(1)

Newton's Divided difference interpolation

Let $f(x)$ be a function defined at two distinct points x_0 and x_1.

Let $p(x)$ be interpolating polynomial of degree '1'.

$\therefore$ ~~~~ $p(x)$ can be expressed as.

$$p(x) = a_0 x + a_1 \quad \text{--- (1)}$$

As it is interpolating polynomial

$$\therefore \quad f(x_0) = p(x_0) = a_0 x_0 + a_1 \quad \text{--- (2)}$$

$$f(x_1) = p(x_1) = a_0 x_1 + a_1 \quad \text{--- (3)}$$

Eliminating a_0 and a_1 from above three eq^ns we get

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$$\begin{array}{ccc} (+) & (-) & (+) \\ \end{array}$$

$$\begin{vmatrix} p(x) & x & 1 \\ f(x_0) & x_0 & 1 \\ f(x_1) & x_1 & 1 \end{vmatrix} = 0$$

Expanding with R_1

$$p(x)\begin{vmatrix} x_0 & 1 \\ x_1 & 1 \end{vmatrix} - x\begin{vmatrix} f(x_0) & 1 \\ f(x_1) & 1 \end{vmatrix} + 1\begin{vmatrix} f(x_0) & x_0 \\ f(x_1) & x_1 \end{vmatrix} = 0$$

$$\Rightarrow p(x)[x_0 - x_1] - x[f(x_0) - f(x_1)] + f(x_0).x_1 - x_0.f(x_1) = 0$$

$$\Rightarrow p(x)(x_0 - x_1) = x[f(x_0) - f(x_1)] - f(x_0).x_1 + x_0 f(x_1)$$

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$$p(x)[x_0 - x_1] = x f(x_0) - x f(x_1) - f(x_0)\cdot x_1 + x_0 \cdot f(x_1)$$

Adding ~~$x_0 f(x_0)$~~ and subtracting $x_0 f(x_0)$ in RHS.

$$p(x)[x_0 - x_1] = x f(x_0) - x f(x_1) - f(x_0)\cdot x_1 + x_0 f(x_1) - x_0 f(x_0) + x_0 f(x_0)$$

$$= x_0 \cdot f(x_0) - x_1 f(x_0) + x_0 [f(x_1) - f(x_0)] - x[f(x_1) - f(x_0)]$$

$$= f(x_0)[x_0 - x_1] + (x_0 - x)[f(x_1) - f(x_0)]$$

$$\Rightarrow p(x) = \frac{f(x_0)\cdot(x_0 - x_1) + (x_0 - x)\cdot[f(x_1) - f(x_0)]}{x_0 - x_1}$$

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$$P(x) = \frac{f(x_0)\,\cancel{(x_0-x_1)}}{\cancel{(x_0-x_1)}} + \frac{(x_0-x)\cdot[f(x_1)-f(x_0)]}{(x_0-x_1)}$$

$$P(x) = f(x_0) + \frac{(x_0-x)\cdot[f(x_1)-f(x_0)]}{-[x_1-x_0]}$$

$$\Rightarrow P(x) = f(x_0) + (x-x_0)\cdot\frac{[f(x_1)-f(x_0)]}{[x_1-x_0]}$$

$$\therefore P(x) = f(x_0) + (x-x_0)\cdot\frac{f(x_1)-f(x_0)}{x_1-x_0}$$

$$\Rightarrow \boxed{P(x) = f(x_0) + (x-x_0)\cdot f[x_0,x_1]}$$

Where $\boxed{f[x_0,x_1] = \frac{f(x_1)-f(x_0)}{x_1-x_0}}$

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which is called 1st divided difference of $f(x)$ at x_0, x_1.

It is also expressed as

$$\boxed{f_1(x) = f(x_0) + (x-x_0)\, f[x_0, x_1]}$$

which is also called Newton's first divided difference interpolation formula.

Similarly Newton's 2nd divided difference interpolation formula is

$$\boxed{\begin{aligned} f_2(x) &= f(x_0) + (x-x_0)\, f[x_0, x_1] \\ &\quad + (x-x_0)(x-x_1)\, f[x_0, x_1, x_2] \end{aligned}}$$

where $f[x_0, x_1, x_2] = \dfrac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$

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Here $f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Proceeding in this way the nth divided difference interpolation formula is

$$\boxed{\begin{aligned} f_n(x) = P(x) &= f(x_0) + (x - x_0) \cdot f[x_0, x_1] \\ &+ (x - x_0)(x - x_1) f[x_0, x_1, x_2] \\ &+ (x - x_0)(x - x_1)(x - x_2) f[x_0, x_1, x_2, x_3] \\ &+ \cdots \\ &+ (x - x_0)(x - x_1) \cdots (x - x_{n-1}) f[x_0, x_1 \cdots x_n] \end{aligned}}$$

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where

$$f[x_0, x_1, x_2 \cdots x_n] = \frac{f[x_1, x_2 \cdots x_n] - f[x_0, x_1 \cdots x_{n-1}]}{x_n - x_0}$$

Divided Difference Table :-

X	f(X)	1st D.D	2nd D.D	3rd D.D	4th D.D
x_0	$f(x_0)$				
		$f[x_0, x_1]$			
x_1	$f(x_1)$		$f[x_0, x_1, x_2]$		
		$f[x_1, x_2]$		$f[x_0, x_1, x_2, x_3]$	
x_2	$f(x_2)$		$f[x_1, x_2, x_3]$		$f[x_0, x_1, x_2, x_3, x_4]$
		$f[x_2, x_3]$		$f[x_1, x_2, x_3, x_4]$	
x_3	$f(x_3)$		$f[x_2, x_3, x_4]$		
		$f[x_3, x_4]$			
x_4	$f(x_4)$				

Q Construct a Newton's divided difference table from the following data

x	0	2	4	6	8
y	-18	6	54	174	414

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Sol:

x	f(x)	1st D.D Δf	2nd D.D $\Delta^2 f$	3rd D.D $\Delta^3 f$	4th D.D $\Delta^4 f$
0	-18				
		12			
2	6		3		
		24		1	
4	54		9		0
		60		1	
6	174		15		
		120			
8	414				

Explaination

1st D.D

$$\frac{6-(18)}{2-0}=12$$

$$\frac{54-6}{4-2}=24$$

$$\frac{174-54}{6-4}=60$$

$$\frac{414-174}{8-6}=120$$

2nd D.D

$$\frac{24-12}{4-0}=3$$

$$\frac{60-24}{6-2}=9$$

$$\frac{120-60}{8-4}=15$$

3rd D.D

$$\frac{9-3}{6-0}=1$$

$$\frac{15-9}{8-2}=1$$

4th

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Q. Find the interpolating polynomial of degree '2' by using Newton's divided ~~~~ difference formula; Also find $f(2)$ from the following table.

Sol

	x_0	x_1	x_2
x	0	1	3
$f(x)$	1	3	55
	$f(x_0)$	$f(x_1)$	$f(x_2)$

pts
is $(0,1), (1,3), (3,55)$

Sol Given that

x	0	1	3
y	1	3	55

D.D. Table

	x	$f(x)$	1st D.D	2nd D.D
x_0	0	1		
			2	
x_1	1	3		8
			26	
x_2	3	55		

(1D)

W.R.T Newton's Divided Difference formula.

$$\boxed{\begin{aligned} f_n(x) &= f(x_0) + (x-x_0)\, f[x_0,x_1] \\ &\quad + (x-x_0)(x-x_1)\, f[x_0,x_1,x_2] + \dots \\ &\quad + (x-x_0)(x-x_1)\cdots(x-x_{n-1})\, f[x_0,x_1,\dots,x_n] \end{aligned}}$$

Here

$$f_2(x) = f(x_0) + (x-x_0)\, f[x_0,x_1] + (x-x_0)(x-x_1)\, f[x_0,x_1,x_2]$$

Putting the value from the table

$$f_2(x) = 1 + (x-0)\cdot 2 + (x-0)(x-1)\cdot 8$$

$$= 1 + 2x + (x^2-x)8$$

$$= 1 + 2x + 8x^2 - 8x$$

$$\boxed{f_2(x) = 8x^2 - 6x + 1}$$

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which is required interpolating polynomial.

we have to find f(2)

putting $x=2$ we get

$f_2(2) = 8(2)^2 - 6.2+1$

$= 32 - 12 + 1 = 21$

$\therefore \boxed{f(2) = 21}$

Using Newton's divided difference formula find the value of f(4) from the given datas.

$f(0) = 2, \ f(1) = 3, \ f(2) = 12$

$f(5) = 3587$

(12)

Sol: Given that

$f(0) = 2,\ f(1) = 3,\ f(2) = 12,\ f(15) = 3587$

$f(x_0) = 2,\ f(x_1) = 3,\ f(x_2) = 12,$
$f(x_3) = 3587$

D.D Table

	x	$f(x)$	1st Δf	2nd $\Delta^2 f$	3rd $\Delta^3 f$
x_0	0	2			
			1		
x_1	1	3		4	
			9		1
x_2	2	12		19	
			275		
x_3	15	3587			

W.K.T Newton's divided difference formula

$$\boxed{\begin{aligned} f_n(x) &= f(x_0) + (x - x_0) f[x_0, x_1] \\ &+ (x - x_0)(x - x_1) f[x_0, x_1, x_2] + \cdots \\ &+ (x - x_0)(x - x_1) \cdots (x - x_{n-1}) f[x_0, x_1, \ldots, x_n] \end{aligned}}$$

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Here

$$f_3(x) = f(x_0) + (x-x_0).f[x_0, x_1] + (x-x_0)(x-x_1) . f[x_0, x_1, x_2] + (x-x_0)(x-x_1)(x-x_2) f[x_0, x_1, x_2, x_3]$$

Putting all values from the table we get

$$f_3(x) = 2 + (x-0).1 + (x-0)(x-1).4 + (x-0)(x-1)(x-2).1$$

$$= 2 + x + (x^2-x).4 + x(x^2-3x+2).$$

$$= 2 + x + 4x^2 - 4x + x^3 - 3x^2 + 2x$$

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$$\boxed{f_3(x) = x^3 + x^2 - x + 2}$$

$$f_3(4) = 4^3 + 4^2 - 4 + 2 = 78$$

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HW

Q Using Newton's divided difference interpolation formula find the value of f (2) from the following table.

X	3	4	7	9	10
Y	5	8	11	13	17

Ans $\boxed{f(2) = 1}$