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## Dolittle's Method :-

To solve the system of linear eqn by LU decomposition (Dolittle method) we use following steps.

**Step - I** :- Express the given system of linear eqn

$$\boxed{A \cdot X = B} \quad \text{--- (1)}$$

Find A, X, B

**Step - II** :- Again eqn (1) can be expressed as

$$\boxed{LUX = B} \quad \text{--- (2)}$$

where $\boxed{A = LU}$

**Step - III** :- Taking $\boxed{A = LU}$ --- (3)

where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$

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$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

from eqn 3 find all elements of L and U.

Step - IV :- Taking $\boxed{LY = B}$

where $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

Here $\boxed{UX = Y}$

find $y_1, y_2, y_3$ of Y.

Step - V :- ~~Again~~ Next taking

$\boxed{UX = Y}$

find the value of X.

i.e we get the value of $x_1, x_2, x_3$.

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**Q** Solve the system of linear eq$^n$ by LU decomposition method. 'OR' by Doolittle's method.

$$2x_1 + x_2 + 4x_3 = 12$$
$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 - x_3 = 33.$$

**Sol$^n$** Given that

$$2x_1 + x_2 + 4x_3 = 12 \quad \text{--- } (1)$$
$$8x_1 - 3x_2 + 2x_3 = 20 \quad \text{--- } (2)$$
$$4x_1 + 11x_2 - x_3 = 33 \quad \text{--- } (3)$$

The above eq$^n$'s are written as

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$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}_{3\times 3} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}_{3\times 1}$$

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$$\boxed{A \cdot X = B} \quad \text{--- } (4)$$

④

where
$$A = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}_{3\times3}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1} \qquad B = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}_{3\times1}.$$

Now eqn ④ can be written as:

$$\boxed{LUX = B} \text{ --- } ⑤$$

Here $\boxed{A = LU}$

i.e. $$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

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Comparing both sides we get

$\boxed{u_{11} = 2}$

$l_{21} \cdot u_{11} = 8$

$\Rightarrow l_{21} \cdot (2) = 8$

$\boxed{l_{21} = 4}$

$l_{31} u_{11} = 4$

$l_{31} \cdot (2) = 4$

$\boxed{l_{31} = 2}$

$\boxed{u_{12} = 1}$

$l_{21} \cdot u_{12} + u_{22} = -3$

$4 \cdot 1 + u_{22} = -3$

$\boxed{u_{22} = -7}$

$l_{31} \cdot u_{12} + l_{32} u_{22} = 11$

$2 \cdot 1 + l_{32} (-7) = 11$

$-7 l_{32} = 9$

$\boxed{l_{32} = -\frac{9}{7}}$

$\boxed{u_{13} = 4}$

$l_{21} \cdot u_{13} + u_{23} = 2$

$4 \cdot 4 + u_{23} = 2$

$\boxed{u_{23} = -14}$

$l_{31} \cdot u_{13} + l_{32} u_{23} + u_{33} = -1$

$2 \cdot 4 + -\frac{9}{7} \cdot -14 + u_{33} = -1$

$8 + 18 + u_{33} = -1$

$\boxed{u_{33} = -27}$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix}$$

(6)

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix}$$

Taking $\boxed{LY = B}$ where $\boxed{Y = UX}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix}_{3\times3} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}_{3\times1} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}_{3\times1}$$

$$\Rightarrow \begin{bmatrix} y_1 \\ 4y_1 + y_2 \\ 2y_1 - \frac{9}{7}y_2 + y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\Rightarrow \boxed{y_1 = 12}$$

$$4y_1 + y_2 = 20$$

$$\Rightarrow 4 \times 12 + y_2 = 20$$

$$y_2 = 20 - 48$$

$$\Rightarrow \boxed{y_2 = -28}$$

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$$2y_1 - \frac{9}{7}y_2 + y_3 = 33$$

$$\Rightarrow 2\cdot(12) - \frac{9}{7}\cdot(-28) + y_3 = 33$$

$$\Rightarrow 24 + 36 + y_3 = 33$$

$$y_3 = 33 - 60 = -27$$

$$\boxed{y_3 = -27}$$

$$\therefore \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

Now taking $\boxed{Ux = y}$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

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$$2x_1 + x_2 + 4x_3 = 12 \quad — (A)$$

$$-7x_2 - 14x_3 = -28 \quad — (B)$$

$$-27x_3 = -27 \quad — (C)$$

From (C)

$$\boxed{x_3 = 1}$$

putting the value of $x_3$ in eqn (B)

$$-7x_2 - 14(1) = -28$$

$$-7x_2 = -28 + 14 = -14$$

$$x_2 = -14/-7 = 2$$

$$\boxed{x_2 = 2}$$

Again
putting the value of $x_2, x_3$ in eqn (A)

$$2x_1 + 2 + 4(1) = 12$$

$$2x_1 = 12 - 6 = 6$$

$$x_1 = 6/2 = 3$$

$$\boxed{x_1 = 3}$$

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$$\therefore \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

**H.W** Using LU decomposition method (Dolittle's method) solve the following system of linear eqn.

$$2x_1 + x_2 + x_3 = 7$$

$$x_1 + 2x_2 + x_3 = 8$$

$$x_1 + x_2 + 2x_3 = 9$$

**Ans** $x_1 = 1, \; x_2 = 2, \; x_3 = 3$



$$\therefore \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$