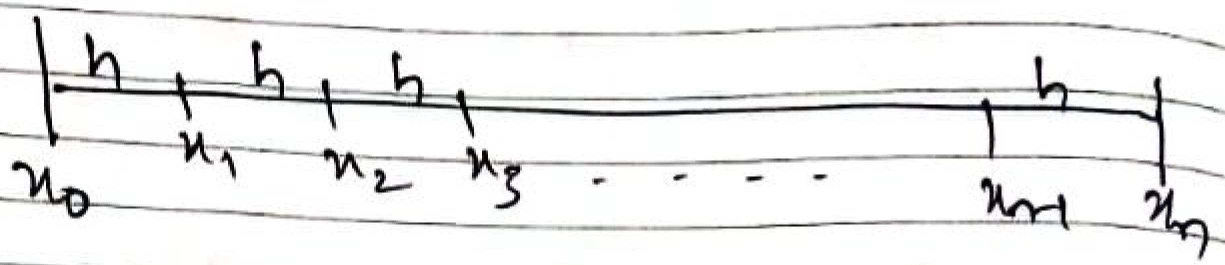
(1)

Interpolation / Interpolating polynomial

Let $x_0, x_1, x_2, \ldots x_n$ are $(n+1)$ pts or nodes, which are equi spaced.



$h \rightarrow$ step size / step length.

The process of finding intermediate value of a function from a set of its value at specific points is called <u>interpolation</u>. which is given in the following table.

x	x_0	x_1	x_2	$\ldots$	x_{n-1}	x_n
$f(x)$	f_{x_0}	$f(x_1)$	$f(x_2)$		$f(x_{n-1})$	$f(x_n)$
	y_0	y_1	y_2		y_{n-1}	y_n

In other words we can say the process of finding the values of y corresponding to x within the given range (x_0, x_n) is called interpolation.

Q. Define Extrapolation.

Ans. It is the process of finding the values of y corresponding to x outside the range (x_0, x_n),

i.e. which does not belong to (x_0, x_n).

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Q. Derive Lagrange's interpolating polynomial.

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Lagrange's linear interpolation

Let $f(x)$ be defined at two distinct points x_0, x_1.

Let $P(x)$ be a linear interpolating polynomial to $f(x)$ in the given interval $[x_0, x_n]$.

Since $P(x)$ is a polynomial of degree 1. Then $P(x)$ can be expressed as

$$\boxed{P(x) = a_0 x + a_1} \text{ --- } (1)$$

As it is interpolating polynomial to $f(x)$.

$$\therefore \boxed{f(x_0) = P(x_0) = a_0 x_0 + a_1} \text{ --- } (2)$$

Similarly

$$\boxed{f(x_1) = P(x_1) = a_0 x_1 + a_1} \text{ --- } (3)$$

Eliminating a_0, a_1 from above three eqn's we get

$$\begin{array}{c} \begin{matrix} (+) & (-) & (+) \end{matrix} \\ \begin{vmatrix} p(x) & x & 1 \\ f(x_0) & x_0 & 1 \\ f(x_1) & x_1 & 1 \end{vmatrix} = 0 \end{array}$$

$$\Rightarrow p(x) \begin{vmatrix} x_0 & 1 \\ x_1 & 1 \end{vmatrix} - x \begin{vmatrix} f(x_0) & 1 \\ f(x_1) & 1 \end{vmatrix} + 1 \begin{vmatrix} f(x_0) & x_0 \\ f(x_1). & x_1 \end{vmatrix} = 0$$

$$\Rightarrow p(x)\left[x_0 - x_1\right] - x\left[f(x_0) - f(x_1)\right] + 1\left[f(x_0)\,x_1 - f(x_1)\,x_0\right] = 0$$

$$\Rightarrow p(x)\left[x_0 - x_1\right] = x\left[f(x_0) - f(x_1)\right] - \left[f(x_0).x_1 - f(x_1).x_0\right]$$

$$= x f(x_0) - x f(x_1) - f(x_0).x_1 + f(x_1).x_0$$

M T W T F S S M T W
1 2 3 4 5 6 7 8 9 10
15 16 17 18 19 20 21 22 23 24
29 30 31

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$$= [x - x_1] f(x_0) - f(x_1)[x - x_0]$$

$$\therefore P(x) = \frac{(x-x_1) f(x_0) - (x-x_0) \cdot f(x_1)}{x_0 - x_1}$$

$$\Rightarrow P(x) = \frac{(x-x_1) \cdot f(x_0)}{x_0 - x_1} - \frac{(x-x_0) f(x_1)}{x_0 - x_1}$$

$$\Rightarrow P(x) = \frac{x - x_1}{x_0 - x_1} \cdot f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

$$\therefore P(x) = l_0(x) f(x_0) + l_1(x) \cdot f(x_1)$$

where $\boxed{l_0(x) = \frac{x - x_1}{x_0 - x_1}}$

$$\boxed{l_1(x) = \frac{x - x_0}{x_1 - x_0}}$$

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It can be expressed as

$$\boxed{f_1(x) = p(x) = l_0(x) f(x_0) + l_1(x) \cdot f(x_1)}$$

which is called Lagrange's linear interpola

(6)

Similarly Lagrange's ~~[illegible]~~ quadratic interpolation is given by

$$\boxed{f_2(x) = p(x) = l_0(x).f(x_0) + L_1(x).f(x_1) + L_2(x).f(x_2)}$$

where

$$\boxed{l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}}$$

$$\boxed{l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}}$$

$$\boxed{l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}}$$

where $f(x)$ is defined at pts x_0, x_1, x_2

proceeding in this way we get

Lagrange's interpolation of degree ~~order~~ 'n' which is

given by

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$$f_n(x) = P(x) = l_0(x)\cdot f(x_0) + l_1(x)\cdot f(x_1) + l_2(x)\cdot f(x_2) + \cdots + l_n(x)\cdot f(x_n)$$

where

$$l_0(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}$$

$\vdots$

$$l_n(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}$$

Here $f(x)$ is defined at $x_0, x_1, x_2 \cdots x_n$.

Ex — Using Lagrange's interpolation find the interpolating polynomial from the datas

$$f(0) = 1,\ f(1) = 3,\ f(3) = 55$$

(8)

Sol? Given that

$f(0) = 1$, $f(1) = 3$, $f(3) = 55$

i.e $f(x_0) = 1$, $f(x_1) = 3$, $f(x_2) = 55$

Here $x_0 = 0$, $x_1 = 1$ $x_2 = 3$

Now

	x_0	x_1	x_2
x	0	1	3
$f(x)$	1	3	55
	$f(x_0)$	$f(x_1)$	$f(x_2)$

we know Lagrange's interpolating polynomial of ~~......~~ degree 2

$$\boxed{f_2(x) = P(x) = l_0(x).f(x_0) + l_1(x).f(x_1) + l_2(x).f(x_2)}$$

(A)

where $l_0(x) = \dfrac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$

$l_1(x) = \dfrac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$

$l_2(x) = \dfrac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$

(9)

Now we have $f(x_0), f(x_1), f(x_2)$ we only find $l_0(x), l_1(x), l_2(x)$

Now $l_0(x) = \dfrac{(x-1)(x-3)}{(0-1)(0-3)} = \dfrac{x^2-4x+3}{3}$

$l_1(x) = \dfrac{(x-0)(x-3)}{(1-0)(1-3)} = \dfrac{x^2-3x}{-2}$

$l_2(x) = \dfrac{(x-0)(x-1)}{(3-0)(3-1)} = \dfrac{x^2-x}{6}$

Putting all the values in eq^n (A) we get

$f_2(x) = p(x) = \dfrac{x^2-4x+3}{3}\cdot(1) + \dfrac{x^2-3x}{-2}\cdot(3)$

$+ \dfrac{x^2-x}{6}\cdot(55)$

$= \dfrac{x^2-4x+3}{3} - \dfrac{(3x^2-9x)}{2} + \dfrac{55x^2-55x}{6}$

(10)

$$= \frac{2[x^2-4x+3]-3(3x^2-9x)+55x^2-55x}{6}$$

$$= \frac{2x^2-8x+6-9x^2+27x+55x^2-55x}{6}$$

$$= \frac{48x^2-36x+6}{6} = \frac{\cancel{6}(8x^2-6x+1)}{\cancel{6}}$$

$\therefore$ $\boxed{f_2(x) = 8x^2-6x+1}$

which is required interpolating polynomial.

Q Using L.I., find interpolating polynomial from the following table.

x	-1	0	2	5
y	9	5	3	15

(11)

Sol Given that

x	-1	0	2	5
y	9	5	3	15

Here $x_0 = -1$, $x_1 = 0$, $x_2 = 2$, $x_3 = 5$

$f(x_0) = 9$, $f(x_1) = 5$, $f(x_2) = 3$, $f(x_3) = 15$

From the data it is clear that we get a cubic polynomial i.e. a polynomial of degree atmost 3 or less.

$$f_3(x) = p(x) = l_0(x).f(x_0) + l_1(x).f(x_1) + l_2(x).f(x_2) + l_3(x).f(x_3) \quad \text{(A)}$$

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where

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

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$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

Now we have $f(x_0)$, $f(x_1)$, $f(x_2)$, $f(x_3)$ only we find the value of $l_0(x)$, $l_1(x)$, $l_2(x)$, $l_3(x)$.

Now
$$l_0(x) = \frac{(x-0)(x-2)(x-5)}{(-1-0)(-1-2)(-1-5)}$$

$$= \frac{x(x^2-7x+10)}{(-1)(-3)(-6)}$$

$$\therefore \boxed{l_0(x) = -\frac{(x^3-7x^2+10x)}{18}}$$

$$l_1(x) = \frac{(x-(-1))(x-2)(x-5)}{(0-(-1))(0-2)(0-5)}$$

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$$= \frac{(x+1)(x-2)(x-5)}{1 \cdot (-2)(-5)}$$

$$= \frac{(x+1)(x^2-7x+10)}{10}$$

$$= \frac{x^3-7x^2+10x+x^2-7x+10}{10}$$

$$\boxed{l_1(x) = \frac{x^3-6x^2+3x+10}{10}}$$

$$l_2(x) = \frac{(x-(-1))(x-0)(x-5)}{(2-(-1))(2-0)(2-5)}$$

$$= \frac{(x+1)\,x\,(x-5)}{3 \cdot 2(-3)} = \frac{(x^2+x)(x-5)}{-18}$$

$$= \frac{x^3+x^2-5x^2-5x}{-18}$$

$$\boxed{l_2(x) = -\frac{(x^3-4x^2-5x)}{18}}$$

$$l_3(x) = \frac{(x-(-1))(x-0)(x-2)}{(5-(-1))(5-0)(5-2)}$$

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$$= \frac{(x+1)\cdot x\cdot(x-2)}{6\cdot 5\cdot 3} = \frac{(x^2+x)(x-2)}{90}$$

$$= \frac{x^3+x^2-2x^2-2x}{90}$$

$$\therefore\ \boxed{l_3(x) = \frac{x^3-x^2-2x}{90}}$$

putting all values in eqn (A)
we get.

$$f_3(x) = f(x) = -\frac{(x^3-7x^2+10x)}{\cancel{18}_{\,2}}\cdot(\cancel{9})$$

$$+\frac{x^3-6x^2+3x+10}{\cancel{18}_{\,2}}\cdot(\cancel{5})$$

$$+\left\{-\frac{(x^3-4x^2-5x)}{\cancel{18}_{\,6}}\cdot(\cancel{3})\right\}$$

$$+\frac{x^3-x^2-2x}{\cancel{90}_{\,6}}\cdot(\cancel{15})$$

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$$f_3(x) = -\frac{(x^3-7x^2+10x)}{2} + \frac{x^3-6x^2+3x+10}{2}$$

$$-\frac{(x^3-4x^2-5x)}{6} + \frac{x^3-x^2-2x}{6}$$

$$= \frac{-3(x^3-7x^2+10x)+3(x^3-6x^2+3x+10) - x^3+4x^2+5x+x^3-x^2-2x}{6}$$

$$= \frac{\cancel{-3x^3}+21x^2-30x+\cancel{3x^3}-18x^2+9x+30 \cancel{-x^3}+4x^2+5x\cancel{+x^3}-x^2-2x}{6}$$

$$= \frac{6x^2-18x+30}{6} = \frac{\cancel{6}(x^2-3x+5)}{\cancel{6}}$$

$$\boxed{f_3(x) = p(x) = x^2-3x+5}$$

which is required interpolating polynomial.

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HW

Q-1 Using Lagrange's interpolation formula find the polynomial from the following table

x	-1	0	2	3
y	-8	3	1	12

Also find f(1).

Ans 2

Q-2 find the interpolating polynomial from the data

x	0	1	3	4	5
y	0	1	8	256	625

by using Lagrange's interpolation polynomial also find f(2).

Ans 16