Circuits

What *is* circuits?

- A branch of maths concerning the voltage and current of electronic and electrical systems.
- An Idealisation of real behaviours.
- For example, $V = IR$ only works under certain circumstances, so the idealisation of $V = IR$ means it is useful but doesn't represent reality. The main problem is that as a component dissipates energy it heats up, increasing R.
- Useful for analysis and design.
- Engineering is, after all, design. You need to understand models to design systems effectively.

The Equations - Mathematical theory of circuits

Ideal Components, and their relationship with I and V.

Component	Ideal Relationship
Resistor	$V = IR$
Capacitor	$I = C\frac{dV}{dt}$
Inductor	$V = L\frac{dI}{dt}$

There are also ideal relationships for diodes, opamps and transistors, which will be talked about later on.

Describing how they are joined: The Kirchhoff Laws

The Voltage Law

The sum of the voltages in a loop always sum to zero. Often abbreviated to KVL.

$$\sum_i v_i = 0$$

The Current Law

The sum of all currents flowing into a point is zero. Often abbreviated to KCL.

$$\sum_i I_i = 0$$

And with these ideal equations and laws we can describe any circuit mathematically.

Capacitors

As a first example, lets look at capacitors in series.

For two capacitors, C_1 and C_2 in series, what is their equivalent capacitance C_3, as if they were one component.

This turns out to be this relationship:

$$\frac{1}{C_3} = \frac{1}{C_1} + \frac{1}{C_2}$$

But how is this derived?

First, find the ideal equations:

$$I_1 = C_1 \frac{dV_1}{dt}$$

$$I_2 = C_2 \frac{dV_2}{dt}$$

$$I_3 = C_3 \frac{dV_3}{dt}$$

Next, use KCL to find relationships with current, in this example you might get:

$$I_2 - I_3 = 0 \therefore I_1 = I_2 = I_3$$

And finally use KVL to find relationships with voltage, an example:

$$V_1 + V_2 - V_2 - V_1 = V_1 + V_2 - V_3 \therefore V_3 = V_1 + V_2$$

And from this information we can derive the formula:

$$\frac{I}{C} = \frac{dV}{dt}$$

$$\frac{I_3}{C_3} = \frac{I_1}{C_1} + \frac{I_2}{C_2}$$

$$\frac{1}{C_3} = \frac{1}{C_1} + \frac{1}{C_2}$$

And this technique works though every circuit we will ever analyse.