PPP vs Hexagonal Grid Model for Downlink Cellular Networks

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- Cellular Networks traditionally modeled by hexagonal grid models
- PPP model presents a probabilistic base station deployment

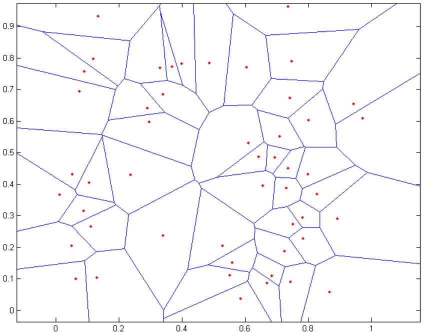


Figure: PPP Tesselation

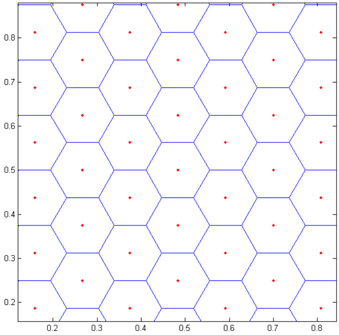


Figure: Hexagonal Tesselation

Poisson Point Process

What is PPP ?

- 2D Spatial point process : Useful model for a random pattern of points in 2-dimensional space
- Poisson point process : Number of points is a Poisson random variable with mean dependent on intensity and area
- Conditionally independent and uniformly distributed

Why PPP in Cellular Networks ?

- Regular Grid Models : Highly idealized, simplistic, not tractable
- PPP : Tractable without compromising on accuracy

System Model

- Standard path loss propagation model, $\alpha > 2$
- Desired signal experiences Rayleigh fading

$$h \sim exp(\mu), \quad \text{where } \frac{1}{\mu} \text{ is transmit power}$$

- Interference can experience any general fading
- Lognormal shadowing for desired signal and interference

Performance Measures : Coverage Probability and Rate

- Coverage Probability :

$$p_c(T, \alpha, \lambda) = \text{Pr}[\text{SINR} > \text{T}] \tag{1}$$

- Average Rate :

$$\tau = \ln(1 + \text{SINR}) \tag{2}$$

$$\text{where, } \ \text{SINR} = \frac{hr^{-\alpha}}{\sigma^2 + \sum g_i R_i^{-\alpha}} \tag{3}$$

Theoretical Expressions for Coverage Probability : General Expression

General expression for coverage probability (Desired Signal : Rayleigh Fading)

$$p_c(T,\lambda,\alpha) = \pi\lambda \int_0^\infty \exp(-\pi\lambda v \beta(T,\alpha) - \mu T \sigma^2 v^{\frac{\alpha}{2}}) dv \qquad (4)$$

where

$$\beta(T,\alpha) = \frac{2(\mu T)^{\frac{2}{\alpha}}}{\alpha} \mathbb{E}\left[g^{2/\alpha} (\Gamma(-2/\alpha, \mu T g) - \Gamma(-2/\alpha)) \right] \qquad (5)$$

Theoretical Expressions for Coverage Probability : Special Cases

Case	Coverage Probability Expression
General Fading, $\alpha = 4$	$\pi\lambda\sqrt{\frac{\pi}{b}}\exp\left(\frac{(\pi\lambda\beta(T,4))^2}{\frac{4T}{SNR}}\right) Q\left(\frac{\pi\lambda\beta(T,4)}{\sqrt{\frac{2T}{SNR}}}\right)$
General Fading, $\sigma = 0$	$\frac{1}{\beta(T,\alpha)}$
Exponential Fading, $\alpha = 4$	$\beta(T) = 1 + \sqrt{T}(\pi/2 - \arctan(1/\sqrt{T}))$

Theoretical Expressions for Average Rate

- General case of desired signal distributed as Rayleigh : three numerical integrations
- $\alpha = 4$: one numerical integration
- No noise : Expression independent of base station density
- No noise, $\alpha = 4$: 2.15 bps/Hz

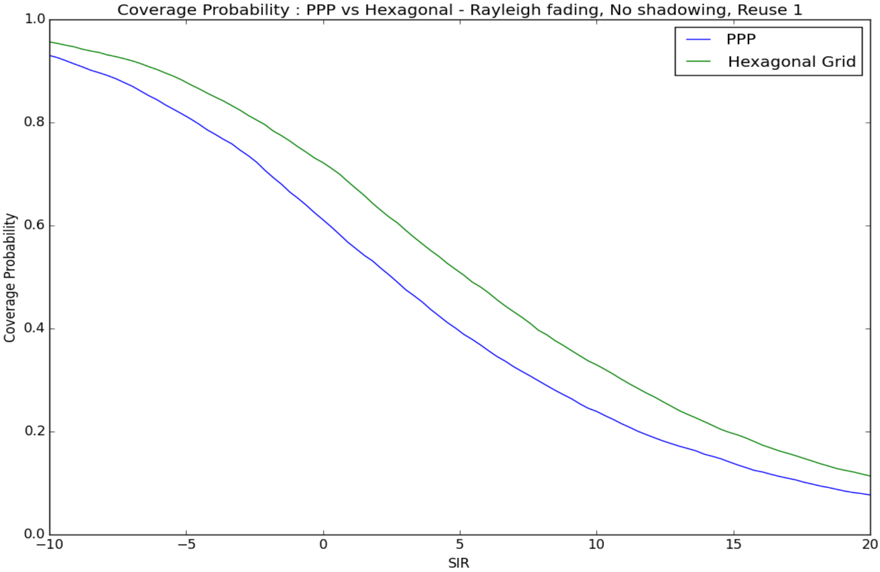
Parameters considered for Simulation of Coverage Probability

Interference distributed as Rayleigh, No noise, $\alpha = 4$, reduces to a simple closed form expression

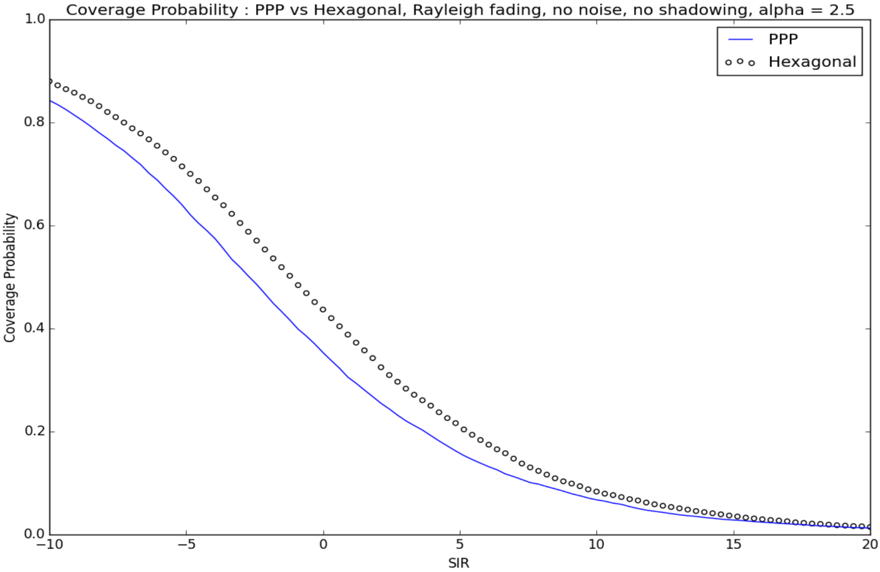
$$p_c(T, \lambda, 4) = \frac{1}{1 + \sqrt{T}(\pi/2 - \arctan(1/\sqrt{T}))} \tag{6}$$

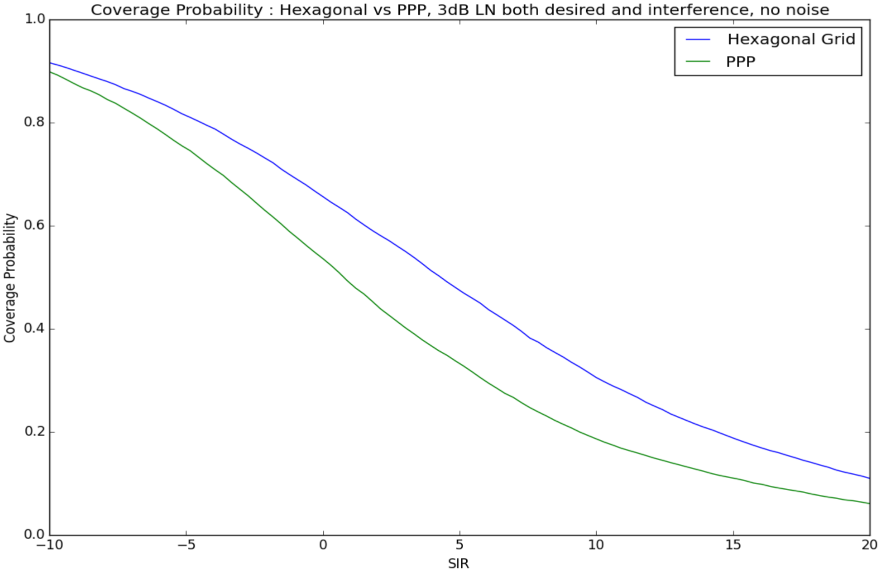
- Transmit Power : 40W
- Base Station density (λ) : 0.25 base stations/km^2
- $\alpha = 4$

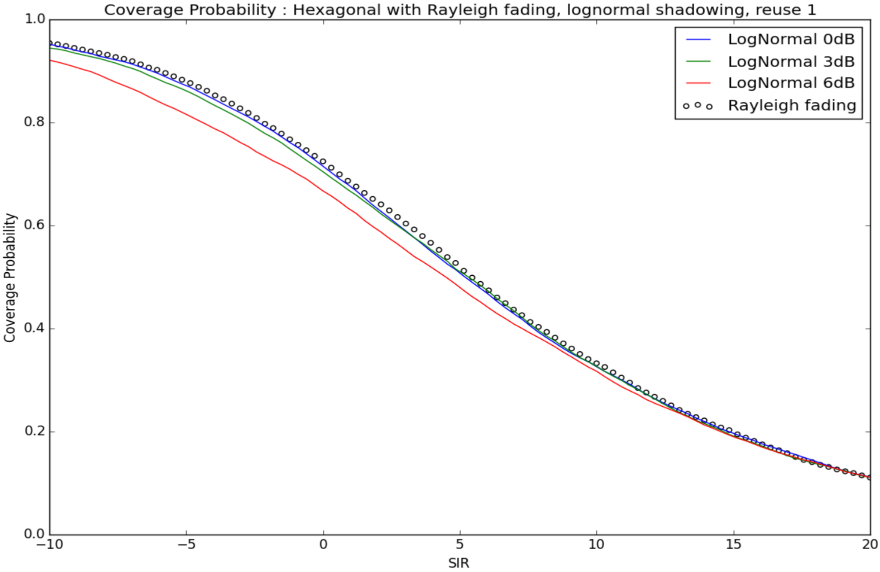
Simulation Results without Frequency Reuse

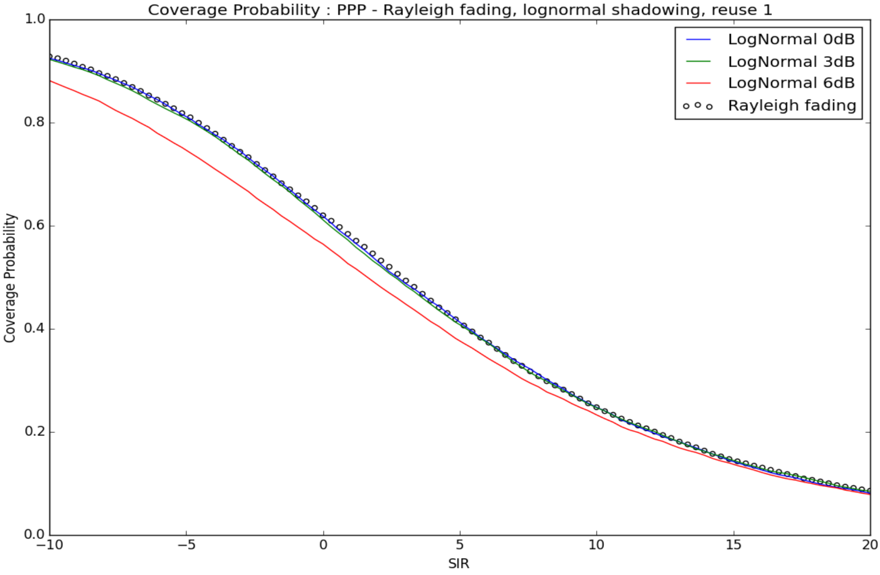


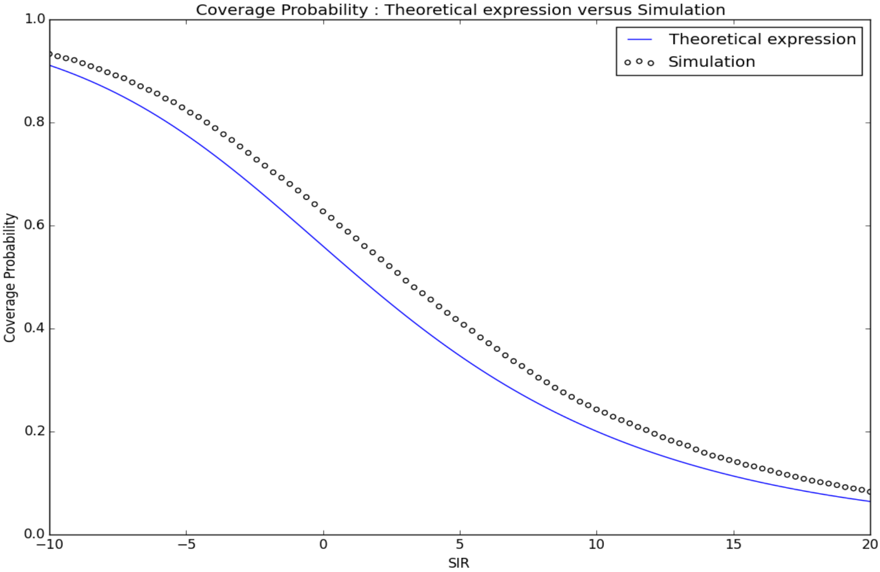
Coverage Probability : PPP vs Hexagonal - Rayleigh fading, No shadowing, Reuse 1
PPP
Hexagonal Grid
Coverage Probability
SIR
1.0
0.8
0.6
0.4
0.2
0.0
−10
−5
0
5
10
15
20

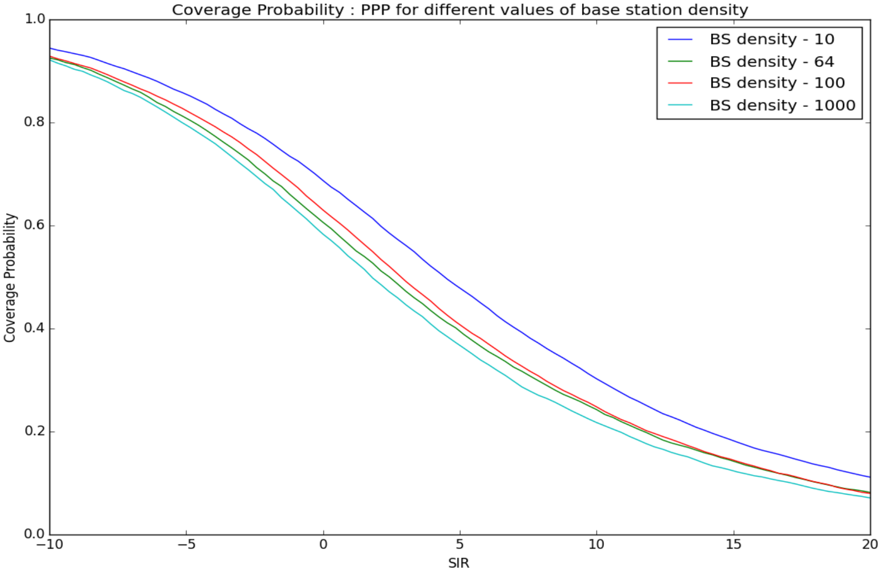

Coverage Probability : PPP vs Hexagonal, Rayleigh fading, no noise, no shadowing, alpha = 2.5
PPP
Hexagonal
Coverage Probability
SIR
1.0
0.8
0.6
0.4
0.2
0.0
−10
−5
0
5
10
15
20


Coverage Probability : Hexagonal vs PPP, 3dB LN both desired and interference, no noise
Hexagonal Grid
PPP
Coverage Probability
1.0
0.8
0.6
0.4
0.2
0.0
−10
−5
0
5
10
15
20
SIR


Coverage Probability : Hexagonal with Rayleigh fading, lognormal shadowing, reuse 1
LogNormal 0dB
LogNormal 3dB
LogNormal 6dB
Rayleigh fading
Coverage Probability
SIR
1.0
0.8
0.6
0.4
0.2
0.0
−10
−5
0
5
10
15
20


Coverage Probability : PPP - Rayleigh fading, lognormal shadowing, reuse 1
LogNormal 0dB
LogNormal 3dB
LogNormal 6dB
Rayleigh fading
Coverage Probability
0.0
0.2
0.4
0.6
0.8
1.0
−10
−5
0
5
10
15
20
SIR

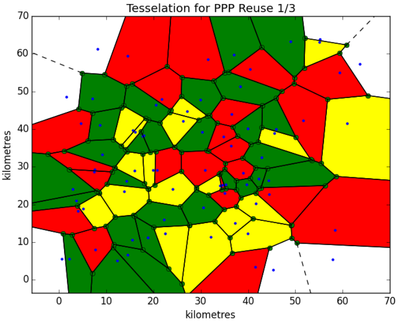

Coverage Probability : Theoretical expression versus Simulation
Theoretical expression
Simulation
Coverage Probability
1.0
0.8
0.6
0.4
0.2
0.0
−10
−5
0
5
10
15
20
SIR

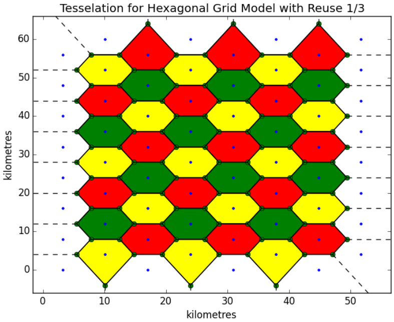

Coverage Probability : PPP for different values of base station density
BS density - 10
BS density - 64
BS density - 100
BS density - 1000
Coverage Probability
SIR
1.0
0.8
0.6
0.4
0.2
0.0
−10
−5
0
5
10
15
20

Discussion of Results

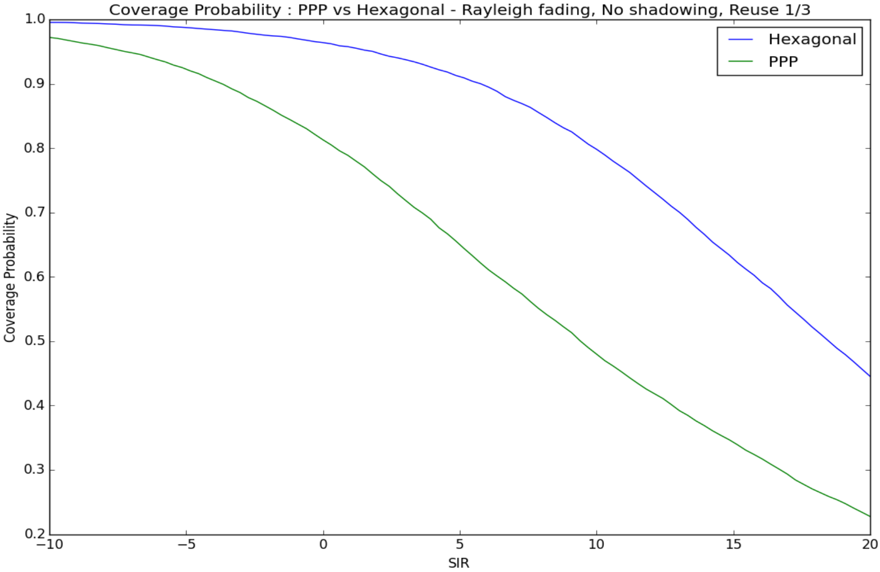
- Not dependent on value of λ as expected
- Lognormal shadowing does not significantly affect accuracy
- Grid model upper bounds PPP
- At a lower value of path exponent, gap is lesser

- Number of frequency bands, $\delta = 3$
- Hexagonal Grid : deterministic frequency allocation
- PPP : random frequency allocation
- Since frequency allocation is random, expression for coverage probability is effectively for a thinned PPP with density λ/δ
- Average rate for PPP is maximized for $\delta = 1$

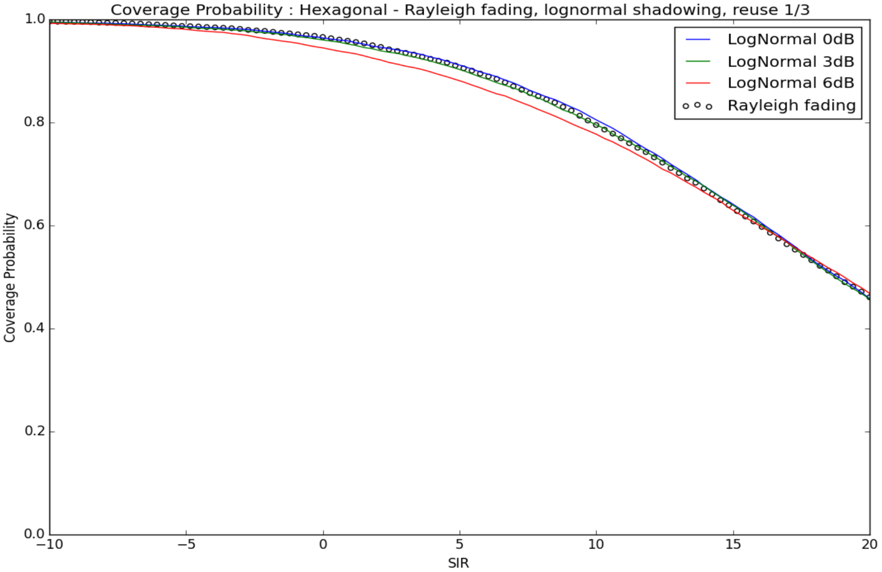


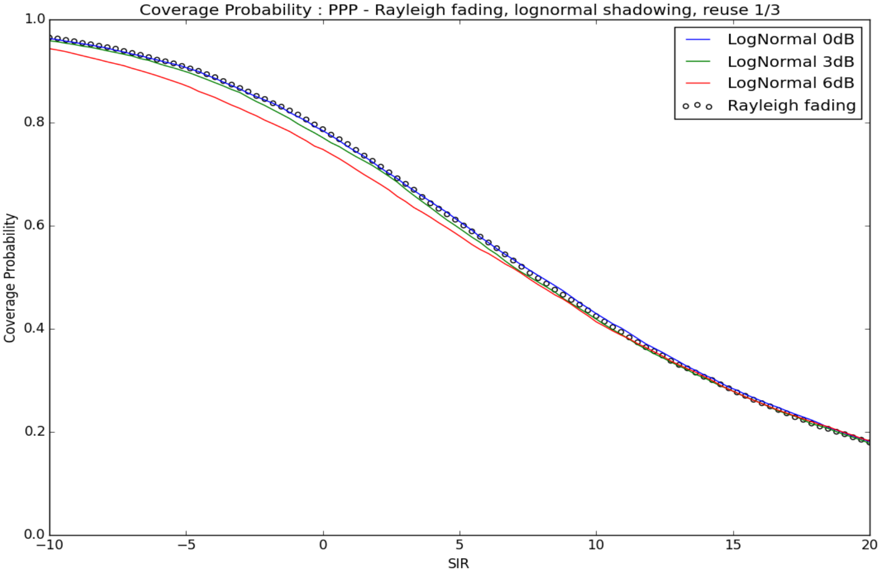


Simulation Results with Frequency Reuse $1/3$



Coverage Probability : PPP vs Hexagonal - Rayleigh fading, No shadowing, Reuse 1/3
Hexagonal
PPP
Coverage Probability
SIR
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
−10
−5
0
5
10
15
20


Coverage Probability : Hexagonal - Rayleigh fading, lognormal shadowing, reuse 1/3
LogNormal 0dB
LogNormal 3dB
LogNormal 6dB
Rayleigh fading
Coverage Probability
0.0
0.2
0.4
0.6
0.8
1.0
−10
−5
0
5
10
15
20
SIR


Coverage Probability : PPP - Rayleigh fading, lognormal shadowing, reuse 1/3
LogNormal 0dB
LogNormal 3dB
LogNormal 6dB
Rayleigh fading
Coverage Probability
SIR
1.0
0.8
0.6
0.4
0.2
0.0
−10
−5
0
5
10
15
20

Discussion of results

- Random frequency allocation gives a higher gap between PPP and grid
- Higher probability of adjacent cells transmitting in same band
- For higher δ, gap between PPP and grid increases further

Summary

- PPP
  - Provides tractable expressions for coverage probability and average rate, even for frequency reuse
  - Tracks a real deployment as accurately as grid-based models
  - Pessimistic bound to a planned deployment ; BSs will be located very near
  - Realistic model compared to traditional grid-based models
  - May capture HetNets, ad hoc deployments and future dense base station placements more accurately
- Hexagonal Grid Based Models
  - Highly idealized, not tractable
  - Optimistic bound; regular geometry optimal from coverage point of view
  - Increasingly inaccurate for HetNets and other ad hoc deployments