Simple Harmonic Motion

Calculating the energy in a mass-spring system

First, we can consider the kinetic energy of the system at time t:

$$E_k(t) = \frac{1}{2}mv^2 = \frac{1}{2}m(\omega x_m \sin(\omega t + \phi))^2$$

$$E_k(t) = \frac{1}{2}kx_m^2\sin^2(\omega t + \phi)$$

Then the potential energy:

$$E_p(t) = \frac{1}{2}kx^2 = \frac{1}{2}k(x_m\cos(\omega t + \phi))^2$$

$$E_p(t) = \frac{1}{2}kx_m^2\cos^2(\omega t + \phi)$$

Therefore, by addition:

$$E_{total} = \frac{1}{2}kx_m^2$$