(1)

Test for μ of Normal distribution (σ^2 known)

Let $x_1, x_2, x_3 \dots x_n$ be a random sample of size 'n' from a large population with mean "μ" and variance "σ^2"

Sample variance $s^2 = \frac{\sigma^2}{n}$

Let 'c' be the critical pt of test

and $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ such that

$$P(z \leq c) = P\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \leq c\right) = \alpha$$

where 'z' is standardized normal random variable.

Test for μ when σ^2 unknown

Priorities

Let $x_1, x_2, x_3 \dots x_n$ be a random sample of size 'n', drawn from a normal population

(2)

with mean "μ" and variance 'σ^2', (unknown)

Then sample mean $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

sample variance $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$

The variate of the statistic 't' is defined as

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$ with $(n-1)$ degrees of freedom

Let c be the critical value at a significant level 'α'

$\therefore$ $P(|t|>c) = \alpha$, $P(|t| \le c) = 1-\alpha$.

(i) If $|t|<c$, then H_0 is accepted

(ii) If $|t|>c$, then H_0 is rejected.

Priorities

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Test for variance (σ^2) of Normal distribution

Let $x_1, x_2, x_3 \dots x_n$ be a random sample of size 'n' with mean $\bar{x}$ and variance s^2 drawn from a population with mean 'μ' and variance i.e σ^2

Then the variate

$$Y = \frac{(n-1)s^2}{\sigma^2}$$ has a chi-square distribution with degrees of freedom $(n-1)$

Let 'C' be the critical value of the test, then

$$P(Y \le c) = \alpha, \text{ and } P(Y > c) = 1-\alpha$$

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Now we have

Priorities

$$\boxed{C^* = \frac{\sigma^2 \cdot Y}{n-1}}$$

If $s^2 < C^*$ then we accept the hypothesis ~~other~~

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otherwise we reject it.

Q1 A random sample of 400 shoes has an average length of 10cm. Can this be considered as a sample from a large population with mean as 10.2 cm and standard deviation 2.25 cm.

Sol Given that

Sample size $n = 400$

Sample mean $\bar{x} = 10cm$

Population mean $\mu = 10.2cm$

Population S.D $\sigma = 2.25cm$

Consider null hypothesis.

Priorities

H_0: the sample is drawn from normal population with $\mu = 10.2$

and $\sigma = 2.25$.

Alternative hypothesis $H_1: \mu \neq 10.2$

Under Null hypothesis.

H_0: the standard normal variable

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{10 - 10.2}{2.25/\sqrt{400}}$$

$$= -1.77$$

$\therefore |z| = 1.77$.

The critical value of z at 5% level of significance is 1.96.

$\therefore |z| < z_\alpha$

As $1.77 < 1.96$

Hence we accept the null hypothesis.

(6)

Q (7) If a sample of 25 tires of a certain kind has a mean life of 37000 miles and standard deviation of 5000 miles. Can the manufacturer claim that the true mean life of such tires is greater than 35000 miles? Set up and test a corresponding hypothesis at the 5% level assuming normality.

Sol^n^ Given that

Sample size $n = 25$

Sample mean $\bar{x} = 37000$

Sample S.D $s = 5000$

Priorities

Population mean $\mu = 35000$

Let $H_0: \mu = \mu_0 = 37000$, $H_1: \mu_1 \leq 35000 < \mu_0$

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Then the variate statistic 't' is given by

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{37000 - 35000}{5000/\sqrt{25}}$$

$$= \frac{2000}{5000} \times 5 = 2$$

$\therefore$ $|t| = 2$

Now $P(|t| > c) = \alpha = 5\%$

or $P(|t| \leq c) = 1 - \alpha = 95\% = 0.95$

The value of 'c' from 't' distribution with degrees of freedom

$(n-1) = 25 - 1 = 24$

i.e $\boxed{c = 1.71}$

As $2 > 1.71$

Priorities

$\therefore$ $|t| > c$

$\therefore$ Hence we have to reject the hypothesis.

(8)

Q (3) Suppose that in operating battery-powered electrical equipment, it is less expensive to replace all batteries at fixed intervals than to replace each battery individually, when it breaks down, provided the standard deviation of the lifetime is less than certain limit, say less than 5 hrs. Set up and apply a suitable test using a sample of 28 values of lifetimes with standard deviation $s = 3.5$ hours and assuming normality, choose $\alpha = 5\%$.

Priorities

Sol Given that

sample size $n = 28$

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Sample S.D $s = 3.5$

Now null hypothesis $H_0: \sigma_0 = 5$

Alternative hypothesis $H_1: \sigma_1 < \sigma_0$

Let the significant level $\alpha = 5\% = 0.05$ and 'c' be the critical point of the test.

Then the variate of the test

$$Y = \frac{(n-1)s^2}{\sigma_0^2}$$

$$\Rightarrow Y = \frac{(28-1).s^2}{(5)^2} = \frac{27.s^2}{25} = 1.08s^2$$

which has χ^2 distribution with

$(n-1) = 28-1 = 27$ degrees of freedom

Priorities

Let 'c' be the critical value of the test then

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$P(Y \le c) = \alpha = 0.05$

$\Rightarrow \boxed{c = 16.2}$

Now $Y = 1.08 s^2$

$\Rightarrow s^2 = \frac{Y}{1.08}$

Hence the critical value corresponding to Y.

$c^* = \frac{16.2}{1.08} = 15$.

But we have $s = 3.5$

$s^2 = 12.25$.

As $12.25 < 15$

i.e. $s^2 < c^*$

Priorities

we accept the hypothesis.

X