Introduction

Outlier detection in time-series data can be a lot more complicated than ordinary multivariate data, owing to the underlying assumption of independent observations of most unsupervised machine-learning algorithms, being violated. In this notebook, we have attempted to adapt an algorithm proposed by Yufeng et. al. (http://dx.doi.org/10.1155/2014/879736) to detect outliers in a **hydrological time-series using sliding window prediction** technique.

Window ¶

A window of observations in the context of a specific time-series data point could be considered as a set of its closest 2k obervations either to the left of it, or k observations to the left and k to the right.

The proposed Algorithm

The algorithm essentially uses a window of 2k observations for each data point and uses this window to predict a confidence interval and an estimate for the data point itself. Once these two have been calculated, it is verified whether the data point falls within the interval or not. If it does not fall within this interval, we classify it as an outlier and replace it with the estimated value. We then move on to the next data-point and repeat the same procedure.

Underlying assumption

The underlying assumption is that the time-series follows a multivariate Gaussian distribution (Normal distribution), each variable having a marginal density function represented by the formulae:

$f(x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x_t-\mu}{2\sigma^2}}$, where μ and σ are unknown parameters of the distribution.

Necessary packages and modules

In [1]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.stats import t
```

Data Ingestion

(i) Use the appropriate url/address.

(ii). Drop Unnecessary features.

In [2]:

```
df = pd.read_csv(r"E:\kritsnam data\CWC-01.csv")
df.drop(columns = ['Flagged'], inplace = True)
df.head()
```

Out[2]:

	Timestamp	Water Level(In mm)
0	2018-09-12 18:36:14	5645.0
1	2018-09-12 18:42:12	5646.0
2	2018-09-12 18:48:10	5642.0
3	2018-09-12 18:53:54	5650.0
4	2018-09-12 19:05:25	5650.0

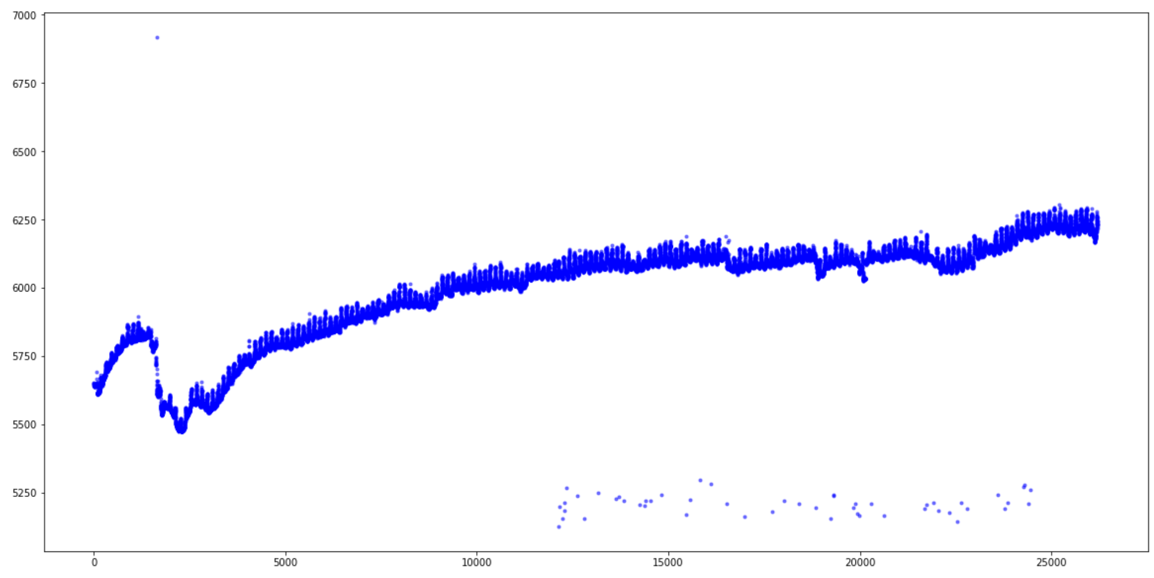
Exploratory/Preliminary Visualization

In [3]:

```
fig_ax = plt.figure(figsize = (20, 10))
ax_fig = fig_ax.add_subplot(111)
ax_fig.plot(df.iloc[:, 1].values, 'b.', alpha = 0.5)
```

Out[3]:

```
[<matplotlib.lines.Line2D at 0xa8f67b8>]
```



The Algorithm

(i). For Historical data

First let us define,

d_t : t^{th} *observation in the time-series.*

$$\eta_t^{(k)} = \{d_{t-k}, d_{t-k+1}, \ldots, d_{t-1}, d_{t+1}, \ldots, d_{t+k-1}, d_{t+k}\}$$

$w_{t-k} \left(\propto \frac{1}{d_t}\right)$: *Weight associated with d_{t-k} based on it's distance from d_t.*

Then calculate,

$$\hat{d}_t = \frac{\sum_{i=t-k}^{t-1} w_i d_i + \sum_{i=t+1}^{t+k} w_i d_i}{\sum_{i=t-k}^{t-1} w_i + \sum_{i=t+1}^{t+k} w_i}$$

$d_t^{(Conf.Int)} = \hat{d}_t \pm t_{\frac{\alpha}{2},2k-1} \hat{s} \sqrt{1 + \frac{1}{2k}}$, where t_α is that point of a t distribution with 2k-1 degrees of freedom, such that α proportion of points of the distribution lie to the right of it and $1-\alpha$ proportion of points lie to the left of it. $\hat{s}$ is the standard deviation of η_t^k

If $\hat{d}_t \notin \left[\hat{d}_t \pm t_{\frac{\alpha}{2},2k-1} \hat{s} \sqrt{1 + \frac{1}{2k}}\right]$ then d_t is deemed an outlier and is replaced by $\hat{d}_t$

Repeat the process for $d_{t+1}, d_{t+2} \ldots$

k and α are parameters of the model.

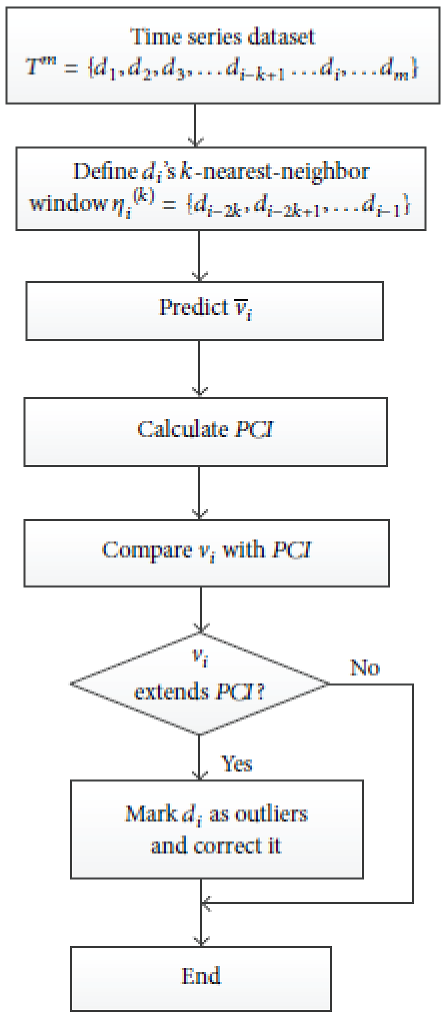

Time series dataset
$T^m = \{d_1, d_2, d_3, \ldots d_{i-k+1} \ldots d_i, \ldots d_m\}$
Define d_i's k-nearest-neighbor window $\eta_i^{(k)} = \{d_{i-2k}, d_{i-2k+1}, \ldots d_{i-1}\}$
Predict $\overline{v}_i$
Calculate PCI
Compare v_i with PCI
v_i extends PCI?
No
Yes
Mark d_i as outliers and correct it
End

Figure 1: Diagram of the proposed outlier detection method.

In [4]:

```
T = df.iloc[:, 1].values
#Function to calulate the estimate and the PCI for each observation in the time-series
def feature_extract(i, T, k, alpha):
    d_original = T[i]
    window_left = T[i-k:i]
    window_right = T[i:i+k]
    wt_right = np.arange(k, 0, -1)
    wt_left = np.arange(1, k+1, 1)
    d_estimate = ((window_left*wt_left).sum() + (window_right*wt_right).sum())/(wt_left
.sum()+wt_right.sum())
    pci_l = d_estimate - t.ppf(alpha, df = 2*k-1)*(np.sqrt(1+1/(2*k)))*np.std(np.hstack
((window_left, window_right)))
    pci_r = d_estimate + t.ppf(alpha, df = 2*k-1)*(np.sqrt(1+1/(2*k)))*np.std(np.hstack
((window_left, window_right)))
    state = 0
    if (d_original<pci_l) or (d_original>pci_r):
        state = 1
    dat = [d_estimate, pci_l, pci_r, state]
    return dat

# Function to compute outliers for an entire data-set
def fin_calc(T, k, alpha):
    l = np.arange(k, len(T)-k, dtype = np.int64)
    fin = map(feature_extract, l, [T]*(len(T)-2*k), [k]*(len(T)-2*k), [alpha]*(len(T)-2
*k))
    df2 = pd.DataFrame(list(fin), columns = ['Estimated', 'Lower', 'Upper', 'Status'],
index = range(k, len(T)-k))
    return df2
```

In [5]:

```
k = 6
alpha = 0.95
df2 = fin_calc(T, k, alpha)
```

In [6]:

```
df2.head()
```

Out[6]:

	Estimated	Lower	Upper	Status
6	5644.428571	5636.803890	5652.053253	0
7	5643.500000	5635.520784	5651.479216	0
8	5642.476190	5634.210206	5650.742175	0
9	5641.571429	5633.305444	5649.837414	0
10	5640.857143	5633.679972	5648.034314	0

In [7]:

```
df2 = pd.merge(df, df2, 'right', left_index = True, right_index = True)
```

In [8]:

```
df2.head()
```

Out[8]:

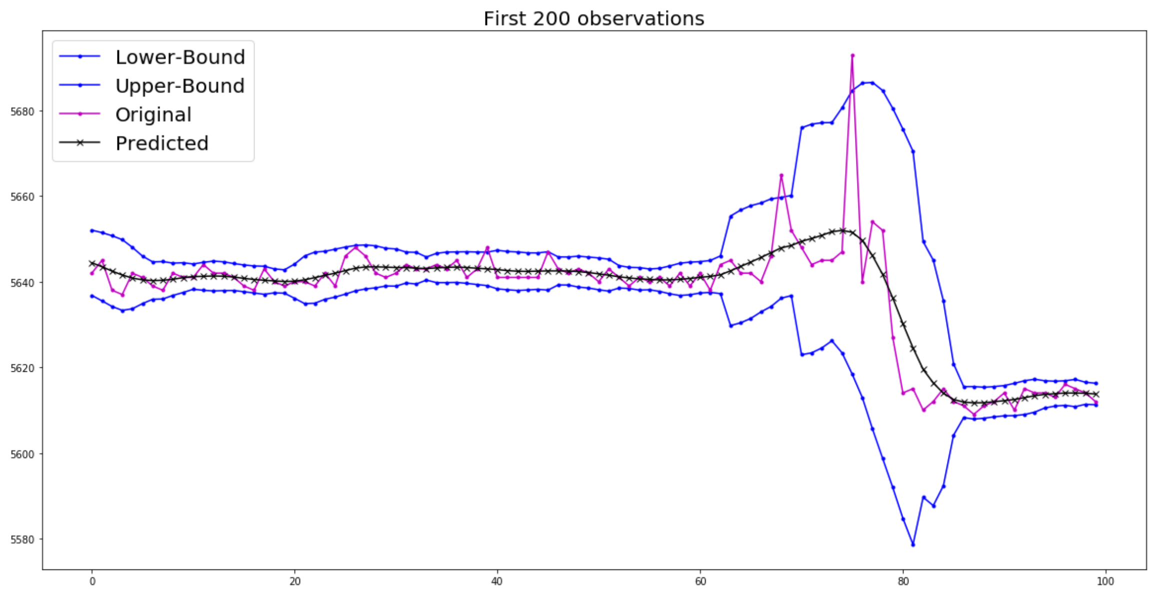
	Timestamp	Water Level(In mm)	Estimated	Lower	Upper	Status
6	2018-09-12 19:16:53	5642.0	5644.428571	5636.803890	5652.053253	0
7	2018-09-12 19:22:35	5645.0	5643.500000	5635.520784	5651.479216	0
8	2018-09-12 19:29:07	5638.0	5642.476190	5634.210206	5650.742175	0
9	2018-09-12 19:34:52	5637.0	5641.571429	5633.305444	5649.837414	0
10	2018-09-12 19:40:37	5642.0	5640.857143	5633.679972	5648.034314	0

In [9]:

```
fig, ax = plt.subplots(1, 1, figsize = (20, 10))
ax.plot(df2.iloc[:, 3].values[0:100], 'b.-', label = "Lower-Bound")
ax.plot(df2.iloc[:, 4].values[0:100], 'b.-', label = "Upper-Bound")
ax.plot(df2.iloc[:, 1].values[0:100], 'm.-', label = "Original")
ax.plot(df2.iloc[:, 2].values[0:100], 'k-x', label = "Predicted")
ax.set_title('First 200 observations', size = 20)
ax.legend(loc = "upper left", fontsize = 20)
```

Out[9]:

```
<matplotlib.legend.Legend at 0xa9ac6a0>
```

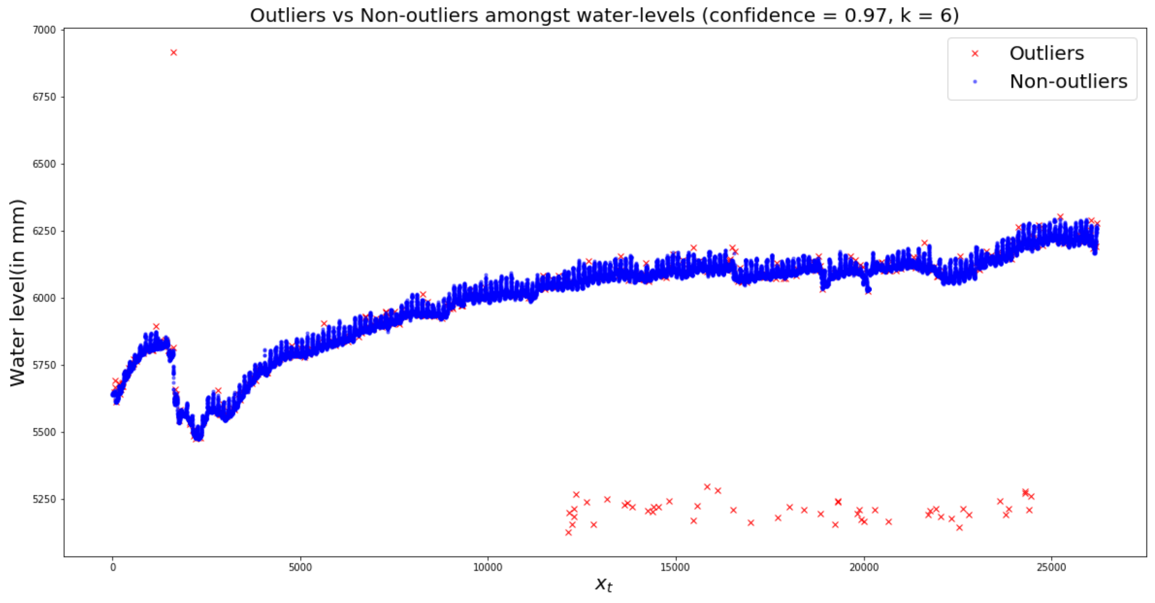


In [10]:

```
fig2 = plt.figure(figsize = (20, 10))
ax = fig2.add_subplot(111)
ax.plot(df2[df2.iloc[:, -1]==1].iloc[:, 1], 'rx', alpha = 1, label = 'Outliers')
ax.plot(df2[df2.iloc[:, -1]==0].iloc[:, 1], 'b.', alpha = 0.5, label = 'Non-outliers')
ax.legend(loc = 1, fontsize = 20)
ax.set_title('Outliers vs Non-outliers amongst water-levels (confidence = 0.97, k = 6)'
, size = 20)
ax.set_ylabel('Water level(in mm)', size = 20)
ax.set_xlabel('$x_t$', size = 20)
```

Out[10]:

```
Text(0.5, 0, '$x_t$')
```



(ii). For new observations.

Let us first define,

d_t : t^{th} *observation in the time-series.*

$\eta_t^{(k)} = \{t_{t-2k}, t_{t-2k+1}, \ldots, t_{t-1}\}$

$w_{t-k}\left(\propto \frac{1}{d_t}\right)$: *Weight associated with* d_{t-k} *based on it's distance from d_t.*

Then we calulate,

$$\hat{d}_t = \frac{\sum_{i=t-2k}^{t-1} w_i d_i}{\sum_{i=t-2k}^{t-1} w_i}$$

$d_t^{(Conf.Int)} = \hat{d}_t \pm t_{\frac{\alpha}{2},2k-1}\,\hat{s}\sqrt{1+\frac{1}{2k}}$, where t_α is that point of a t distribution with 2k-1 degrees of freedom, such that α proportion of points of the distribution lie to the right of it and $1-\alpha$ proportion of points lie to the left of it. $\hat{s}$ is the standard deviation of η_t^k

If $\hat{d}_t \notin \left[\hat{d}_t \pm t_{\frac{\alpha}{2},2k-1}\,\hat{s}\sqrt{1+\frac{1}{2k}}\right]$ then d_t is deemed an outlier and is replaced by $\hat{d}_t$

Repeat the process for $d_{t+1}, d_{t+2}\ldots$ k and α are parameters of the model.

In [11]:

```
def new_obs_pred(d_obs, T, k, alpha):
    d_original = d_obs[1]
    window_left = df.iloc[:, 1].values[len(df)-2*k:len(df)]
    wt_left = np.arange(1, 2*k+1)
    d_estimate = sum(window_left*wt_left)/sum(wt_left)
    pci_l = d_estimate - t.ppf(alpha, df = 2*k-1)*(np.sqrt(1+1/(2*k)))*np.std(window_le
ft)
    pci_r = d_estimate + t.ppf(alpha, df = 2*k-1)*(np.sqrt(1+1/(2*k)))*np.std(window_le
ft)
    state = 0.0
    if (d_original<pci_l) or (d_original>pci_r):
        state = 1.0
    dat = [d_obs[0], d_original, d_estimate, pci_l, pci_r, state]
    return dat
```

In [12]:

```
d_obs = df.iloc[len(df)-k]
ob = new_obs_pred(d_obs, T, 6, 0.97)
```

Out[12]:

```
Timestamp            2019-04-02 18:27:08
Water Level(In mm)                  6239
Name: 26198, dtype: object
```

In [14]:

```
df2 = df2.append(pd.Series(ob, index = df2.columns), ignore_index = True)
df.tail(5)
```

Out[14]:

	Timestamp	Water Level(In mm)
26199	2019-04-02 18:37:20	6232.0
26200	2019-04-02 18:47:32	6233.0
26201	2019-04-02 18:57:44	6231.0
26202	2019-04-02 19:07:56	6224.0
26203	2019-04-02 19:18:08	6218.0

Applying the algorithm on multiple devices.

In [15]:

```
a = []
file = ['02', '03', '05', '06', '07', '08', '09', '10', '11', '12']
for i in range(0, 10):
    a.append((pd.read_csv(r'E:\kritsnam data\CWC-{}.csv'.format(file[i]))).drop(columns
= 'Flagged'))
#for i in a:
#    print(i.head())
```

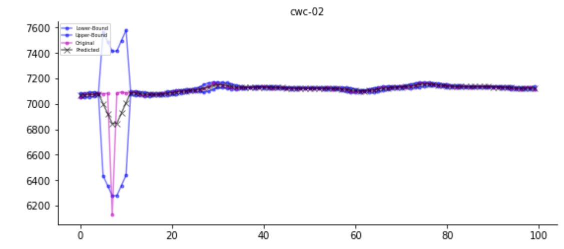
In [16]:

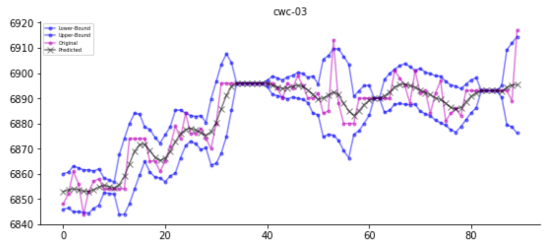
```
k = 3
alpha = 0.90
for i, elem in enumerate(a):
    T = elem.iloc[:, 1].values
    d_1 = fin_calc(T, k, alpha)
    d_1 = pd.merge(elem, d_1, left_index = True, right_index = True)
    a[i] = d_1
```

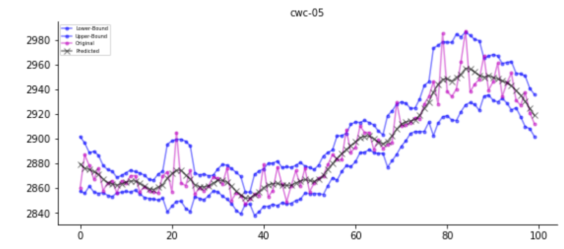
Visualization of the confidence intervals along with the original and predicted observations.

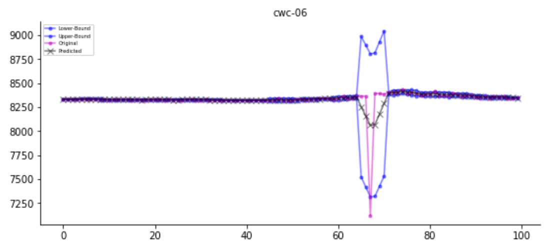
In [40]:

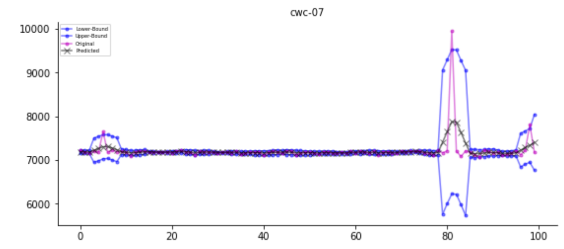
```
fig3, ax = plt.subplots(5, 2, figsize = (20, 20))
fig3.subplots_adjust(top = 2, bottom = 1, hspace = 0.5)
c = 0
for i in ax:
    for j in i:
        j.spines['top'].set_visible(False)
        j.spines['right'].set_visible(False)
        j.plot(a[c].iloc[:, 3].values[:100], 'b.-', label = "Lower-Bound", alpha = 0.5)
        j.plot(a[c].iloc[:, 4].values[:100], 'b.-', label = "Upper-Bound", alpha = 0.5)
        j.plot(a[c].iloc[:, 1].values[:100], 'm.-', label = "Original", alpha = 0.5)
        j.plot(a[c].iloc[:, 2].values[:100], 'k-x', label = "Predicted", alpha = 0.5)
        j.set_title('cwc-{}'.format(file[c]), size = 10)
        j.legend(loc = "upper left", fontsize = 5)
        c+=1
```

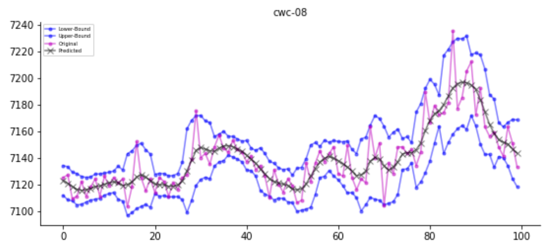


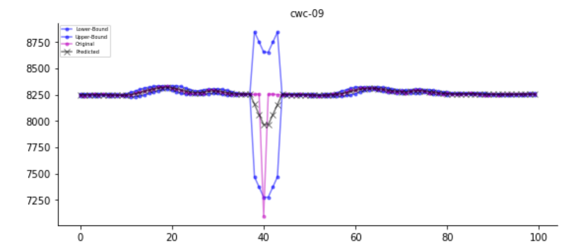


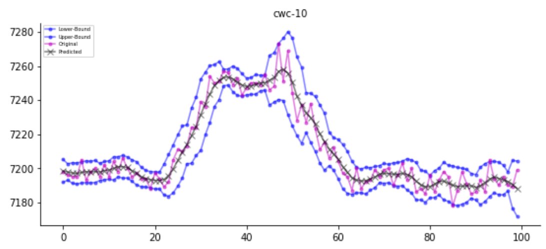


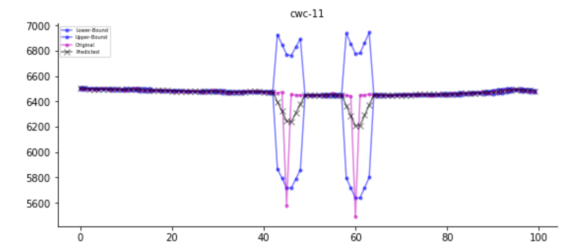


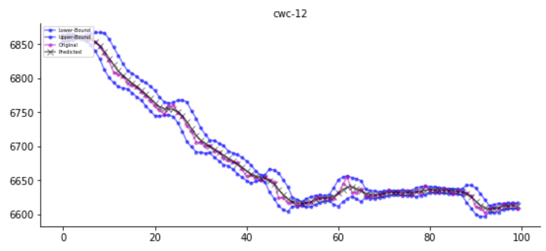








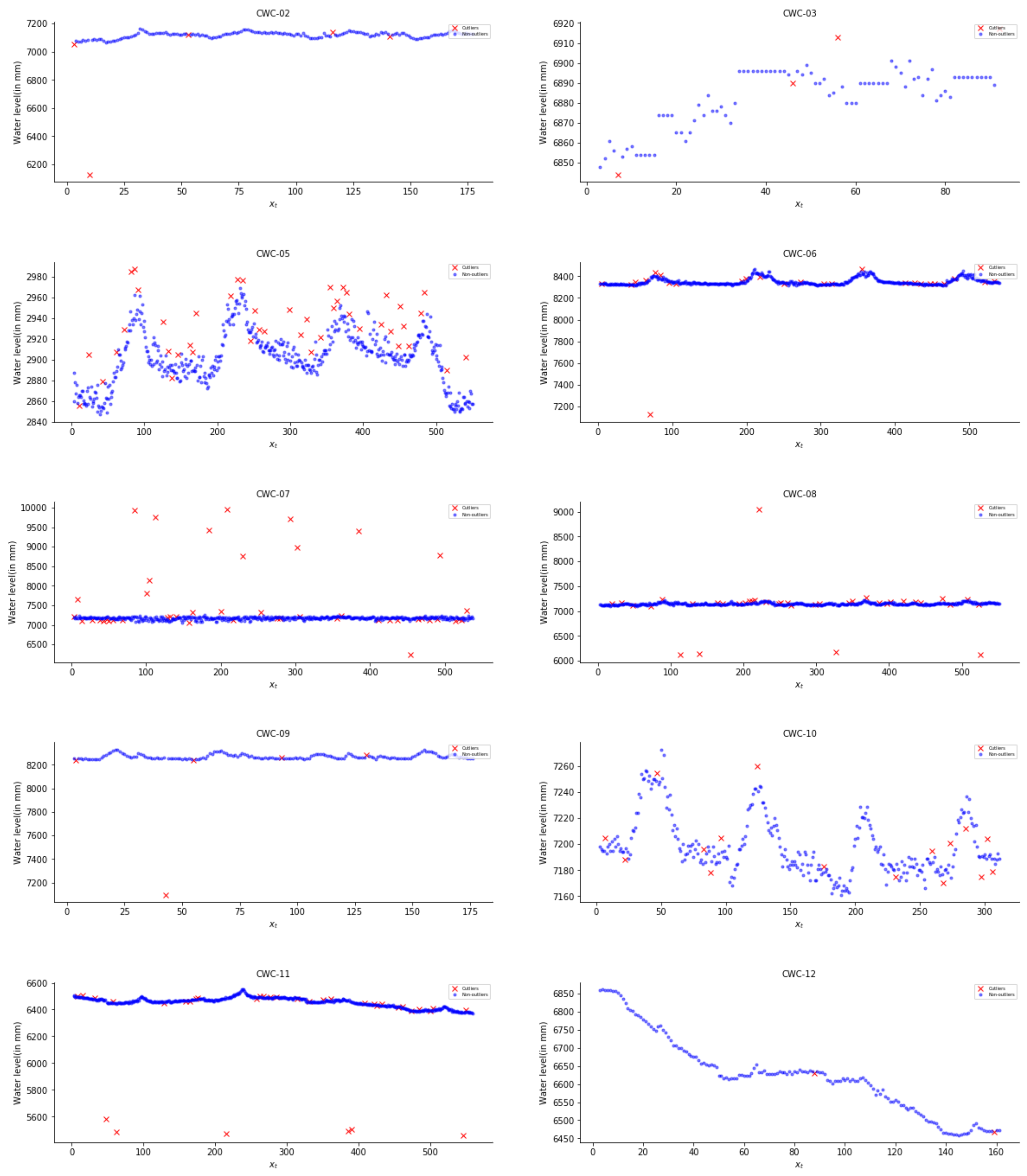




Visualization of the data points with outliers marked in red.

In [39]:

```
fig4, ax1 = plt.subplots(5, 2, figsize = (20, 20))
fig4.subplots_adjust(top = 1, bottom = 0.1, hspace = 0.5)
c = 0
for i in ax1:
    for j in i:
        j.spines['top'].set_visible(False)
        j.spines['right'].set_visible(False)
        j.plot(a[c][a[c].iloc[:, -1]==1].iloc[:, 1], 'rx', alpha = 1, label = 'Outlier
s')
        j.plot(a[c][a[c].iloc[:, -1]==0].iloc[:, 1], 'b.', alpha = 0.5, label = 'Non-ou
tliers')
        j.legend(loc = 1, fontsize = 5)
        j.set_title('CWC-{}'.format(file[c]), size = 10)
        j.set_ylabel('Water level(in mm)', size = 10)
        j.set_xlabel('$x_t$', size = 10)
        c+=1
```


CWC-02
CWC-03
CWC-05
CWC-06
CWC-07
CWC-08
CWC-09
CWC-10
CWC-11
CWC-12
Water level(in mm)
x_t
Outliers
Non-outliers