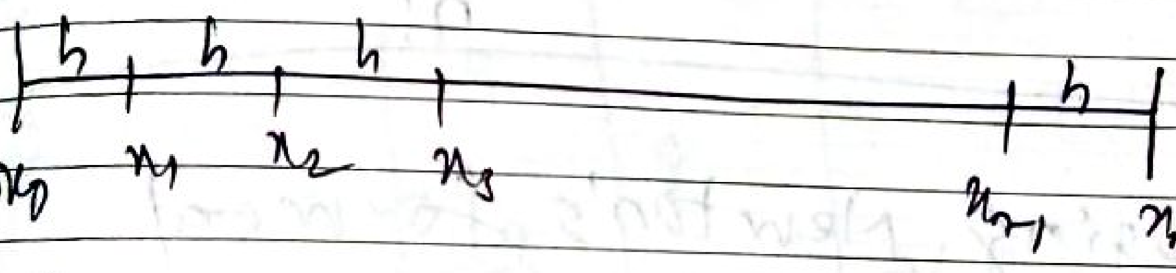
Newton's forward difference interpolation formula

Let us consider $f(x)$ be a function which is defined at $(n+1)$ pts such as $x_0, x_1, x_2, \ldots x_n$ which are equispaced. i.e. step size between two pts is equal to 'h'.



Here $x_n = x_0 + nh \Rightarrow \boxed{h = \dfrac{x_n - x_0}{n}}$

For Newton's forward difference interpolation formula

taking $x = x_0 + uh$

$$\boxed{h = \frac{x - x_0}{u}}$$

Then Newton's forward difference interpolation is given by

$$f_n(x) = p(x) = f(x_0) + u\,\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \cdots + \frac{u(u-1)(u-2)\cdots[u-(n-1)]}{n!}\Delta^n f(x_0)$$

Ex. Using Newton's forward difference interpolation find $f(x)$ at $x = 1$, from the following table.

x	0	2	4	6
y	13	27	49	73

(3)

Sol^n. Given that

	x_0	x_1	x_2	x_3
x	0	2	4	6
$y=f(x)$	13	27	49	73
	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$

Forward difference Table

x	$y=f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$
0	13			
		14		
2	27		8	
		22		-6
4	49		2	
		24		
6	73			

we have to find $f(1)$.

Here $\boxed{x=1}$, $h=2$

$$u = \frac{x-x_0}{h} = \frac{1-0}{2} = \frac{1}{2}$$

13 14 15 16 17 18 19 20 21 22
27 28 29 30 31

4

$u = \frac{1}{2}$

W.K.T Newton's forward difference interpolation

$$\boxed{f_3(x) = f(x_0) + u\,\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(x_0)}$$

$$\therefore f_3(x) = 13 + \frac{1}{2}(14) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\cdot 8 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\cdot(-6)$$

$$= 13 + 7 + \frac{\left(\frac{1}{2}\right)\cdot\left(\frac{-1}{2}\right)}{2\times 1}(8) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\cdot\left(-\frac{3}{2}\right)}{3\times 2\times 1}\cdot(-6)$$

5

$$= 13 + 7 - \frac{1}{8}.(8) + \left(\frac{3}{8}\right)(-1)$$

$$= 13 + 7 - 1 - \frac{3}{8}$$

$$= 19 - \frac{3}{8} = 18.625$$

$$\therefore \boxed{f_3(x) = 18.625}$$

Q. Find the value of $f(0.5)$ by using Newton's forward difference interpolation from the following table.

x	0	1	2	3	4
y	1	7	23	55	109

Sol^n Given that

	x_0	x_1	x_2	x_3	x_4
x	0	1	2	3	4
$y = f(x)$	1	7	23	55	109
	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$

Forward difference Table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
0	1				
		6			
1	7		10		
		16		6	
2	23		16		0
		32		6	
3	55		22		
		54			
4	109				

Here $x = 0.5$ $h = 1$

$$u = \frac{x - x_0}{h} = \frac{0.5 - 0}{1} = 0.5 = \frac{1}{2}$$

W.K.T Newton's forward interpolation formula

$$f_4(x) = f(x_0) + u\,\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0)$$

$$+ \frac{u(u-1)(u-2)}{3!} \Delta^3 f(x_0) + \frac{u(u-1)(u-2)(u-3)}{4!} \Delta^4 f(x_0)$$

$$\therefore f_4(x) = 1 + \frac{1}{2} \cdot 6 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!} \cdot 10$$

$$+ \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3 \times 2 \times 1} \cdot 6$$

$$+ \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)}{4 \times 3 \times 2 \times 1} \cdot 0$$

$$= 1 + 3 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2} \times 10$$

$$+ \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6} \cdot 6 + 0$$

$$= 1 + 3 + \left(-\frac{5}{4}\right) + \frac{3}{8}$$

$$= 4 - 1.25 + 0.375$$

$$= 3.125$$

$$\therefore \boxed{f_4(0.5) = 3.125}$$

Newton's Backword difference interpolation formula

Let $f(x)$ be a function which is defined at $(n+1)$ pts such as $x_0, x_1, x_2 \cdots x_n$ which are equispaced.

Now taking $x = x_n + uh$

$$\Rightarrow x - x_n = uh$$

$$\Rightarrow \boxed{u = \frac{x - x_n}{h}}$$

Then the Newton's backword difference interpolation formula is given by

(9)

$$f_n(u) = f(x_n) + u\,\nabla f(x_n) + \frac{u(u+1)}{2!}\nabla^2 f(x_n) + \frac{u(u+1)(u+2)}{3!}\nabla^3 f(x_n) + \cdots + \frac{u(u+1)(u+2)\cdots[u+(n-1)]}{n!}\nabla^n f(x_n)$$

Q. Using Newton's Backward difference interpolation formula, find the value of $f(8)$ from the following data

x	1	3	5	7
$y=f(x)$	12	35	73	128

Sol^n Given that

	x_0	x_1	x_2	x_3
x	1	3	5	7
$f(x)$	12	35	73	128
	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$

(10)

Backward difference table

x		$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
x_0	1	12			
x_1	3	35	23		
x_2	5	73	38	15	
x_3	7	128	55	17	2

Here $\boxed{h = 2}$

we have to find $f(8)$

W.K.T

$$\boxed{u = \frac{x - x_n}{h}}$$

Here $x_n = x_3 = 7$, $x = 8$

$\therefore$ $u = \frac{8-7}{2} = \frac{1}{2}$

(11)

we know Newton's Backward difference interpolation formula

$$f_3(x) = f(x_3) + u\nabla f(x_3) + \frac{u(u+1)}{2!}\nabla^2 f(x_3)$$
$$+ \frac{u(u+1)(u+2)}{3!}\nabla^3 f(x_3)$$

$$\therefore f_3(8) = 128 + \frac{1}{2}(55) + \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)}{2\times 1}\cdot(17)$$
$$+ \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right)}{3\times 2\times 1}\cdot(2)$$

$$= 128 + \frac{55}{2} + \frac{3}{8}\cdot(17)$$
$$+ \frac{15/8}{3}$$

$$= 128 + \frac{55}{2} + \frac{51}{8} + \frac{5}{8}$$

$$= 162.5$$

$$\therefore \boxed{f(8) = 162.5}$$

(12)

Q Using Newton's Backward difference interpolation formula find the value of $f(3.5)$ from the following data.

x	-2	0	2	4	6
$f(x)$	3	1	6	18	29

Soln Given that

	x_0	x_1	x_2	x_3	x_4
x	-2	0	2	4	6
$f(x)$	3	1	6	18	29
	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$

we have to find $f(3.5)$

i.e. $\boxed{x = 3.5}$

Backward difference Table

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
-2	3				
0	1	-2			
2	6	5	7		
4	18	12	7	0	
6	29	11	4	-8	-8

Here $\boxed{h = 2}$

W.K.T

$$\boxed{u = \frac{x - x_n}{h}}$$

Here $x_n = x_4 = 6$

$$\therefore u = \frac{3.5 - 6}{2} = -5/4$$

21

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(14)

$$\boxed{u = -5/4}$$

W.K.T

$$f_4(x) = f(x_4) + u \nabla f(x_4) + \frac{u(u+1)}{2!} \nabla^2 f(x_4)$$

$$+ \frac{u(u+1)(u+2)}{3!} \nabla^3 f(x_4)$$

$$+ \frac{u(u+1)(u+2)(u+3)}{4!} \nabla^4 f(x_4)$$

$$\therefore f_4(3.5) = 29 + \left(-\frac{5}{4}\right) \cdot (11)$$

$$+ \frac{\left(-\frac{5}{4}\right)\left(-\frac{5}{4}+1\right)}{2 \times 1} (-1)$$

$$+ \frac{\left(-\frac{5}{4}\right)\left(-\frac{5}{4}+1\right)\left(-\frac{5}{4}+2\right)}{3 \times 2 \times 1} (-8)$$

$$+ \frac{\left(-\frac{5}{4}\right)\left(-\frac{5}{4}+1\right)\left(-\frac{5}{4}+2\right)\left(-\frac{5}{4}+3\right)}{4 \times 3 \times 2 \times 1} \cdot (-8)$$

15

$$= 29 - \frac{55}{4} + \left(-\frac{5}{4}\right)\cdot\left(-\frac{1}{4}\right)\cdot\left(-\frac{1}{2}\right)$$

$$+ \left(-\frac{5}{4}\right)\left(-\frac{1}{4}\right)\left(\frac{3}{4}\right)\cdot\left(-\frac{4}{3}\right)$$

$$+ \left(-\frac{5}{4}\right)\cdot\left(-\frac{1}{4}\right)\left(\frac{3}{4}\right)\cdot\left(\frac{7}{4}\right)\cdot\left(-\frac{1}{3}\right)$$

$$= 28 - \frac{55}{4} - \frac{5}{32} - \frac{5}{16} - \frac{35}{256}$$

$$\boxed{f_4(3.5) = 13.645}$$

$$\therefore \boxed{f(3.5) = 13.645}$$