

Damped SHM

After a mass starts moving its amplitude gradually decreases with time because of the resistive forces due to the air. The time period does not change as the amplitude decreases, but the time period is larger than an equivalent system with no resistive forces.

Hooke's Law: $F_{\text{spring}} = -kx$ Damping: $F_{\text{damping}} = -bv$ Newton II: $F = ma = \frac{md^2x}{dt^2}$

Equation of a damped SHM system:

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

$$Ae^{\frac{-bt}{2m}} \cos(\omega't + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

So, therefore if $b = 0$ then $\omega = \omega'$, but if $b \ll \sqrt{km}$ then $\omega' \approx \omega$.

The total mechanical energy of a damped system decreases as it does work against resistive forces.

$$U(t) = \frac{1}{2}kA^2e^{-\frac{bt}{m}} \cos^2(\omega't + \phi)$$

$$K(t) \approx \frac{1}{2}kA^2e^{-\frac{bt}{m}} \sin^2(\omega't + \phi)$$

$$E(t) = U(t) + K(t) \approx \frac{1}{2}kA^2e^{-\frac{bt}{m}}$$