7

$$= -\frac{1}{8}\left[-4 - 1.1084 - 3(1.04)\right] = \frac{8.22}{8}$$

$$\boxed{x_2^{(2)} = 1.028}$$

$$x_3^{(2)} = \frac{1}{9}\left[12 - 2x_1^{(2)} - x_2^{(2)}\right]$$

$$= \frac{1}{9}\left[12 - 2(1.1084) - 1.028\right]$$

$$\boxed{x_3^{(2)} = \cancel{0.986}\ 0.974}$$

Now 3rd approximations are

$$x_1^{(3)} = \frac{1}{8}\left[8 - 2x_2^{(2)} + 2x_3^{(2)}\right]$$

$$= \frac{1}{8}\left[8 - 2(1.028) + 2(0.974)\right]$$

$$\boxed{x_1^{(3)} = 0.9865}$$

$$\cancel{x_2^{(3)} = \frac{1}{8}\left[-4 - 0.986 - 3(0.97\right.}$$

$$x_2^{(3)} = -\frac{1}{8}\left[-4 - x_1^{(3)} - 3 \cdot x_3^{(2)}\right]$$

$$= -\frac{1}{8}\left[-4 - 0.9865 - 3(0.974)\right]$$

(8)

$$\boxed{x_2^{(3)} = 0.9886}$$

$$x_3^{(3)} = \frac{1}{9}\left[12 - 2x_1^{(3)} - x_2^{(3)}\right]$$

$$= \frac{1}{9}\left[12 - 2(0.9865) - 0.9886\right]$$

$$= 1.004$$

$$\therefore \boxed{x_3^{(3)} = 1.004}$$

Now $x_1^{(3)} = 0.9865$, $x_2^{(3)} = 0.9886$

$x_3^{(3)} = 1.004$.

Then the 4th approximation are

$$x_1^{(4)} = \frac{1}{8}\left[8 - 2x_2^{(3)} + 2x_3^{(3)}\right]$$

$$= \frac{1}{8}\left[8 - 2(0.9886) + 2(1.004)\right]$$

$$= 1.004$$

$$\boxed{x_1^{(4)} = 1.004}$$

9

$$x_2^{(4)} = -\frac{1}{8}\left[-4 - x_1^{(4)} - 3x_3^{(3)}\right]$$

$$= \frac{1}{8}\left[4 + 1.004 + 3(1.004)\right]$$

$$= 1.002$$

$$\boxed{x_2^{(4)} = 1.002}$$

$$x_3^{(4)} = \frac{1}{9}\left[12 - 2x_1^{(4)} - x_2^{(4)}\right]$$

$$= \frac{1}{9}\left[12 - 2(1.004) - 1.002\right]$$

$$= \frac{1}{9}\left[8.99\right] = 0.999 \approx 1.00$$

$$\therefore \boxed{x_3^{(4)} = 1.00}$$

$$\therefore \boxed{x_1 = 1,\ x_2 = 1,\ x_3 = 1}$$ Ans

verification only for you (not write in exam copy)

eqn① $8 - 2 - 2 = 8$

eqn② $1 - 8 + 3 = -4$

eqn③ $2 + 1 + 9 = \ldots$

(10)

Q. Using Gauss-Seidel method solve the following system of linear eqn.

$$x+y+3z=6$$
$$x+3y+z=8$$
$$2x+y+z=5$$

Soln Given that

$$x+y+3z=6 \quad \text{--- (1)}$$
$$x+3y+z=8 \quad \text{--- (2)}$$
$$2x+y+z=5 \quad \text{--- (3)}$$

The above eqns are not satisfying diagonally dominant / Convergent Condition.

So we rearrange the eqns

$$2x+y+z=5 \quad \text{--- (a)}$$
$$x+3y+z=8 \quad \text{--- (b)}$$
$$x+y+3z=6 \quad \text{--- (c)}$$

(11)

In the eqn (a), (b), (c)

$|a_{11}| \geq |a_{12}| + |a_{13}|$

$|2| \geq |1| + |1|$

i.e $2 = 2$

Next $|a_{22}| \geq |a_{21}| + |a_{23}|$

$|3| \geq |1| + |1|$

$\Rightarrow 3 > 2$

and $|a_{33}| \geq |a_{31}| + |a_{32}|$

$|3| \geq |1| + |1|$

$\Rightarrow 3 > 2$

Since condition of convergen is satisfied then apply Gauss-Seidel formula.

(12)

Here

$$x = \frac{1}{a_{11}}[b_1 - y - z]$$

$$y = \frac{1}{a_{22}}[b_2 - x - z]$$

$$z = \frac{1}{a_{33}}[b_3 - x - y]$$

Here $a_{11} = 2$, $a_{22} = 3$, $a_{33} = 3$

Taking $x^{(0)} = 0$, $y^{(0)} = 0$, $z^{(0)} = 0$.

1st approximation

$x^{(1)} = \frac{1}{2}[5 - 0 - 0]$

$= \frac{5}{2} = 2.5$

$\boxed{x^{(1)} = 2.5}$

$y^{(1)} = \frac{1}{3}[8 - 2.5 - 0]$

$\boxed{y^{(1)} = 1.83}$

$z^{(1)} = \frac{1}{3}[6 - 2.5 - 1.83]$

$\boxed{z^{(1)} = 0.557}$

2nd approximation

$x^{(2)} = \frac{1}{2}[5 - 1.83 - 0.557]$

$\boxed{x^{(2)} = 1.307}$

$y^{(2)} = \frac{1}{3}[8 - 1.307 - 0.557]$

$\boxed{y^{(2)} = 2.045}$

$z^{(2)} = \frac{1}{3}[6 - 1.307 - 2.045]$

$\boxed{z^{(2)} = 0.883}$

(13)

3rd approximation

$x^{(3)} = \frac{1}{2}[5 - 2.045 - 0.883]$

$\boxed{x^{(3)} = 1.036}$

$y^{(3)} = \frac{1}{3}[8 - 1.036 - 0.883]$

$\boxed{y^{(3)} = 2.027}$

$z^{(3)} = \frac{1}{3}[6 - 1.036 - 2.027]$

$\boxed{z^{(3)} = 0.979}$

4th approximation

$x^{(4)} = \frac{1}{2}[5 - 2.027 - 0.979]$

$\boxed{x^{(4)} = 0.997}$

$y^{(4)} = \frac{1}{3}[8 - 0.997 - 0.979]$

$\boxed{y^{(4)} = 2.008}$

$z^{(4)} = \frac{1}{3}[6 - 0.997 - 2.008]$

$\boxed{z^{(4)} = 0.998}$

5th approximation

$x^{(5)} = \frac{1}{2}[5 - 2.008 - 0.998]$

$\boxed{x^{(5)} = 0.997}$

$\simeq 1.00$

$\boxed{x = 1}$

$y^{(5)} = \frac{1}{3}[8 - 0.997 - 0.998]$

$\boxed{y^{(5)} = 2.002}$

$\simeq 2$

$\boxed{y = 2}$

$z^{(5)} = \frac{1}{3}[6 - 0.997 - 2.002]$

$\boxed{z^{(5)} = 1.000}$

$\boxed{z = 1}$

(14)

Verification

eqⁿ ① $2(1)+2+1=5$

eqⁿ ② $1+3(2)+1=8$

eqⁿ ③ $1+2+3(1)=6$

$\therefore$ we conclude that

$\boxed{x=1}$, $\boxed{y=2}$, $\boxed{z=1}$.

Solve the system of linear eqⁿ by LU Decomposition method.

Let us consider the system of linear eqⁿ

$$a_{11}x + a_{12}y + a_{13}z = b_1$$

$$a_{21}x + a_{22}y + a_{23}z = b_2$$

$$a_{31}x + a_{32}y + a_{33}z = b_3$$

15

The system of linear eqn can be written in matrix form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times1}$$

$$\Rightarrow \boxed{A \cdot X = B} \quad \text{--- } (1)$$

where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1}$ $\quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times1}$

Now eqn (1) can be expressed as

$$\boxed{LUX = B} \quad \text{--- } (2)$$

1b

where $\boxed{A = LU}$

where $L \rightarrow$ lower triangular matrix

$U \rightarrow$ upper triangular matrix.

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}_{3\times 3}$$

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}_{3\times 3}$$

LU Decomposition

Dolittle's method

Crout's method