①

Newton's forward and Backward Difference

Let $f(x)$ be a function which is defined at $(n+1)$ pts such as $x_0, x_1, x_2 \cdots x_n$.

Now $x_1 = x_0 + h$

$$x_2 = x_1 + h = x_0 + h + h = x_0 + 2h$$

$$x_3 = x_0 + 3h$$

$$x_4 = x_0 + 4h$$

$$\vdots$$

$$x_n = x_0 + nh$$

$$\therefore \quad nh = x_n - x_0$$

$$\boxed{h = \frac{x_n - x_0}{n}}$$

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1 2 3 4 5 6 7 8 9 10 11 12
13 14 15 16 17 18 19 20 21 22 23 24 25 26
27 28 29 30 31

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Now we have difference operator.

(I) Forward difference operator (Δ)

(II) Backward difference operator (∇)

Forward difference operator (Δ)

It is defined as

$$\boxed{\Delta f(x_i) = f(x_i + h) - f(x_i)}$$

for $i = 0, 1, 2 \cdots n$

Taking $i = 0$

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$$\Delta f(x_0) = f(x_0 + h) - f(x_0)$$
$$= f(x_1) - f(x_0)$$

$$\therefore \boxed{\Delta f(x_0) = f(x_1) - f(x_0)}$$

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③

Taking $i=1$

$$\Delta f(x_1) = f(x_1+h) - f(x_1)$$

$$= f(x_2) - f(x_1)$$

$$\therefore \boxed{\Delta f(x_1) = f(x_2) - f(x_1)}$$

Taking $i=2$

$$\boxed{\Delta f(x_2) = f(x_3) - f(x_2)}$$

proceeding similarly

for $i=n$ we get

$$\boxed{\Delta f(x_n) = f(x_{n+1}) - f(x_n)}$$

Forrward difference Table :-

Table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
x_0	$f(x_0)$				
		$\Delta f(x_0)$			
x_1	$f(x_1)$		$\Delta^2 f(x_0)$		
		$\Delta f(x_1)$		$\Delta^3 f(x_0)$	
x_2	$f(x_2)$		$\Delta^2 f(x_1)$		$\Delta^4 f(x_0)$
		$\Delta f(x_2)$		$\Delta^3 f(x_1)$	
x_3	$f(x_3)$		$\Delta^2 f(x_2)$		
		$\Delta f(x_3)$			
x_4	$f(x_4)$				

This table is also known as diagonal difference table.

Ex Construct a forrward difference table from following data.

x	2	4	6	8	10
y	5	10	17	29	50

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Also find $\Delta f(2)$, $\Delta f(6)$, $\Delta^2 f(6)$, $\Delta^2 f(4)$, $\Delta^3 f(2)$, $\Delta^4 f(2)$.

Solⁿ Table :-

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
2	5				
		5			
4	10		2		
		7		3	
6	17		5		1
		12		4	
8	29		9		
		21			
10	50				

$\Delta f(2) = 5$, $\Delta f(6) = 12$

$\Delta^2 f(6) = 9$, $\Delta^2 f(4) = 5$

$\Delta^3 f(2) = 3$ $\Delta^4 f(2) = 1$

(6)

Q Construct a forward difference table from the following data

x	1	2	3	4	5
$y=f(x)$	4	6	9	12	17

find $\Delta f(3)$, $\Delta^2 f(3)$, $\Delta^2 f(1)$

Soln $\Delta^3 f(1)$, $\Delta^3 f(2)$, $\Delta^4 f(1)$.

Table :-

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	4				
		2			
2	6		1		
		3		-1	
3	9		0		3
		3		2	
4	12		2		
		5			
5	17				

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Now $\Delta f_5(3) = 3$, $\Delta^2 f(3) = 2$

$\Delta^2 f(1) = 1$, $\Delta^3 f(1) = -1$

$\Delta^3 f(2) = 2$, $\Delta^4 f(1) = 3$

Backward difference operator (∇)

It is defined as

$$\boxed{\nabla f(x_i) = f(x_i) - f(x_i - h)}$$

for $i = 1, 2, 3 \ldots n$

Taking $i = 1$

$$\nabla f(x_1) = f(x_1) - f(x_1 - h)$$

$$= f(x_1) - f(x_0) \quad (\text{As } x_1 = x_0 + h)$$

$$\therefore \boxed{\nabla f(x_1) = f(x_1) - f(x_0)}$$

M	T	W	T	F	S	S	M	T	W	T
					1	2	3	4	5	6
10	11	12	13	14	15	16	17	18	19	20
24	25	26	27	28	29	30				

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9 Taking $i=2$

10 $$\nabla f(x_2) = f(x_2) - f(x_2 - h)$$

11 $$= f(x_2) - f(x_1) \quad (\because x_2 = x_1 + h$$

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1 $$\therefore \boxed{\nabla f(x_2) = f(x_2) - f(x_1)}$$

2

Taking $i=3$

3

4 $$\boxed{\nabla f(x_3) = f(x_3) - f(x_2)}$$

5

6 proceeding similarly for

7 $i=n$ we get

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$$\boxed{\nabla f(x_n) = f(x_n) - f(x_{n-1})}$$

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Backword difference Table

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
x_0	$f(x_0)$				
x_1	$f(x_1)$	$\nabla f(x_1)$			
x_2	$f(x_2)$	$\nabla f(x_2)$	$\nabla^2 f(x_2)$		
x_3	$f(x_3)$	$\nabla f(x_3)$	$\nabla^2 f(x_3)$	$\nabla^3 f(x_3)$	
x_4	$f(x_4)$	$\nabla f(x_4)$	$\nabla^2 f(x_4)$	$\nabla^3 f(x_4)$	$\nabla^4 f(x_4)$

This table is also called horizontal difference table.

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Ex Construct a backward difference table from the following data.

x	1	3	5	7	9
y	8	12	21	36	62

Find the value of $\nabla f(3)$, $\nabla^2 f(7)$, $\nabla^3 f(9)$, $\nabla^4 f(9)$.

Soln Table:-

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
1	8				
3	12	4			
5	21	9	5		
7	36	15	6	1	
9	62	26	11	5	4

From table it clear that

(11)

$\nabla f(3) = 4$, $\nabla^2 f(7) = 6$

$\nabla^3 f(9) = 5$, $\nabla^4 f(9) = 4$

Q. Construct a Backwardword difference table from the following data

x	1	2	3	4	5
y	4	6	9	12	17

Also find $\nabla f(2)$, $\nabla^2 y(3)$, $\nabla^2 f(4)$, $\nabla^3 f(5)$, $\nabla^4 f$...

Soln

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
1	4				
2	6	2			
3	9	3	1		
4	12	3	0	-1	
5	17	5	2	2	3

(12)

From the ~~table~~ it is clear that

$\nabla f(2) = 2$

$\nabla^2 f(3) = 1$

$\nabla^2 f(4) = 0$

$\nabla^3 f(5) = 2$

$\nabla^4 f(5) = 3$