(1)

Newton's Iterative formula

Q-(1) Obtain Newton's Iterative formula for finding $\sqrt{N}$, where N is positive real numbers. Hence find the value of $\sqrt{142}$.

Sol^n Let $x = \sqrt{N}$

squaring both sides

$$x^2 = N$$

$$\Rightarrow x^2 - N = 0 \quad \text{—— (1)}$$

which is in the form of

$$f(x) = 0$$

Here $f(x) = x^2 - N$ —— (2)

$f'(x) = 2x$ —— (3)

we know N-R method

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}} \quad \text{—— (A)}$$

for $k = 0, 1, 2 \ldots$

$$f(x_k) = x_k^2 - N$$

$$f'(x_k) = 2x_k$$

Then from relation (A) we get

$$x_{k+1} = x_k - \left(\frac{x_k^2 - N}{2x_k}\right)$$

$$= x_k - \left(\frac{x_k^2}{2x_k} - \frac{N}{2x_k}\right)$$

$$= x_k - \frac{x_k}{2} + \frac{N}{2x_k}$$

$$= \frac{2x_k - x_k}{2} + \frac{N}{2x_k}$$

$$= \frac{x_k}{2} + \frac{N}{2x_k}$$

$$\boxed{x_{k+1} = \frac{1}{2}\left[x_k + \frac{N}{x_k}\right]}$$

for $k = 0, 1, 2$

Which is called Newton's iterative formula for finding $\sqrt{N}$.

we have to find $\sqrt{142}$

i.e. $\sqrt{142} = \sqrt{N}$

Here $N = 142$

Let $x_0 = 12$

$\therefore$ First approximation is

$k = 0$

$$x_1 = \frac{1}{2}\left[x_0 + \frac{N}{x_0}\right]$$

$$\Rightarrow x_1 = \frac{1}{2}\left[12 + \frac{142}{12}\right]$$

$$\Rightarrow \boxed{x_1 = 11.9167}$$

Taking $k=1$

The 2nd approximation

$$x_2 = \frac{1}{2}\left[x_1 + \frac{N}{x_1}\right]$$

$$= \frac{1}{2}\left[11.9167 + \frac{142}{11.9167}\right]$$

$$\boxed{x_2 = 11.9164}$$

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Taking $\underline{k=2}$

The 3rd approximation

$$x_3 = \frac{1}{2}\left[x_2 + \frac{N}{x_2}\right]$$

$$= \frac{1}{2}\left[11.9164 + \frac{142}{11.9164}\right]$$

$$\boxed{x_3 = 11.9164}$$

Now $|x_3 - x_2| = 0.0000$

$\therefore \sqrt{142} = 11.9164$. (Ans)

Q Obtain N.I.F and find the value $\sqrt{20}$ which is Correct upto two decimal Places

Q Obtain N.I.F and find the value $\sqrt{29}$ which is correct upto five decimal places

Obtain the N.I.F for finding the value of $\frac{1}{N}$ where N is a non-zero real number and calculate the value of $\frac{1}{26}$ which is correct upto four decimal places.

Ans Let $x = \frac{1}{N}$

$\Rightarrow N = \frac{1}{x}$

$\Rightarrow \frac{1}{x} - N = 0$ —— ①

which is in the form of

$f(x) = 0$

Here $f(x) = \frac{1}{x} - N$ —— ②

$f'(x) = -\frac{1}{x^2}$ —— ③

we know N-R method

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}}$$ —— Ⓐ

for $k = 0, 1, 2 \ldots$

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$$\therefore f(x_k) = \frac{1}{x_k} - N$$

$$f'(x_k) = -\frac{1}{x_k^2}$$

From relation (A) we get

$$x_{k+1} = x_k - \left[\frac{\frac{1}{x_k} - N}{-\frac{1}{x_k^2}}\right]$$

$$= x_k + x_k^2\left[\frac{1}{x_k} - N\right]$$

$$= x_k + x_k - Nx_k^2$$

$$= 2x_k - Nx_k^2$$

$$\boxed{x_{k+1} = x_k\left[2 - Nx_k\right]}$$

for $k = 0, 1, 2 \ldots$

which is called Newton's iterative formula. for finding the value of $\frac{1}{N}$.

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Now we have to find $\frac{1}{26}$

Here $N = 26$

Let $x_0 = 0.04$

we have

$$\boxed{x_{k+1} = x_k [2 - N x_k]}$$

$k = 0, 1, 2 \ldots$

Putting $k = 0$

1st Approximation

$$x_1 = x_0 [2 - N x_0$$

$$x_1 = 0.04 [2 - 26(0.04)]$$

$$\boxed{x_1 = 0.0384}$$

2nd approximation

$$x_2 = x_1 [2 - N x_1]$$

$$= 0.0384 [2 - 26(0.0384)]$$

$$\boxed{x_2 = 0.0385}$$

②

$k=2$

3rd approximation

$$x_3 = x_2(2 - Nx_2)$$

$$= 0.0385\,[2 - 26(0.0385)]$$

$$\boxed{x_3 = 0.0385}$$

$$\therefore |x_3 - x_2| = 0.0000$$

$$\therefore \boxed{\frac{1}{26} = 0.0385}$$

Which is correct up to four decimal places.

Q. Obtain N.I.F for finding the value of $\frac{1}{N}$ and calculate the value of $\frac{1}{31}$ which is correct up to four decimal places.