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System of linear eqns:—
[Gauss-Seidel method]

Let us consider a system of linear eqns.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \quad \text{—— (1)}$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \quad \text{—— (2)}$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \quad \text{—— (3)}$$

The above system of linear eqn can be written ~~as~~ in matrix form

$$\underset{3\times 3}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}} \cdot \underset{3\times 1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{3\times 1}{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}$$

c R

$$\Rightarrow \boxed{A \cdot X = B}$$

where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$

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$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} \quad \text{and} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times 1}$$

Now to solve the system of linear eqn by Gauss-Siedel method we use the following steps.

i.e. we have to find the value of x_1, x_2, x_3.

Working Rule :-

Step - I :- check the given system of linear eqn satisfy convergent condition.

i.e. diagonally dominant.

$$\therefore \ |a_{11}| \geq |a_{12}| + |a_{13}|$$

$$|a_{22}| \geq |a_{21}| + |a_{23}|$$

$$|a_{33}| \geq |a_{31}| + |a_{32}|$$

Step-II :- If the given eq^ns are not satisfying Convergent condition, then rearrange the eq^ns such that they ~~are~~ satisfy convergent condition.

Step-III :- Applying initial approximation

$$x_1 = \frac{1}{a_{11}}\left[b_1 - a_{12}x_2 - a_{13}x_3\right]$$

$$x_2 = \frac{1}{a_{22}}\left[b_2 - a_{21}x_1 - a_{23}x_3\right]$$

$$x_3 = \frac{1}{a_{33}}\left[b_3 - a_{31}x_1 - a_{32}x_2\right]$$

Step-IV :- Taking the initial approximat $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$

Step-V :- Repeat the procedure until getting two approximation are equal (nearly equal).

Q Using Gauss-Seidel method solve the following system of linear eqn.

$8x_1 + 2x_2 - 2x_3 = 8$

$x_1 - 8x_2 + 3x_3 = -4$

$2x_1 + x_2 + 9x_3 = 12$

Sol Given that

$8x_1 + 2x_2 - 2x_3 = 8$ —— (1)

$x_1 - 8x_2 + 3x_3 = -4$ —— (2)

$2x_1 + x_2 + 9x_3 = 12$ —— (3)

check it is diagonally dominant

$\boxed{|a_{11}| \geq |a_{12}| + |a_{13}|}$

$\therefore |8| > |2| + |-2|$

i.e. $8 > 2+2$

$\boxed{|a_{22}| \geq |a_{21}| + |a_{23}|}$

$\therefore |-8| > |1| + |3|$

$8 > 1+3$

$\boxed{|a_{33}| \geq |a_{31}| + |a_{32}|}$

$\therefore |9| > |2| + |1|$ i.e. $9 > 2+1$

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$$x_1 = \frac{1}{a_{11}}\left[b_1 - a_{12}x_2 - a_{13}x_3\right]$$

$$x_2 = \frac{1}{a_{22}}\left[b_2 - a_{21}x_1 - a_{23}x_3\right]$$

$$x_3 = \frac{1}{a_{33}}\left[b_3 - a_{31}x_1 - a_{32}x_2\right]$$

$$\therefore \quad x_1 = \frac{1}{8}\left[8 - 2x_2 + 2x_3\right] \quad \text{— (a)}$$

$$x_2 = \frac{1}{-8}\left[-4 - x_1 - 3x_3\right] \quad \text{— (b)}$$

$$x_3 = \frac{1}{9}\left[12 - 2x_1 - x_2\right] \quad \text{— (c)}$$

Taking initial approximation

$$x_1^{(0)} = 0, \quad x_2^{(0)} = 0, \quad x_3^{(0)} = 0$$

From (a), (b) (c) we get the 1st approximation

$$x_1^{(1)} = \frac{1}{8}\left[8 - 2(0) + 2(0)\right] = 1$$

$$\boxed{x_1^{(1)} = 1}$$

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$$x_2^{(1)} = \frac{1}{-8}\left[-4 - 10 - 3(1)\right] = \frac{-5}{-8}$$

$$\boxed{x_2^{(1)} = 0.625}$$

$$x_3^{(1)} = \frac{1}{9}\left[12 - 2(1) - \frac{5}{8}\right] = \frac{1}{9}\left[10 - \frac{5}{8}\right]$$

$$= \frac{75}{72} = 1.04$$

$$\boxed{x_3^{(1)} = 1.04}$$

$\therefore$ The 1st approximations are

$$x_1^{(1)} = 1, \quad x_2^{(1)} = 0.625, \quad x_3^{(1)} = 1.04$$

The 2nd approximation are

$$x_1^{(2)} = \frac{1}{8}\left[8 - 2x_2^{(1)} + 2x_3^{(1)}\right]$$

$$= \frac{1}{8}\left[8 - 2(0.625) + 2(1.04)\right]$$

$$\boxed{x_1^{(2)} = 1.104}$$

$$x_2^{(2)} = -\frac{1}{8}\left[-4 - x_1^{(2)} - 3x_3^{(1)}\right]$$

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$$= -\frac{1}{8}\left[-4 - 1.104 - 3(1.04)\right]$$

$$\boxed{x_2^{(2)} = 1.027}$$

$$x_3^{(2)} = \frac{1}{9}\left[12 - 2x_1^{(2)} - x_2^{(2)}\right]$$

$$= \frac{1}{9}\left[12 - 2(1.104) - 1.027\right]$$

$$x_3^{(2)} = 0.986$$

Now 3rd approximations are

$$x_1^{(3)} = \frac{1}{8}\left[8 - 2x_2^{(2)} + 2x_3^{(2)}\right]$$

$$= \frac{1}{8}\left[8 - 2(1.028) + 2(0.981)\right]$$

$$\boxed{x_1^{(3)} = 0.986}$$

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$\cancel{x_2^{(3)}} = \frac{1}{\ } [-4$