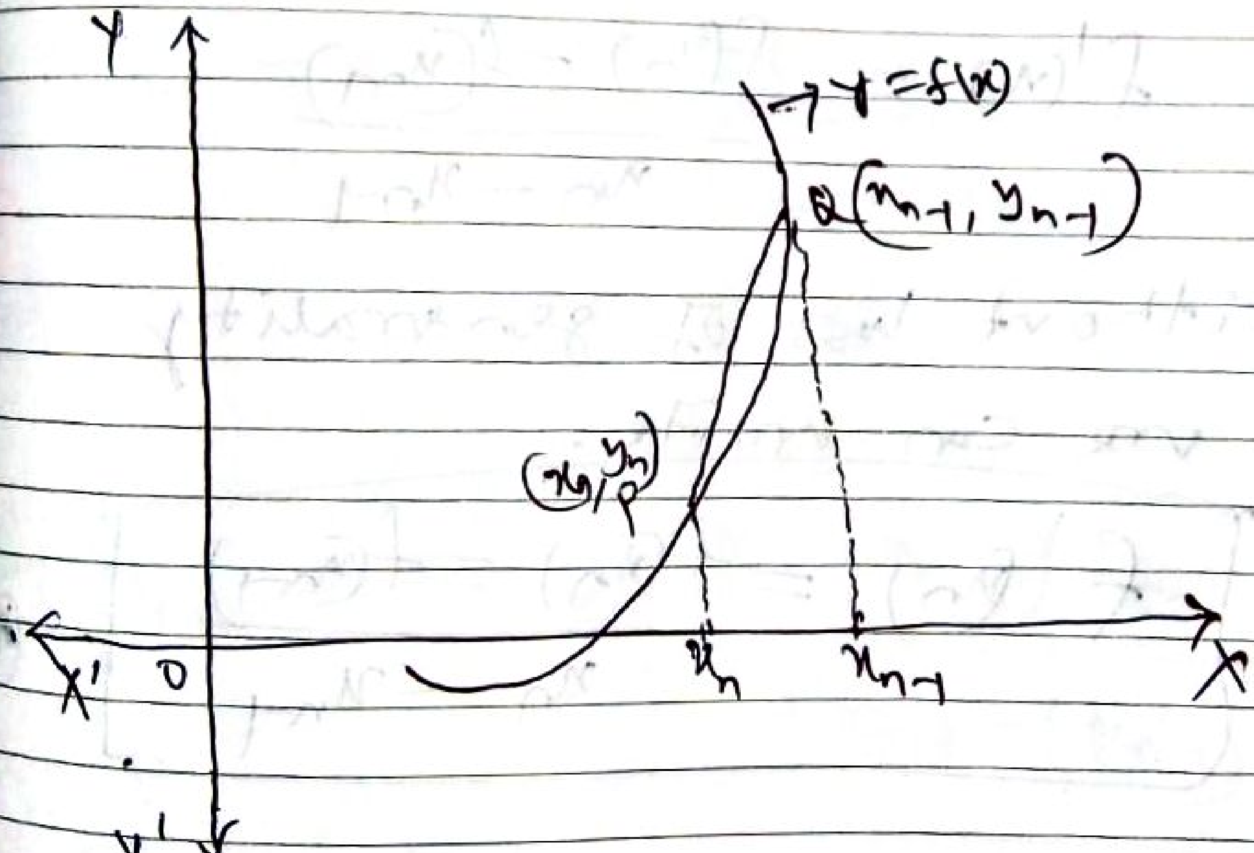
Q. Derive Secant Method.

Ans. Let us consider a curve $y=f(x)$ which cuts x-axis at a point. Let P and Q are two points on the curve $y=f(x)$.

The Co-ordinate of P and Q are (x_n, y_n) and (x_{n-1}, y_{n-1}) respectively.



$$\text{Slope of } PQ = \frac{y_n - y_{n-1}}{x_n - x_{n-1}}$$

M	T	W	T	F	S	S	M	T	W	T	F	S	S
	1	2	3	4	5	6	7	8	9	10	11	12	13
14	15	16	17	18	19	20	21	22	23	24	25	26	27
28	29	30	31										

(2)

since $y = f(x)$

$$\therefore \; y_n = f(x_n)$$

$$y_{n-1} = f(x_{n-1})$$

$$\text{slope} = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

i.e. $$\frac{dy}{dx} = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

$$f'(x) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

without loss of generality we can write

$$\boxed{f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}} \quad \text{— (A)}$$

③

We know N-R method

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

for $n=0,1,2,3\ldots$

putting the value of $f'(x_n)$ in the above formula we get

$$x_{n+1} = x_n - \frac{f(x_n)}{\frac{f(x_n)-f(x_{n-1})}{x_n - x_{n-1}}}$$

$$\Rightarrow \boxed{x_{n+1} = x_n - \frac{(x_n - x_{n-1})\, f(x_n)}{f(x_n) - f(x_{n-1})}}$$

for $n=1,2,3\ldots$

Which is known secant method

(4)

It is also called **chord method**

Note :- It is also expressed as

$$\boxed{x_{k+1} = x_k - \frac{(x_k - x_{k-1})\, f(x_k)}{f(x_k) - f(x_{k-1})}}$$

for $k = 1, 2, 3, \ldots$

(*) Working Rule :-

To find the roots of an eqn $f(x) = 0$ by secant method we use the following steps.

Step-(1) :- Take two initial approximation x_0 and x_1

Step-II :- find $f(x_0)$ and $f(x_1)$

Step-III Apply the formula

$$\boxed{x_{n+1} = x_n - \frac{(x_n - x_{n-1})\, f(x_n)}{f(x_n) - f(x_{n-1})}}$$

for $n=1, 2, 3 \ldots$

Here taking $n=1$ we get the ~~a~~ first approximation x_2.

Step-IV :- Repeat the procedure until getting desired decimal place accuracy.

OR

error tolerance is negligible.