23 Nov

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Factorial :- The product of first 'n' natural numbers is called "n factorial".

It is denoted by $\lfloor n$ or $n!$

It is defined as

$$n! = 1 \times 2 \times 3 \times 4 \times \cdots \times (n-1) \times n$$

or

$$\lfloor n = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1.$$

Also it is expressed as

$$\boxed{n! = n \times (n-1)!}$$

Ex

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$$

Note :- $\boxed{0! = 1}$, $\boxed{1! = 1}$

Q. Define permutation.

(2)

Ans An arrangement of objects by taking some at a time or all at a time.

Let 'n' and 'r' two non-negative integers. Such that $0 \leq r \leq n$.

The permutation of n different objects taken 'r' at a time is denoted by '^nP_r' or $P(n,r)$.

It is defined as

$$\boxed{^nP_r \text{ or } P(n,r) = \frac{n!}{(n-r)!}}$$

When $r=0$

$$^nP_0 \text{ or } P(n,0) = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$$

$$\therefore \boxed{^nP_0 = 1}$$

(3)

When $r=n$

$$^nP_n \text{ or } P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1}$$

$$\therefore \boxed{^nP_n = n!}$$

Note :- We can not evaluate factorial of negative integers and fractions.

i.e. ~~Ex~~ $(-5)!$ not defined

$\left(\frac{2}{3}\right)!$ not defined.

Combination :- The selection of objects by taking some at a time or all at a time is called combination.

It is also called grouping of objects

(4)

The combination of n different objects taken 'r' at a time is denoted by $C(n,r)$ or ~~[illegible]~~ $({}^nC_r)$ or $\binom{n}{r}$.

It is defined as

$$\boxed{{}^nC_r \text{ or } C(n,r) = \frac{n!}{r!(n-r)!}}$$

when $0 \le r \le n$.

when $r=0$

$${}^nC_0 = \frac{n!}{0!(n-0)!} = \frac{n!}{1!n!} = 1$$

$$\boxed{{}^nC_0 = 1}$$

when $r=n$

$${}^nC_n = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = 1$$

$\therefore$ $\boxed{{}^nC_n = 1}$.

$\therefore$ $\boxed{{}^nC_0 = {}^nC_n = 1}$.

~~Note~~ If ${}^nC_x = {}^nC_y$ then

$\boxed{x = y}$ or $\boxed{x + y = n}$.

~~Q~~ A bag consists of 9 red balls, 7 white balls and 4 black balls. If 2 balls are drawn at random what is the probability that

(I) both are red.

(II) both are white

(III) both are black

(IV) One is white other is red

(V) one is white other is black

(VI) one is white

(6)

(VII) one is red

(VIII) one is black

(IX) both are same colour.

Soln Given that bag consists of
9 red, 7 white and 4 black balls.

$\therefore$ ~~Total no. of balls~~ $= 9+7+4$
$= 20$ balls.

~~A/Q 2 balls are selected random~~
out of 20 balls.

$\therefore |S| = {}^{20}C_2$

(I) First we have to find probability
of getting both are red balls.

There are 9 red. balls in the
bag.

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we have to choose 2 ~~bad~~ red balls out of 9 red ball.

Let that event is A.

$\therefore |A| = {}^{9}C_{2}$

$$\therefore P(A) = \frac{|A|}{|S|} = \frac{{}^{9}C_{2}}{{}^{20}C_{2}} = \frac{\frac{9!}{2!(9-2)!}}{\frac{20!}{2!(20-2)!}}$$

$$= \frac{\frac{9\times8\times7!}{7!}}{\frac{20\times19\times18!}{18!}} = \frac{9\times8}{20\times19} = \frac{18}{95}$$

$$\therefore \boxed{P(A) = \frac{18}{95}}$$

(ii) we have to find the probability of getting both are white balls.

There are 7 white balls in the bag.

we choose 2 white balls from 7 white balls.

(6)

Let that event is B.

$\therefore |B| = {}^7C_2.$

$\therefore P(B) = \frac{|B|}{|S|} = \frac{{}^7C_2}{{}^{20}C_2} = \frac{\frac{7!}{2!(7-2)!}}{\frac{20!}{2!(20-2)!}}$

$= \frac{\frac{7\times6\times\cancel{5!}}{\cancel{5!}}}{\frac{20\times19\times\cancel{18!}}{\cancel{18!}}}$

$= \frac{7\times\cancel{6}^{3}}{\cancel{20}_{10}\times19} = \frac{21}{190}$

(iii) We have to find the probability of both are black balls.

There are 4 black balls.

we choose 2 black balls out of 4 black balls.

Let that event is 'C'

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$$|C| = {}^4C_2$$

$$P(C) = \frac{|C|}{|S|} = \frac{{}^4C_2}{{}^{20}C_2} = \frac{4!/2!(4-2)!}{\frac{20!}{2!(20-2)!}}$$

$$= \frac{4!/2!}{20!/18!} = \frac{4\times3\times\cancel{2!}/\cancel{2!}}{20\times19\times\cancel{18!}/\cancel{18!}}$$

$$= \frac{\cancel{4}\times3}{{}_5\cancel{20}\times19} = \frac{3}{95}$$

iv P (one is white and other is red)

$$= P(W_1, R_1) = \frac{{}^7C_1 \times {}^9C_1}{{}^{20}C_2}$$

$$= \frac{\frac{7!}{1!(7-1)!} \times \frac{9!}{1!(9-1)!}}{\frac{20!}{2!(20-2)!}}$$

$$= \frac{7\times\cancel{6!}}{\cancel{6!}} \times \frac{9\times\cancel{8!}}{\cancel{8!}} \Big/ \frac{\cancel{20}^{10}\times19\times\cancel{18!}}{\cancel{2}\times1\cdot\cancel{18!}}$$

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$$= \frac{7\times 9}{10\times 19} = \frac{63}{190}$$

(v) P(one is white and other is black)

$$= P(W_1, B_1) = \frac{{}^7C_1 \times {}^4C_1}{{}^{20}C_2}$$

$$= \frac{7\times 4}{\frac{20\times 19}{2\times 1}} = \frac{28}{190} = \frac{14}{95}$$

$$\boxed{P(WB) = \frac{14}{95}}$$

(vi) We have to find probability one is white.

i.e. other is either black or red.

$$P(W_1) = \frac{{}^7C_1 \times {}^{13}C_1}{{}^{20}C_2} = \frac{7\times 13}{190} = \frac{91}{190}$$

(11)

(VII) we have to find probability of one red.

$\therefore$ Other is either black or white.

$$P(B_1) = \frac{{}^{9}C_1 \times {}^{11}C_1}{{}^{20}C_2} = \frac{99}{190}$$

(VIII) we have to find probability of one is black.

$\therefore$ Other is either red or white.

$$P(B_1) = \frac{{}^{4}C_1 \times {}^{16}C_1}{{}^{20}C_2} = \frac{4 \times 16}{190}$$

$$= \frac{32}{95}$$

(11) we have to find the probability of ~~find~~ getting both balls are same ~~#~~ colour.

$\therefore$ either both balls are red or both are white or both are black.

$$P(2R / 2W / 2B)$$

$$= \frac{{}^{9}C_2}{{}^{20}C_2} + \frac{{}^{7}C_2}{{}^{20}C_2} + \frac{{}^{4}C_2}{{}^{20}C_2}$$

$$= \frac{18}{95} + \frac{21}{190} + \frac{3}{95}$$

$$= \frac{36 + 21 + 6}{190} = \frac{63}{190}$$

Q. In a single throw of 2 dice find the probability of sum of two faces is either 8 or 11.

Ans Since 2 dices are thrown once

$$S = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$$
$$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$$
$$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$$
$$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$$
$$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$$
$$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$$

$$|S| = 36$$

Let A be the event sum of two faces is '8'

(14)

$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$|A| = 5$

$\therefore P(A) = \frac{|A|}{|S|} = \frac{5}{36}$

Let B be the event sum of two faces is 11.

$B = \{(5,6), (6,5)\}$, $|B| = 2$

$P(B) = \frac{|B|}{|S|} = \frac{2}{36} = \frac{1}{18}$

Here A and B are mutually exclusive events.

$A \cap B = \phi$

$\rightarrow \boxed{P(A \cap B) = 0}$

(15)

$\therefore \boxed{P(A \cup B) = P(A) + P(B)}$

$\Rightarrow P(A \cup B) = \frac{5}{36} + \frac{1}{18} = \frac{5+2}{36} = \frac{7}{36}$

$\therefore \boxed{P(A \cup B) = \frac{7}{36}}$

Q In a single throw of two dice find the probability of getting even number on the first face or sum of '8' both faces.

Sol:- Since two dices are thrown once, then the sample space is

$S = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$

- - - - - - - - -

- - - - - - - - -

$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$

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$|S| = 36$.

Let A be the event first face is even number

$A = \{ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$

$|A| = 18$

$P(A) = \frac{|A|}{|S|} = \frac{18}{36}$

Let B be the event sum of two faces is 8

$B = \{ (2,6), (3,5), (4,4), (5,3), (6,2) \}$

$|B| = 5$

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$$\therefore \quad P(B) = \frac{|B|}{|S|} = \frac{5}{36}$$

$$A \cap B = \{(2,6), (4,4), (6,2)\}$$

$$|A \cap B| = 3$$

$$P(A \cap B) = \frac{|A \cap B|}{|S|} = \frac{3}{36}$$

W.K.T Addition theorem

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$$

$$= \frac{18 + 5 - 3}{36} = \frac{\cancel{20}^{5}}{\cancel{36}_{9}} =$$

$$\boxed{P(A \cup B) = \frac{5}{9}}$$