The oblique impact of elastic spheres

Article *in* Wear · June 1976

DOI: 10.1016/0043-1648(76)90201-5

Wear,

THE OBLIQUE IMPACT OF ELASTIC SPHERES

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(Received October 10. 1975)

Summary

A solution is developed for the oblique impact of an elastic sphere on a half-space. The Hertzian theory of impact is used for the normal components of force and velocity, and it is assumed that the contact area comprises sticking and slipping regions, in the latter of which the coefficient of friction is constant. The mixed boundary value problem for the tangential tractions and displacements is reduced to a system of simultaneous equations by dividing the contact area into a set of concentric annuli.

The trajectory of the sphere depends on only two non-dimensional parameters: one related to the angle of incidence and the other to the radius of gyration. A simple rigid body theory of impact gives a reasonable approximation to the more exact method at low and high angles of incidence, but is unsatisfactory at intermediate values.

1. Introduction

This paper is concerned with the non-colinear impact of two elastic spheres of similar materials. It is convenient to restrict attention to the case of a sphere impacting a half-space, since the more general problem presents no new features. It will be assumed that the coefficient of sliding friction μ between the surfaces is a constant and that friction is the only source of energy dissipation (and hence that the coefficient of restitution in a normal impact is unity).

The elementary approach to this problem is to neglect the elastic displacements of the solids, in which case only two conditions can be distinguished: (i) sliding and (ii) rolling of the sphere on the plane. Sliding will generally occur at the start of the impact, but an opposing frictional force will be set up tending to reduce the sliding velocity. If this velocity reaches zero during the impact, sliding will give way to rolling and the frictional force will fall instantaneously to zero. Once rolling is established, it will persist to the end of the impact.

The elastic displacements during impact will generally be small in comparison with the dimensions of the solids, but they introduce new possibilities which cannot be accommodated within the elementary theory.

The work done in producing tangential displacements is stored as elastic strain energy in the solids and is therefore recoverable under suitable circumstances. Also, the tangential displacement is not necessarily constant throughout the contact area and it is possible to generate conditions in which some regions are in sliding contact whilst others are stuck. For example, Mindlin [1] treats the case in which two spheres are pressed together with a constant normal force P and then subjected to a tangential force F. If $F > \mu P$ sliding occurs throughout the contact region, but for lower values of F a central circular region remains stuck whilst sliding or microslip occurs in the surrounding annulus.

During an impact, conditions are considerably more complex than this, since the contact area is continually changing. When it is increasing, new regions of contact will be laid down free of tangential stress, but changes in the applied tangential force will involve a redistribution of stress. In a more recent paper, Mindlin and Deresiewicz [2] considered a wide range of problems involving varying normal and tangential forces and emphasised that the response of the system to small changes depends upon the whole previous loading history. For this reason, they restricted their attention to cases in which the initial state was reached by applying first the normal force and then the tangential force.

In the impact problem, the tangential loading history is not known *a priori* since it depends on the interaction between the compliance of the contact and the motion of the spheres. This difficulty can be overcome in a numerical solution by advancing through the period of the impact in small discrete time increments. Starting from known values of normal and tangential velocity, the displacements in a time increment can be found and these define the boundary conditions of the instantaneous contact problem. The changes in velocity components during the time increment are then found from the contact forces by considerations of momentum.

In a previous publication [3] the method of Mindlin and Deresiewicz was used to solve each of the incremental contact problems in such a procedure. However, with this method the state after the nth step is described as the sum of n irreducible components. In other words, the previous history of the system has to be continuously available at each step and this places a limit on the number of time increments which can conveniently be used.

The previous history of the system only influences the instantaneous behaviour in so far as it determines the locked-in tangential displacements in those regions of the contact area which are stuck. Hence, if the distribution of tangential displacement (and/or traction) can be approximated by a suitable series of functions, the amount of information carried through the procedure will be independent of the number of time increments used. This is the method used in this paper. The results from the two methods have been found to agree closely in cases to which both can be applied.

2. Solution

We divide the potential contact region by a series of n equi-spaced concentric circles of radius ai/n ($i = 1,..., n$) which define the limits of a series of tangential traction distributions such that the total traction in the direction of the tangential motion at a radius r is

$$f(r) = \sum_{i=j}^{n} f_i \left(1 - \frac{n^2 r^2}{a^2 i^2}\right)^{1/2} \tag{1}$$

where j is the smallest integer greater than nr/a. Any series form could have been used for the distribution of traction but this representation, owing something to Mindlin's analytic solutions [1, 2], possesses the two virtues of giving analytically tractable expressions for tangential displacements and of including the exact solution for the condition of gross slip as a special case. An integral form of eqn. (1) has been explored to some advantage by Segedin [4] in connection with the normal indentation problem.

One equation for determining the n coefficients f_i can be obtained from each of the n annuli. In stick regions the tangential displacement due to $f(r)$ is prescribed, whilst in slip regions $f(r) = \pm\ \mu p(r)$, where $p(r)$ is the local normal contact pressure.

We therefore assume a provisional division into stick and slip regions, solve the appropriate equations, and test the solution to see whether the initial assumption was correct. In stick regions the tangential traction must be below the limits at which slip occurs, whereas in slip regions the relative incremental displacement must be in the correct sense for the assumed frictional traction. If these tests fail in any region, the assumption in that region is changed and a new solution is obtained. Convergence is rapid.

2.1. The normal contact problem

Since the materials of the two solids are similar, the symmetry of the system guarantees that the normal contact problem is unaffected by tangential tractions and hence the Hertzian theory can be used. If, at some instant during the impact, the relative normal approach at the contact is u_z, the contact radius will be

$$b = (Ru_z)^{1/2} \tag{2}$$

whilst the contact pressure distribution will be

$$\left.\begin{aligned} p(r) &= \frac{2G(b^2 - r^2)^{1/2}}{\pi R(1-\nu)} \qquad & 0 \leqslant r \leqslant b \\ &= 0 & r > b \end{aligned}\right\} \tag{3}$$

where R is the radius of the sphere and G, ν are respectively the modulus of rigidity and Poisson's ratio for the material [5]. This corresponds to a total force

$$P = \frac{4b^3 G}{3R(1-\nu)} \tag{4}$$

Assuming that P remains constant over a small time increment δt, the change in the normal component of velocity v_z during the time increment will be

$$\delta v_z = -\frac{4b^3 G\delta t}{3MR(1-\nu)} \tag{5}$$

where M is the mass of the sphere. If v_z is assumed constant during the time increment, the change in the relative normal approach will be

$$\delta u_z = v_z \delta t \tag{6}$$

Note that the duration of the impact is given by

$$\begin{aligned} t &= \frac{4\,\Gamma(2/5)\,\pi^{1/2}\,C^2 R}{5\,\Gamma(9/10)\,v_1} \\ &\doteqdot 2.9432\,C^2 R/v_1 \end{aligned} \tag{7}$$

whilst the maximum contact radius is

$$a = CR \tag{8}$$

where

$$C = \left\{\frac{15Mv_1^2\,(1-\nu)}{16GR^3}\right\}^{1/5} \tag{9}$$

and v_1 is the normal component of the velocity of the sphere at incidence [5] . Equation (7) provides a useful check on the accuracy of the numerical integration procedure based on eqns. (2), (5) and (6).

2.2. *The tangential contact problem*

We first find the tangential displacements* due to a distribution of traction of the form

$$\left.\begin{aligned} f(r) &= (1 - r^2/b^2)^{1/2} \qquad 0 \leqslant r \leqslant b \\ &= 0 \qquad\qquad\qquad\quad r > b \end{aligned}\right\} \tag{10}$$

obtaining

$$\left.\begin{aligned} u_x &= \frac{\pi}{16Gb}\,\{2(2-\nu)(2b^2 - r^2) + \nu r^2 \cos 2\theta\} \qquad 0 \leqslant r \leqslant b \\ &= \frac{1}{8Gb}\left[2(2-\nu)\left\{(2b^2 - r^2)\arcsin\left(\frac{b}{r}\right) + b(r^2 - b^2)^{1/2}\right\} + \right. \\ &\quad \left. + \nu\left\{r^2 \arcsin\left(\frac{b}{r}\right) + \frac{b(3b^2 - r^2)(r^2 - b^2)^{1/2}}{r^2}\right\}\cos 2\theta \right] \qquad r > b \end{aligned}\right\} \tag{11}$$

*The term displacement is used to refer to the combined displacement of the two solids.

in the direction of the traction and

$$\left.\begin{aligned} u_y &= \frac{\pi\nu r^2 \sin 2\theta}{16Gb} \qquad 0 \leqslant r \leqslant b \\ &= \frac{\nu}{8Gb}\left\{ r^2 \arcsin\left(\frac{b}{r}\right) + \frac{b(3b^2 - r^2)(r^2 - b^2)^{1/2}}{r^2} \right\} \sin 2\theta \\ &\qquad\qquad r > b \end{aligned}\right\} \quad (12)$$

in the tangential direction perpendicular to the traction. The polar coordinate θ is measured from the positive x axis. The expressions for $0 \leqslant r \leqslant b$ are given by Mindlin [1] and those for $r > b$ can be found by a similar method. The corresponding expressions for the traction distribution of eqn. (1) are readily obtained by writing ai/n for b in eqns. (11) and (12).

It will be observed that these expressions contain non-axisymmetric terms in $\sin 2\theta$ and $\cos 2\theta$ and therefore it is not generally possible to satisfy the prescribed tangential displacement conditions using a series of the form of eqn. (1). However, it would be possible to do so if these terms could be neglected. This procedure can be justified on the following grounds.

(i) The terms in question are small. For example, in the first of eqns. (11) the term in $\cos 2\theta$ adds at the worst points only $\pm 9\%$ to the corresponding terms in r^2, for $\nu = 0.3$.

(ii) In certain circumstances the offending terms are self-cancelling. For example, if the incremental problem involves a central stuck region surrounded by an annulus in microslip (as in all the cases treated by Mindlin and Deresiewicz [2] and most of those treated here), the process of neutralising the radius-dependent axisymmetric terms in the stick region conveniently neutralises the non-axisymmetric terms as well.

(iii) The additional stress distribution required for an exact solution would be self-equilibrating and would therefore not affect the trajectory of the sphere except in so far as the change in tangential stress caused a change in the division of stick and slip regions.

(iv) The changes envisaged in (iii) would make the stick–slip boundary non-circular and the problem would become intractable.

We therefore propose to neglect the terms in $\cos 2\theta$ and $\sin 2\theta$ in eqns. (11) and (12).

2.3. Development of a set of equations

In stick regions the tangential displacement is prescribed, being the sum of the displacement of the sphere during the time increment and the value before that time increment. Write $u_x(j)$ for the value of u_x before the given time increment at a radius $(j - ½)a/n$, $j = 1,\ldots, n$. The points so defined are the midpoints of the annular rings produced by the division of the contact area.

After the same time increment,

$$\frac{(2-\nu)a}{4G}\sum_{i=1}^{n} A_{ij}\, f_i = u_x(j) + v_x\,\delta t \tag{13}$$

in all stick regions, where v_x is the instantaneous value of the tangential velocity at the point of contact and

$$\left.\begin{aligned} A_{ij} &= \frac{\pi\{2i^2-(j-½)^2\}}{2in} && j=1,\ldots,i \\ &= \frac{\{2i^2-(j-½)^2\}\arcsin\{i/(j-½)\}}{in} + \frac{\{(j-½)^2-i^2\}^{1/2}}{n} && \\ & && j=i+1,\ldots,n \end{aligned}\right\} \tag{14}$$

from eqns. (1), (2) and (11).

Equation (13) ceases to apply in slip regions, where instead

$$\left.\begin{aligned} \sum_{i=j}^{n} f_i\left\{1-\frac{(j-½)^2}{i^2}\right\}^{1/2} &= \pm\,\mu p\left\{\frac{a(j-½)}{n}\right\} \\ &= \pm\,\frac{2\mu G}{\pi R(1-\nu)}\left\{b^2-\frac{a^2(j-½)^2}{n^2}\right\}^{1/2} && j<\frac{bn}{a}+\frac{1}{2} \\ &= 0 && j\geqslant\frac{bn}{a}+\frac{1}{2} \end{aligned}\right\} \tag{15}$$

from eqn. (3). The sign taken depends upon the direction of the anticipated relative motion.

Annuli outside the contact region ($r > b$) can be treated as slip regions for the purpose of this calculation. Alternatively, the condition

$$f_i = 0 \qquad i > \frac{bn}{a}+\frac{1}{2} \tag{16}$$

which has precisely the same effect can be imposed. This saves computing time by reducing the size of the remaining matrix.

Every region must be either stuck or slipping. Hence, if we assume a provisional division into stick and slip regions, eqns. (13) and (15) will yield n equations for the n unknowns f_i.

2.4. Tests of stick and slip regions

When the set of eqns. (13) and (15) has been solved, the stick regions are tested to ensure that the required tangential traction does not exceed that at which sliding begins, *i.e.*

$$-\left\{b^2 - \frac{a^2(j-½)^2}{n^2}\right\}^{1/2} < \frac{\pi R(1-\nu)}{2\mu G} \sum_{i=j}^{n} f_i \left\{1 - \frac{(j-½)^2}{i^2}\right\}^{1/2}$$

$$< + \left\{b^2 - a^2 \frac{(j-½)^2}{n^2}\right\}^{1/2} \tag{17}$$

from eqn. (15). If either inequality fails, it is assumed that the region will slip in the appropriate sense.

In the slip regions it is necessary to verify that the relative motion is in the required sense, *i.e.*

$$s(j)\left\{u_x(j) + v_x\delta t - \frac{(2-\nu)a}{4G} \sum_{i=1}^{n} A_{ij} f_i\right\} > 0 \tag{18}$$

where $s(j)$ is the sign of the frictional traction taken in eqn. (15). If this test fails, it is assumed that the region in question will stick.

If any changes in the assumed division are necessitated by the tests (17) and (18), the solution of Section 2.3 is repeated.

2.5. Change of tangential velocity

When a satisfactory solution has been obtained for the tangential contact problem, the change in velocity during the ensuing time increment is found from considerations of momentum. The tangential force F produces a change in angular and linear velocity so that

$$\delta v_x = -\frac{F\delta t}{M} - \frac{FR^2\delta t}{I} \tag{19}$$

where I is the moment of inertia of the sphere about its centre.

It should be emphasised that the velocity v_x as here defined refers to the vicinity of the contact area. Thus, if the sphere has an angular velocity v and its centre has a tangential velocity ω, the velocity v_x will be

$$v_x = v + \omega R \tag{20}$$

The history of the impact depends upon v and ω only in so far as they determine the value of v_x. Greater generality is therefore achieved by working in terms of v_x.

Equation (19) is conveniently written

$$\delta v_x = -\frac{F\delta t}{M}\left(1 + \frac{1}{K^2}\right) \tag{21}$$

where K $(=(I/MR^2)^{1/2})$ is a non-dimensional radius of gyration. For a homogeneous solid sphere, $K^2 = 2/5$ and $1 + 1/K^2 = 7/2$.

The tangential force, found by integrating eqn. (1) over the contact area, is

$$F = \frac{2\pi a^2}{3} \sum_{i=1}^{n} i^2 \frac{f_i}{n^2} \tag{22}$$

When the new tangential velocity has been found, the new values of tangential displacement $u_x(j)$ are substituted for the old, and the procedure is repeated for the next time increment.

3. Non-dimensional formulation

A considerable increase in generality can be achieved by a suitable non-dimensional formulation of the problem. The contact radius never exceeds the value CR (eqn. (8)) and therefore this is used as the radius a of the outermost circle.

The following non-dimensional quantities are also defined:

$$\delta\tau = \frac{\delta t}{t} = \frac{v_1 \delta t}{2.9432\, C^2 R} \tag{23}$$

$$\beta = b/CR \tag{24}$$

$$\phi_i = (1-\nu) f_i / GC\mu \tag{25}$$

$$\zeta = u_z / C^2 R \tag{26}$$

$$\xi(j) = \frac{2(1-\nu)u_x(j)}{\mu(2-\nu)C^2 R} \tag{27}$$

$$V = v_z / v_1 \tag{28}$$

$$\psi = \frac{2(1-\nu)v_x}{\mu(2-\nu)v_1} \tag{29}$$

3.1. Controlling equations

With this notation, the controlling eqns. (2), (5), (6), (13), (15) and (20) respectively take the simplified forms

$$\beta = \zeta^{1/2} \tag{30}$$

$$\delta V = -3.6790\, \beta^3 \delta\tau \tag{31}$$

$$\delta\zeta = 2.9432\, V \delta\tau \tag{32}$$

$$\frac{1}{2} \sum_{i=1}^{n} A_{ij}\, \phi_i = \xi(j) + 2.9432\, \psi \delta\tau \tag{33}$$

$$\frac{\pi}{2}\sum_{i=j}^{n}\left\{1-\frac{(j-\frac{1}{2})^2}{i^2}\right\}^{1/2}\phi_i = \pm\left\{\beta^2-\frac{(j-\frac{1}{2})^2}{n^2}\right\}^{1/2} \tag{34}$$

and

$$\delta\psi = -\frac{3.6790\pi(1-\nu)(1+1/K^2)\delta\tau}{2-\nu}\sum_{i=1}^{n}\frac{i^2\phi_i}{n^2} \tag{35}$$

whilst the tests of stick and slip regions (17) and (18) become respectively

$$-\left\{\beta^2-\frac{(j-\frac{1}{2})^2}{n^2}\right\}^{1/2} < \frac{\pi}{2}\sum_{i=j}^{n}\left\{1-\frac{(j-\frac{1}{2})^2}{i^2}\right\}^{1/2}\phi_i$$

$$< +\left\{\beta^2-\frac{(j-\frac{1}{2})^2}{n^2}\right\}^{1/2} \tag{36}$$

and

$$s(j)\left\{\xi(j)+2.9432\,\psi\delta\tau-\frac{1}{2}\sum_{i=1}^{n}A_{ij}\phi_i\right\}>0 \tag{37}$$

3.2. Initial conditions

At the beginning of the impact, $\zeta = \xi = \beta = 0$ and $\gamma = 1$. The only independent variable at input for a given system is ψ_1 — the initial non-dimensional tangential velocity — whilst the system itself is characterised by only one further parameter

$$\chi = \frac{(1-\nu)(1+1/K^2)}{2-\nu} \tag{38}$$

which features in eqn. (35).

The local angle of incidence α at the contact area is given by

$$\tan\alpha = v_x/v_1 \tag{39}$$

and hence ψ_1 is proportional to the ratio between $\tan\alpha$ and the coefficient of friction μ; therefore ψ_1 is referred to as the non-dimensional angle of incidence.

The value of ψ_1 determines whether or not the impact commences in gross slip. During gross slip the distribution of traction must be

$$f(r) = \frac{2G\mu(b^2-r^2)^{1/2}}{\pi R(1-\nu)} \tag{40}$$

from eqn. (3), and the tangential displacement at the centre of the contact circle due to this traction is

$$u_x = \frac{\mu(2-\nu)b^2}{2R(1-\nu)} = \frac{\mu(2-\nu)u_z}{2(1-\nu)} \tag{41}$$

from eqns. (11) and (2).

If the sphere is on the point of sticking,

$$v_x = \mathrm{d}u_x/\mathrm{d}t \tag{42}$$

$$= \frac{\mu(2-\nu)v_z}{2(1-\nu)} \tag{43}$$

from eqn. (41). The initial value of v_z is v_1 and hence the impact will commence with a central stuck region if

$$v_x < \frac{\mu(2-\nu)v_1}{2(1-\nu)}$$

i.e.

$$\psi_1 < 1 \tag{44}$$

If the angle of incidence is sufficiently large,the whole impact may take place in gross slip. In this case the tangential force is at all times μ multiplied by the normal force and the final tangential velocity will be

$$v_x = \frac{\mu(2-\nu)v_1\psi_1}{2\,(1-\nu)} - 2\mu v_1 \left(1 + \frac{1}{K^2}\right) \tag{45}$$

whilst the final normal velocity is $-v_1$. Applying the condition (43) and using eqn. (38), gross slip will occur throughout the cycle if

$$\psi_1 > 4\chi - 1 \tag{46}$$

4. Results

4.1. General characteristics

Numerical results have been obtained for a wide range of values of ψ_1 and χ and may be qualitatively described as follows.

(i) $\psi_1 \leqslant 1$

For $\psi_1 < 1$ the sphere sticks at the beginning of the impact, as shown above (condition (44)). As the sphere penetrates the half-space and the contact area grows, successive annuli are laid down in a stress-relieved state and there is no microslip. However, once the midpoint of the impact is passed, the contact area shrinks and the tangential elastic recovery of the surfaces causes an annulus of microslip to be established surrounding a central stuck region. This annulus spreads inwards as the cycle proceeds until eventually the whole contact area slides, *i.e.* gross slip is established.

(ii) $1 < \psi_1 < 4\chi - 1$

In this intermediate range of angles of incidence, the impact com-

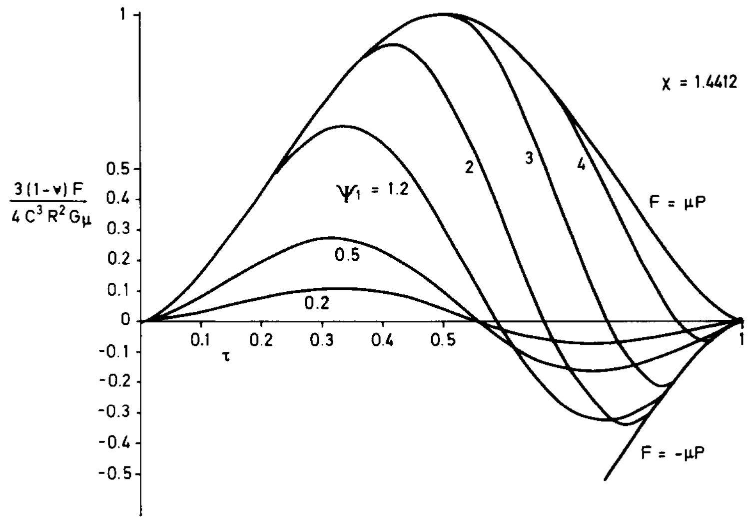

Fig. 1. The variation of tangential force during the impact for a homogeneous solid sphere of Poisson's ratio 0.3 (χ = 1.4412) at various values of the non-dimensional local angle of incidence ψ_1.

mences in gross slip but stick occurs at some point during the cycle. The change appears to take place very rapidly; less than 2% of the impact duration is required to pass from complete slip to complete stick.

The subsequent history of the impact is similar to that described in (i), with a reversion to gross slip before exit. As ψ_1 is increased the slip–stick transition occurs at progressively later points in the cycle, until the entire impact takes place in gross slip when $\psi_1 \geqslant 4\chi - 1$.

4.2. Variation of tangential force during the cycle

Figure 1 shows the variation of the tangential and normal forces during the cycle at various values of ψ_1. The value of χ is fixed at 1.4412, corresponding to a homogeneous solid sphere with $\nu = 0.3$.

The forces are plotted on such a scale as to make F and P coincide when $F = \mu P$, *i.e.* in gross slip. At low angles of incidence, the tangential force shows a complete cycle of oscillation with a reversal in direction shortly after the maximum penetration is reached. In effect, the deformable surfaces act as a spring in the tangential direction which is compressed during the first quarter cycle and which overshoots its equilibrium position on recovery because of the inertia of the sphere. At the midpoint of the cycle, the local tangential velocity is opposite in direction to that at incidence.

This tangential oscillation is cut short by the occurrence of gross slip when F reaches the value $-\mu P$ and the direction of slip is opposite to the relative tangential velocity at incidence. The surface of the half-space, having

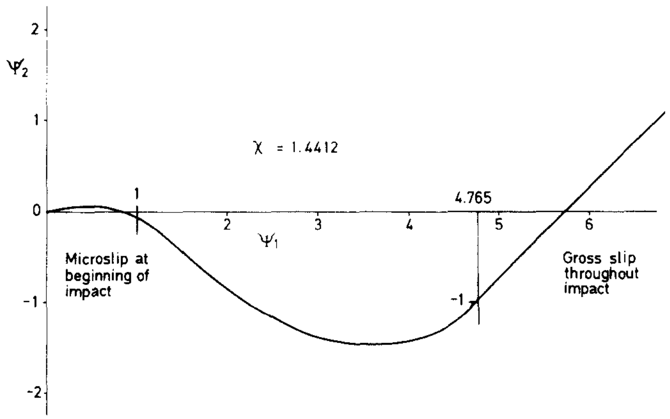

Fig. 2. The non-dimensional local angle of reflexion ψ_2 as a function of the corresponding angle of incidence ψ_1 for a homogeneous solid sphere of Poisson's ratio 0.3.

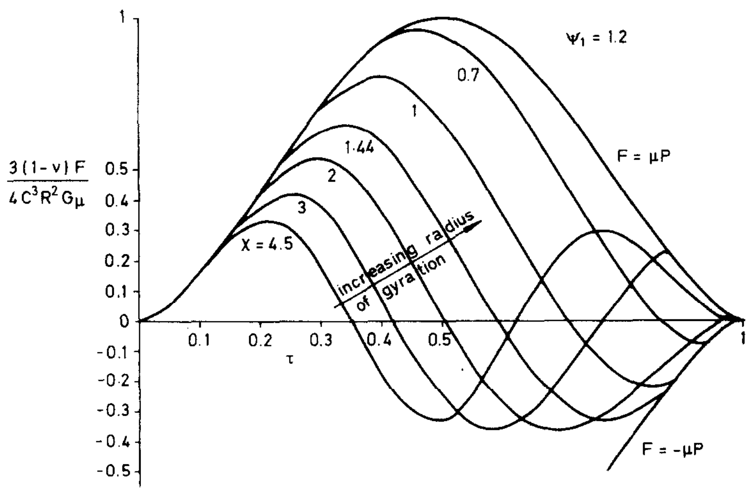

Fig. 3. The variation of tangential force during the impact at a local angle of incidence ψ_1 = 1.2 for spheres of various radii of gyration.

passed through a nearly complete cycle of oscillation, is now moving in the same direction as the local incident tangential velocity of the sphere and at a greater speed than the sphere itself (which has been retarded during the impact).

When a larger angle of incidence is used, the start of the cycle is delayed by the initial period of gross slip, but the same general type of behaviour is observed.

4.3. Local tangential velocity at exit

From the applied dynamics viewpoint, the conditions during the cycle are of less importance than those at exit. Two questions of particular interest are:

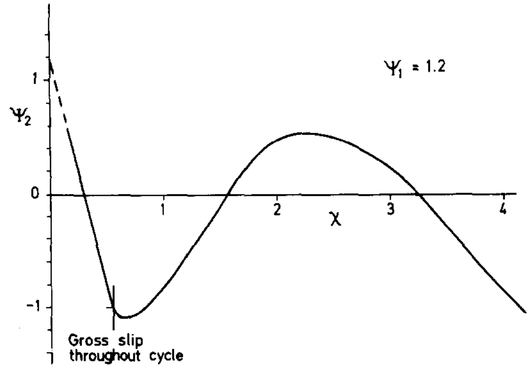

Fig. 4. The non-dimensional local angle of reflexion as a function of χ for a local angle of incidence $\psi_1 = 1.2$.

(i) how does the local angle of reflexion depend upon the local angle of incidence? and

(ii) how does the answer to (i) differ from that obtained from the simple rigid body theory?

Figure 2 shows the value of ψ at exit ($=\psi_2$) as a function of ψ_1 for a homogeneous solid sphere with $\nu = 0.3$. The function ψ_2 is a non-dimensional local angle of reflexion in the same sense that ψ_1 is a local angle of incidence (see Section 3.2). A positive angle of reflexion is one in which the tangential velocity retains the same sense.

The rigid body theory of impact agrees with the more exact theory for $\psi_1 \geqslant 4\chi$, but predicts $\psi_2 = 0$ for $\psi_1 < 4\chi$. This is a reasonable approximation when $\psi_1 \leqslant 1$ (see Fig. 2), but in the range $1 < \psi_1 < 4\chi$ the error can be considerable. The elastic recovery of the surfaces enables negative local angles of reflexion to be obtained, whereas the minimum value predicted from rigid body considerations is zero, corresponding to the condition of rolling at exit. It is also notable that gross slip occurs throughout the cycle at lower values of ψ_1 than those predicted by the simple theory, since tangential elastic recovery of the surfaces can maintain relative motion even when the sphere has been brought to rest.

4.4. *The effect of radius of gyration*

Once gross slip ceases, the tangential compliance and the inertia of the sphere act very much like a non-linear spring–mass system. Therefore the natural frequency of this system may be expected to depend upon the radius of gyration of the sphere.

This prediction is confirmed by the results presented in Fig. 3. As χ is increased (and hence K is reduced), the number of reversals in tangential force during the cycle increases.

The final period of gross slip can be in either direction depending upon the ratio between the period of tangential oscillation and the duration of the impact. This ratio also determines the sign of the angle of reflexion as shown in Fig. 4, where ψ_2 is plotted against χ for an angle of incidence of

1.2. However, the range of values of χ obtainable with a solid sphere of varying density is comparatively small.

Acknowledgments

The authors thank Dr. K. L. Johnson of Cambridge University Engineering Laboratory for his advice and constructive criticism.

Nomenclature

a	maximum radius of contact area
b	instantaneous radius of contact area
C	$\{15\, Mv_1^2\, (1-\nu)/16\, GR^3\}^{1/5}$
$f(r)$	tangential traction
F	tangential force
G	modulus of rigidity
I	moment of inertia of the sphere about its centre
K	$(I/MR^2)^{1/2}$
M	mass of the sphere
n	number of concentric annuli
$p(r)$	normal contact pressure
P	normal force
r, θ	polar coordinates
R	radius of the sphere
$s(j)$	direction of slip
t	duration of the impact
δt	time increment
u_x, u_y	combined tangential displacements of the surfaces
u_z	relative normal approach
v_x, v_z	local tangential and normal velocities
v_1	initial value of v_z
V	non-dimensional parameter defined by eqn. (28)
α	angle of incidence
μ	coefficient of friction
ν	Poisson's ratio
$\beta, \zeta, \xi, \tau, \phi_i, \psi, \chi$	non-dimensional parameters defined by eqns. (23) - (29), (38)

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