07 sept 20

(1)

Note:- The rate of convergence of secant method is 1.618 / 1.62

Q Using secant method find a real root of the eqn $x^2 - 3 = 0$ which is correct upto 3 decimal places.

Soln Given that

$$x^2 - 3 = 0 \quad \text{--- (1)}$$

which is in the form of

$$f(x) = 0$$

Here $f(x) = x^2 - 3$

putting $x = 0$

$$f(0) = -3 < 0$$

Again $x = 1$

$$f(1) = -2 < 0$$

Taking $x = 2$ $f(2) = 1 > 0$

(2)

$\therefore$ root lies between 1 and 2...

Let us choose $x_0 = 1$, $x_1 = 2$.

which are initial approximation.

We know that secant method

$$\boxed{x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} \cdot f(x_k)}$$

for $k = 1, 2, 3 \ldots$

Taking $k = 1$

$$\boxed{x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_1)}$$

Now $f(x_0) = f(1) = 1^2 - 3 = -2$

$f(x_1) = f(2) = 2^2 - 3 = 1$

$\therefore x_2 = 2 - \frac{2-1}{1-(-2)} \cdot 1$

$= 2 - \frac{1}{3} = \frac{5}{3} = 1.6667$

③

$$\therefore \boxed{x_2 = 1.6667}$$

Taking $K=2$

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} \cdot f(x_2)$$

$$= 1.6667 - \frac{1.6667 - 2}{f(1.6667) - f(2)} \cdot f(1.667)$$

Now $f(x_2) = f(1.6667) = (1.6667)^2 - 3$

$$= 2.7779 - 3$$

$$= -0.2221$$

$$\therefore x_3 = 1.6667 - \frac{-0.3333}{-0.2221 - 1} \cdot (-0.2221)$$

$$= 1.6667 - \frac{0.3333}{1.2221} \cdot (-0.2221)$$

$$= 1.6667 + \frac{0.3333}{1.2221} (0.2221)$$

$$= 1.6667 + (0.2727)(0.2221)$$

4

$$= 1.6667 + 0.0606 = 1.7273$$

$$\therefore \boxed{x_3 = 1.7273}$$

Taking $\underline{k=3}$

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} \cdot f(x_3)$$

$$= 1.7273 - \frac{1.7273 - 1.6667}{f(1.7273) - f(1.6667)} \cdot f(1.7273)$$

Here $f(1.7273) = (1.7273)^2 - 3$

$$= 2.9836 - 3$$

$$= -0.0164$$

$$\therefore x_4 = 1.7273 - \frac{0.0606}{-0.0164 - (-0.2221)} (-0.0164)$$

$$= 1.7273 - \frac{0.0606}{-0.0164 + 0.2221} \cdot (-0.0164)$$

$$= 1.7273 - \frac{0.0606}{0.2057} (-0.0164)$$

M T W T F S S M T W T F
1 2 3 4 5 6 7 8 9
12 13 14 15 16 17 18 19 20 21 22 23
26 27 28 29 30 31

5

9 $= 1.7273 + (0.2946)\cdot(0.0164)$

10 $= 1.7273 + 0.0048$

11 $= 1.7321$

12 $\therefore \boxed{x_4 = 1.7321}$

1 Taking $\underline{K=4}$

2

3 $$x_5 = x_4 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} \cdot f(x_4)$$

4 $$= 1.7321 - \frac{1.7321 - 1.7273}{f(1.7321) - f(1.7273)} \cdot f(1.7321)$$

5

6 $$= 1.7321 - \frac{0.0048}{f(1.7321) - (-0.0164)} \cdot f(1.7321)$$

7

Now $f(1.7321) = (1.7321)^2 - 3$

$= 3.0002 - 3$

$= 0.0002$

(6)

$$\therefore x_5 = 1.7321 - \frac{0.0048}{0.0002 + 0.0164} \cdot (0.0002)$$

$$= 1.7321 - \frac{0.0048}{0.0166}(0.0002)$$

$$= 1.7321 - (0.2892)(0.0002)$$

$$= 1.7321 - 0.0001$$

$$= 1.732$$

$$\therefore \boxed{x_5 = 1.732}$$

$$\therefore |x_5 - x_4| = 0.0001.$$

$$\therefore \text{root} = 1.732.$$

which is correct up to required decimal places.

H.W

Q.1 Using Secant method, find a real root of $x^3 - 2x - 5 = 0$ which is correct up to three ~~significant figures~~.

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Q-2 Using secant method find the value of r_2 starting with $x_0 = 1$ and $x_1 = 2$, which is correct upto three-significant figures.

Q Using secant method find the root of an eqn $x - \cos x = 0$ where $x_0 = 0.5$ and $x_1 = 1$.

Soln Given that

$$x - \cos x = 0 \quad \text{---} \quad (1)$$

which is in the form of

$$f(x) = 0$$

Here $f(x) = x - \cos x$

$$f(x_0) = f(0.5) = 0.5 - \cos(0.5)$$

$$= 0.5 - 0.8776$$

$$= -0.3776$$

8

$f(1) = 1 - \cos 1 = 1 - 0.5403$
$= 0.4597$

We know secant method

$$\boxed{x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} \cdot f(x_k)}$$

for $k = 1, 2, 3 \ldots$

Taking $\underline{k=1}$

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1)$$

$$= 1 - \frac{1 - 0.5}{0.4597 - (-0.3776)} \cdot (0.4597)$$

$$= 1 - \frac{0.5}{0.4597 + 0.3776} \cdot 0.4597$$

$$= 1 - \frac{0.5}{0.8373} (0.4597)$$

$$= 1 - (0.5972)(0.4597)$$

$$= 1 - 0.2745 = 0.7255$$

M T W T F S S M T W T F S
AUGUST
1 2 3 4 5 6 7 8 9
12 13 14 15 16 17 18 19 20 21 22 23 24
26 27 28 29 30 31

9

$$\therefore \boxed{x_2 = 0.7255}$$

$$f(x_2) = f(0.7255) = 0.7255 - \cos(0.7255)$$

$$= 0.7255 - 0.7482$$

$$= -0.0227$$

Taking $k = 2$

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$$

$$= 0.7255 - \frac{0.7255 - 1}{-0.0227 - 0.4597} (-0.0227)$$

$$= 0.7255 - \frac{-0.2745}{-(0.0227 + 0.4597)} (-0.0227)$$

$$= 0.7255 - \frac{0.2745}{0.4824} (-0.0227)$$

$$= 0.7255 + (0.5690)(0.0227)$$

$$= 0.7255 + 0.0129$$

$$\boxed{x_3 = 0.7384}$$

Taking $k = 3$

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} \cdot f(x_3)$$

$$x_4 = 0.7384 - \frac{0.7384 - 0.7255}{f(0.7384) - f(0.7255)} \cdot f(0.7384)$$

Now

$$f(0.7384) = 0.7384 - \cos(0.7384)$$

$$= 0.7384 - 0.7395$$

$$= -0.0011$$

$$x_4 = 0.7384 - \frac{0.0129}{-0.0011 - (-0.0227)} \cdot (-0.0011)$$

$$= 0.7384 - \frac{0.0129}{0.0216} \cdot (-0.0011)$$

$$= 0.7384 + \frac{(0.0129)}{0.0216} \cdot (0.0011)$$

$$= 0.7384 + (0.5972)(0.0011)$$

$$= 0.7384 + 0.00067$$

$$\boxed{x_4 = 0.7391}$$

$$\therefore |x_4 - x_3| = |0.7391 - 0.7384|$$

$$= 0.0807$$

$$\therefore \boxed{\text{root} = 0.738}$$

which is correct upto 3 decimal places.