fig. 22.7,

$$I_{a1} = \frac{E_a}{X_1 + \dfrac{Z_0 Z_2}{Z_0 + Z_2}} = \frac{1 + j0}{j0.25 - \dfrac{j0.35 \times j0.10}{j0.35 + j0.10}}$$

$$= \frac{1}{j0.25 + j0.0778} = \frac{1}{j0.3278} = -j3.05 \text{ p.u.}$$

$$V_{a1} = V_{a2} = V_{c0} \qquad \text{...for LL-G Fault.}$$

$$= E_a - I_{a1} X_1$$

$$= 1 + j0 - (-j3.05)\,(j0.25) = 1 - 0.763 = 0.237 \text{ p.u.}$$

$$I_{a2} = \frac{-V_{a2}}{jX_2} = -\frac{0.237}{j0.35} = j0.68 \text{ p.u.}$$

$$I_{a0} = \frac{-V_{a0}}{X_{g0}} = -\frac{-0.237}{j0.10} = j2.37$$

$$I_a = I_{a1} + I_{a2} + I_{a0} = -j3.05 + j0.68 + j2.37 = 0$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$
$$= j2.37 + (-0.5 - j0.866)(-j3.05) + (-0.5 + j0.866)(+j0.68)$$
$$= -3.229 + j3.55 = 4.81 \angle 132.5 \text{ p.u.}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$
$$= j2.37 + (-0.5 + j0.866)(-j3.05) + (-0.5 - j0.866)(j0.68)$$
$$= j2.37 + j1.525 + 2.64 - j0.34 + 0.589$$
$$= 3.229 + j3.555 = 4.81 \angle 47.5° \text{ p.u. (Amp.)}$$

$$I_n = 3I_{a0} = 3 \times j2.37 = j7.11 \text{ p.u.}$$

or
$$I_n = I_b + I_c$$
$$= -3.229 + j3.555 + 3.229 + j3.555 = j7.11 \text{ p.u. (Check)}$$

Phase voltages $V_a = V_{a1} + V_{a2} + V_{a0} = 3V_{a1}$

$$= 3 \times 0.237 = 0.711 \text{ p.u.}$$
$$V_b = V_c = 0$$

Line to line voltages

$$V_{ab} = V_a - V_b = 0.711 \text{ p.u.}$$
$$V_{bc} = V_b - V_c = 0$$
$$V_{ca} = V_c - V_a = -0.711 \text{ p.u.}$$

Base voltage $1 \text{ p.u.} = \frac{11}{\sqrt{3}} \text{ kV since } E_a = 1 + j0 = 11/1.73 = 6.35 \text{ kV}$

$$V_{ab} = 0.711 \times \frac{11}{\sqrt{3}} = 4.52 \angle 0° \text{ kV} \quad \textbf{Ans.}$$

$$V_{ca} = -0.711 \times \frac{11}{\sqrt{3}} = 4.52 \angle 180° \text{ kV}$$

Base Current $= \frac{\text{Base kVA}}{\sqrt{3}\text{ Base kV}} = \frac{25{,}000}{\sqrt{3} \times 11} = 1310 \text{ Amp.}$

$$|I_b| = 4.81 \times 1310 = 6300 \text{ A} \angle 132.5°$$
$$|I_c| = 4.81 \times 1310 = 6300 \text{ A} \angle 47.5°$$
$$I_a = 0 \qquad \textbf{Ans.}$$
$$I_n = I_b + I_c = 7.11 \times 1310 \equiv 9320 \text{ A}$$

Example 22.4 (B) Neutral Reactor

In Ex. 22.4 (A), the Neutal to Ground Circuit has reactance of 0.1 p.u. instead of zero. [Add $X_n = 0.1$ p.u. in Fig. 22.6].

Calculate Fault Current following through the Neutral Reactor

Solution. Ref. Fig. 22.7.

Now $X_0 = X_{g0} + 3X_n$

where X_0 = Total Eq. Zero Seq. Reactance

X_{g0} = Zero Seq. Reactance of Generator

X_n = Reactance in Neutral to Ground Circuit.

X_1 and X_2 remains unchanged.

In this example, $X_{g0} = j0.1$ p.u. ; $X_n = j0.1$ p.u.

$$X_0 = j0.1 + 3 \times j0.1 = j0.4 \text{ p.u.}$$

$$I_{a1} = \frac{E_a}{X_1 + \frac{X_0 X_2}{X_0 + X_2}} = \frac{1 + j0}{j0.25 + \frac{j0.4 \times j0.35}{j0.4 + j0.35}}$$

$$= \frac{1 + j0}{j0.25 + j0.187} = \frac{1 + j0}{j0.437} = -j2.29 \text{ p.u.}$$

$$V_{a1} = V_{a2} = V_{a0} = E_a - I_{a1} X_1$$
$$= (1 - j0) - (-j2.29)(j0.25) = 1 - 0.572 = 0.428 \text{ p.u.}$$

$$I_{a2} = \frac{-V_{a2}}{X_2} = \frac{-V_{a1}}{X_2} = \frac{-0.428}{j0.35} = j1.225 \text{ p.u.}$$

$$I_{a0} = \frac{-V_{a0}}{X_0} = \frac{-0.428}{j0.4} = j1.072$$

$$I_a = I_{a1} + I_{a2} + I_{a0}$$
$$= -j2.29 + j1.225 + j1.072 \approx 0 \text{ (check)}$$

$$I_b = I_{a0} + a^2 I_a + a I_{a2} = -3.04 + j0.6045$$

$$I_c = I_{a0} + a I_a + a^2 I_{a2} = +3.04 + j0.6045$$

$$I_n = I_b + I_c + I_a = -3.04 + j0.6045 + 3.04 + j0.6045 = j1.2090 \text{ p.u.}$$

Base current was 1310 A.

Hence current in Neutral-to-ground Reactor = 1.209 × 1310 = **1570 Amp. Ans.**

Note. Reason for Reactance Grounding. Ex. 22.4 (A) Neutral-to-Ground Current without neutral reactor was 9320 A. Ex. 22.4 (B) Neutral-to-Ground Current with neutral reactor was 1570 A. Reactance in Neutral to-Ground circuit reduces fault current. Hence modern practice is in favour of reactance grounding. [Ref. Sec. 33.6]

Example 22.5. *A 3-phase, 10 MVA star connected alternator having solid neutral earthing supplies a feeder. The per unit reactances are as follows :*

Generator : $X_1 = j0.16, X_2 = j0.08, X_0 = j0.06$

Feeder : $X_1 = j0.1, X_2 = j0.1, X_0 = j0.3.$

Determine fault current and line to neutral voltages at the generator terminals for a double line to ground fault at the other end of the feeder. Generator rated voltage is 11 kV.

Solution.

Total $X_1 = j0.26, X_2 = j0.18, X_0 = j0.36.$

Let $E_a = 1 + j0$ p.u. be reference. Fault occurs between b, c and ground, Figs. 22.6 and 22.7.

$$I_{a1} = \frac{1 + j0}{j0.26 + \frac{j0.18 \times j0.36}{j0.18 + j0.36}} = -j2.63 \text{ p.u.}$$

or
$$V_{a1} = E_a - I_{a1} Z_1 = 1 - (-j2.63)(j0.26) = 1 - 0.684 = 0.316$$

$$I_{a2} = \frac{-V_{a2}}{Z_2} = \frac{-0.316}{j0.18} = j1.75 \text{ p.u.}$$

$$I_{a0} = \frac{-0.316}{j0.36} = j0.875 \text{ p.u.}$$

Check : $I_{a2} + I_{a0} + I_{a1} = j1.75 + j0.875 - j2.63 = j2.625 - j2.63$

is approximate $= 0$

∴ $I_a = 0$ (Check) since fault is on b, c.

$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0} = -3.8 + j1.32 = 4.02 \angle 160° \text{ p.u.}$$

$$I_c = a I_{a2} + a^2 I_{a2} + I_{a0} = 3.8 + j0.32 = 4.02 \angle 20° \text{ p.u.}$$

Base current $= \dfrac{10 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 5.25 \times 10^2 \text{ A} = 225 \text{ A}$

$I_b = 4.02 \times 524 = 2110\angle 160°$

$I_c = 4.02 \times 524 = 2110\angle 20° \text{ A}$

Fault current $= I_b + I_c$

$= -3.8 + j1.32 - 3.8 + j1.32$

$= j2.64 \text{ p.u.} = j2.64 \times 524 = 1385 \text{ A}\angle 90°$

or $I_a = I_b + I_c = 3I_a = j0.875 \times 3 = j2.625 \text{ p.u.}$

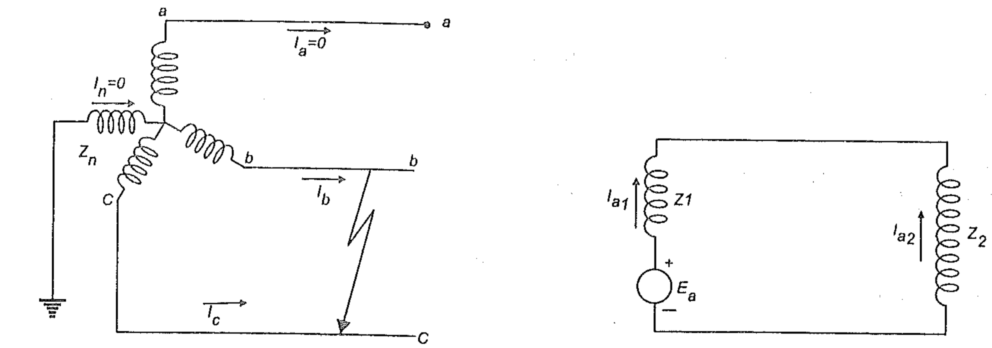
This current flows through ground.

Voltage at the terminal of generator

$$V_{a1} = E_1 - I_{a1} Z_1 = (1 + j0) - (-j2.63)\,(j0.16) = 0.58\angle 0° \text{ p.u.}$$
$$V_{a2} = -I_{a2}Z_2 = -j1.75 \times j0.08 = 0.14\ \angle 0° \text{ p.u.}$$
$$V_{a0} = 0 - I_{a0}Z_0 = -j0.875 \times j0.06 = 0.0525\angle 0° \text{ p.u.}$$
$$V_a = V_{a0} + V_{a1} + V_2 = 0.77 \text{ p.u. } \angle 0° = 4.9\ \angle 0° \text{ kV}$$
$$V_b = a^2V_{a1} + aV_{a2} + V_{a0} = -0.306 - j0.383 \text{ p.u.} = 3.12\angle 232° \text{ kV}$$
$$V_c = aV_{a1} + a^2V_{a2} + V_{a0} = 0.306 + j0.382 \text{ p.u.} = 3.12\ \angle 128° \text{ kV.}$$

22.6. LINE TO LINE FAULT ON UNLOADED ALTERNATOR (GENERATOR)

Let a, b, c be the three terminals of a generator whose neutral is grounded through an impedance Z_n. A fault occurs between lines b and c (Fig. 22.8). We have to determine the current and voltages for the fault condition, neglecting load current.



(*a*) Condition of fault (*b*) Connection of sequence network.

Fig. 22.8. *L*/*L* Fault.

Taking a look at the circuit conditions (Fig. 22.8) we can directly say the following :

$I_a = 0$

$I_n = 0$. Since the fault does not involve earth.

$V_b = V_c$

$I_b = -I_c$ means $I_c = -I_b$

Fault does not involve ground. Putting these conclusions in the equations we proceed as follows :

$$I_{a0} = \frac{I_n}{3} = 0$$
$$I_{a0} = -I_{a0}Z_0 = 0.$$

No zero sequence network for *L-L* fault need be considered.

$$I_{a1} = \frac{1}{3}\,[I_a + aI_b + a^2I_c]$$
$$I_{a2} = \frac{1}{3}\,[I_a + a^2I_b + aI_b]$$

Since $I_a = 0$ and $I_b = -I_c$, we get

$$I_{a1} = \frac{1}{3}\,(a - a^2)\,I_b$$
$$I_{a2} = \frac{1}{3}\,(a^2 - a)\,I_b$$

$\therefore$ $I_{a1} = -I_{a2}$.

To get these conditions the sequence networks of the generator are connected in parallel as shown in Fig. 22.2 (*b*).

Given the sequence impedances, we proceed as follows :

$$I_{a1} = \frac{E_a}{Z_1 + Z_2}$$
$$I_{a1} = -I_{a2}$$
$$I_{a0} = 0.$$

Thus $I_{a0}\, I_{a1}, I_{a3}$ are known from which I_a, I_b, I_c can be determined.

$$V_{a1} = E_a - I_{a1}Z_1 = V_{a2}$$
$$V_{a0} = 0,$$

From V_{a2}, V_{a1}, V_{a2} the voltages can be determined.

Example 22.6. *Part* (*c*) of example 20.1.

Given : *11 kV ; 25,000 kVA generator*

$X_1 = j0.25, X_2 = j0.35, X_0 = j0.1$ *p.u.*

Line to the fault on terminal b, c.

Solution.

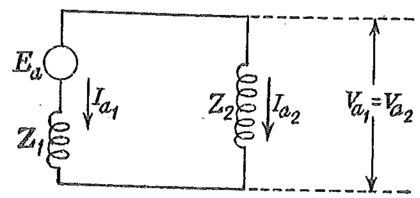


Fig. 22.9 of Ex. 22.6.

Let $E_a = 1$ p.u. $= 1 + j0 = \dfrac{11}{\sqrt{3}}$ kV

$$I_{a1} = -I_{a2} = \frac{E_0}{X_1 + X_2} = \frac{1 + j0}{j0.25 + j0.35} = -j1.667 \text{ p.u.}$$

$I_{a0} = 0$. Line to line fault does not involved ground. Hence $I_n = 0$. Hence $I_a = 0$

$$I_{a0} = 0$$
$$I_a = I_{a0} + I_{a2} + I_{a2} = 0 - j1.667 + j1.667 = 0$$
$$I_b = I_{a0} + a^2 i_{a1} + aI_{a2},$$
$$= 0 + (-0.5 - j0.866)\,(-j1.667) + (-0.5 + j0.866)\,(j1.667) = -2.892 \text{ p.u.}$$
$$I_c = -I_b = 2.892 \text{ p.u.}$$

Base current $= \dfrac{\text{Base kVA}}{\sqrt{3}\ \text{Base kV}} = \dfrac{25{,}000}{\sqrt{3} \times 11} = 1310 \text{ A}$

$I_B = 2.892 \times 1310 = -3786$ A

$I_C = +3786$ A

Voltages $V_{a1} = V_{a2} = -I_{a2}\,X_2 = -(j\,0.35)\,(+j1.667) = 0.584$ p.u.

Also $V_{a1} = E_a - Ia_1\,X_1 = (1 + j0) - (-j1.667)\,(j0.25) = 1 - 0.416 = 0.584$ p.u.

$$V_{a0} = 0$$
$$V_a = V_{a0} + V_{a1} + V_{a2} = 0.584 + 0.584 = 1.168\angle 0° \text{ p.u.}$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$
$$= 0 + (-0.5 - j0.866)(0.584) + (-0.5 + j0.866)(0.584) = -0.584 \text{ p.u.}$$
$$V_b = V_c = -0.584 \text{ p.u.}$$
$$V_{ab} = V_a - V_b = 1.168 + 0.584 = 1.752 \angle 0° \text{ p.u.}$$
$$V_{bc} = V_b - V_c = -0.584 + 0.584 = 0$$
$$V_{ca} = V_c - V_a = -0.584 - 1.168 = 1.752 \angle 180° \text{ p.u.}$$

Voltage in kV

$$\left.\begin{aligned} V_{ab} &= 1.752 \times \frac{11}{\sqrt{3}} = 11.15 \angle 0° \text{ kV} \\ V_{bc} &= 0 \\ V_{ca} &= 11.15 \angle 180° \text{ kV} \end{aligned}\right\} \textbf{Ans.}$$

Fault current $I_b = 3786 \text{ Amp.} = I_c$

Example 22.7. *A 3-phase generator has $X_1 = 0.15$ p.u., $X_2 = 0.15$ p.u. is with solidly grounded neutral. Calculate the ratio of the line currents for line-to-line fault to three phase fault.*

Solution.

Let e.m.f. $= E_a = 1 + j0$ p.u.

(I) Line-to-line fault on phases b, c.

$$I_{a1} = \frac{E_a}{X_1 + X_2} = \frac{1 + j0}{j0.15 + j0.15} = \frac{1}{j0.3}$$

Fault current $= I_a = a^2 I a_1 + a I_{a2} = (a^2 - a) I_{a1}$

$$= -1.732\, I_{a1} = 1.732 \left(\frac{1}{j0.3}\right) = j5.78 \text{ p.u.}$$

3-phase fault current

$$I_a = \frac{E_a}{X_1} = \frac{1}{j0.15} = -6.66 \text{ p.u.}$$

Ratio $\dfrac{5.78}{6.66} = \dfrac{\text{Line to line fault}}{\text{3 phase fault}} = \mathbf{0.868.}$ **Ans.**

Example 22.8. *A generator has the following sequence reactances : $X_1 = 60\%$, $X_2 = 25\%$ and $X_0 = 15\%$.*

(a) Calculate percentage reactance that should be added in the generator neutral such that the current for single line to ground fault does not exceed the rated current.

(b) Calculate value of resistance to be connected to neutral to achieve the same purpose.

Solution.

(*a*) $E_a = 1$ p.u. Let X_a be p.u. reactance added in the neutral connection.

Fault current for single line to ground fault

$$= 3\left(\frac{1 + j0}{X_1 + X_2 + X_0}\right) = \frac{3}{0.6 + 0.25 + 0.15 + 3X_n} = \frac{3}{1.00 + 3X_n}$$

To limit fault current to rated current, *i.e.* 1 p.u., we must get

$$1 = \frac{3}{1 + 3X_n}. \qquad \text{Hence } 3X_n = 2$$

$$X_n = \frac{2}{3} = 0.66 \text{ p.u.} = 66.6\%.$$

(*b*) Let resistance *r* p.u. be added to neutral connection

$$I_f = \frac{3E_a}{X_1 + X_2 + X_0} = \frac{3E_a}{X_1 + X_2 + X_{g0} + 3r} = \frac{3}{[3r + j1.0]}$$

$I_f = 1$ since current is to be limited to 1

$$1 = \frac{3}{[3r + j1]}$$
$$|3r + j1| = 3$$
$$\sqrt{1^2 + 9r^2} = 3$$
$$9r^2 = 9 - 1 = 8$$
$$r^2 = \frac{8}{9}$$
$$r = \frac{\sqrt{8}}{3} = \frac{2.85}{3} = 0.95 \text{ p.u.}$$

Hence resistance to be added in neutral to ground circuit to achieve the same purpose is 95 per cent.

Example 22.9. *Three alternators have identical constants given by $X_d' = 21\%$, $X_2 = 12\%$, $X_0 = 10\%$ are operating in parallel.*

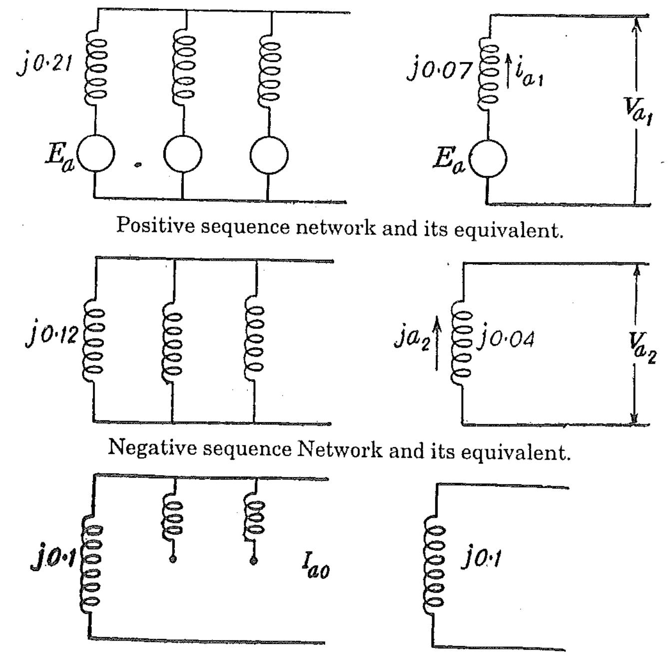
Neutral of only one is grounded solidly. Other machines have ungrounded neutral.

(1) Find short circuit current for line to ground fault.

(2) Determine the ratio in which the alternators contribute to the fault mentioned above

(3) How does a 3-phase short circuit current compare with line to ground fault current ?

Solution. Draw Thevenin's equivalents of three networks. Note that the zero sequence networks of ungrounded generators are open. Hence zero sequence component is contributed only by generator 1.



Positive sequence network and its equivalent.

Negative sequence Network and its equivalent.

Zero sequence Network and its equivalent.

Fig. of Ex. 22.9.

(*a*) For single line to ground fault, connect the equivalent networks in series

$$I_a = \frac{3E_a}{X_1 + X_2 + X_0} = \frac{3 \times 1}{j0.07 + j0.04 + j0.1} = \frac{3}{j0.21} = \mathbf{14.3 \text{ p.u.}} \quad \textbf{Ans.}$$

(*b*) **Contributions of Generators.** Only generator 1 whose neutral is grounded contributes to zero sequence component.

Total $I_{a0} = \dfrac{14.3}{3} = 4.77$ p.u.

Total $I_{a1} = \dfrac{14.3}{3} = 4.77$ p.u.

Total $I_{a2} = \dfrac{14.4}{3} = 4.77$ p.u.

Contributions

I_{a0} completely by generator $1 = 4.77$ p.u.

I_{a1} equally by 3 generators each $= 1.59$ p.u.

I_{a2} equally by 3 generators each $= 1.59$ p.u.

Contribution of generator I

$$= I_{a0} + I_{a1} + I_{a2} = 4.77 + 1.59 + 1.59 = 7.86 \text{ p.u.}$$

Contribution of generators II and III

$$= I_{a1} + I_{a2} = 1.59 + 1.59 = 3.18$$

Ratio of share = *7.86 : 3.18 : 3.18*

10 : 4.05 : 4.05

Generators 1, 2, 3 will share the short circuit currents in proportion 10 : 4.05 : 4.05.

(*c*) 3-phase fault current

$$= \frac{E_a}{X_{1eq}} = \frac{1}{j0.07} = -j14.3 \text{ p.u.}$$

Line to ground fault current

$$= \frac{3E_a}{X_1 + X_2 + X_0} = \frac{3 \times 1}{-j0.21} = -j14.3 \text{ p.u.}$$

Hence the two currents are equal.

Example 22.10. *Compare the fault currents of a generator with three-phase fault current.*

Solution. Let *a*, *b*, *c* be the terminals.

$$E_a = 1 \text{ p.u.}$$

For fault on terminal *a* to ground.

Single line to ground fault.

$$I_f = \frac{3E_a}{X_1 + X_2 + X_0} = \frac{3}{X_1 + X_2 + X_0} \quad \text{...(I)}$$

3-phase fault

$$I_f = \frac{E_a}{X_1} = \frac{1}{X_1} \quad \text{...(II)}$$

Line to line fault between B, C.

$$I_{a1} = \frac{E_a}{X_1 + X_2}$$

$$I_{a2} = \frac{E_a}{X_1 + X_2}$$

$$I_a = (a^2 - a)\frac{E_a}{X_1 + X_2} = 1.732\frac{E_a}{X_1 + X_2} \quad \text{...(III)}$$

$$= \frac{1.732}{X_1 + X_2}$$

Ratio of fault currents

$$\frac{L\text{–}G \text{ fault}}{3 \text{ phase fault}} = \frac{\text{(I)}}{\text{(II)}} = \frac{3X_1}{X_1 + X_2 + X_0}$$

$$\frac{L\text{–}L \text{ fault}}{3 \text{ phase fault}} = \frac{\text{(III)}}{\text{(II)}} = \frac{1.732\, X_1}{X_1 + X_2}$$

Summary

1. The generator positive sequence network consists of an e.m.f. source in series with reactance, which is transient/sub-transient or steady state reactance.

2. The negative sequence network has only negative sequence reactance.

3. The zero sequence network consists of $X_{g0} + 3X_n$, where X_n is the reactance in neutral connection.

These three networks connected as follows :

(*i*) *Single line to ground fault :* $I_{a0} = I_{a1} = I_{a2}$. Connect networks in series.

(*ii*) *Line to line fault :* $I_{a0} = 0$, $V_{a0} = 0$, $I_{a1} = -I_{a2}$.

Positive sequence network in parallel to negative sequence network.

(*iii*) *2 LG fault :* Connect the three networks in parallel

$$V_{a1} = V_{a2} = V_{a0}.$$

QUESTIONS

1. Define positive sequence impedance, negative sequence impedance, zero sequence impedance. Derive expressions for fault currents on an unloaded generator for single line to ground fault, line to line fault and 3 phase fault.

2. A single line to ground fault occurs on a cable connected to a 10,000 kVA, 3 phase, alternator with solidly earthed neutral. The positive negative and zero impedances of the generator are

$$0.5 + j4.7 \text{ ohms}, 0.2 + j0.6, j0.43 \text{ ohms}$$

respectively. The corresponding line values of cable are

$$0.36 + j0.25, 0.36 + j0.25, 2.9 + j0.95 \text{ ohms}$$

respectively. Line voltage 6.6 kV, calculate

(*i*) fault current,

(*ii*) voltages of sound lines to earth point.

3. Three 6600 V, 10,000 kVA, 3 phase alternators are connected in a parallel each has $X_1 = 0.15$ p.u., $X_2 = 0.75$ p.u., $X_0 = 0.30$ p.u. an earth fault occurs on one bus bar. Calculate fault current if

(*a*) all the alternators have solid neutral earthing :

(*b*) if one alternator neutral is solidly earthed ;

(*c*) if all the neutrals are isolated.

4. Derive the expressions for the ratios given below for a generator on no load.

(*a*) $\dfrac{\text{Line to ground fault current}}{\text{Line to line fault current}}$ (*b*) $\dfrac{\text{Line to line fault current}}{\text{3-phase fault current}}$

[**Hint.** Refer Ex. 22.10]

5. A 20,000 kVA, 13.8 kV generator has the following reactances :

Direct axis sub-transient reactance = 0.25 p.u.
Negative sequence reactance = 0.35 p.u.
Zero sequence reactance = 0.1 p.u.
Neutral is solidly earthed.

Fault occurs when the generator is on no load and rated terminal voltage. Calculate the fault currents and line to line voltage for the following :

(*a*) Line to ground fault (*b*) Line to line fault (*c*) 2L-G fault.

[**Ans.** (*a*) 3585 A ; 8.05 kV ; 15.73 kV ; 8.05 kV
(*b*) 2420 A ; 13.95 kV, 0 kV, 13.95 kV
(*c*) 4025 A ; 4025 A, 5.66 kV, 0 kV, 5.66 kV]

6. A generator has positive sequence reactance of 0.25 p.u. Calculate the p.u. reactance to be connected in series to limit the fault current for 3 phase fault to rated current.

7. A 3-phase generator has the following reactance :

$$X_1 = 0.20 \text{ p.u.}, X_2 = 0.20 \text{ p.u.}, X_0 = 0.1 \text{ p.u.}$$

reactance connected to neutral 0.3 p.u. calculate p.u. fault current for a

(*a*) Single line to ground fault

(*b*) Three-phase fault.

8. Calculate the single line to ground fault current for the generator if the neutral is solidly grounded. Given $X_1 = 0.58$ p.u., $X_3 = 0.25$ p.u., $X_0 = 0.1$ p.u.

9. Two generators rated 11 kV, 100 kVA having $X_1 = 0.15$, $X_3 = 0.12$, $X_0 = 0.1$ p.u. are operating in parallel a single line to ground fault occurs on the bus bar. Calculate the fault current if

(*a*) Both generator neutrals are solidly earthed ;

(*b*) only one generator neutral is solidly earthed ;

(*c*) both neutrals are isolated. (**Hint.** Refer Ex. 22.9)

10. For a generator the ratio of fault current for line to line fault and three phase faults is 0.866. The positive sequence reactance is 0.15 p.u. Calculate negative sequence reactance. (**Hint.** Refer Ex. 22.10)

11. A fault occurs on an unloaded generator. The zero sequence component of fault current for a single line to ground fault has to magnitudes of 100 Amperes. Calculate the current in the neutral to ground connection.

12. A 3 phase 132 kV system can be represented by a solidly earthed source, feeding a 132/33 kV star delta transformer whose star point is solidly earthed. An earth fault occurs on one of the 132 kV terminals when 33 kV side is not connected to load, determine the fault current to earth and current in transformer delta winding. The sequence reactances based on 100 MVA base are as follows :

Source : P.S. Reactance 20%
N.S. Reactance 15%
Zero seq. Reactance 10%
Transformer : 15% reactance [**Ans.** 3200 A, 985 A]

23

Faults on Power Systems

Sequence Networks — Connections of Transformers — Connections of Sequence Networks — Single Line to Ground Fault — Line to Line Fault — Double Line to Ground Fault on Power Systems — Solved Examples.

23.1. Sequence Networks

The positive sequence network was considered in analysing symmetrical faults. In positive sequence network only positive sequence voltages, positive sequence impedance and positive sequence current are effective. Positive sequence network is same as impedance or reactance diagram. The negative sequence network is one in which the negative sequence voltages, negative sequence currents and negative sequence reactances exist. Negative sequence networks are very much like positive sequence networks but differ in the following aspects :

(1) Normally there are no negative sequence e.m.f. sources.

(2) Negative sequence impedances of rotating machine is generally different from their positive sequence impedances.

The phase displacement of transformer banks for negative sequence is of opposite sign to that of positive sequence.

The zero sequence network differs greatly from the positive sequence, negative sequence networks in the following aspects :

(1) Z.S. Reactance of transmission lines is higher than that for positive sequence.

(2) Equivalent circuits for transformers are different.

(3) The neutral grounding should be considered in zero sequence network.

Zero sequence networks. As the zero sequence currents in three phases (I_{a0}, I_{b0}, I_{c0}) are equal and of same phase, three systems operate like single phase as regards zero sequence currents. Zero sequence currents flow only if return path is available through which circuit is completed.

Case I. **Star Connections**

Star connection without neutral wire or neutral ground. Zero sequence currents have no return path and, therefore, the zero sequence network is open. Beyond the neutral point the zero sequence currents find infinite impedance hence no zero sequence current flows [Fig. 23.1 (*a*)].

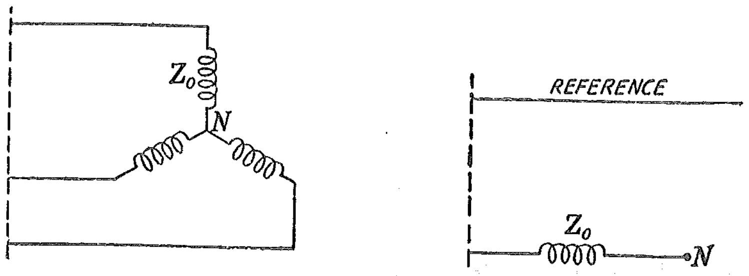
Case II. **Star connection with solid grounding of neutral**

The zero sequence current flows through the ground connection. Hence in the zero sequence network the point N is connected directly to the reference bus [Fig. 23.1 (*b*)].

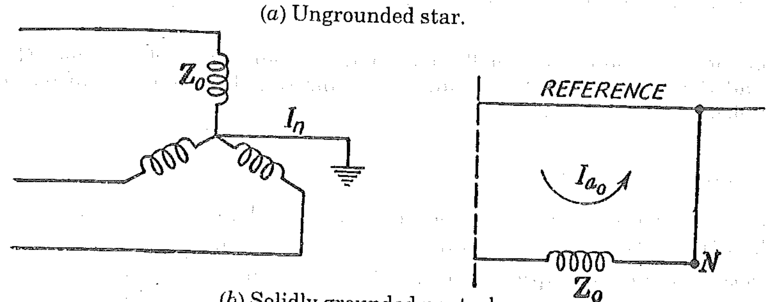
Case III. **Star connection with impedance grounding.** The neutral current $I_n = 3I_{a0}$ flows through the impedance Z_n connected between the neutral and ground. In the zero sequence network impedance $3Z_n$ is connected between N and reference, current flowing being, I_{a0}.

Case IV. Delta Connection

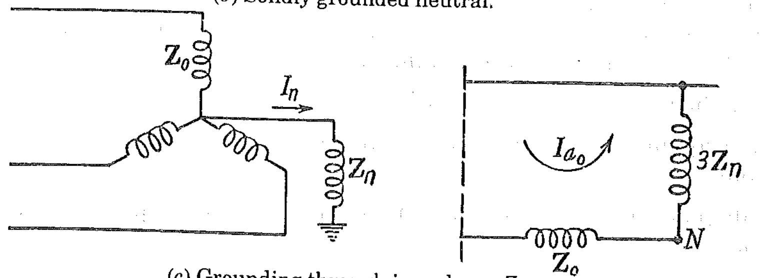
In delta connection the return neutral path is absent. Hence the zero sequence currents have no road to go ahead, they find the road to be suddenly stopped with infinite impedance ahead. How-



(a) Ungrounded star.



(b) Solidly grounded neutral.



(c) Grounding through impedance Z_n.

Fig. 23.1. Zero sequence equivalent circuits.

ever zero sequence currents may circulate in closed delta, if any zero sequence voltages are induced in the delta (Fig. 23.2).

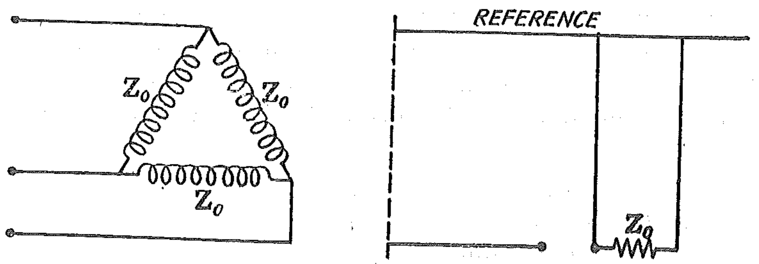


Fig. 23.2. Zero sequence circuit of delta.

Three phase transformer banks. The philosophy followed above for star and delta connection is valued for corresponding transformer connection. The zero sequence networks for different transformer connections are given in Fig. 23.3.

Example 23.1. *Fig. 23.4 represents a simple power system. Draw the positive sequence network, negative sequence network and zero sequence network.*

Connections of sequence networks. In chapter 22, we studied the unsymmetrical faults on an unloaded generator. The method consisted of connecting the sequence networks according to the type of fault. The procedure is extended to power systems. The three sequence networks are drawn. The fault point is indicated on the networks. The Thevenins's equivalents are drawn for each se-

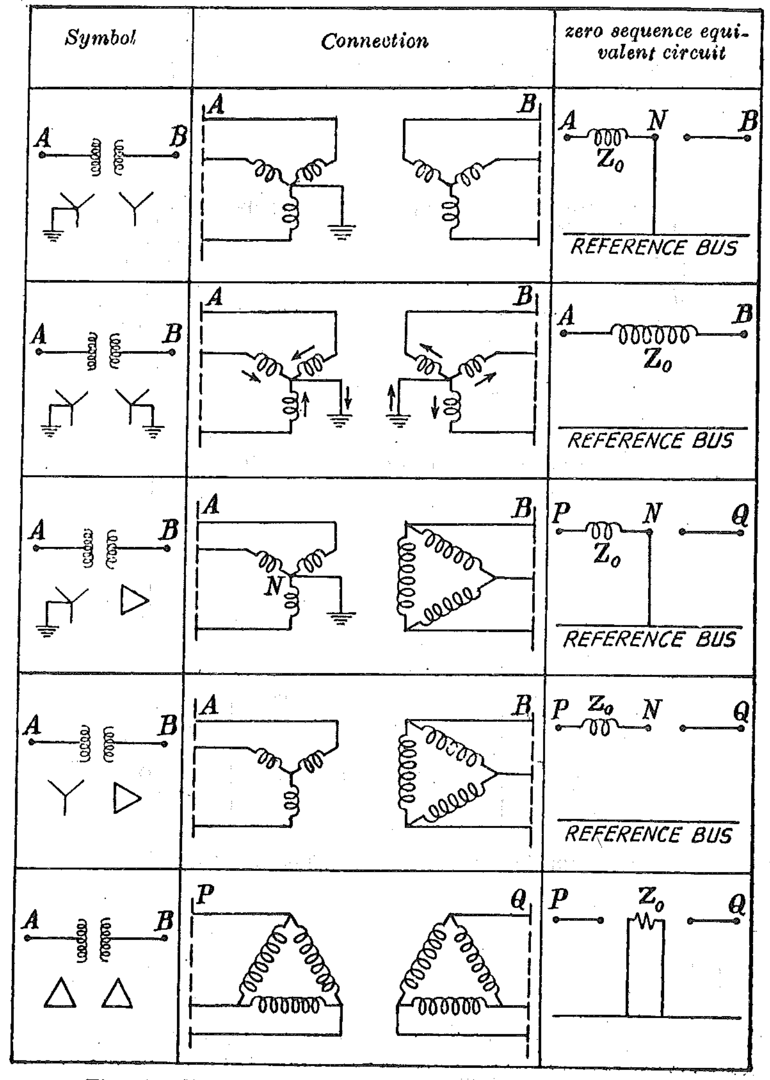


Fig. 23.3. Zero-Sequence Equivalent Circuits for Transformers.

quence network. Next, the three equivalent networks are connected in the same manner as connection of networks of single generator, *i.e.*

(1) Three Thevenin's equivalent networks are connected in series for single line to ground fault.

(2) The positive sequence equivalent and negative sequence equivalent are connected in parallel for line to line fault.

(3) The three equivalent networks are connected in parallel for double line to ground fault.

The sequence components of currents and voltage are calculated in the same way as those of generator (Ref. Ch. 22).

Example 23.2. *A synchronous generator (G) is connected synchronous motor (M). Both machine are rated at 1250 kVA, 600 V, with reactance $X'' = X_2 = 10\%$, $X_0 = 4\%$.*

Neutrals of both the machines are solidly grounded ; draw the sequence networks. Neglect reactance of busbars.

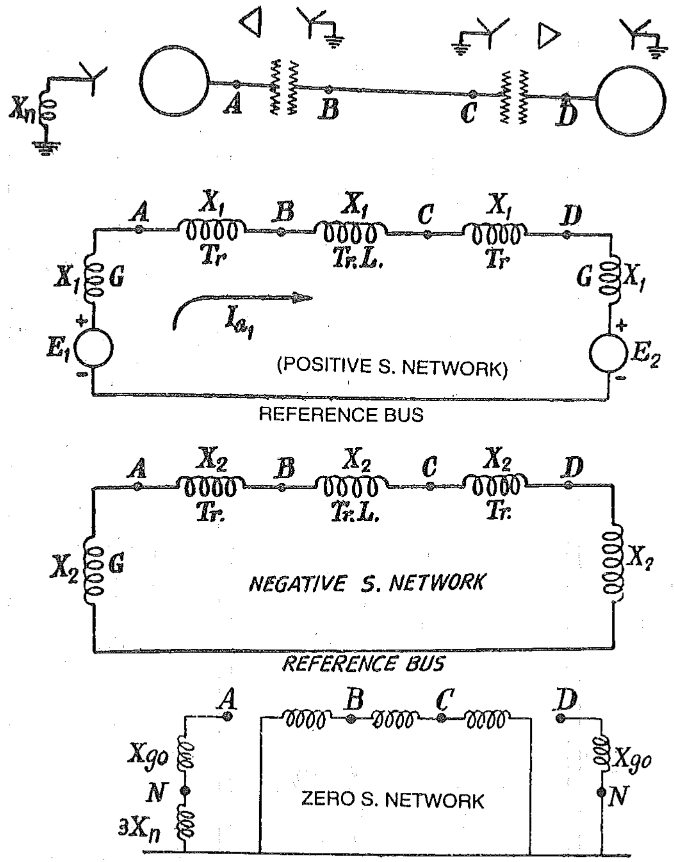
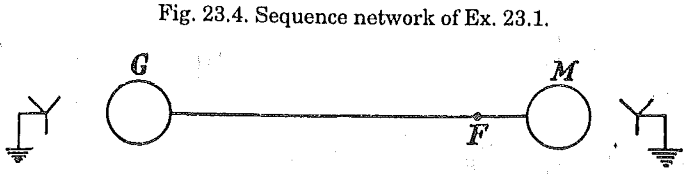
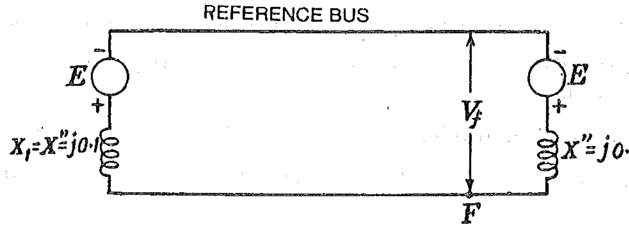


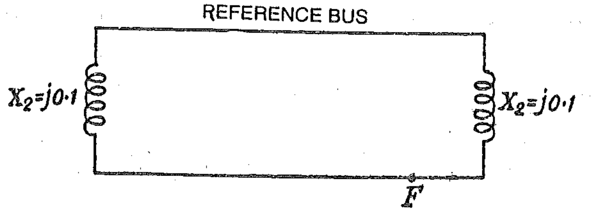
Fig. 23.4. Sequence network of Ex. 23.1.



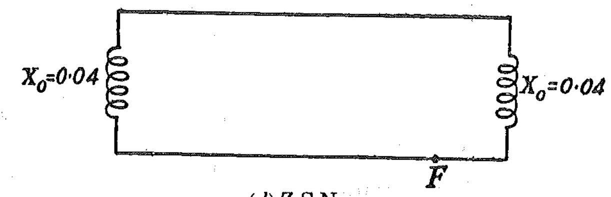
(*a*) Single line diagram.



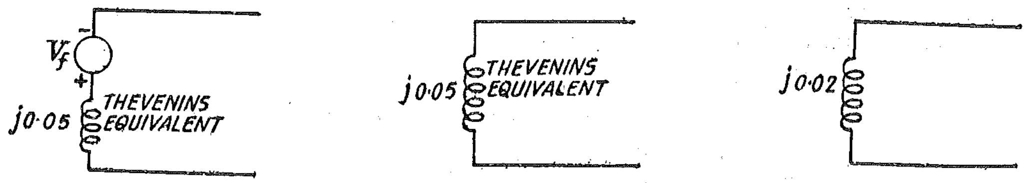
(*b*) P.S.N.



(*c*) N.S.N.



(*d*) Z.S.N.



(*e*) Thevenin's equivalent of P.S.N. (*f*) Thevenin's equivalent of N.S.N. (*g*) Thevenin's equivalent of Z.S.N.

Fig. 23.5. For Ex. 23.2.

A fault F occurs near the terminals of the motor. Draw the Thevenin's equivalents of the sequence networks. Neglect load current.

Solution.

Base kVA = 1250 kVA

Base Voltage = 600 V

Thevenin's equivalent reactance is obtained as follows :

The e.m.f. sources are replaced by short-circuit links. The reactance of the network looked from fault points is calculated which is Thevenin's equivalent reactance. For example in P.S.N., as seen from *F*, there are two reactances of $j\,0.1$ p.u. each. These are in parallel. Hence Thevenin's equivalent reactance is $j0.05$ (*e*).

Example 23.3. *A single line to ground fault occurs at 'F', of Example 23.2. Calculate fault current.*

Solution. For single *line* to ground fault connect the three equivalent networks in series, for fault on phase *a*.

$$I_{a2} = \frac{V_f}{X_1 + X_2 + X_0}$$

$$= \frac{1 + j0}{j0.5 + j0.5 + j0.02} = \frac{1}{j1.02} = -j\,0.98 \text{ p.u. Amp.}$$

$$I_{a1} = I_{a2} = I_{a0}$$

$$I_a = 3I_{a1} = 3 \times 0.98 \text{ p.u. Amp.} = 2.94 \text{ p.u. Amp.}$$

Base current in Amp.

$$= \frac{\text{Base kVA}}{\sqrt{3} \times \text{Base kV}} = \frac{1250}{\sqrt{3} \times 0.6} = 1200 \text{ Amp.}$$

∴ The fault current

$$= 1200 \times 2.94 = 3525 \text{ Amp. (r.m.s.)}$$

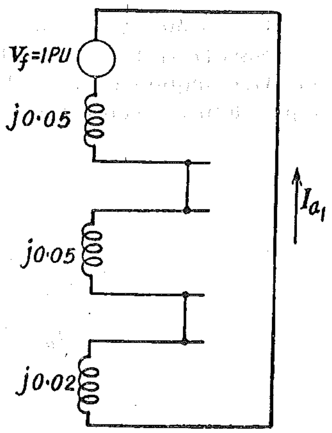


Fig. 23.6 for Ex. 23.3.

Example 23.4. *A double line to ground fault occurs at point F of the system given in Example 23.2. Calculate fault current.*

Solution. For double line to ground fault the Thevenin's equivalent networks of the three sequence networks are to be connected in parallel.

$$I_{a1} = \frac{V_f}{X_1 + \left[\dfrac{1}{\dfrac{1}{X_2} + \dfrac{1}{X_0}}\right]}$$

$$= \frac{V_f}{X_1 + \dfrac{X_2 X_0}{X_2 + X_0}} = \frac{V_f}{j0.05 + \dfrac{j0.05 \times j0.02}{j0.05 + j0.02}}$$

$$= \frac{V_f}{j0.05 + j0.0143} = \frac{V_f}{j0.0643} = \frac{1}{j0.0643} = -j15.55 \text{ p.u.}$$

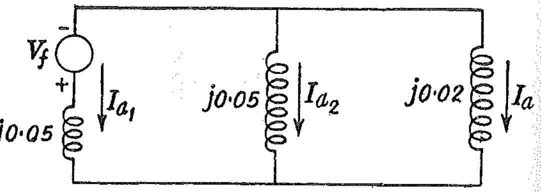


Fig. 23.7 for Ex. 23.4.

$$V_{a1} = V_f - I_{a1} X_1$$
$$= 1 + j0 - (-j15.55)\,(j0.05) = 1 - 0.775 = 0.225 \text{ p.u.}$$
$$I_{a2} = \frac{V_{a1}}{X_2} = \frac{-0.225}{j0.05} = +4.5 \text{ p.u.}$$
$$I_{a0} = \frac{-V_{a1}}{X_0} = \frac{-0.22}{j0.02} = j11.00 \text{ p.u.}$$

$\therefore$
$$I_{a1} = -j15.5$$
$$I_{a2} = +j4.5$$
$$I_{a0} = +j11.00$$
$$I_a = I_{a0} + I_{a1} + I_{a2} = 0 \text{ (Check)}$$

Fault current I_b for double line to ground fault is :

$$I_b = I_{a0} + a^2 I_{a2} + a I_{a1}$$
$$= +j11.00 + (-0.5 - j0.866)\,(j4.5) + (-0.5 + j0.866)\,(-j15.5)$$
$$= +j11.00 + [-j2.25 + 3.9] + [+j7.75 + 13.6]$$
$$= 17.5 + j16.5 = 22.7 \text{ p.u.}$$
$$I_b = 22.7 \times 1200 = 27{,}200 \text{ Amp.}$$

Example 23.5. *A line to line fault occurs on the system is Example 23.2. Calculate the fault current.*

Solution. For line to line fault, the zero sequence network is out of question, ignore it. The positive sequence networks Thevenin's equivalent and negative sequence networks Thevenin's equivalent are connected in parallel.

$$I_{a1} = \frac{V_f}{X_1 + X_2}$$
$$= \frac{1 + j0}{j0.05 + j0.05} = \frac{1}{j0.1} = -j10 \text{ p.u. Amp.}$$
$$I_{a2} = -I_{a1} = +j10 \text{ A}$$
$$I_a = 0 + j10 - j10 = 0$$
$$I_{a0} = 0$$

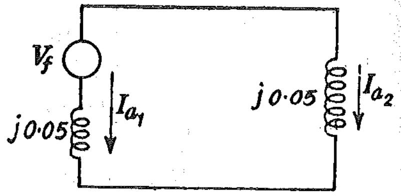


Fig. 23.8 for Ex. 23.5.

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$
$$= 0 + (-0.5 - j0.866)\,(-j10) + (-0.5 + j0.866)\,(+j10)$$
$$= +j5 - j8.66 - j5 - j8.66 = -j17.32 \text{ p.u.}$$
$$I_c = a I_{a1} + a^2 I_{a2} = j17.32$$

Fault current I_c = 17.32 p.u. = 17.32 × 1200 = 20,800 Amp.

Example 23.6. *A single line to ground fault occurs at point P of the system shown in Fig. 23.9. Find the sub-transient fault current neglecting pre-fault current. Both machines are synchronous and rated 1250 kVA, 1600 volts with reactances $X'' = X_2 = 10\%$. Each transformer is rated 1250 kVA,*

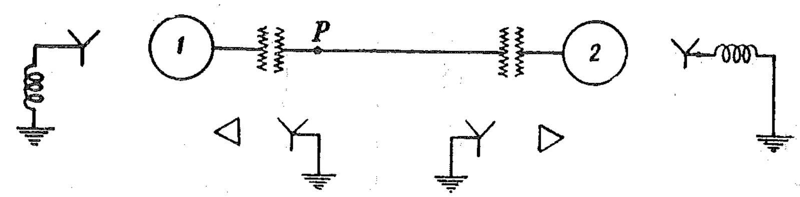


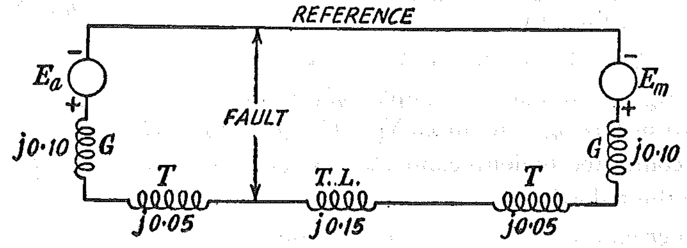
Fig. 23.9 for Ex. 23.6.

600—4160V, with leakage reactance 5%. Reactances of transmission line are $X_1 = X_2 = 15\%$, $X_0 = 50\%$ on 1250 kVA and 4.16 kV (Base Values).

Solution. Positive Sequence Network. Thevenin's equivalent impedance looked from the fault, shunting the e.m.f. sources.

$$Z_{th} = \frac{j0.15 \times j0.30}{j0.15 + j0.3} = \frac{-0.045}{j0.45} = j0.10 \text{ p.u.}$$

The load current is neglected. Hence the voltage at fault point is same as E_{a1} *i.e.*, 1 p.u. Hence Thevenin's equivalent of positive sequence networks is as follows :



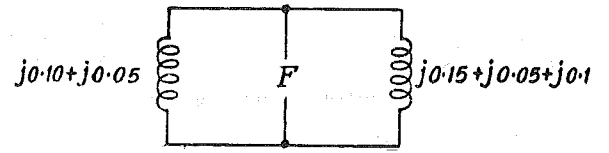


Fig. 23.10 for Ex. 23.6.

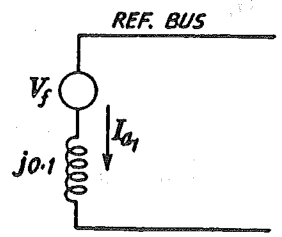


Fig. 23.11

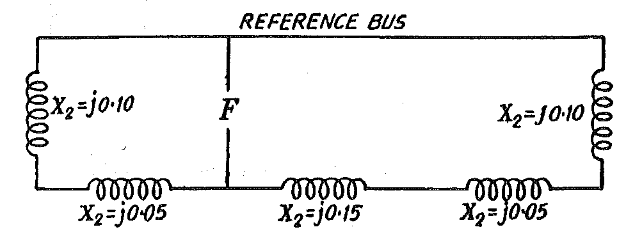


Fig. 23.12

Negative Sequence Network. The e.m.f. sources in PSN are absent in NSF. The PS reactances are replaced by *NS* reactance. Thevenin's equivalent is obtained by calculating equivalent reactance between fault points, after shunting the voltage sources.

$$Z_{th} = \frac{j0.15 \times j0.30}{j0.15 + j0.30} = \frac{-0.045}{j0.45} = j0.1$$

Zero Sequence Network

$$Z_{th} = \frac{j0.05 \times j0.55}{j0.05 + j0.55} = j\frac{0.0275}{0.6} = j0.0459 \text{ p.u.}$$

For the single line to ground fault connect the three equivalent networks in series.

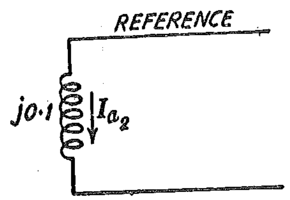


Fig. 23.13. N.S.N.

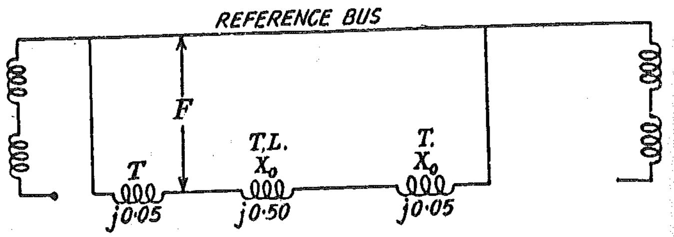


Fig. 23.14. Z.S.N.

$$I_{a1} = \frac{E}{X_1 + X_2 + X_0}$$

$$I_{a1} = I_{a2} = I_{a0}$$

$$= \frac{1 + j0}{j0.1 + j0.1 + j0.0459} = \frac{1}{j0.2459} = -j4.06 \text{ p.u.}$$

$$I_a = 3I_{a1} = -j3 \times 4.06 = -j12.18 \text{ p.u.}$$

$$\text{Base current} \quad = \frac{\text{Base kVA}}{\sqrt{3} \times \text{Base kV}} = \frac{1250}{\sqrt{3} \times 0.6} = 1200$$

$\therefore$ Fault current = 12.8 × 1200 = **15360 A.** **Ans.**

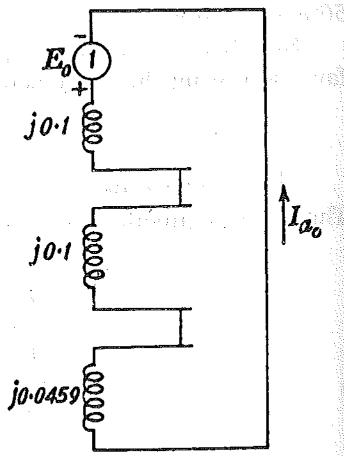
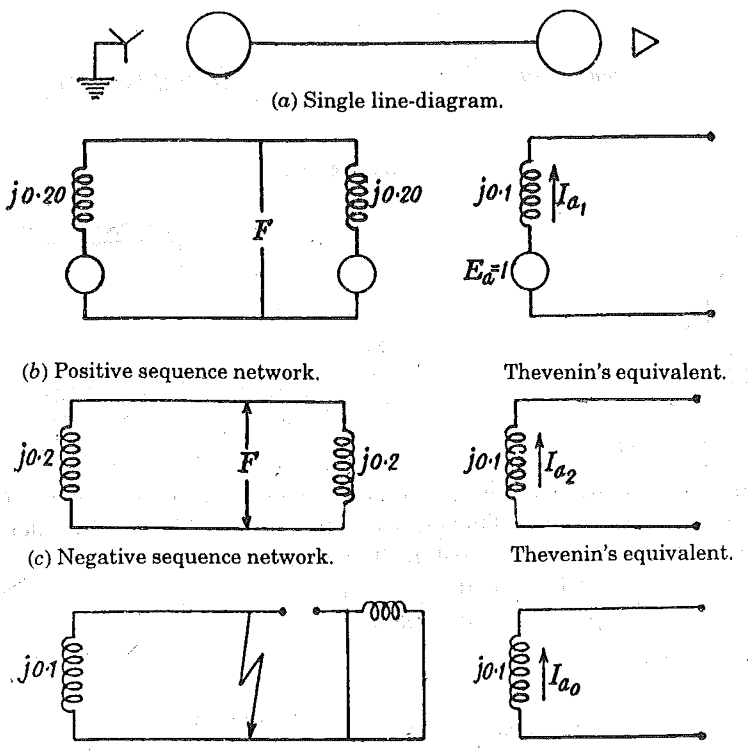


Fig. 23.15.

Example 23.7. Fig. 13.16 shows a simple system in which a star connected generator having reactances $X_1 = 0.20$ p.u., $X_2 = 0.2$ p.u., $X_0 = 0.1$ p.u. is connected to delta connected motor. Calculate the fault current for the following cases, neglecting load current.

(1) Single line to ground fault at the motor terminal.



(a) Single line-diagram.

(b) Positive sequence network. Thevenin's equivalent.

(c) Negative sequence network. Thevenin's equivalent.

(d) Zero sequence network. Thevenin's equivalent.

Fig. 23.16 for Ex. 23.7.

(2) Line to line fault at motor terminal.

Ratings are as follow :

Generator 11 kV, 1500 kVA.

Motor 11 kV, 1500 kVA.

Per unit reactances of motor are same as that of generator. Generator neutral is solidly grounded. Reactance of tie bar is negligible.

Solution. *Case I.* **Single line to ground fault (*L-G*).** Connect the three equivalents of sequence networks in series as in Fig. 23.17.

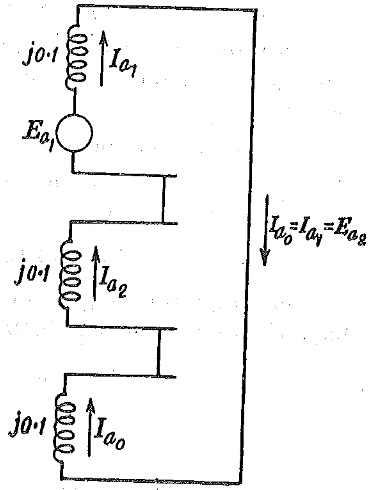


Fig. 23.17. Connection for *L-G* fault.

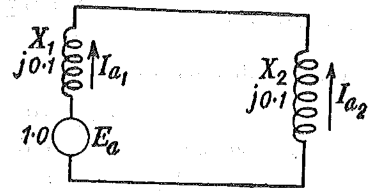
$$I_a = 1 + j0 \text{ p.u.}$$

$$I_a = 3I_{a1}$$

$$= \frac{3E_a}{X_1 + X_2 + X_0} = \frac{3\,(1 + j0)}{j0.1 + j0.1 + j0.1}$$

$$= \frac{3}{j0.3} = -j10 \text{ p.u.} = 10 \times \frac{1500}{\sqrt{3} \times 11} = 787 \text{ Amp.}$$

Case II. **L-L Fault.** Connect the positive and negative sequence reactance equivalent parallel.



$$I_{a1} = -I_{a2}$$

$$= \frac{E_a}{X_1 + X_2} = \frac{1 + j0}{j0.2} = j5 \text{ p.u.}$$

$$I_{a0} = 0$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

$$I_a = I_{a0} + a^2 I_{a1} + aI_{a2}$$

$$= (a^2 - a)\, I_{a1} = (1.732)\, 5 = 8.660 \text{ p.u.}$$

$$= 8.66 \times \frac{1500}{\sqrt{3} \times 11} \text{ Amp.}$$

$$= 8.66 \times 78.7 = 683 \text{ Amp.}$$

Example 23.8. *A 3-phase 37.5 MVA, 33 kV alternator having $X_1 = j0.18$ p.u., $X_2 = j0.12$ p.u. and $X_0 = j0.10$ p.u. based on its ratings is connected to a 33 kV overhead transmission line having the following reactances : $X_1 = 6.6$ ohms, $X_2 = 6.3$ ohms and $X_0 = 12.6$ ohms per conductor. A single line to ground fault occurs at the remote end of the transmission line. The alternator star point is solidly earthed. Calculate the fault current and phase voltages.*

Solution. Select base kVA and base kV.

Let Base MVA (3 phase) = 37.5 MVA

Base kV (line to line) = 33 kV.

$$\text{Base impedance} \quad = \frac{(\text{Base kV})^2 \times 1000}{\text{Base kVA}} = \frac{(33)^2 \times 1000}{37.5 \times 1000} = 29.0 \text{ ohms}$$

P.u. reactances of transmission line are

$$X_1 = \frac{j6.3}{29.0} = j0.2165 \text{ p.u.}$$

$$X_2 = \frac{j6.3}{29.0} = j0.217 \text{ p.u.}$$

$$X_0 = \frac{j12.6}{29.0} = j0.434 \text{ p.u.}$$

P.u. reactances of generator are :

$$X_1 = j0.18 \text{ p.u.}$$
$$X_2 = j0.12 \text{ p.u.}$$
$$X_0 = j0.10 \text{ p.u.}$$

Per unit reactances between generator and fault, is the sum of the generator p.u. reactances and transmission line p.u. reactances.

Total p.u. reactances are, therefore,

$$X_1 = j0.18 + j0.2170 = j0.397 \text{ p.u.}$$
$$X_2 = j0.12 + j0.2170 = j0.337 \text{ p.u.}$$
$$X_0 = j0.10 + j0.434 = j0.534 \text{ p.u.}$$

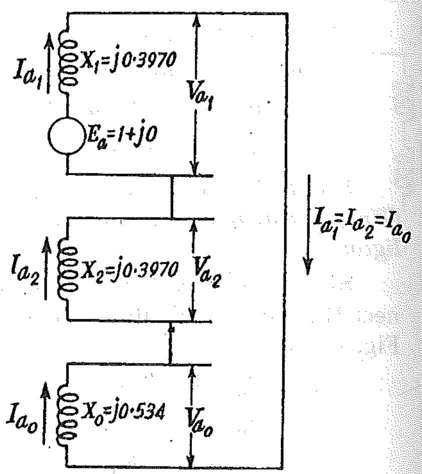


Fig. 23.19. Connection of sequence networks of Ex. 23.8.

For single line to ground fault connect the three sequence networks in series (Fig. 23.19).

$$I_{a1} = I_{a2} = I_{a0}$$
$$= \frac{E_a}{X_1 + X_2 + X_{g0}} = \frac{E_a}{X_1 + X_2 + X_{g0} + 3X_n}$$

Here X_n, *i.e.* the reactance between neutral point and ground is zero, since the earthing (grounding) is solid one.

Hence $$I_{a1} = \frac{1 + j0}{j0.3970 + j0.3370 + j0.534} = \frac{1 + j0}{j1.268} = -j0.790 \text{ p.u.}$$

$$I_a = I_{a1} + I_{a2} + I_{a0} = 3I_{a1} = 3 \times (-)j0.790 = -j2.370 \text{ p.u.}$$

Base current $$= \frac{\text{Base MVA}}{\sqrt{3}\text{ Base MV}} = \frac{37.5}{\sqrt{3} \times 0.033} = 656 \text{ Amp.}$$

Hence fault current I_a

$$= -j2.370 \times 656 = -j1557 \text{ Amp.}$$

Sequence voltages line to neutral at the terms of the alternator.

$$V_{a1} = E_a - I_{a1} X_1$$
$$= (1 + j0) - (-j0.791)(j0.18) = 0.858 + j0 \text{ p.u.}$$
$$V_{a2} = -I_{a2}X_2 = -I_{a1}X_2$$
$$= -(-j0.791)(j0.12) = -0.0948 + j0 \text{ p.u.}$$
$$V_{a0} = -I_{a0}X_0 = -I_{a1}X_0$$
$$= -(-j0.791)(j0.1) = -0.0791 + j0 \text{ p.u.}$$
$$V_a = V_{a1} + V_{a2} + V_{a0}$$
$$= 0.6841 \angle 0° \text{ p.u.} = 0.684 \times 33/\sqrt{3} = 13 \text{ kV}$$
$$V_b = a^2 V_{a1} + aV_{a2} + V_{a0} = 0.945 \angle -119° \text{ p.u.} = 0.945 \times \frac{33}{\sqrt{3}} = 18 \text{ kV} \angle -119°$$

Since $$E_a = \frac{33}{\sqrt{3}} = 1 \text{ p.u. (phase)}$$

$$V_0 = aV_a + a^2V_{a2} + V_{a0} = 0.945 \angle +119° \text{ p.u.} = 18 \angle +119° \text{ kV.}$$

Example 23.9. *A generator transformer unit shown in Fig. 23.20 is on no load and the voltage on H.T. side is 70 kV when a fault occurs at F. Calculate fault currents if the fault is*

(1) 3 phase fault ;

(2) Line to line fault ;

(3) Single line to earth fault. [*Note* : Change in Voltage]

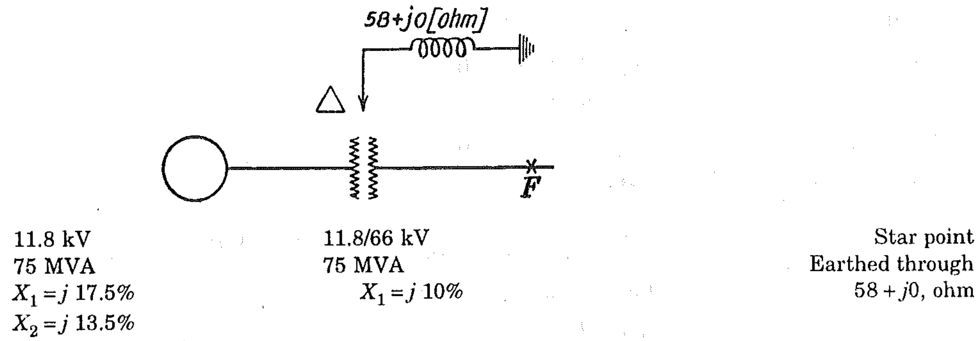


Fig. 23.20. of Ex. 23.9.

Solution. Let base MVA 100

p.u. reactances to this base are

$$X_{g1} = j0.175 \times \frac{100}{75} = j0.234 \text{ p.u.}$$
$$X_{g2} = j0.135 \times \frac{100}{75} = j0.180 \text{ p.u.} \quad \text{(of Generator)}$$
$$X_1 = j0.10 \times \frac{100}{75} = j\ 0.133 \text{ p.u.} \quad \text{(of transformer)}$$

Zero sequence impedance of the neutral earthing resistor is given by

$$Z_{n0} = 3Z_n = 58 \times 3 = 174 + j0 \text{ ohm}$$

Base current $$= \frac{\text{Base MVA} \times 10^3}{\sqrt{3} \times \text{Base kV}} = \frac{100 \times 10^6}{\sqrt{3} \times 66{,}000} = 875 \text{ A}$$

Base voltage phase to neutral $= \frac{66{,}000}{\sqrt{3}} = 38{,}100$ V

Base impedance on 100 MVA, 66 kV base

$$= \frac{38{,}100}{875} = 43.5 \text{ ohm}$$

or Base impedance $$= \frac{(66)^2 \times 1000}{100{,}000} = 43.5 \text{ ohm}$$

$$\text{P.u. } Z_{g0} = \frac{174 + j0}{43.5} = 3.99 + j0 \text{ p.u.}$$

Pre-fault voltage at $F = 70$ kV. **Note the change in voltage**

$$= \frac{70}{60} = 1.060 \text{ p.u.}$$

Note :
$$E_a = 1.060 + j0 \text{ p.u.}$$
$$X_1 = j0.367 \text{ p.u.}$$
$$X_2 = j0.313 \text{ p.u.}$$
$$Z_0 = 3.99 + j0.133 \text{ p.u.}$$

Case I. **Three phase fault** [Note use of a^2, a]

$$I_{a1} = I_a = \frac{E_a}{X_1} = \frac{1.06}{j0.367}$$
$$I_a = -j2.89 \text{ p.u.}$$
$$I_b = a^2 I_{a1} = -2.50 + j1.445 \text{ p.u.}$$
$$I_c = aI_{a1} = 2.50 + j1.445 \text{ p.u.}$$

Multiplying I_a, I_b, I_c by base current 875 A,

$$I_a = 2526 \underline{/ -90^\circ} \text{ A}$$
$$I_b = 2526 \underline{/ +150^\circ} \text{ A}$$
$$I_c = 2526 \underline{/ +30^\circ} \text{ A}$$

Case II. **Line to line fault on lines b, c.**

$$I_{a1} = \frac{E_a}{X_1 + X_2} = \frac{1.060 + j0}{j0.680} = -j1.560 \text{ p.u.}$$
$$I_{a2} = -I_{a1} = j1.560 \text{ p.u.}$$
$$I_{a0} = 0$$
$$I_a = I_{a0} + I_{a1} + I_{a2} = 0 \text{ (check)}$$
$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0} = -2.7 + j0 \text{ p.u.}$$
$$I_c = a I_{a1} + a^2 I_{a2} + I_{a0} = +2.7 + j0 \text{ p.u.} = 2.7 \times 874 \text{ A} = 2360 \text{ A.}$$

Case III. **Single line to ground fault on line a.**

$$I_{a1} = I_{a2} = I_{a0}$$
$$= \frac{E_a}{X_1 + X_2 + Z_0} = \frac{1.060 + j0}{3.98 + j0.052} \text{ p.u.} = 0.255 - j0.052 \text{ p.u.}$$
$$I_a = 3I_{a0} = 0.765 - j0.156 \text{ p.u.}$$
$$I_b = a^2 I_{a1} + a I_{a2} + I_{a2} = 0$$
$$I_c = a I_{a1} + a^2 I_{a2} + I_{a0} = 0$$

I_a in amperes $= 874 \times I_a$ p.u. $= 683 \underline{/ -11.30^\circ}$.

Summary

For faults on power systems, the following procedure is adopted :

(1) Draw single line diagram of power system.

(2) Choose same kVA base for complete system. Choose different kV bases for each voltage level. Convert the reactances to p.u. reactances.

(3) Draw positive sequence network, obtain its Thevenin's equivalent. Repeat it for negative and zero sequence networks.

(4) For *L-G* fault connect the Thevenin's equivalents of the three circuits in series, proceed as in the case of fault on generator.

(5) Connect the positive sequence and negative sequence equivalent in parallel for line to line fault.

(6) Connect the three equivalent networks in parallel for double line ground fault.

The sequence components are calculated and from them, the fault current and voltages are calculated.

QUESTIONS

1. A double line to ground fault occurs on lines *b* and *c* at the point *P* in the circuit whose diagram is shown below. Find subtransient current in phase *a* of the machine. 1. Neglect prefault current. Both machines are rated at 1250 kVA, 600 V with reactances $X'' = X_2 = 10\%$; $X_0 = 4\%$ each three phase transformer is rated 1250 kV, 600 V delta/4160 V star with leakage reactances 5%. The reactance of transmission line is $X_1 = 15\%$, $X_2 = 15\%$, $X_0 = 50\%$ based on 1250 kVA, 4.16 kV = bases. Point *P* at centre of line.

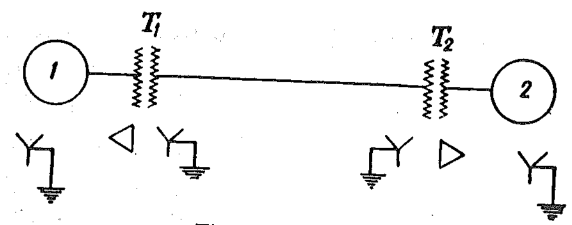


Fig. 23.21 of Q. 1.

2. A 5000 kVA, 13.8 kV generator is star-connected and grounded through a reactance of 2.5%. The reactances of generator are $X'' = X_2 = 10\%$ and $X_0 = 2.5\%$. The generator supplies a delta connected motor rated 2500 kVA, 13.8 kV with reactances $X'' = X_2 = 20\%$, $X_0 = 10\%$. A single line to ground fault occurs near the motor, find the initial symmetrical fault current. Neglect load current.
3. A 11 kV, 15 MVA generator having $X_1 = 20\%$, $X_2 = 20\%$, $X_0 = 10\%$ is connected to transformer rated 11/33 kV 15 MVA having $X_1 = 5\%$. The transformer is delta connected on L.T. side and star connected on H.T. side. Neutral of generator and transformer is solidly earthed ; calculate fault currents for
   (*a*) Single line to ground fault on H.T. side L.T. side.
   (*b*) Double line to ground fault on H.T. side, L.T. side.
   (*c*) 3-phase fault on H.T. side, L.T. side.
4. A generator transformer unit shown in Fig. 23.22 is supplying to a h.t. line. A fault occurs in the line at point *F*. Calculate the fault current for
   (*a*) 3 phase fault
   (*b*) *L-G* fault
   (*c*) *L-L* fault.
   (*d*) 2 *L-G* fault.

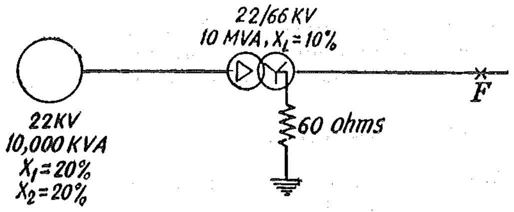


Fig. 23.22.

5. Three generators are operating in parallel.
   G_1 : 15 MVA, 12 kV, $X_1 = X_2 = 20\%$, $X_0 = 10\%$
   G_2 : 15 mVA, 11 kV, $X_1 = X_2 = 20\%$, $X_0 = 10\%$
   G_3 : 10 MVA, 11kV, $X_1 = X_2 = 20\%$, $X_0 = 10\%$.

   Neutral of G_1 grounded solidly. Neutral of G_3 is grounded through a reactance of 5% based on generator ratings. A fault occurs on generator bus. Calculate fault current for
   (*a*) *L-G* fault. (*b*) 2 *L-G* fault. (*c*) *L-L* fault.
6. A star connected synchronous motor is connected directly to a generator by means of bus bar of negligible reactances.

   Reactances of generator and motor are as follows :

   $$X_1 = 0.2 \text{ p.u.}, X_2 = 0.2 \text{ p.u.}, X_0 = 0.1 \text{ p.u.}$$

   Both rated at 11 kV, 1500 kVA.

   Generator star connected with solid neutral earth.

   Motor Delta connected.

   Calculate fault current for terminal *L-L* fault, *L-G* fault. [**Ans.** 787 A, 683 A]

24

Use of A.C. Network Analyser and Digital Computer in Fault Calculations

Introduction — Problems on Network Analyzer — Description of A.C. Network Analyzer — Procedure of short circuit study on network analyzer — Digital Computer — Organization processes for solving engineering problem — Short circuit studies on digital computers

24.1. INTRODUCTION

Fault calculation of simple systems can be carried out with a calculator. Modern power systems are large and complex as they consist of generating stations, transmission lines, load centres etc. Fault calculations of such systems by direct means is laborious and time consuming. A.C. Network, Analyser, also called A.C. Calculating Board is being used for power system studies since 1929. A.C. Network Analyzer is in fact a small scale, single phase replica of the power system. It consists of component such as generators, transformers, load units impedance units, capacitors etc. which correspond to the components of the actual power system. The components of the network analyzed are connected such that it represents the power systems under study. For fault calculations, the fault is applied at different points of the analyzer and the fault currents and voltages are noted. The corresponding readings in actual values are obtained on multiplication by the scale factor. Besides fault calculations, the A.C. Network analyzer is used for stability studies, load flow studies, economic operation studies etc. But the use of Network Analyzer is restricted to the problem in power system alone. Digital computers are now being used for solving almost all the power system problems. No new Network Analyzers are installed any more. Digital computers are versatile computing devices, which can be used for solving a variety of technological, engineering, scientific, commercial and management problems. This chapter gives only an introduction the use of network analyzer and digital computer for fault calculations.

HVDC simulator is used for simulating various abnormal conditions in HVDC system and associated AC Networks.

24.2. A.C. NETWORK ANALYZER (A.C. CALCULATING BOARD)

(A) Problems that can be solved on an A.C. network analyzer.

1. Load studies.
2. Stability studies.
3. Special circuit problems.
4. Short circuit studies.

We will study the short circuit problems briefly. These include :

(*a*) Maximum short circuit current at different location for determining the short circuit duties of the circuit-breakers.

(*b*) Bus voltage during short circuits.

(*c*) Maximum and minimum fault currents and voltages for determining the relay settings.

(*d*) Effect of various types of neutral grounding on behaviour of power systems, etc.

(B) Description. Most A.C. network analyzer operate on 400 Hz or 480 Hz, though there are some, operating on 60 Hz or 10,000 Hz. Higher frequency permits smaller size of reactors.

An A.C. network analyzer consists of a number of independent single phase units such as generator units, variable resistors, reactors and capacitors, auto-transformers etc. The units can be arranged, adjusted and connected to have a circuit which represents the system under study. Sensitive measuring instruments ammeters, voltmeters, wattmeters, varmeters are provided for making electrical measurements at any point of system. For symmetrical fault calculations a single phase network is employed. For unsymmetrical fault the method of symmetrical components is used in which three sequence networks are involved. The nominal voltage or base voltage for a typical network analyzer is 50 volts and base current 50 milliamperes. Thereby base power is 2.5 watts and base impedance is 1000 ohms. All adjusting dials and instruments are marked in per unit of these base quantities. The 400 Hz supply is obtained from a motor generator set, or frequency converters.

Note. Figures in bracket give number of units in a particular network analyzer.

Generator Units (16). These represent the e.m.f. sources of actual power system. These are provided with independent phase shifter and voltage regulator. Each generator unit is equipped with voltmeter, varmeter. The output is single phase.

Line impedance Units (76). These consist of variable resistors and reactor connected in series and are used for representing transmission lines.

Load Impedance Units (50). These are adjustable resistors and reactors which are connected either in series or parallel to represent loads. The load impedance units have higher impedance rating than the line units.

Auto Transformer Units (32). Represent transformer.

Capacitor Units (48). Represent capacitances of cables, overhead transmission lines, capacitors, synchronous condensers.

Synchronous Impedance Units (16). Consist of adjustable resistors and reactors connected in series intended to represent synchronous impedances of machines.

Mutual Transformer Units (8). These are 1:1 ratio transformers which represent mutual reactance between parallel transmission lines which are not connected at the ends.

Master Instrument System. Consists of measuring instruments, metering selector panel, selector switches, etc.

(C) Procedure of fault calculation (Brief).

— To represent the power system by positive, negative and zero sequence network, chosen to common base MVA.
— To represent the network by network analyzer by choosing appropriate units mentioned in Sec. 23.3 (B) and connecting them such that the system is represented.
— The voltages and loads are adjusted to represent load condition.
— The fault is applied by plugging in the fault plug at desired point.
— The values of voltages and currents at various locations are measured.
— The multiplying factors are used to obtain the values corresponding to the represented system.

Three phase Faults. The network analyzer is arranged to represent the system under study. The voltages and loads are adjusted to obtain load balance for desired loading condition. Faults are applied by plugging the fault plug at a desired point of the system. The measurement of fault current is done by pressing the keys on the instrument panels.

Unsymmetrical Faults. The sequence networks (Refer Ch. 22) are arranged on the network analyzer. Very often the positive and negative sequence networks are further simplified with the assumption that they are identical. The study involving line to line faults do not require zero sequence network.

24.3. DIGITAL COMPUTERS

Digital computer may be defined as device which compute by arithmetic processes. The basic processes employed are addition and subtraction. These processes are also used in successive approximation or interaction to achieve function values, integration, solution of algebraic equations and linear and non-linear differential equations. Fundamental principles of the digital computer were set forth by the English Mathematician Charles Babbage in 1940. After the period 1944 several computers are built. High speed electronic computer such as IBM 7090, [International Business Machines Corporation] has the following execution speeds (including time required to take a number from or to put a number into core storage).

Addition	4.36μ sec.
Multiplication	25.3μ sec.
Division	31.0μ sec.

Digital computer is versatile computing device being increasingly used by power system engineers.

24.4. ORGANIZATION OF A DIGITAL COMPUTERS*

Fig. 24.1 illustrates the components of a digital computer, which are commonly known as input, control, memory, arithmetic and output. The operator supplies instructions (programme) to the input unit of the computer in the form of punched cards or type written instructions. The instructions are stored in 'memory'. The stored programme includes both 'instruction' words and 'data' words. The 'instruction' states the operation, to be performed such as multiplication, division, addition, printing or so. Each work of the instruction consists of two parts. Type of operation mentioned above and secondly 'address' *i.e.* the storage location in the memory for the data to be used. The instructions are in the binary form when they reach the control. The control separates the operation code and the addresses according to their position in the instruction word. The control unit issues orders (electrical pulses) to withdraw the data numbers from the memory and place them in appropriate perform registers. The control then activates the necessary circuits to operation the specified arithmetic operations. When the operation is completed, the next instruction word is taken up by the control from the memory and the specified operation is performed again. The operations are performed at a high speed until the instructions are carried out. The answer are printed or typed.

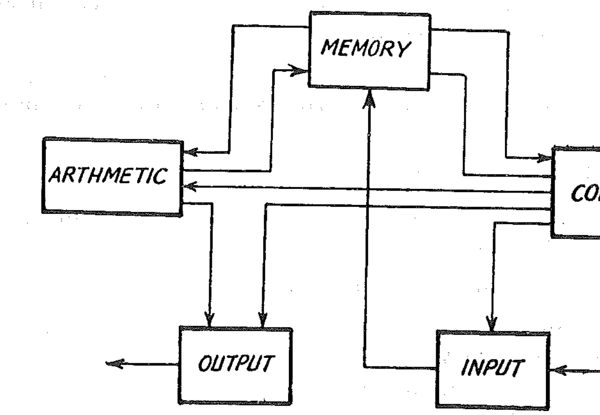


Fig. 24.1. Components of Digital Computer.

24.5. PROCESS OF SOLVING ENGINEERING PROBLEMS ON DIGITAL COMPUTERS

The process involves several steps, which include the following :

1. **Problems definition.** The problem to be solved is defined, its objectives are determined. Data required for input are acquired.

2. **Mathematical formulation.** The problem is developed into mathematical equations.

3. **Selection of solution technique.** The formulation of most of the engineering problems involves mathematical expressions, such as non-linear differential equations, trigonometric functions which cannot be directly evaluated by the computer since the computer can perform only four basic arithmetic operation namely addition, subtraction, multiplication and division. Numerical techniques are employed for solution of the problem.

* Ref. Ch. 46 for Microprocessors and Ref. Sec. 46.2 for Terms and Definitions.

4. **Programme Design.** The important aspects of programme design are :

(*a*) Sequence of logical steps by which particular problem is solved.

(*b*) Allocation of memory.

(*c*) Access of data.

(*d*) Assignment of inputs.

The objectives are primarily to develop a procedure which eliminates unnecessary repetitive calculations and remains within the capability of computer. The programme design is usually prepared in the form of a diagram called *Flow Chart.*

5. **Programming.** A digital computer is supplied a series of instructions consisting of operation codes and addresses which it is able to interpret and execute. In addition to the arithmetic input output instructions, logical instruction are available which are used in direct sequence of calculations. The programme can be developed by using computer instructions in actual and symbolic form or can by written in generalized programme language.

6. **Programme Verification.** There are chances of errors while developing a complete programme. Therefore, a systematic and series must be performed to ensure correctness of problem formulation, method of solution and operation of programme.

24.6. (I) SHORT CIRCUIT STUDIES ON DIGITAL COMPUTER

A typical short circuit programme which is designed to calculate fault currents needs positive sequence and zero sequence impedance matrices. The input data describing the system is specified using the names of the power stations, names of sub-stations, data of system components, voltage levels, reactances etc. The first programme assigns sequential bus number and then rearranges the network data to facilitate formation of positive sequence and zero sequence impedance matrices. During this phase, extensive data checks are performed. Next, the positive sequence bus impedance matrix is formed. This matrix is temporarily stored on an auxiliary storage device to provide space in memory for the next programme segment. Then zero sequence impedance matrix is formed and positive sequence matrix is retrieved for use in fault calculations. Since these matrices are symmetrical, only the diagonal elements and the upper of diagonal element need to be formed and stored. The sequences steps are shown in Fig. 24.2. The short-circuit MVA's are calculated for each bus and tabulated with corresponding station name. The following results are obtained :

1. Total 3-phase and line to ground fault current.

2. Contribution for the above from each line to connected to the fault bus.

3. Currents when the lines connected to fault bus are opened one by one.

4. Zero sequence driving point reactances for faulted bus.

(II) Nodal Interactive Method

In this method, interaction of entire network are required for each fault condition. The basis of the node interactive method of briefly explained below :

The node equations are formulated by applying Kirchoff's current law. The node equations put in a systematic form provide an excellent method for computer solution. Consider a circuit Fig. 24.3. With certain simplification, this circuit is represented by Fig. 24.4. Series reactances have been combined. Capacitors are added at each high voltage bus of original system. The e.m.f. source with

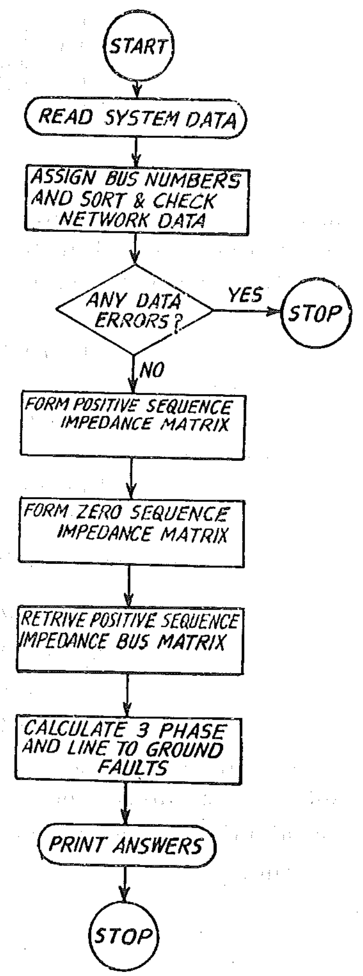


Fig. 24.2. Simplified flow chart for a short-circuit programme.

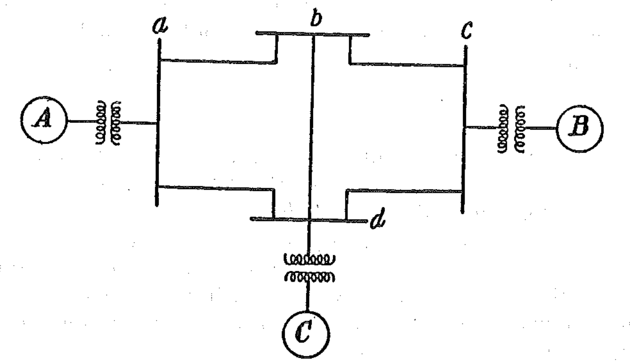


Fig. 24.3. System for one line diagram.

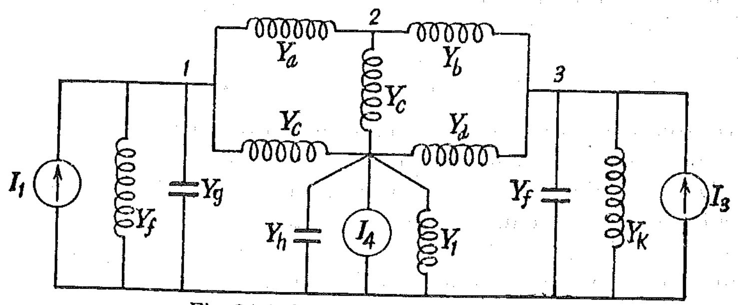


Fig. 24.4. Circuit for node equation.

their series impedance are replaced by their equivalent current sources and shunt impedances. Nodes are designated by numbers 1, 2, 3, 4. Applying Kirchhoff's current laws to the current at node 1, current entering into the node from the source is equal to current going away from the nodes, *i.e.*

$$V_1 (Y_f + Y_g) + (V_1 - V_2) Y_a + (V_1 - V_4) Y_c = I_1$$

For node 2,

$$0 = (V_2 - V_1) Y_a + (V_2 - V_3) Y_b + (V_2 - V_4) Y_c$$

rearranging,

$$V_1 (Y_f + Y_g + Y_a + Y_c) - V_2 Y_a - V_4 Y_c = I_1$$
$$- V_1 Y_a + V_2 (Y_a + Y_b - Y_c) - V_3 Y_b - V_4 Y_c = 0$$

Similar equations are obtained for nodes 3 and 4. The equations are put in the following standard form :

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$$
$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4$$
$$I_4 = Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4$$

where Y_{11}, Y_{22}, Y_{33}, Y_{44} are called Self Admittances at the respective nodes which are sum of admittances terminating on the node. The other admittances such as Y_{12}, etc. are obtained by adding the admittances connected directly between the nodes 1, 2 and are given a negative sign.

Thus $Y_{12} = - Y_r$, $Y_{23} = - Y_r$, likewise. The equations given above are put in the form

$$I_k = \sum_{m=1}^{m=M} Y_{k_m} V_m$$

where M is the number of independent nodes (number of busses).

For solution on computer, one equation is written for each of buses at which the voltage is unknown. Load current is neglected and all internal voltages are assumed to be equal. If the voltage of faulty bus is zero, voltages are computed by load studies or assumed equal to V_f, the equations can be written as

$$Y_{kk}V_k + \sum_{m=1}^{m=M} Y_{km}V_m = 0$$

With $\quad I_k = 0$, and $m \neq k$

A set of simultaneous equations is formed as equations are written for all the nodes where voltage is unknown. The equations are then solved by interactive process. Initial values are assumed for all unknown voltages. A correction value is found for the voltage at the first node based on unknown and assumed voltages at other nodes. This corrected value is used for subsequent buses. The computer repeats the computation until the correction at each bus is less than the required precision mark.

Another approach is by forming impedance-matrix. The matrix operations are formed on the impedances of branches that form loops of the network. After this a short-circuit matrix is obtained. The output information is obtained by means of arithmetic operations.

QUESTIONS

1. Describe a typical A.C. Network analyzer. How is it used for fault calculations ?
2. Describe the producer of fault calculations on an A.C. Network analyzer.
3. Describe the set-up of digital computer*.
4. Explain the procedure of fault calculation on digital computer.

* Ref. Sec. 46.2 for Terms and Definitions related with Digital Computers and Microprocessors.
Ref. Sec. 47.13 for 'HVDC Simulator' used for analysing HVDC Transmission System and associated AC Systems.