(1)

Crout's Method :-

To solve the system of linear eqⁿ by LU decomposition (Crout's Method) we use following steps.

Step - (I) :- Express the given system of linear eqⁿ in to matrix form

$$\boxed{AX = B} \quad \text{—— (1)}$$

Find A , X , B

Step - (II) :- Again ~~taking~~ eqⁿ (1) can be written as

$$\boxed{LUX = B} \quad \text{—— (2)}$$

Where $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$ → lower triangular matrix

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$$U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{upper triangular matrix.}$$

Step-III :- Taking $\boxed{A = LU}$ — 3

find all elements of L and U.

Step-IV :- Taking $\boxed{LY = B}$ — 4

where $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

Also Here $\boxed{UX = Y}$.

find y_1, y_2, y_3 from 4

③

Step (V) :- Taking $Ux = Y$

find the value of X.

i.e we have to find x_1, x_2, x_3.

Q Solve the system of linear eqn by using LU decomposition method (Crout's method)

$$2x_1 + x_2 + 4x_3 = 12$$
$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$

Soln Given that

$$2x_1 + x_2 + 4x_3 = 12 \quad \text{--- (1)}$$
$$8x_1 - 3x_2 + 2x_3 = 20 \quad \text{--- (2)}$$
$$4x_1 + 11x_2 - x_3 = 33 \quad \text{--- (3)}$$

(4)

The system of linear eqⁿ can be expressed into matrix form.

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\Rightarrow \boxed{A \cdot X = B} \quad \text{— (4)}$$

where $A = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}_{3\times 3}$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} \qquad B = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}_{3\times 1}$$

Now eqⁿ (1) can be expressed as

$$\boxed{LUX = B} \quad \text{— (5)}$$

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where $\boxed{A = LU}$

$$\therefore \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \cdot \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12}+l_{22} & l_{21}u_{13}+l_{22}u_{23} \\ l_{31} & l_{31}u_{12}+l_{32} & l_{31}u_{13}+l_{32}u_{23}+l_{33} \end{bmatrix}$$

Comparing both sides we get

$\boxed{l_{11} = 2}$

$\boxed{l_{21} = 8}$

$\boxed{l_{31} = 4}$

$l_{11}u_{12} = 1$

$2 \cdot u_{12} = 1$

$\boxed{u_{12} = 1/2}$

$l_{21} \cdot u_{12} + l_{22} = -3$

$\cancel{8}^{4} \cdot \frac{1}{\cancel{2}} + l_{22} = -3$

$\boxed{l_{22} = -7}$

$l_{11}u_{13} = 4$

$2 \cdot u_{13} = 4$

$\boxed{u_{13} = 2}$

$l_{21} \cdot u_{13} + l_{22}u_{23} = 2$

$8 \cdot 2 + (-7) \cdot u_{23} = 2$

$u_{23} = \cancel{-14}/-7$

$\boxed{u_{23} = 2}$

b

$l_{31}\cdot u_{12} + l_{32} = 11$

$\cancel{4}^{2}\cdot\frac{1}{\cancel{2}} + l_{32} = 11$

$l_{32} = 11-2$

$\boxed{l_{32} = 9}$

$l_{31}\cdot u_{13} + l_{32}u_{23} + l_{33} = -1$

$\Rightarrow 4\cdot 2 + 9\cdot 2 + l_{33} = -1$

$l_{33} = -1-26$

$\boxed{l_{33} = -27}$

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{33} & l_{33} \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Now taking

$\boxed{LY = B}$ where

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

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$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2y_1 \\ 8y_1 - 7y_2 \\ 4y_1 + 9y_2 - 27y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\Rightarrow \quad 2y_1 = 12 \Rightarrow \boxed{y_1 = 6}$$

$$8y_1 - 7y_2 = 20$$

$$\Rightarrow 8 \cdot 6 - 7 \cdot y_2 = 20$$

$$-7y_2 = 20 - 48 = -28$$

$$\boxed{y_2 = 4}$$

and $4y_1 + 9y_2 - 27y_3 = 33$

$$\Rightarrow 4 \cdot 6 + 9 \cdot 4 - 27y_3 = 33$$

$$\Rightarrow -27y_3 = 33 - 60 = -27$$

(8)

$$\boxed{y_3 = 1}$$

$$\therefore \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$$

Now taking $\boxed{UX = Y}$

$$\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$$

$$x_1 + \frac{x_2}{2} + 2x_3 = 6 \quad \text{——— (a)}$$

$$x_2 + 2x_3 = 4 \quad \text{——— (b)}$$

$$\boxed{x_3 = 1} \quad \text{——— (c)}$$

From (b) $\quad x_2 + 2(1) = 4$

$$x_2 = 4 - 2 = 2$$

$$\boxed{x_2 = 2}$$

9 10 11 12 13 14 15 16 17
23 24 25 26 27 28 29 30 31

(9)

From (a) $x_1 + \frac{x_2}{2} + 2x_3 = 6$

$\Rightarrow x_1 + \frac{2}{2} + 2(1) = 6$

$\Rightarrow x_1 + 1 + 2 = 6$

$\Rightarrow x_1 = 6 - 3 = 3$

$\boxed{x_1 = 3}$

$\therefore \boxed{x_1 = 3}, \boxed{x_2 = 2}, \boxed{x_3 = 1}$

$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

H.W

Q: Using LU decomposition (Crout's method) solve following system of linear eqn.

$2x_1 + x_2 + x_3 = 7$

$x_1 + 2x_2 + x_3 = 8$

$x_1 + x_2 + 2x_3 = 9$

Ans $x_1 = 1$, $x_2 = 2$, $x_3 = 3$