①

Successive over Relaxation Method [SOR Method].

Consider the system of linear eqn

$$a_{11}x + a_{12}y + a_{13}z = b_1$$

$$a_{21}x + a_{22}y + a_{23}z = b_2$$

$$a_{31}x + a_{32}y + a_{33}z = b_3$$

The above system of linear eqn can be written in matrix form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times1}$$

$$\Rightarrow \boxed{A \cdot X = B} \quad \text{--- ①}$$

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Where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

In G. S. method

$$x^{(k+1)} = \frac{1}{a_{11}}\left[b_1 - a_{12}y^{(k)} - a_{13}z^{(k)}\right]$$

$$y^{(k+1)} = \frac{1}{a_{22}}\left[b_2 - a_{21}x^{(k+1)} - a_{23}z^{(k)}\right]$$

$$z^{(k+1)} = \frac{1}{a_{33}}\left[b_3 - a_{31}x^{(k+1)} - a_{32}y^{(k+1)}\right]$$

for $k = 0, 1, 2, \ldots$

③

where $x^{(0)}, y^{(0)}, z^{(0)}$ are initial approximation chosen by us.

Similarly let 'w' is the relaxation parametre.

Also $x^{(0)}, y^{(0)}, z^{(0)}$ are initial approximations. Then

$$x^{(k+1)} = (1-w)\, x^{(k)} + \frac{w}{a_{11}}\left[b_1 - a_{12} y^{(k)} - a_{13} z^{(k)}\right]$$

$$y^{(k+1)} = (1-w)\, y^{(k)} + \frac{w}{a_{22}}\left[b_2 - a_{21} x^{(k+1)} - a_{23} z^{(k)}\right]$$

$$z^{(k+1)} = (1-w) \cdot z^{(k)} + \frac{w}{a_{33}}\left[b_3 - a_{31} x^{(k+1)} - a_{32} y^{(k+1)}\right]$$

for $k = 0, 1, 2, \ldots$

which is called SOR method.

where the relaxation parameter "w" lies between 1 and 2.

i.e $\boxed{1 < w < 2}$

Here we put the value of x, y, z successively like G.S. method.

Sometimes initial approximation are given in the questions otherwise we take it.

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Note :- (I) w is non-negative.

(II) when $0 < w < 1 \Rightarrow$ under relaxation

(III) when $1 < w < 2 \Rightarrow$ over relaxation

(IV) when $w = 1 \Rightarrow$ G.S. method.

Q Solve the system of linear eqⁿˢ by SOR method, taking relaxation parametre $w = 1.25$ and initial approximation as $x^{(0)} = y^{(0)} = z^{(0)} = 1$.

$$4x + 3y = 14$$

$$3x + 4y - z = 12$$

$$-y + 4z = 6$$

(6)

Sol^n Given that

$$4x + 3y = 14 \quad \text{--- (1)}$$

$$3x + 4y - z = 12 \quad \text{--- (2)}$$

$$-y + 4z = 6 \quad \text{--- (3)}$$

Also given relaxation parametre $w = 1.25$

Initial approximation are $x^{(0)} = 1$, $y^{(0)} = 1$, $z^{(0)} = 1$.

We know by SOR method

$$x^{(k+1)} = (1-w)x^{(k)} + \frac{w}{a_{11}}\left[b_1 - a_{12}y^{(k)} - a_{13}z^{(k)}\right]$$

$$y^{(k+1)} = (1-w)y^{(k)} + \frac{w}{a_{22}}\left[b_2 - a_{21}x^{k+1} - a_{23}z^{(k)}\right]$$

$$z^{(k+1)} = (1-w)z^{(k)} + \frac{w}{a_{33}}\left[b_3 - a_{31}x^{k+1} - a_{32}y^{k+1}\right]$$

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where $k = 0, 1, 2, \ldots$

Here

$$x^{(k+1)} = (1-w)x^{(k)} + \frac{w}{4}\left[14 - 3y^{(k)}\right]$$

$$y^{(k+1)} = (1-w)y^{(k)} + \frac{w}{4}\left[12 - 3x^{(k+1)} + z^{(k)}\right]$$

$$z^{(k+1)} = (1-w)z^{(k)} + \frac{w}{4}\left[6 + y^{(k+1)}\right]$$

for $k = 0, 1, 2, \ldots$

Taking $k = 0$, we get 1st approximation

$$x^{(1)} = (1 - 1.25)x^{(0)} + \frac{1.25}{4}\left[14 - 3y^{(0)}\right]$$

$$= (-0.25).(1) + 0.3125\left[14 - 3\right]$$

$$\Rightarrow \boxed{x^{(1)} = 3.1875}$$

$$y^{(1)} = (1 - 1.25)y^{(0)} + \frac{1.25}{4}\left[12 - 3x^{(1)} + z^{(0)}\right]$$

$$= (-0.25).(1) + 0.3125\left[12 - 3(3.1875) + 1\right]$$

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$$=(-0.25)+0.3125[3.4375]$$

$$\Rightarrow \boxed{y^{(1)} = -0.824}$$

$$z^{(1)} = (1-1.25)z^{(0)} + \frac{1.25}{4}[6+y^{(1)}]$$

$$=(-0.25)(1)+0.3125[6+(-0.824)]$$

$$\Rightarrow \boxed{z^{(1)} = 1.3675}$$

Taking k=1, we get 2nd approx^n

$$x^{(2)} = (-0.25)\cdot(3.1875)+0.3125[14-3(-0.824)]$$

$$\Rightarrow \boxed{x^{(2)} = 4.351}$$

$$y^{(2)} = (-0.25)\cdot(-0.824)+0.3125[12-3(4.351)+1.3675]$$

$$\Rightarrow \boxed{y^{(2)} = 0.304}$$

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$$z^{(2)} = (-0.25)(1.3675) + 0.3125[6 + 0.304]$$

$$\Rightarrow \boxed{z^{(2)} = 1.628}$$

Taking k = 2, we get 3rd approxn.

$$x^{(3)} = (-0.25)(4.351) + 0.3125[14 - 3(0.304)]$$

$$\Rightarrow \boxed{x^{(3)} = 3}$$

$$y^{(3)} = (-0.25)(0.304) + 0.3125[12 - 3(3) + 1.628]$$

$$\Rightarrow \boxed{y^{(3)} = 1.37}$$

$$z^{(3)} = (-0.25)(1.628) + (0.3125)[6 + 1.37]$$

$$\Rightarrow \boxed{z^{(3)} = 1.9}$$

(10)

9 Taking k=3, we get 4th approx^n.

10 $$x^{(4)} = (-0.25).3 + 0.3125\left[14 - 3(1.37)\right]$$

11

$$\Rightarrow \boxed{x^4 = 2.34}$$

1 $$y^{(4)} = (-0.25)(1.37) + 0.3125\left[12 - 3(2.34) + 1.9\right]$$

2

3 $$\Rightarrow \boxed{y^{(4)} = 1.81}$$

4

5 $$z^{(4)} = (-0.25)(1.9) + 0.3125\left[6 + 1.81\right]$$

6

7 $$\Rightarrow \boxed{z^{(4)} = 1.97}$$

Taking k=4, we get 5th approx^n.

$$x^{(5)} = (-0.25).(2.34) + 0.3125\left[14 - 3(1.81)\right]$$

(II)

$$\Rightarrow \boxed{x^{(5)} = 2.1}$$

$$y^{(5)} = (-0.25)(1.81) + 0.3125[12 - 3(2.1) + 1.97]$$

$$\Rightarrow \boxed{y^{(5)} = 1.94}$$

$$z^{(5)} = (-0.25)(1.97) + 0.3125[6 + 1.94]$$

$$\Rightarrow \boxed{z^{(5)} = 1.98}$$

Taking $k=5$, we get 6th approx^n

$$x^{(6)} = (-0.25)\cdot(2.1) + 0.3125[14 - 3(1.94)]$$

$$\Rightarrow \boxed{x^{(6)} = 2.03}$$

$$y^{(6)} = (-0.25)(1.94) + 0.3125[12 - 3(2.03) + 1.98]$$

$$\Rightarrow \boxed{y^{(6)} = 1.98}$$

$$z^{(6)} = (-0.25)(1.98) + 0.3125[6 + 1.98]$$

$$\Rightarrow \boxed{z^{(6)} = 1.99}$$

Taking $K=6$, we get 7th approx^n.

$$x^{(7)} = (-0.25)\cdot(2.03) + (0.3125)[14 - 3(1.98)$$

$$\Rightarrow \boxed{x^{(7)} = 2.01}$$

$$y^{(7)} = (-0.25)(1.98) + 0.3125[12 - 3(2.01) + 1.99]$$

$$\Rightarrow \boxed{y^{(7)} = 1.99}$$

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$$z^{(7)} = (-0.25)(1.99) + 0.3125[6 + 1.99]$$

$$\boxed{z^{(7)} = 1.99}$$

Taking $k=7$, we get 8th approxm

$$x^{(8)} = (-0.25)(2.01) + 0.3125[14 - 3(1.99)]$$

$$\boxed{x^{(8)} = 2.01} \simeq 2$$

$$y^{(8)} = (-0.25)\cdot(1.99) + 0.3125[12 - 3(2.01) + 1.99]$$

$$\boxed{y^{(8)} = 1.99} \simeq 2$$

$$z^{(8)} = (-0.25)\cdot(1.99) + 0.3125[6 + 1.99]$$

$$\boxed{z^{(8)} = 1.99} \simeq 2$$

$\boxed{x = 2}$, $\boxed{y = 2}$, $\boxed{z = 2}$

(14)

9 Q. Solve the system of linear eqn
10 by using method of SOR.
11 Where relaxation parameter
$w = 1.25$ and initial
12 approximations are

1 ~~$x_1^{(0)} = 1, \; x_2^{(0)} = 1, \; x_3^{(0)} = 1$~~

2 $x_1^{(0)} = 1, \; x_2^{(0)} = 1, \; x_3^{(0)} = 1$

3

$4x_1 + 3x_2 = 24$

4 $3x_1 + 4x_2 - x_3 = 30$

5 $-x_2 + 4x_3 = -24$

6

7 Soln Given that

$4x_1 + 3x_2 = 24$ —— (1)

$3x_1 + 4x_2 - x_3 = 30$ —— (2)

$-x_2 + 4x_3 = -24$ —— (3)

(15)

Also given relaxation parametre $w = 1.25$.

Initial approximations are

$x_1^{(0)} = 1, \; x_2^{(0)} = 1, \; x_3^{(0)} = 1.$

We know by SOR method

$$x_1^{(k+1)} = (1-w)\,x_1^{(k)} + \frac{w}{a_{11}}\left[b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}\right]$$

$$x_2^{(k+1)} = (1-w)\,x_2^{(k)} + \frac{w}{a_{22}}\left[b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}\right]$$

$$x_3^{(k+1)} = (1-w)\,x_3^{(k)} + \frac{w}{a_{33}}\left[b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}\right]$$

for $k = 0, 1, 2, \ldots$

Here

$$x_1^{(k+1)} = (1-w)\,x_1^{(k)} + \frac{w}{4}\left[24 - 3x_2^{(k)}\right]$$

$$x_2^{(k+1)} = (1-w)\,x_2^{(k)} + \frac{w}{4}\left[30 - 3x_1^{(k+1)} + x_3^{(k)}\right]$$

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$$x_3^{(k+1)} = (1-w)\,x_3^{(k)} + \frac{w}{4}\left[-24 + x_2^{(k+1)}\right]$$

for $k = 0, 1, 2, \ldots$

Taking $K=0$ we get 1st approximation

$$x_1^{(1)} = (1-1.25).1 + \frac{1.25}{4}\left[24 - 3.(1)\right]$$

$$= -0.25 + 0.3125\,[21]$$

$$\boxed{x_1^{(1)} = 6.3125}$$

$$x_2^{(1)} = (1-1.25).1 + \frac{1.25}{4}\left[30 - 3(6.3125) + 1\right]$$

$$= -0.25 + 0.3125\,[31 - 18.9375]$$

$$\boxed{x_2^{(1)} = 3.52}$$

$$x_3^{(1)} = (1-1.25).1 + \frac{1.25}{4}\left[-24 + 3.52\right]$$

$$= -0.25 + 0.3125\,[-24 + 3.52]$$

$$\boxed{x_3^{(1)} = -6.65}$$

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Taking K=1, we get 2nd approximation

$$x_1^{(2)} = (-0.25)(6.3125) + 0.3125[24 - 3(3.52)]$$

$$= (-0.25)(6.3125) + 4.2$$

$$= -1.578 + 4.2$$

$$\boxed{x_1^{(2)} = 2.62}$$

$$x_2^{(2)} = (-0.25)(3.52) + 0.3125[30 - 3(2.62) + (-6.65)]$$

$$= (-0.25)(3.52) + 4.841$$

$$\boxed{x_2^{(2)} = 3.96}$$

$$x_3^{(2)} = (-0.25)(-6.65) + 0.3125[-24 + 3.96]$$

$$\boxed{x_3^{(2)} = -4.6}$$

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Taking $K=2$ we get 3rd approx^m.

$$x_1^{(3)} = (-0.25)(2.62) + (0.3125)[24 - 3(3.96)]$$

$$\boxed{x_1^{(3)} = 3.1325}$$

$$x_2^{(3)} = (-0.25)(3.96) + (0.3125)[30 - 3(3.1325) + (-4.6)]$$

$$\boxed{x_2^{(3)} = 4.01}$$

$$x_3^{(3)} = (-0.25)(-4.6) + (0.3125)[-24 + 4.01]$$

$$\boxed{x_3^{(3)} = -5.1}$$

Taking $K=3$ we get 4th approx^m.

$$x_1^{(4)} = (-0.25)(3.1325) + (0.3125)[24 - 3(4.01)]$$

$$\boxed{x_1^{(4)} = 2.96}$$

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$$x_2^{(4)} = (-0.25)(4.01) + (0.3125)\left[30 - 3(2.96) + (-5.1)\right]$$

$$\boxed{x_2^{(4)} = 4.004}$$

$$x_3^{(4)} = (-0.25)(-5.1) + (0.3125)\left[-24 + 4.004\right]$$

$$\boxed{x_3^{(4)} = -4.97}$$

Taking $K=4$ we get 5th approx^m

$$x_1^{(5)} = (-0.25)(2.96) + (0.3125)\left[24 - 3(4.004)\right]$$

$$\boxed{x_1^{(5)} = 3.01}$$

$$x_2^{(5)} = (-0.25)(4.004) + 0.3125\left[30 - 3(3.01) + (-4.97)\right]$$

$$\boxed{x_2^{(5)} = 3.999}$$

$$x_3^{(5)} = (-0.25)(-4.97) + 0.3125\left[-24 + 3.999\right]$$

$$\boxed{x_3^{(5)} = -5.01}$$

Taking k=5, we get 6th approximation

$$x_1^{(6)} = (-0.25)(3.01) + (0.3125)\left[24 - 3(3.999)\right]$$

$$\boxed{x_1^{(6)} = 2.99}$$

$$x_2^{(6)} = (-0.25)\cdot(3.999) + (0.3125)\left[30 - 3(2.99) + (-5.01)\right]$$

$$\boxed{x_2^{(6)} = 4.01}$$

$$x_3^{(6)} = (-0.25)(-5.01) + (0.3125)\left[-24 + 4.01\right]$$

$$\boxed{x_3^{(6)} = -4.99}$$

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Taking $k=6$ we get 7th approx^{n}.

$$x_1^{(7)} = (-0.25)(2.99) + (0.3125)[24 - 3(4.01)]$$

$$\boxed{x_1^{(7)} = 2.99} \simeq 3$$

$$x_2^{(7)} = (-0.25)(4.01) + (0.3125)[30 - 3(2.99) + (-4.99)]$$

$$\boxed{x_2^{(7)} = 4.01} \simeq 4$$

$$x_3^{(7)} = (-0.25)(-4.99) + (0.3125)[-24 + 4.01]$$

$$\boxed{x_3^{(7)} = -4.99} \simeq -5$$

Now we observe that 6th and 7th approx^{n}s are same.
We stop the procedure.

$\therefore$ The final result is $\boxed{x_1 = 3}$, $\boxed{x_2 = 4}$, $\boxed{x_3 = -5}$

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HW

Q Solve the following system of linear eqⁿ by using SOR method where the relaxation parametre $w = 1.25$ and initial approximations are

$$x_1^{(0)} = 0, \quad x_2^{(0)} = 0, \quad x_3^{(0)} = 0.$$

$$3x_1 - x_2 + x_3 = -1$$

$$-x_1 + 3x_2 - x_3 = 7$$

$$x_1 - x_2 + 3x_3 = -7$$

Ans $\boxed{x_1 = 1}$, $\boxed{x_2 = 2}$, $\boxed{x_3 = -2}$