Basic Formulas

$\int dx = x$

$\int x^n dx = \dfrac{x^{n+1}}{n+1}\ (n \neq -1)$

$\int \dfrac{1}{x}\, dx = \log|x|$

$\int \cos\, dx = \sin x$

$\int \sin x dx = -\cos x$

$\int \sec^2 x dx = \tan x$

$\int \operatorname{cosec}^2 x dx = -\cot x$

$\int \sec x \tan x dx = \sec x$

$\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x$

$\int e^x dx = e^x$

$\int \dfrac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x$

$\int \dfrac{1}{\sqrt{1+x^2}}\, dx = \tan^{-1} x$

$\int \dfrac{1}{x\sqrt{x^2-1}}\, dx = \sec^{-1} x$

$\int a^x dx = \dfrac{a^x}{\log_e a},\ a > 0$

$\int \tan x\ dx = \log|\sec x| + c$

$\int \cot x\ dx = \log|\sin x| + c$

$$\int \sec x\ dx = \log|\sec x + \tan x| + c = \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$$

$$\int \operatorname{cosec} x\ dx = \log|\operatorname{cosec} x - \cot x| + c = |\tan x_2| + c$$

Method of Substitution

Type 1 $\int \dfrac{f'(x)}{f(x)}\, dx = \log|f(x)| + c$

Type 2 $\int [f(x)]^n\ f'(x)\ dx = \dfrac{f(x)^{n+1}}{n+1} + c$

Type 3 $\int f[g(x)]g'(x)dx$. In this type, we substitute $g(x) = t$, then, Hence integral reduces to $\int f(t)\, dt$

Type 4 $P(x).(ax+b)^n$ or $\dfrac{P(x)}{(ax+b)^n}$ where $P(x)$ is a polynomial in x and n is a positive rational number.

Working Rule : Put $z = ax + b$

Type 5 $\sin f(x)$ or $\cos f(x)$ then put $z = f(x)$

Type 6 : $\int \sin^m x \cos^n x dx$ Working Rule :

(i) If power of sin x is odd positive integer, put $z = \cos x$

(ii) If power of cos x is odd positive integer, put $z = \sin x$

(iii) If powers of both sin x and cos x are odd positive integers, put $z = \sin x$ or $z = \cos x$.

(iv) If powers of neither cos x nor sin x is odd positive integer, see the sum of powers of sin x and cos x.

(a) If the sum of powers is even negative integer, put $z = \tan x$.

(b) If the sum of powers $(m + n)$ is even positive integer and m, n are integers, express the integrand as the algebraic sum of sines and cosines of multiple angles.

Type 7 : $\int \tan^m x \sec^n x\,dx$ or $\int \cot^m x \operatorname{cosec}^n x\,dx$

. For $\int \tan^m x \sec^n x\,dx$: similar can be derived for the other pair

(i) If power of sec$x$ is even positive integer, put $z = \tan x$.

(ii) If power of sec$x$ is not even positive integer, then see the power of tan$x$.

(a) If power of tan$x$ is odd positive integer, put $z = \sec x$.

(b) If power of tan$x$ is even positive integer, then put $\sec^2 x - 1$ in place of $\tan^2 x$ and then substitute $z = \tan x$.

(iii) If power of tan$x$ is zero and power of sec$x$ is odd positive integer greater than 1, then method of integration by parts is used.

SOME STANDARD SUBSTITUTIONS

1. $\sqrt{a^2 - x^2}$ or $\dfrac{1}{\sqrt{a^2 - x^2}}$ $x = a\sin\theta$ or $a\cos\theta$

2. $\dfrac{1}{(a^2 + x^2)}$ or $\dfrac{1}{\sqrt{a^2 + x^2}}$ $x = a\tan\theta$ or $a\cot\theta$

3. $\dfrac{1}{\sqrt{x^2 - a^2}}$ or $\sqrt{x^2 - a^2}$ $x = a\sec\theta$ or $a\operatorname{cosec}\theta$

4. $\sqrt{\dfrac{a - x}{a + x}}$ or $\sqrt{\dfrac{a + x}{a - x}}$ $x = a\cos 2\theta$

5. $\sqrt{\dfrac{x}{a - x}}$ or $\sqrt{\dfrac{a - x}{x}}$ $x = a\sin^2\theta$ or $x = a\cos^2\theta$

6. $\sqrt{\dfrac{x}{a + x}}$ or $\sqrt{\dfrac{1 + x}{x}}$ $x = a\tan^2\theta$ or $x = a\cot^2\theta$

7. $\sqrt{\left(\dfrac{x - a}{b - x}\right)}$ or $\sqrt{(x - a)(b - x)}$

$x = a\cos^2\theta + b\sin^2\theta$

8. $\sqrt{\dfrac{(x - a)}{(x - b)}}$ or $\sqrt{(x - a)(x - b)}$

$x = a\sec^2\theta - b\tan^2\theta$

9. $\dfrac{1}{\sqrt{(x - a)(x - b)}}$ $x - a = t^2$ or $x - b = t^2$

SOME STANDARD FORMULAS DERIVED FROM SUBSTITUTION

Set-I

1. $\displaystyle\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left|\frac{x - a}{x + a}\right| + c$

2. $\displaystyle\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a + x}{a - x}\right| + c$

3. $\displaystyle\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

Set-II

1. $\displaystyle\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$

2. $\displaystyle\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + c$

3. $\displaystyle\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + c$

4. $\displaystyle\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$

Set-III **1.** $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{2} + c$

2. $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

3. $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

INTEGRATION BY PARTS

Integral of product of two functions

= (1st function) × (Integral of 2nd function) – Integral of {(differential of 1st function) × Integral of 2nd function}

In symbols : $\int f(x).\,g(x)dx = f(x).\int g(x)dx - \int\left\{\frac{d}{dx} f(x).\int g(x)dx\right\}dx$

or $\int u.vdx = u\int vdx - \int u'(\int vdx)dx$

where **I** stands for Inverse circular function

L stands for Logarthmic function

A stands for Algebraic function

T stands for Trigonometrical functions

and **E** stands for Exponential function

(ii) If both the functions are trigonometrical, take that function as v whose integral is simpler.

(iii) If both the functions are algerbraic take that functions as u whose d c is simpler.

Standard Forms derived using By Parts

(i) $\int e^x\left[f(x) + f'(x)\right]dx = e^x f(x) + c$

(ii) $\int e^{mx}\left[mf(x) + f'(x)\right]dx = e^{mx} f(x) + c$

(iii) $\int [xf'(x) + f(x)]dx = x f(x) + c$

(iv) $\int e^{ax}\sin(bx + c)\,dx = e^{ax}\frac{\sin\left(bx + c - \tan^{-1}\frac{b}{a}\right)}{\sqrt{a^2 + b^2}} + c$

(v) $\int e^{ax}\cos(bx + c)\,dx = e^{ax}\frac{\cos\left(bx + c - \tan^{-1}\frac{b}{a}\right)}{\sqrt{a^2 + b^2}} + c$

METHOD : INTEGRATION BY PARTIAL FRACTION

We have divided this method into 2 types, depending upon the denominator.

1. If denominator has non repeated factors
2. If denominator has repeating factors

Type 1 : For non-Repeating roots

When denominator can be expressed as non repeating factors

i.e. $D(x) = (x - \alpha_1)(x - \alpha_2)...$ (for linear factors)

$= (ax + bx + c)(px^2 + qx + r)...$ (for quadratic factors)

Type 2

When repeating factors are present i.e. when denominator is of the form

$D(x) = (x - \alpha)^{k1}(x - \beta)^{k2}$ ... {for linear factor}

$= (ax^2 + bx + c)^{k1}(px^2 + qx + c)^k$ {for quadratic

(1) If function is linear.

i.e. $$\frac{N(x)}{(x-a)(x-b)^2(x-c)^3} = \frac{A}{(x-a)} + \frac{B_1}{(x-b)} + \frac{B_2}{(x-b)^2} + \frac{C_1}{(x-c)} + \frac{C_2}{(x-c)^2} + \frac{C_3}{(x-c)^3}$$

(2) If function has quadratic factors

i.e of the form $$\frac{N(x)}{(ax^2+bx+c)(px^2+qx+c)^2} = \frac{Ax+B}{ax^2+bx+c} + \frac{P_1x+Q_1}{\left(px^2+qx+r\right)} + \frac{P_2x+Q_2}{\left(px^2+qx+r^2\right)}$$

INTEGRATION OF RATIONAL & IRRATIONAL FUNCTIONS

Integral of the form $\int \frac{dx}{ax^2+bx+c}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ **and** $\int \sqrt{ax^2+bx+c}\,dx$

For evaluating such integral we make the coefficients of x^2 in $ax^2 + bx + c$ as one. Complete the square by adding and subtracting the square of half of the coefficient of x to get the form

$$a\left[\left(x+\frac{b}{2a}\right)^2 + \left(\frac{c}{a}-\frac{b^2}{4a^2}\right)\right]$$

Integrals of the form $\int \frac{px+q}{ax^2+bx+c}\,dx$, $\int \frac{px+q}{\sqrt{ax^2+bx+c}}\,dx$ and $(px+q)\sqrt{ax^2+bx+c}\,dx$

For evaluating such integrals we choose suitable constants A and B such that

$$px+q = A\left[\frac{d}{dx}(ax^2+bx+c)\right] + B$$

Integrals of the Form : $\int \frac{px^2 + qx + r}{\sqrt{ax^2 + bx + c}} dx$

For evaluating such integrals we choose suitable constants, A, B and C such that

$$px^2 + qx + r = A\,(ax^2 + bx + c) + B\left(\frac{d}{dx}(ax^2 + bx + c)\right) + C$$

Integrals of the form : $\int \frac{x^2 \pm 1}{x^4 + kx^2 + 1} dx$

For evaluating such integrals, divide the numerator and denominator by x^2. Complete the square of denominator to get the form $\left(x + \frac{1}{x}\right)^2 + a$ or $\left(x - \frac{1}{x}\right)^2 + a$

Then the integral can be evaluated by using the method of substitution.

Special Integration

Type I $\int \frac{x^2 + \sqrt{q}}{x^4 + px^2 + q}$ Divide numerator & denominator by x^2

Type II $\int \frac{dx}{x^4 + px^2 + q}$ write this in form $\frac{1}{2\sqrt{q}} \int \frac{\left(x^2 + \sqrt{q}\right) - \left(x^2 - \sqrt{q}\right)}{x^4 + px^2 + q}$

Type III $\int \frac{x^2 + r}{x^4 + px^2 + q} dx$ express $x^2 + r$ as $x^2 + r = l\left(x^2 + \sqrt{q}\right) + m\left(x^2 - \sqrt{q}\right)$

where $l + m = 1$ & $\sqrt{q}\,(l - m) = r$

Integration of Trigonometric Functions

Type I $\int \frac{dx}{a + b\cos x}$ or $\int \frac{dx}{a + b\sin x}$ or $\int \frac{dx}{a + b\cos x + c\sin x}$

Working Rule : Put $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$ and $\cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

whichever is needed and then put $z = \tan\frac{x}{2}$

Type II $\int \frac{\sin x}{a \sin x + b \cos x} dx$, $\int \frac{\cos x}{a \sin x + b \cos x} dx$, or $\int \frac{p \sin x + q \cos x}{c \sin x + b \cos x} dx$

Step – 1 : Put Numerator = A (dinominator) + B (derivative of denominator.) where $a \neq 0$, $b \neq 0$

Step – 2 : Then equate the coefficients of sinx and cosx to find A and B.

Type 3. $\int \frac{a \sin x + b \cos x + c}{p \sin x + q \cos x + r}$

(i) Write Numerator = λ (Diff. of denominator) + μ (Denominator) + v

i.e. $a \sin x + b \cos x + c = \lambda (p \cos x - q \sin x) + \mu (p \sin x + q \cos x + r) + v$

Type 4.

$$\int \frac{1}{a \sin^2 x + b \cos^2 x} dx . \int \frac{1}{a + b \sin^2 x} dx, \int \frac{1}{a + b \cos^2 x} dx, \int \frac{1}{(a \sin x + b \cos x)^2} dx, \int \frac{1}{a + b \sin^2 x + \cos^2 x} dx$$

(i) Divide numerator and denominator both by $\cos^2 x$

(ii) Replace $\sec^2 x$, if any, in denominator by $1 + \tan^2 x$

(iii) Put $\tan x = t$ so that $\sec^2 x \, dx = dt$

This substitution reduces the integral in the form $\int \frac{1}{at^2 + bt + c} dt$

Integrals of the form : $\int \frac{dx}{\mathbf{P} \sqrt{\mathbf{Q}}}$

where P and Q are linear or quadratic expression in x

1. Q is linear and P is linear or quadratic., For evaluating such integrals, put $Q = t^2$

2. Q is quadratic and P is linear., For evaluating such integrals, put $P = \frac{1}{t}$

3. Both P and Q are pure quadratic., For evaluating such integrals, put $x = \frac{1}{t}$.

Integration of Irrational Functions

<u>*Types of functions (intergrand)*</u>	<u>*Approach*</u>
1. $f\left\{x, \left(\frac{ax + b}{cx + d}\right)^{a/n}\right\} (a, b, c, d, \alpha, n \in \mathbf{R})$	Substitute : $\frac{ax + b}{cx + d} = t^n$
2. $f\left\{x, (ax + b)^{a/n}, (ax + b)^{\beta/m}\right\}$	$ax + b = t^p$, where p is L.C.M. of m and n.

3. $f\left\{\left(x \pm \sqrt{a^2 + x^2}\right)^n\right\}$ Workrule : $x \pm \sqrt{a^2 + x^2} = t$

4. $\sqrt{\frac{1}{x^m (a + bx)^p}}$ $m + p \in N,\ m + p > 1$ Workrule : $a + bx = tx$

5. $\sqrt{\frac{1}{\{L_1(x)\}^m \{L_2(x)\}^n}}$

(i) If $n > m$; $\frac{L_1(x)}{L_2(x)} = t$

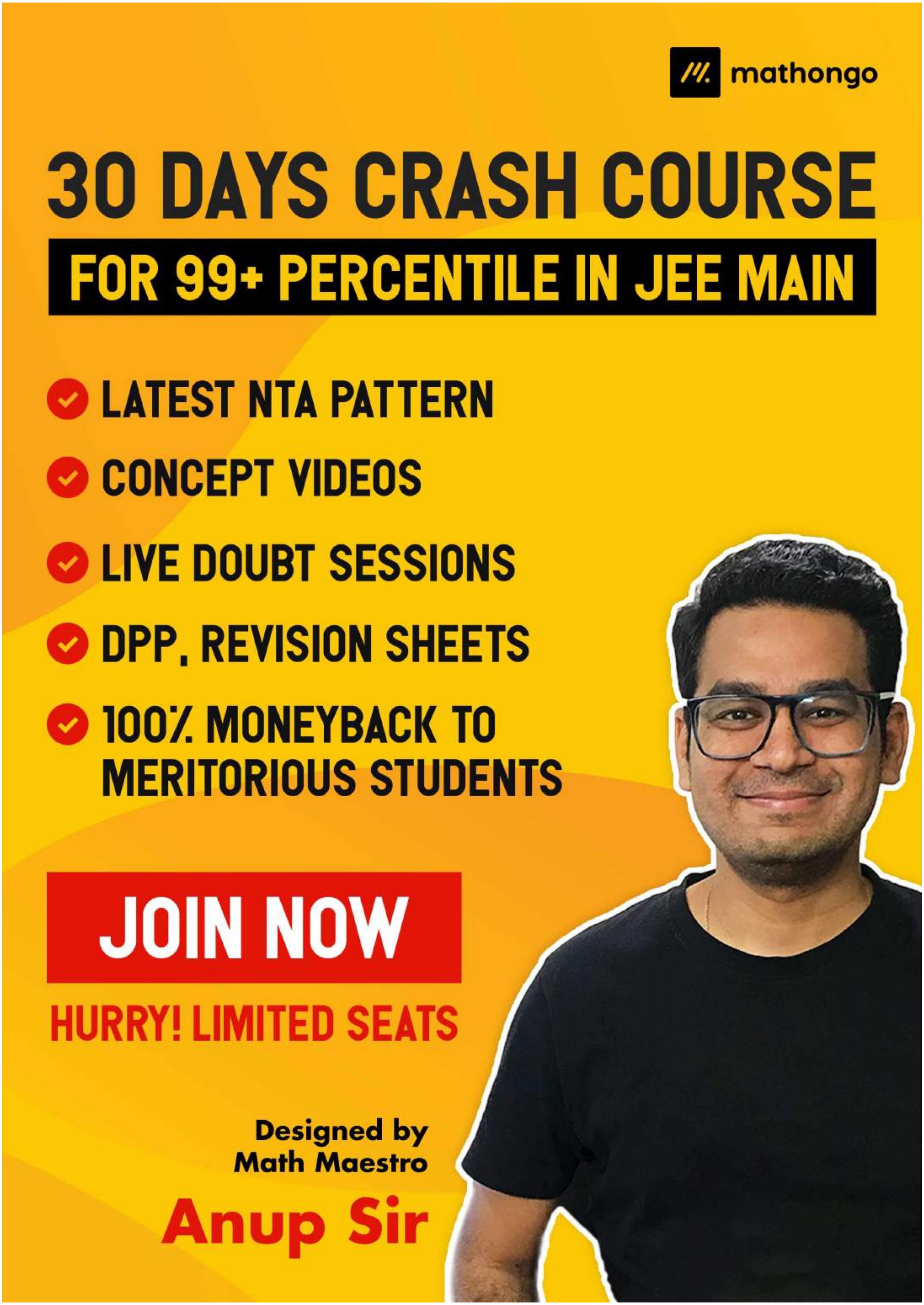
(ii) If $n < m$; $\frac{L_2(x)}{L_1(x)} = t$

6. $x^m(a + bx^n)^p dx$

(i) If $p \in I$, substitute $x = t^s$
where s is L.C.M. of denominator of m & n.

(ii) If $\frac{m+1}{n}$ is an Integer, substitute $a + bx^n = t^s$
is the denominator of fraction p.

(iii) If $\frac{m+1}{n} + p$ substitute $ax^{-n} + b = t^s$
s is denominator of rational number p.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