Project 2 - Regression and CART

Group 3

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Introduction to the Heart Dataset

Observing the data:

##		Age	Sex	ChestPain	RestBP	Chol	Fbs	${\tt RestECG}$	${\tt MaxHR}$	ExAng	Oldpeak	Slope	Ca
##	1	63	1	typical	145	233	1	2	150	0	2.3	3	0
##	2	67	1	asymptomatic	160	286	0	2	108	1	1.5	2	3
##	3	67	1	asymptomatic	120	229	0	2	129	1	2.6	2	2
##	4	37	1	nonanginal	130	250	0	0	187	0	3.5	3	0
##	5	41	0	nontypical	130	204	0	2	172	0	1.4	1	0
##	6	56	1	nontypical	120	236	0	0	178	0	0.8	1	0
##			Tł	nal AHD AHDBi	nary								
##	1	1 fixed No			0								
##	2	2 normal Yes			1								
##	3	3 reversable Yes		1									
##	4	normal No		0									
##	5	normal No		0									
##	6	o normal No		0									

Describing the data, and (1) identify Y and X.

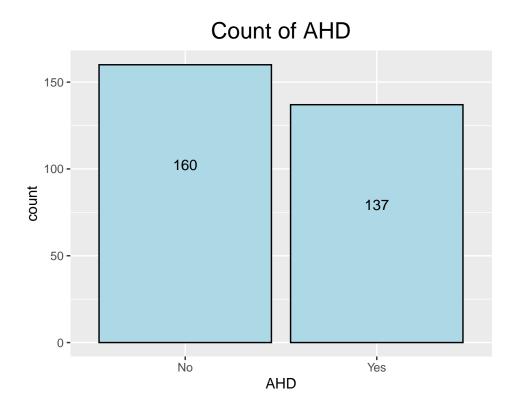
The Heart dataset considers AHD, their binary relationship of having heart disease or not, and their other associated properties. The dataset contains 303 observations and 14 attributes.

More to be added/edited

Exploratory Data Analysis

Count of Binary Outcome

The following is a simple barplot of the count for the binary AHD outcome. We can see there are some more "No", than "Yes".



Correlation matrix

No correlation among AHD Binary exceeds 0.5, naturally.



Logistic Model

Logistic Model

Constructing a logistic model for AHD based on other features. training and testing sets were separated randomly in a 70/30 split. To construct a final logistic model, features with the highest p values were removed until the remaining attributes were all significant. Some summaries were skipped over for brevity.

Splitting the data and setting seed.

```
set.seed(1)
indices <-sample(1:nrow(data), 0.7 * nrow(data), replace = TRUE)
training <-data[indices,]
test <-data[-indices,]</pre>
```

Carrying out the feature selection and yielding a final regression model.

```
# Entire qlm fit for numeric data
glm.fit <- glm(AHDBinary ~ Age+Sex+RestBP+Chol+Fbs+RestECG+MaxHR+ExAng+Oldpeak+Slope+Ca, data = traini</pre>
summary(glm.fit)
##
## Call:
  glm(formula = AHDBinary ~ Age + Sex + RestBP + Chol + Fbs + RestECG +
       MaxHR + ExAng + Oldpeak + Slope + Ca, family = binomial,
##
##
       data = training)
##
## Deviance Residuals:
##
      Min
                1Q
                     Median
                                  3Q
                                          Max
## -2.8883 -0.5778 -0.1832
                              0.4322
                                        2.6547
##
## Coefficients:
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -4.427167
                          3.212744 -1.378 0.168203
                          0.029954 -1.558 0.119350
## Age
              -0.046653
                                    3.376 0.000735 ***
## Sex
               1.770038
                          0.524263
## RestBP
               0.024170
                          0.014339
                                    1.686 0.091854 .
## Chol
               0.012568
                          0.004439 2.831 0.004637 **
## Fbs
              -1.254899
                          0.670752 -1.871 0.061361
                          0.218381 -0.357 0.721158
## RestECG
               -0.077942
## MaxHR
              -0.027156
                         0.012998 -2.089 0.036686 *
## ExAng
               1.792944
                          0.503486
                                   3.561 0.000369 ***
## Oldpeak
               0.037720
                          0.245289 0.154 0.877785
                                    3.001 0.002694 **
## Slope
               1.234978
                          0.411564
## Ca
               1.695576
                          0.362786 4.674 2.96e-06 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
##
      Null deviance: 282.89 on 206 degrees of freedom
## Residual deviance: 154.98 on 195 degrees of freedom
## AIC: 178.98
##
## Number of Fisher Scoring iterations: 6
```

```
# Considering non-numeric categorical data
glm.fit.cat <- glm(AHDBinary ~ChestPain+Thal, data = training, family = binomial)</pre>
summary(glm.fit.cat)
##
## Call:
## glm(formula = AHDBinary ~ ChestPain + Thal, family = binomial,
##
      data = training)
##
## Deviance Residuals:
               1Q
                   Median
                                 3Q
## -2.1222 -0.5724 -0.1844
                           0.4715
                                      2.8576
## Coefficients:
                     Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                                  0.7777
                                          1.142
                                                  0.2536
                       0.8879
                                  0.4452 -4.113 3.91e-05 ***
## ChestPainnonanginal -1.8308
## ChestPainnontypical -3.8667
                                  0.8128 -4.757 1.96e-06 ***
## ChestPaintypical
                      -1.0123
                                  0.6070 - 1.668
                                                  0.0954
                                  0.8114 -1.340
## Thalnormal
                      -1.0871
                                                  0.1803
## Thalreversable
                       1.2529
                                  0.8419 1.488
                                                 0.1367
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
      Null deviance: 282.89 on 206 degrees of freedom
## Residual deviance: 176.17 on 201 degrees of freedom
## AIC: 188.17
## Number of Fisher Scoring iterations: 5
# Altogether
glm.fit <- glm(AHDBinary ~ Age+Sex+RestBP+Chol+Fbs+RestECG+MaxHR+ExAng+Oldpeak+Slope+Ca+ChestPain+Thal
summary(glm.fit)
##
## Call:
## glm(formula = AHDBinary ~ Age + Sex + RestBP + Chol + Fbs + RestECG +
      MaxHR + ExAng + Oldpeak + Slope + Ca + ChestPain + Thal,
##
      family = binomial, data = training)
##
## Deviance Residuals:
                1Q
                    Median
                                 3Q
                                         Max
## -3.0063 -0.4291 -0.1109
                             0.2791
                                      2.5940
## Coefficients:
                      Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                     -8.522904 3.965361 -2.149 0.031608 *
## Age
                     2.758 0.005816 **
## Sex
                      1.837955 0.666409
## RestBP
                      0.026716 0.015309
                                          1.745 0.080956 .
## Chol
```

```
## Fbs
                    -1.015934 0.808116 -1.257 0.208695
## RestECG
                   ## MaxHR
                   -0.012725 0.015260 -0.834 0.404342
                    1.294977 0.600853
                                       2.155 0.031144 *
## ExAng
                   -0.045660 0.303666 -0.150 0.880479
## Oldpeak
## Slope
                    1.370356  0.518486  2.643  0.008218 **
                    1.700924 0.407042 4.179 2.93e-05 ***
## ChestPainnonanginal -2.045326  0.619620  -3.301  0.000964 ***
## ChestPainnontypical -3.158668 0.935319 -3.377 0.000733 ***
## ChestPaintypical
                 -1.983980 0.763728 -2.598 0.009383 **
## Thalnormal
                    1.010428 1.224139 0.825 0.409134
## Thalreversable
                     2.097811 1.203044 1.744 0.081202 .
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
##
      Null deviance: 282.89 on 206 degrees of freedom
## Residual deviance: 123.51 on 190 degrees of freedom
## AIC: 157.51
##
## Number of Fisher Scoring iterations: 6
# Removing highest param (Oldpeak)
glm.fit <- glm(AHDBinary ~ Age+Sex+RestBP+Chol+Fbs+RestECG+MaxHR+ExAng+Slope+Ca+ChestPain+Thal, data =
summary(glm.fit)
##
## Call:
  glm(formula = AHDBinary ~ Age + Sex + RestBP + Chol + Fbs + RestECG +
      MaxHR + ExAng + Slope + Ca + ChestPain + Thal, family = binomial,
##
      data = training)
##
##
## Deviance Residuals:
      Min
           1Q
                  Median
                              3Q
                                     Max
                                   2.5968
## -3.0063 -0.4277 -0.1120 0.2755
##
## Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
##
## (Intercept)
                    -8.579505 3.950732 -2.172 0.029884 *
## Age
                    -0.011101 0.033006 -0.336 0.736619
## Sex
                    0.026199 0.014899
## RestBP
                                       1.758 0.078675
                    0.010847 0.004786 2.266 0.023432 *
## Chol
## Fbs
                   -1.005109 0.807074 -1.245 0.212994
## RestECG
                    ## MaxHR
                    2.153 0.031331 *
## ExAng
                    1.285547 0.597140
## Slope
                    1.347793 0.496717
                                      2.713 0.006660 **
                    1.688378 0.398209
                                       4.240 2.24e-05 ***
## ChestPainnonanginal -2.052046  0.619517  -3.312  0.000925 ***
## ChestPainnontypical -3.136389 0.921067 -3.405 0.000661 ***
## ChestPaintypical
                  ## Thalnormal
                    1.040552 1.205186 0.863 0.387920
```

```
## Thalreversable
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
      Null deviance: 282.89 on 206 degrees of freedom
## Residual deviance: 123.53 on 191 degrees of freedom
## AIC: 155.53
##
## Number of Fisher Scoring iterations: 6
# Removing highest param (Age?!)
glm.fit <- glm(AHDBinary ~ Sex+RestBP+Chol+Fbs+RestECG+MaxHR+ExAng+Slope+Ca+ChestPain+Thal, data = tra</pre>
# Removing highest param (RestECG)
glm.fit <- glm(AHDBinary ~ Sex+RestBP+Chol+Fbs+MaxHR+ExAng+Slope+Ca+ChestPain+Thal, data = training, f</pre>
# Removing highest params (MaxHR)
glm.fit <- glm(AHDBinary ~ Sex+RestBP+Chol+Fbs+ExAng+Slope+Ca+ChestPain+Thal, data = training, family =</pre>
# Removing rest of nonsignificant params in order (skipping ahead)
glm.fit5 <- glm(AHDBinary ~ Sex+Chol+ExAng+Slope+Ca+ChestPain+Thal, data = training, family = binomial
summary(glm.fit5)
##
## Call:
## glm(formula = AHDBinary ~ Sex + Chol + ExAng + Slope + Ca + ChestPain +
##
      Thal, family = binomial, data = training)
##
## Deviance Residuals:
                    Median
      Min
                1Q
                                 3Q
                                        Max
## -2.8391 -0.4416 -0.1502 0.2739
                                      2.5677
## Coefficients:
##
                      Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                     -7.075262 2.146256 -3.297 0.000979 ***
## Sex
                      1.560495 0.581312 2.684 0.007265 **
## Chol
                      0.009321 0.004352 2.142 0.032189 *
                      1.448660 0.550374 2.632 0.008485 **
## ExAng
## Slope
                      1.210650 0.447351 2.706 0.006804 **
## Ca
                      ## ChestPainnonanginal -2.184537   0.594776   -3.673   0.000240 ***
## ChestPainnontypical -3.058687   0.840206   -3.640   0.000272 ***
## ChestPaintypical
                   -1.884966 0.702410 -2.684 0.007284 **
                      0.964779 1.132919 0.852 0.394443
## Thalnormal
## Thalreversable
                      2.236792 1.108621
                                         2.018 0.043629 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
      Null deviance: 282.89 on 206 degrees of freedom
## Residual deviance: 128.12 on 196 degrees of freedom
```

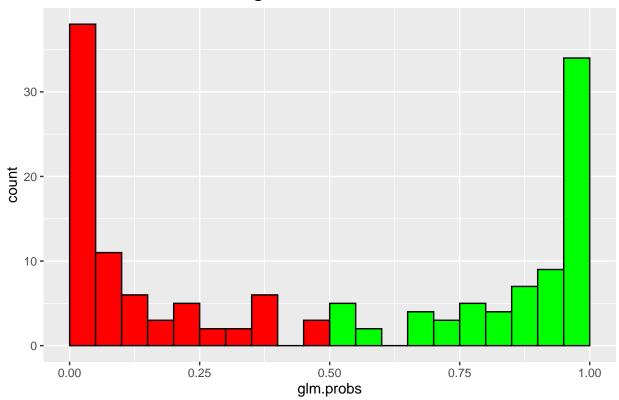
```
## AIC: 150.12
##
## Number of Fisher Scoring iterations: 6
```

Confusion Table

Constructing probability distribution for predictions on each test observation.

```
glm.probs = predict(glm.fit5, test, type = "response")
#The first 10 predicted probabilities
glm.probs[1:10]
                                                                           7
##
             2
                                      4
                                                  5
                         3
## 0.998979144 0.997511014 0.316001177 0.002336079 0.014800821 0.963959548
                         9
                                     10
## 0.462227736 0.958686279 0.975752401 0.213535076
#Visualizing our probability distribution
colors <- c(rep("red",10), rep("green",10))</pre>
probHist = ggplot(mapping = aes(glm.probs)) + geom_histogram(binwidth=0.05,boundary = 0,color="black", :
probHist = probHist + ggtitle("Histogram of Probabilities") + theme(plot.title = element_text(hjust = 0
probHist
```

Histogram of Probabilities



Generate confusion table based off of 0.5 prediction cutoff.

```
#Choosing 0.5 as the cutoff for prediction
glm.pred <- ifelse(glm.probs > 0.5,1,0)
#Constructing the table
glm.table = table(glm.pred,test$AHDBinary)
glm.table

##
## glm.pred 0 1
## 0 65 11
## 1 14 59

Mode-Test Statistics

Calculate classification accuracy and error, sensitivity, specificity, PPV and NPV.
```

```
#Accuracy
table.trace = sum(diag(glm.table))
table.sum = sum(glm.table)
acc = table.trace / table.sum
acc
## [1] 0.8322148
#0.8754209
#error
err = 1 - acc
err
## [1] 0.1677852
#sensitivity
sens = glm.table[1]/(glm.table[1] + glm.table[2])
## [1] 0.8227848
#Specificity
spec = glm.table[4]/(glm.table[4] + glm.table[3])
spec
## [1] 0.8428571
#PPV - Positive Predictive Value
PPV = glm.table[1]/(glm.table[1] + glm.table[3])
PPV
## [1] 0.8552632
```

```
#NPV - Negative Predictive Value
PPV = glm.table[4] / (glm.table[4] + glm.table[2])
PPV
```

[1] 0.8082192

ROC and AUC

Generate ROC and compute AUC for each model

```
test_prob = predict(glm.fit5, newdata = test, type = "response")

test_roc = roc(test$AHDBinary, test_prob)

## Setting levels: control = 0, case = 1

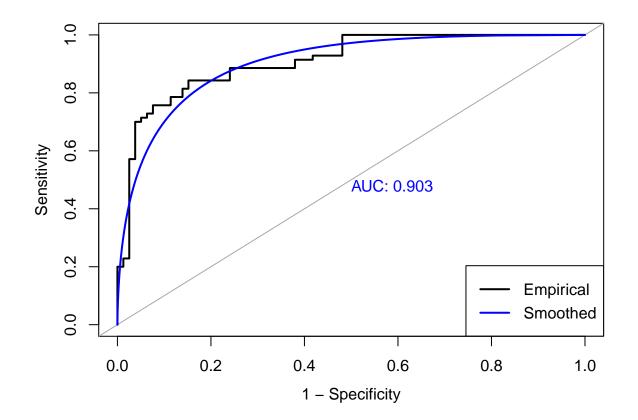
## Setting direction: controls < cases

plot.roc(test_roc, col=par("fg"),print.auc=FALSE,legacy.axes=TRUE,asp =NA)

plot.roc(smooth(test_roc),col="blue",add=TRUE,print.auc=TRUE,legacy.axes = TRUE, asp =NA)

legend("bottomright",legend=c("Empirical","Smoothed"),col=c(par("fg"),"blue"), lwd=2)

abline(v = -coef(glm.fit5)[1] / coef(glm.fit5)[2], lwd = 3)</pre>
```



S sigmoid curve

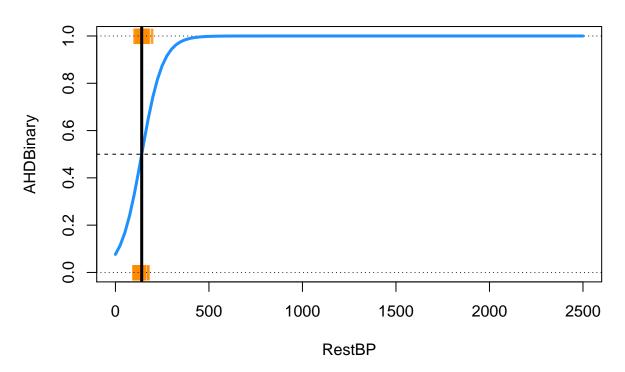
Generate s-curve for Y against one attribute (you can pick any one attribute), and interpret your findings I used RestBP TEMPORARILY. This will be replaced with something more appropriate when we try our CART model.

```
single.glm <- glm(AHDBinary ~ RestBP, data = data, family = "binomial")

plot(AHDBinary ~ RestBP, data = data,col = "darkorange", pch = "|", xlim = c(0, 2500), ylim = c(0, 1),m
   abline(h = 0, lty = 3)
   abline(h = 1, lty = 3)
   abline(h = 0.5, lty = 2)

curve(predict(single.glm, data.frame(RestBP = x), type = "response"),add = TRUE, lwd = 3, col = "dodger"
   abline(v = -coef(single.glm)[1] / coef(single.glm)[2], lwd = 3)</pre>
```

Using Logistic Regression for Classification



CART Model

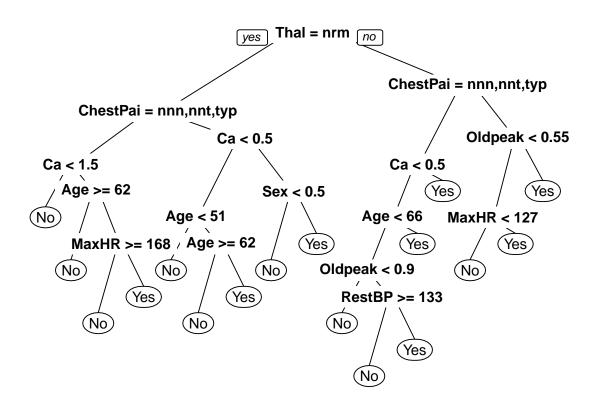
CART MODEL

CART MODEL

```
Heart <-read.csv('https://www.statlearning.com/s/Heart.csv')
set.seed(1)
heart.split <- sample(1:nrow(Heart), size=nrow(Heart) * 0.7)
heart.train <- Heart[heart.split,]
heart.test <- Heart[-heart.split,]</pre>
```

Constructing our splits

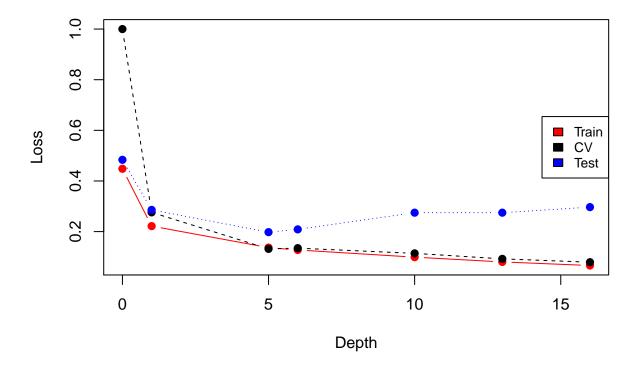
```
class.cart <- rpart(formula = AHD ~ Age+Sex+RestBP+Chol+Fbs+RestECG+MaxHR+ExAng+Oldpeak+Slope+Ca+ChestP
prp(class.cart, roundint = FALSE)</pre>
```



```
cp.class.param <- class.cart$cptable
train.acc <- double(6)
cv.acc <- double(6)
test.acc <- double(6)
for (i in 1:nrow(cp.class.param)) {
   alpha <- cp.class.param[i, 'CP']
   train.cm <- table(heart.train$AHD, predict(prune(class.cart, cp=alpha), newdata = heart.train, type='
   train.acc[i] <- 1-sum(diag(train.cm))/sum(train.cm)
   cv.acc[i] <- cp.class.param[i, 'xerror'] * cp.class.param[i, 'rel error']
   test.cm <- table(heart.test$AHD, predict(prune(class.cart, cp=alpha), newdata = heart.test, type='cla
   test.acc[i] <- 1-sum(diag(test.cm))/sum(test.cm)
}</pre>
```

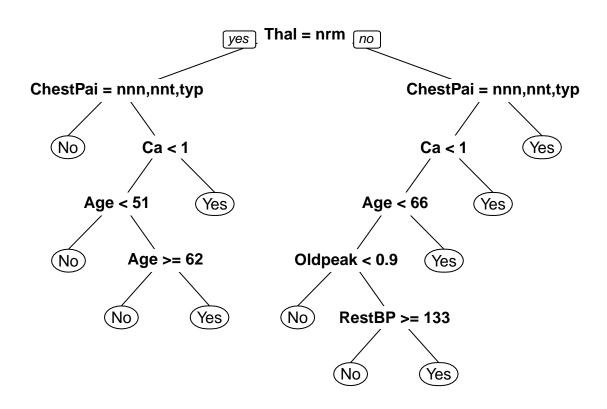
Train cv and test accuracy.

```
matplot(cp.class.param[,'nsplit'], cbind(train.acc, cv.acc, test.acc), pch=19, col=c("red", "black", "b
legend("right", c('Train', 'CV', 'Test') ,col=seq_len(3),cex=0.8,fill=c("red", "black", "blue"))
```



Pruning

```
prune.class.trees <- prune(class.cart, cp=cp.class.param[5,'CP'])
prp(prune.class.trees)</pre>
```



Confusion Matrix

Confusion Matrix

##

##

```
conf.mat.tree <- table(heart.test$AHD, predict(prune.class.trees, type = 'class', newdata = heart.test)
conf.mat.tree
##
##
No Yes</pre>
```

Mode-Test Statistics

No 30 17

Yes 8 36

Statistics regarding our confusion table.

```
acc <- sum(diag(conf.mat.tree))/sum(conf.mat.tree)
err <- 1 - acc
sens <- conf.mat.tree[1,1]/(conf.mat.tree[1,1] + conf.mat.tree[2,1])
spec <- conf.mat.tree[2,2]/(conf.mat.tree[2,2] + conf.mat.tree[1,2])
ppv <- conf.mat.tree[1,1]/(conf.mat.tree[1,1] + conf.mat.tree[1,2])
npv <- conf.mat.tree[2,2]/(conf.mat.tree[2,2] + conf.mat.tree[2,1])
c(Accuracy = acc, Error = err, Sensitivity=sens, Specificity = spec, PPV = ppv, NPV = npv)</pre>
```

```
## Accuracy Error Sensitivity Specificity PPV NPV ## 0.7252747 0.2747253 0.7894737 0.6792453 0.6382979 0.8181818
```

ROC

```
library(rpart)
library(ROCR)

test_prob = predict(prune.class.trees, newdata = heart.test, type='class')
test_roc = roc(heart.test$AHD, factor(test_prob, ordered = TRUE))

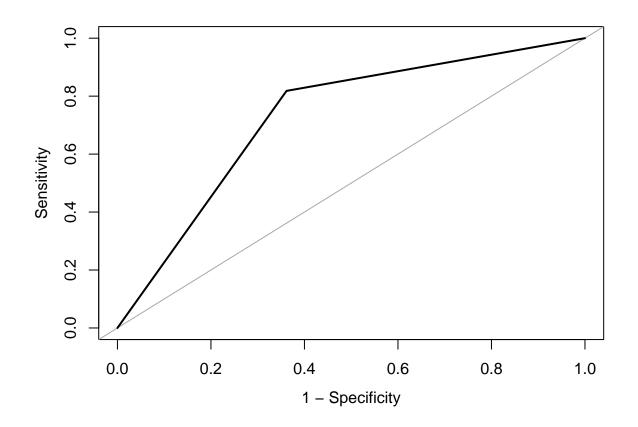
## Setting levels: control = No, case = Yes

## Warning in value[[3L]](cond): Ordered predictor converted to numeric vector.

## Threshold values will not correspond to values in predictor.

## Setting direction: controls < cases

plot.roc(test_roc, col=par("fg"),plot=TRUE,print.auc = FALSE, legacy.axes = TRUE, asp =NA)</pre>
```



```
#pred <- prediction(predict(class.cart, type="prob")[, 2], Heart$AHD)
#plot(performance(pred, "tpr", "fpr"), col="blue", main="ROC AHD")
#abline(0, 1, lty=2)</pre>
```

Random Forests

The decision trees approach suffers from high variance, meaning the results of the tree fitting to the raining set can be quite different depending on the training/test set split. To combat this issue with the decision trees bootstrap aggregation is employed, which is also referred to as "bagging". In this approach many training sets are derived from the population using bootstrap, a separate prediction model using each training set is developed, and the resulting predictions are averaged. For the classification task that we consider in this project instead of averaging the majority vote is taken across the predicted classes. This allows to reduce the variance of the statistical method.

On average, each bagged tree makes use of around two-thirds of the observations. The remaining one-third of the observations not used to fit a given bagged tree are referred to as the out-of-bag (OOB) observations. Testing the predictions on the OOB observations is the foundation for the OOB error estimate. Based on the results shown below approximately 230 trees are sufficient for both errors to stabilize.

Random forests provide an improvement over bagged trees by way of a random tweak that decorrelates the trees. The split is allowed to use only one a subset of predictors. A fresh sample of predictors is taken at each split, and the number of predictors in the subset typically equals the square root of the total number of predictors. The random forest results and a variable importance plot for the Heart data are given below.

```
Heart <-read.csv('Heart.csv')
Heart <- na.omit(Heart) #Remove NA for demo
data1 <- Heart[,-1]
set.seed(490)
split <- sample(1:nrow(data1), size=nrow(data1) * 0.7)
train <- data1[split,]
test <- data1[-split,]
train$AHD = factor(train$AHD)</pre>
```

library(randomForest)

```
## randomForest 4.6-14

## Type rfNews() to see new features/changes/bug fixes.

##

## Attaching package: 'randomForest'

## The following object is masked from 'package:dplyr':

##

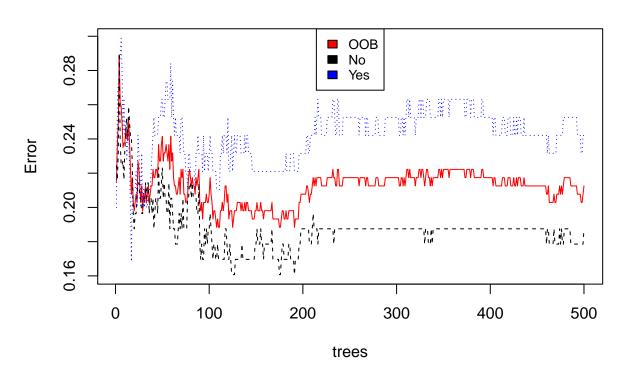
## combine

## The following object is masked from 'package:ggplot2':

##

## margin
```

rf.class



Confusion Table

Carry out prediction with the Random Forest model:

```
rf_classifier <- randomForest(AHD ~ ., data=train, ntree = 500, mtry=round(sqrt(ncol(train)-1)), import
prediction_for_table <- predict(rf_classifier, test[,-14])
c.table <- table(observed=test[,14],predicted=prediction_for_table)
c.table

## predicted
## observed No Yes
## No 43 5</pre>
```

Mode-Test Statistics

Yes 9 33

##

Calculate classification accuracy and error, sensitivity, specificity, PPV and NPV.

```
#Accuracy
table.trace = sum(diag(c.table))
table.sum = sum(c.table)
acc = table.trace / table.sum
acc
## [1] 0.8444444
#0.8754209
#error
err = 1 - acc
err
## [1] 0.1555556
#sensitivity
sens = c.table[1]/(c.table[1] + c.table[2])
## [1] 0.8269231
#Specificity
spec = c.table[4]/(c.table[4] + c.table[3])
## [1] 0.8684211
#PPV - Positive Predictive Value
PPV = c.table[1]/(c.table[1] + c.table[3])
PPV
## [1] 0.8958333
#NPV - Negative Predictive Value
PPV = c.table[4]/(c.table[4] + c.table[2])
PPV
## [1] 0.7857143
```

Thus, the Random Forest classifier achieved the accuracy of $\sim 84~\%$ and corresponding error of $\sim 16~\%$.

ROC and AUC

Generate ROC and compute AUC for Random Forest

```
test_prob = predict(rf_classifier, newdata = test[,-14], type = "class")
#test_prob = predict(prune.class.trees, newdata = heart.test, type='class')
test_roc = roc(test$AHD, factor(test_prob, ordered = TRUE))
```

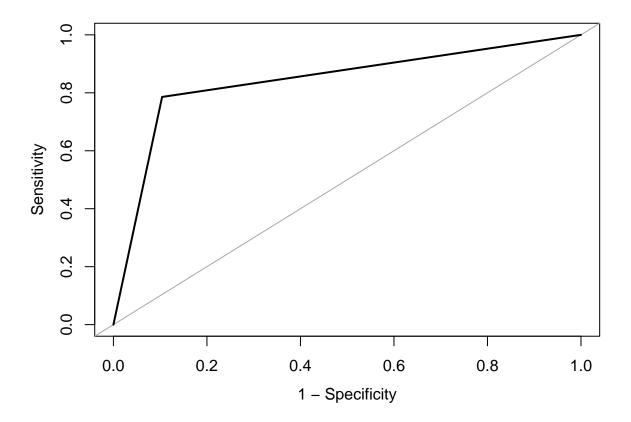
```
## Setting levels: control = No, case = Yes

## Warning in value[[3L]](cond): Ordered predictor converted to numeric vector.

## Threshold values will not correspond to values in predictor.

## Setting direction: controls < cases

plot.roc(test_roc, col=par("fg"),plot=TRUE,print.auc = FALSE, legacy.axes = TRUE, asp =NA)</pre>
```



Comparing Models

	Accuracy	sens	spec	PPV	NPV
Logistic	0.832	0.822	0.842	0.855	0.808
CART	0.725	0.789	0.679	0.638	0.818
Rand Forest	0.844	0.826	0.868	0.895	0.785

Citations

- [1] Index of An Introduction to Statistical Learning/Heart.csv, https://www.statlearning.com/s/Heart.csv.
- [2] Fang, Julia. "CIS490_LS9_21S_Classification_Logistic&ROC&AUC." MyCourses, 2021.

- [3] Fang, Julia. "CIS490_LS10_21S_CART." MyCourses, 2021.
- [4] Fang, Julia. "R_logistic&ROC_21S." MyCourses, 2021.
- [5] Fang, Julia. "R_Trees_S21." MyCourses, 2021.
- [6] Fang, Julia. "Supplement_Reading_BaggingRandomForestBoosting." MyCourses, 2021
- [7] James, G., Witten, D., Hastie, T. and Tibshirani, R., 2013. An introduction to statistical learning (Vol. 112, p. 18). New York: springer.