(1)

Test of significance for single mean

Let $x_1, x_2, x_3 \cdots x_n$ be a random sample of size 'n' from a large sample $X_1, X_2 \cdots X_N$ (of size N) with mean μ and variance σ^2

Then sample mean ($\bar{x}$) and variance (s^2) are

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad , \quad s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Also, the standard error (S.E.) of mean of a random sample of size (n) from a population with variance (σ^2) is $\sigma/\sqrt{n}$

i.e. $$S.E(\bar{x}) = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}}$$

For large sample, the standard normal variate corresponding $\bar{x}$ is given by

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Priorities

(2)

- Under null hypothesis (H_0) that the sample has been drawn from a population with mean 'μ' and variance 'σ^2'
ie there is no significant difference between the sample mean ($\bar{x}$) and population mean (μ)
for large sample the test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

If the population standard deviation 'σ' is not known

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

where 's' is the standard deviation of the sample.

3

Confidence limits for μ

(1) If the LOS is α and Z_α is the critical value then

$$-Z_\alpha < \left| \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \right| < Z_\alpha$$

The limit of population mean 'μ' is given by

$$\bar{x} - Z_\alpha \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

(2) (a) 95% confidence interval for μ at 5% level of significance is

$$\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}$$

Priorities

(b) 99% confidence interval for μ at 1% level of significance is

Devo

(4)

$$\bar{x} - 2.58 \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 2.58 \frac{\sigma}{\sqrt{n}}$$

(3) In sampling from a finite population of size N the corresponding 95% and 99% confidence limit for μ are

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$ [for 95%]

$$\bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$ (for 99%)

Q - (1) The average marks in Mathematics of a sample of 100 students was 51 with a S.D of 6 marks Could this have been a random sample from a population with average marks 50.

Priorities

(5)

sol Given that Sample size $n = 100$

$\bar{x}$ = sample mean = 51

Sample S.D $s = 6$

population mean $\mu = 50$

σ is unknown here.

Consider null hypothesis

H_0 : the sample drawn from a population with $\mu = 50$

$H_1 : \mu \neq 50$

Under H_0,

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{51-50}{6/\sqrt{100}}$$

$$= \frac{1}{6/10} = \frac{10}{6}$$

$$\Rightarrow z = 1.6666 \simeq \cancel{1.667}\ 1.67$$

$$\Rightarrow |z| = 1.67$$

(6)

(2nd)

Conclusion :- Since $|Z| = 1.67 < 1.96$

When Z_α is the significant value of Z at 5% LOS.

$\therefore H_0$ is accepted.

Q-2 A normal population has a mean of 6.8 and S.D 1.5. A sample of 400 members gave a mean of 6.75, is the difference significant.

Soln H_0: There is no significant difference between $\bar{x}$ and μ.

H_1: There is significant difference between $\bar{x}$ and μ.

Priorities

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Given that $\mu = 6.8$

$\sigma = 1.5$

$\bar{x} = 6.75$, $n = 400$

$$\therefore |Z| = \left| \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \right|$$

$$= \left| \frac{6.75 - 6.8}{1.5 / \sqrt{400}} \right|$$

$$= |-0.67| = 0.67$$

Conclusion

$|Z| < Z_\alpha = 1.96$

at 5% level of significance.

$\therefore H_0$ is accepted.

Hence there is no significant difference between $\bar{x}$ and μ.

Q (3) A random sample of 900 members has mean 3.4 cms. Can it be reasonably regarded as a sample from a large ~~sample~~ population of mean 3.2 cms and S.D 2.3 cms.

Soln Given that

n = Sample size = 900

$\bar{x} = 3.4$ = Sample mean

$\mu = 3.2$ (population mean)

$\sigma = 2.3$ (population S.D)

Consider null hypothesis

H_0: Assume the sample is drawn from a large population with mean 3.2 and S.D = 2.3

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$H_1: \mu \neq 3.2$

Under H_0

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{3.4 - 3.2}{2.3/\sqrt{900}}$$

$$= 0.261$$

$$|z| = 0.261$$

Conclusion:

Now $|z| = 0.261 < 1.96$ (z_α)

The significant value of z at 5% LOS.

H_0 is accepted therefore the sample is drawn from the population with $\mu = 3.2$ and S.D 2.3.

(18)

~~Q (4)~~ The mean weight obtained from a random sample of size 100 is 64 gms. The S.D is 3 and the mean wt distribution of the population is 67 gms. at 5% LOS. Also set up 99% confidence limits of the mean weight of the population.

~~Soln~~ Given that

$n = 100, \mu = 67$

$\bar{x} = 64, \sigma = 3$

H_0: There is no significant difference between sample and population mean

H_1: There is significant

(11)

difference between sample mean and population mean.

under H_0

$$|z| = \left|\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right| = \left|\frac{64-67}{3/\sqrt{100}}\right| = |10|$$

$$|z| = 10$$

Conclusion:

since $|z| > 1.96$ (z_α)

the significant value of z at 5% LOS.

H_0 is rejected.

i.e. the sample is not drawn from the population with $\mu = 67$

(ii) 99% confidence limit is

Priorities

given by

$$\bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}} = 64 \pm 2.58 \cdot \frac{3}{\sqrt{100}}$$

$$= 64.774,\ 63.226.$$