Travelling Waves and Wave Equation

Introduction to Waves

Reading: Sections 16.1 though 16.5

At time t the displacement of y at position x is

$$y(x,t) = y_m \sin(kx \pm \omega t)$$

So, for a wavelength of lambda you have a wave number of

$$k = \frac{2\pi}{\lambda}$$

Therefore, by fixing the position at $ x = 0 $ we observe the motion

$$y(0,t) = -y_m \sin(\omega t)$$
$$y(0,t) = y_m \cos\left(\omega t - \frac{\pi}{2}\right)$$

Which is the form of SHM.

We can get the speed of a travelling wave

$$v = \frac{\omega}{k} = \lambda f$$

Waves on a Stretched String

Reading: Sections 16.6-16.7

An element Δl os moving in the arc of a circle so it's acceleration is

$$a = \frac{v^2}{r}$$

Using Newton II ($F = \Delta m a$) and considering the tension on the element

$$F = 2(\tau \sin\theta) \approx 2\theta\tau \approx \tau\frac{\Delta l}{r}$$

The mass of the element

$$\Delta m = \mu \Delta l$$

Then gives

$$\tau \frac{\Delta l}{r} = (\mu \Delta l) \frac{v^2}{r}$$

So to find v

$$v = \sqrt{\tau/\mu}$$

The power delivered to the string

$$P = \vec{F}\ \vec{v}_{transverse}$$

So, you can then say that

$$P = -\tau \frac{dy}{dt} \frac{dy}{dx}$$

Finally,

$$P = \mu v \omega^2 y_m^2 \cos^2(ks - \omega t)$$