Q-① Find the value of $y(0.2)$ and $y'(0.2)$ where $y'' = x\cdot(y')^2 - y^2$ given that $y=1$, $\frac{dy}{dx}=0$ at $x=0$ by using R-K method.

Sol? Given that

$$\frac{d^2y}{dx^2} = x\cdot\left(\frac{dy}{dx}\right)^2 - y^2, \quad y(0)=1, \quad \left.\frac{dy}{dx}\right]_{x=0} = 0$$

$$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = x\cdot\left(\frac{dy}{dx}\right)^2 - y^2, \quad y(0)=1, \quad \left.\frac{dy}{dx}\right)_{x=0} = 0$$

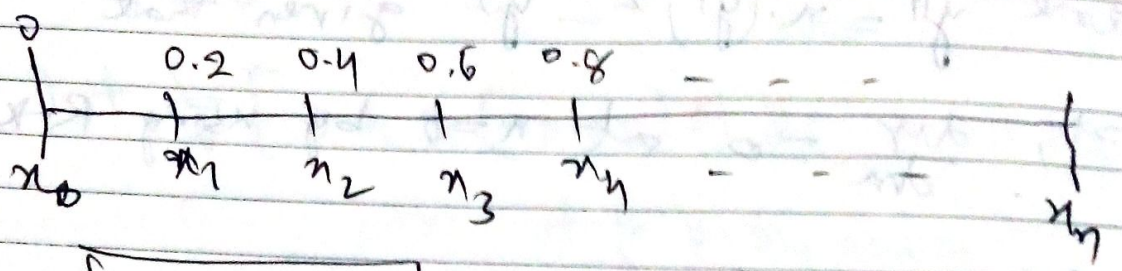
putting $\frac{dy}{dx} = z$

$$\frac{dz}{dx} = xz^2 - y^2, \quad y(0)=1, \quad z(0)=0$$

$$\left.\begin{aligned} \frac{dy}{dx} &= f(x,y,z) = z, \quad y(0)=1 \\ \frac{dz}{dx} &= g(x,y,z) = xz^2 - y^2, \quad z(0)=0 \end{aligned}\right\}$$

(2)

Here $x_0 = 0$, $y_0 = 1$, $z_0 = 0$.



$$h = 0.2$$

We have to find $y(0.2)$ and $y'(0.2)$

i.e. $y(0.2)$ and $z(0.2)$

$\Rightarrow$ $y(x_1)$ and $z(x_1)$

i.e. y_1 and z_1.

We know by R-K method

$$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$z_1 = z_0 + \frac{1}{6}[m_1 + 2m_2 + 2m_3 + m_4]$$

We have to determine K_1, K_2, K_3, K_4 and m_1, m_2, m_3, m_4.

$K_1 = h f(x_0, y_0, z_0)$

$= 0.2 \cdot f(0, 1, 0)$

$= (0.2) \cdot 0$

$\boxed{K_1 = 0}$

$m_1 = h g(x_0, y_0, z_0)$

$= 0.2\, g(0, 1, 0)$

$= 0.2 [0.0 - (1)^2] = -0.2$

$\boxed{m_1 = -0.2}$

$K_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}, z_0 + \frac{m_1}{2}\right)$

$= 0.2 f(0.1, 1, -0.1)$

$= (0.2) \cdot (-0.1) = -0.02$

$\boxed{K_2 = -0.02}$

$m_2 = h g\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}, z_0 + \frac{m_1}{2}\right)$

$= h g(0.1, 1, -0.1)$

$= (0.2)[(0.1) \cdot (-0.1)^2 - (1)^2]$

$= (0.2)[(0.1)(0.01) - 1]$

$\boxed{m_2 = -0.1998}$

$K_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{m_2}{2}\right)$

$= 0.2 f(0.1, 0.99, -0.0999)$

$= (0.2) \cdot (-0.0999)$

$\boxed{K_3 = -0.01998}$

$m_3 = h g\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{m_2}{2}\right)$

$= 0.2\, g(0.1, 0.99, -0.0999)$

$= (0.2)[(0.1) \cdot (-0.0999)^2 - (0.99)^2]$

$\boxed{m_3 = -0.1958}$

(4)

$k_4 = h f(x_0+h, y_0+k_3, z_0+m_3)$

$= 0.2 f(0.2, 1-0.01998, -0.1958)$

$= 0.2(-0.1958)$

$\boxed{k_4 = -0.03916}$

$m_4 = h g(x_0+h, y_0+k_3, z_0+m_3)$

$= h g(0.2, 0.98002, -0.1958)$

$= 0.2[(0.2)(-0.1958)^2 - (0.98002)$

$= -0.19055$

$\boxed{m_4 = -0.19055}$

Now

$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= 1 + \frac{1}{6}[0 + 2(-0.02) + 2(-0.01998) + (-0.03916)]$

$\boxed{y_1 = 0.9801}$

$\Rightarrow \boxed{y(0.2) = 0.9801}$

(5)

$$z_4 = z_0 + \frac{1}{6}[m_1 + 2m_2 + 2m_3 + m_4]$$

$$= 0 + \frac{1}{6}[-0.2 + 2(-0.1998) + 2(-0.1956) + (-0.19055)]$$

$$\boxed{z_4 = -0.1967}$$

$$\therefore z(0.2) = -0.1967$$

$$\therefore \boxed{y'(0.2) = -0.1967}$$

Q-① H.W

Using R-K method order 4, solve the system of diff eqn.

$$\frac{dy}{dx} = 1 + xz, \quad \frac{dz}{dx} = -xy$$

find the value of $y(0.3)$ and $z(0.3)$ where $x=0, y=0, z=1$.

Q-② Using R-K method

Solve $y'' = y + xy'$, $y(0) = 1$, $y'(0) = 0$

find $y(0.2)$ and $y'(0.2)$.

(6)

Using Euler method,

Solve $\frac{dy}{dx} = -\frac{1}{2}y$, $\frac{dz}{dx} = 4 - 0.3z - 0.1y$

$y(0) = 4$, $z(0) = 6$, $h = 0.5$

Find the value of $y(0.5)$ and $z(0.5)$

and also $y(1)$ and $z(1)$.

Solve

$y'' + 0.3y' + 0.1y = 4$

by using Euler method

find the value of $y(0.5)$, $z(0.5)$

$y(1)$ and $z(1)$ where

$y(0) = 4$, $z(0) = 6$, $h = 0.5$.

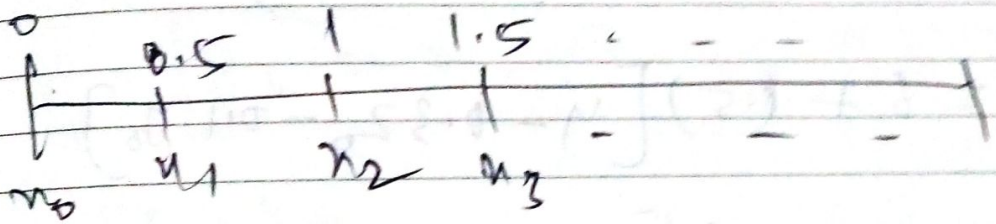
(7)

Sol: Given that

$$\frac{dy}{dx} = -z = f(x, y, z), \quad y(0) = 4$$

$$\frac{dz}{dx} = g(x, y, z) = 4 - 0.5z - 0.1y$$

$z(0) = 6$



Here $x_0 = 0, \quad y_0 = 4, \quad z_0 = 0, \quad z_0 = 6.$

W.K.T Euler method

$$y_{n+1} = y_n + h f(x_n, y_n, z_n)$$

Putting $n = 0$

$$y_1 = y_0 + h f(x_1, y_0, z_0)$$

$$= 4 + (0.5) \cdot \left(-\frac{y_0}{2}\right)$$

$$= 4 + (0.5) \cdot \left(\frac{4}{2}\right) = 3$$

$$\boxed{y_1 = 3}$$

i.e $y(x_1) = 3$ i.e $\boxed{y(0.5) = 3}$

Now $\boxed{z_{n+1} = z_n + h\, g(x_n, y_n, z_n)}$.

~~putting~~ $n = 0$ we get

$$z_1 = z_0 + h\, g(x_0, y_0, z_0)$$

$$= 6 + (0.5).[4 - 0.3z_0 - 0.1y_0]$$

$$= 6 + (0.5)[4 - (0.3).6 - (0.1).(3)]$$

$$= 6 + 0.9 = 6.9$$

$\therefore \boxed{z_1 = 6.9}$

i.e $z(x_1) = 6.9$

$\Rightarrow \boxed{z(0.5) = 6.9}$

16 SUNDAY • 259-106

Now $x_1 = 0.5$, $y_1 = 3$, $z_1 = 6.9$

we have to find y_2 and z_2

i.e $y(1)$ and $z(1)$.

(9)

Taking $h=1$ in Euler's method

$$y_2 = y_1 + h f(x_1, y_1, z_1)$$

$$= 3 + (0.5)\left(-\frac{y_1}{2}\right)$$

$$= 3 + (0.5)\left(-\frac{3}{2}\right) = 2.25$$

$$\therefore \boxed{y_2 = 2.25} \Rightarrow \boxed{y(1) = 2.25}$$

Similarly

$$z_2 = z_1 + h g(x_1, y_1, z_1)$$

$$= 6.9 + 0.5\left[4 - 0.3 z_1 - 0.1 y_1\right]$$

$$= 6.9 + 0.5\left[4 - 0.3(6.9) - 0.1(3)\right]$$

$$\boxed{z_2 = 7.715}$$

$$\therefore \boxed{z(1) = 7.715}$$

(10)

H.W

Q Using Euler method

Solve $y'' + 2y' + \frac{3}{4}y = 0$

$y(0) = 3, \quad y'(0) = -2.5$

when $h = 0.2$

find (i) $y(0.2)$ and $y'(0.2)$.

(ii) $y(0.4)$ and $y'(0.4)$