(1)

Some Imp. problems on probability.

Q-(1) Three unbiased coins are tossed once. Find the probability of getting

(i) all heads (ii) 2 heads
(iii) one head (iv) atleast one head
(v) atleast 2 heads (vi) exactly 2 heads.
(vii) exactly one head (viii) atmost 2 heads,
(ix) no head (x) no tail

Sol^n Given that Three coins are tossed once.

$\therefore \quad |S| = (2)^3 = 8.$

i.e $S = \{ (HHH), (HHT), (HTH), (HTT)$
$(THH), (THT), (TTH), (TTT) \}$

(i) Let A be the event which consists of all heads.

2

i.e $A = \{(HHH)\}$

$|A| = 1$.

$\therefore P(A) = \frac{|A|}{|S|} = \frac{1}{8}$.

(II) Let A be the event consists of 2 heads.

$\therefore A = \{(HHT), (HTH), (THH)\}$

$|A| = 3$

$\therefore P(A) = \frac{|A|}{|S|} = \frac{3}{8}$

(III) Let A be the event consists of one head.

$\therefore A = \{(HTT), (THT), (TTH)\}$

$|A| = 3$. $\therefore P(A) = \frac{|A|}{|S|} = \frac{3}{8}$

3

(iv) Let A be the event which consists of atleast one head.

i.e ~~more than~~ one and more than one.

i.e one head, 2 heads, 3 heads

$\therefore A = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH)\}$

$\therefore |A| = 7$

$P(A) = \frac{|A|}{|S|} = \frac{7}{8}$.

(v) Let A be the event which consists of atleast 2 heads.

i.e 2 heads and more than 2.

$\therefore$ 2 heads and 3 heads.

$\therefore A = \{(HHH), (HHT), (HTH), (THH)\}$

4

$|A| = 4$.

$\therefore P(A) = \frac{|A|}{|S|} = \frac{4}{8} = \frac{1}{2}$

(VI) Let A be the event which consists of exactly 2 heads

$A = \{(HHT), (HTH), (THH)\}$

$\therefore |A| = 3$

$P(A) = \frac{|A|}{|S|} = \frac{3}{8}$

(VII) Let A be the event which consisting of exactly one head.

$A = \{(HTT), (THT), (TTH)\}$

$|A| = 3$

$P(A) = \frac{|A|}{|S|} = \frac{3}{8}$

5

(viii) Let A be the event which consists of at most 2 heads

i.e. (no head, one head, 2 heads)

$A = \{(HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$

$|A| = 7$

$P(A) = \frac{|A|}{|S|} = \frac{7}{8}$.

(ix) Let A be the event which consists of no head.

$A = \{(TTT)\}$

$|A| = 1$

$P(A) = \frac{|A|}{|S|} = \frac{1}{8}$.

(x) Let A be the event which consists of no tail.

$A = \{HHH\} \Rightarrow |A| = 1 \Rightarrow P(A) = \frac{|A|}{|S|} = \frac{1}{8}$

6

Q-2 Two dice are rolled once find the probability of getting following.

(I) Sum of two faces is an even number

(II) Sum of two faces is a prime number

(III) Sum is atleast 10.

(IV) doublet

(V) doublet (an even number)

(VI) Sum is multiple of 3

(VII) multiple of 2 on 1st face and multiple of 3 on 2nd face.

Soln Two dices are rolled once Then the sample space

$$\boxed{|S| = 36 = 6^2}$$

7

$S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$

1) Let A be the event which consists of sum of two faces is an even number.

$A = \{ (1,1), (1,3), (1,5)$
$(2,2), (2,4), (2,6)$
$(3,1), (3,3), (3,5)$
$(4,2), (4,4), (4,6)$
$(5,1), (5,3), (5,5)$
$(6,2), (6,4), (6,6) \}$

$|A| = 18$, $P(A) = \frac{|A|}{|S|} = \frac{18}{36} = \frac{1}{2}$

(II) Let A be the event which consists of sum of two face is a prime

i.e may be 2, 3, 5, 7, 11

$A = \{ (1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5),$
$(3,2), (3,4), (4,1), (4,3), (5,2), (5,6)$
$(6,1), (6,5) \}$

$|A| = 15$

$P(A) = \frac{|A|}{|S|} = \frac{15}{36} = \frac{5}{12}$.

(III) Let A be the event when consists of sum of two faces is atleast 10.

i.e sum is 10, 11, 12.

$A = \{(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)\}$

$|A| = 6$

$P(A) = \frac{|A|}{|S|} = \frac{6}{36} = \frac{1}{6}$

(IV) Let A be the ~~eve~~ event which consists of doublet (same face)

$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$|A| = 6$

$P(A) = \frac{|A|}{|S|} = \frac{6}{36} = \frac{1}{6}$

(V) Let A be the event which consists of doublet an even number.

$A = \{(2,2), (4,4), (6,6)\}$

$|A| = 3$

$P(A) = \frac{|A|}{|S|} = \frac{3}{36} = \frac{1}{12}$

18

VI Let A be the event which consists of sum is multiple of 3

$$A = \{(1,2),(1,5),(2,1),(2,4),(3,3),(3,6),(4,2),(4,5),(5,1),(5,4),(6,3),(6,6)\}$$

$$|A| = 12$$

$$P(A) = \frac{|A|}{|S|} = \frac{12}{36} = \frac{1}{3}$$

VII Let A be the event 1st face is multiple of '2' and 2n face is multiple of '3'

$$A = \{(2,3),(2,6),(4,3),(4,6),(6,3),(6,6)\}$$

$$|A| = 6$$

$$P(A) = \frac{|A|}{|S|} = \frac{6}{36} = \frac{1}{6}$$

Q-(3) find the probability that a leap year selected at random which will contain 53 sunday.

Soln A leap year consists of 366 day.

There 52 weeks and 2 days as (366/7)

As 52 weeks contain 52 sundays.

we have to find probability of getting 53 sundays.

Now the remaining 2 days are

= { (Sun, mon), (Mon, Tue), (Tue, wed) (wed, Thu) (Thu, Fri), (Fri, Sat) (Sat, Sun) }

∴ The sample space (S) = 7

Let A be the event which consists of days in which 53th sunday will appear.

(12)

i.e. $A = \{(\text{sun}, \text{mon}), (\text{sat}, \text{sun})\}$

$\therefore |A| = 2$

$P(A) = \frac{|A|}{|S|} = \frac{2}{7}$.

Q-(4) Find the probability of getting 53 sundays in general year.

Sol^n Generally 1 year consists of 365 days

i.e. it contains 52 weeks and 1 day. (As $364/7$

52 weeks contains 52 Sundays.

we have to find probability of getting 53 sunday.

That will come from remaining one day.

i.e. The sample space

$S = \{ Sun, Mon, Tue, Wed, Thu, Fri, Sat \}$

$|S| = 7$

~~Let~~ A be the event which consists of 53th Sunday.

i.e $A = \{ Sun \}$

$|A| = 1$

$\therefore P(A) = \frac{|A|}{|S|} = \frac{1}{7}$