3rd sem ①

Q. Obtain Newton's iterative formula for finding $\frac{1}{\sqrt{N}}$ where N is a positive real number and find the value of $\frac{1}{\sqrt{14}}$ which is correct up to four decimal places.

Ans Let $x = \frac{1}{\sqrt{N}}$

$\Rightarrow$ Squaring bothsides

$\Rightarrow x^2 = \frac{1}{N}$

$\Rightarrow x^2 - \frac{1}{N} = 0$ —— (1)

Which is in the form of

$f(x) = 0$

Here $f(x) = x^2 - \frac{1}{N}$ —— (2)

$f'(x) = 2x$ —— (3)

We know N-R method

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}}$$ —— (A) for $k = 0, 1, 2$

Now $f(x_k) = x_k^2 - \frac{1}{N}$

$$f'(x_k) = 2x_k$$

From relation (A) we get

$$x_{k+1} = x_k - \frac{x_k^2 - \frac{1}{N}}{2x_k}$$

$$= \frac{2x_k^2 - x_k^2 + \frac{1}{N}}{2x_k}$$

$$= \frac{x_k^2 + \frac{1}{N}}{2x_k}$$

$$= \frac{1}{2}\left[\frac{x_k^2}{x_k} + \frac{\frac{1}{N}}{x_k}\right]$$

$$= \frac{1}{2}\left[x_k + \frac{1}{N x_k}\right]$$

$$\therefore \boxed{x_{k+1} = \frac{1}{2}\left[x_k + \frac{1}{N x_k}\right]}$$

for $k = 0, 1, 2 \ldots$

which is called Newton's iterative formula.

(3)

Now we have to find the value of $\frac{1}{\sqrt{14}}$

Here $\frac{1}{\sqrt{14}} = \frac{1}{\sqrt{N}}$

Now $\boxed{N=14}$

Let $x_0 = 0.25$ $\qquad \frac{1}{\sqrt{16}} = \frac{1}{4} = 0.25$

We have N.I.F

$$\boxed{x_{k+1} = \frac{1}{2}\left[x_k + \frac{1}{Nx_k}\right]}$$

for $k=0,1,2\ldots$

Taking $k=0$

$\therefore$ 1st approximation

$$x_1 = \frac{1}{2}\left[x_0 + \frac{1}{Nx_0}\right]$$

$$= \frac{1}{2}\left[0.25 + \frac{1}{14(0.25)}\right]$$

$$\boxed{x_1 = 0.26785}$$

(4)

Taking $k=1$

the 2nd approximation is

$$x_2 = \frac{1}{2}\left(x_1 + \frac{1}{Nx_1}\right)$$

$$= \frac{1}{2}\left[0.26785 + \frac{1}{14.(0.26785)}\right]$$

$$\boxed{x_2 = 0.26726}$$

Taking $k=2$

The 3rd approximation is

$$x_3 = \frac{1}{2}\left[x_2 + \frac{1}{Nx_2}\right]$$

$$= \frac{1}{2}\left[0.26726 + \frac{1}{14(0.26726)}\right]$$

$$\boxed{x_3 = 0.26726}$$

$$\therefore |x_3 - x_2| = 0.00000$$

$$\therefore \frac{1}{\sqrt{14}} = 0.26726 = 0.2673$$

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Q. Obtain Newton's iterative formula for finding the value of $\sqrt[3]{N}$ where N is the real number and find the value of $\sqrt[3]{24}$ which is correct upto four decimal place.

Ans Let $x = \sqrt[3]{N}$

Raising power 3 both sides

$\Rightarrow x^3 = N$

$\Rightarrow x^3 - N = 0$ —— (1)

Which is in the form of

$f(x) = 0$

$\therefore f(x) = x^3 - N$ —— (2)

$f'(x) = 3x^2$ —— (3)

we know N-R method

$$\boxed{x_{K+1} = x_K - \frac{f(x_K)}{f'(x_K)}} \text{ —— (A)}$$

for $K = 0, 1, 2 \ldots$

(6)

Now $f(x_k) = x_k^3 - N$

$$f'(x_k) = 3x_k^2$$

from relation (A) we get

$$x_{k+1} = x_k - \frac{x_k^3 - N}{3x_k^2}$$

$$= \frac{3x_k^3 - x_k^3 + N}{3x_k^2}$$

$$= \frac{2x_k^3 + N}{3x_k^2}$$

$$= \frac{1}{3}\left[\frac{2x_k^3 + N}{x_k^2}\right]$$

$$= \frac{1}{3}\left[\frac{2x_k^3}{x_k^2} + \frac{N}{x_k^2}\right]$$

$$\boxed{x_{k+1} = \frac{1}{3}\left[2x_k + \frac{N}{x_k^2}\right]}$$

for $k = 0, 1, 2 \cdots$

 which is Newton's iterative formula

Now we have to find $\sqrt[3]{24}$

now $\sqrt[3]{24} = \sqrt[3]{N}$

Here $N = 24$

Let $x_0 = 3$

As $(27)^{1/3} = 3$

we have N.I.F.

$$\boxed{x_{k+1} = \frac{1}{3}\left[2x_k + \frac{N}{x_k^2}\right]}$$

for $k = 0, 1, 2 \ldots$

Taking $k=0$

The 1st approximation

$$x_1 = \frac{1}{3}\left[2x_0 + \frac{N}{x_0^2}\right]$$

$$= \frac{1}{3}\left[2(3) + \frac{24}{(3)^2}\right] = \frac{8.66667}{3}$$

$$\boxed{x_1 = 2.88889}$$

Taking $k=1$

The 2nd approximation is

$$x_2 = \frac{1}{3}\left[2x_1 + \frac{N}{x_1^2}\right]$$

$$= \frac{1}{3}\left[2(2.88889) + \frac{24}{(2.88889)^2}\right]$$

$$\boxed{x_2 = 2.88451}$$

Taking $k=2$

3rd approximation is

$$x_3 = \frac{1}{3}\left[2x_2 + \frac{N}{x_2^2}\right]$$

$$= \frac{1}{3}\left[2(2.88451) + \frac{24}{(2.88451)^2}\right]$$

$$\boxed{x_3 = 2.8845}$$

$\therefore \; |x_3 - x_2| = 0.0000$

$\therefore \; \sqrt[3]{24} = 2.8845$

Q. Obtain N.I.F for finding $\sqrt[3]{N}$ by using N-R method where N is +ve real number and $\sqrt[3]{10}$ which is correct upto 4 decimal places. $\sqrt[3]{15} = 2.449$. ($x_0 = 2$)

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Note — To find $\sqrt[R]{N}$ by using N–R method we get

$$x_{K+1} = \frac{1}{R}\left[(R-1)x_K + \frac{N}{x_K^{(R-1)}}\right]$$

$R = 3, 4, 5, 6, \ldots$

Suppose $R=4$

$$x_{K+1} = \frac{1}{4}\left[3x_K + \frac{N}{x_K^3}\right]$$

$R=5$

$$x_{K+1} = \frac{1}{5}\left[4x_K + \frac{N}{x_K^4}\right]$$

and so on.