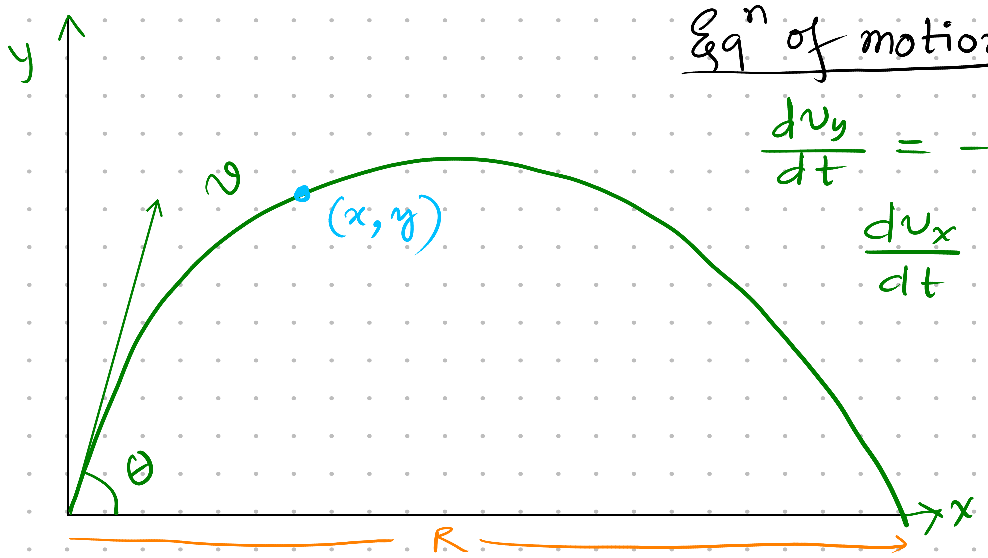
Find the angle at which the area bound by the projectile will be maximum.



Eq^n of motion:

$$\frac{dv_y}{dt} = -g$$

$$\frac{dv_x}{dt} = 0.$$

$x = v\cos\theta \cdot t$

$\Rightarrow t = \dfrac{x}{v\cos\theta}$

$y = v\sin\theta\, t - \frac{1}{2} g t^2$

$y = \tan\theta \cdot x - \dfrac{g\, x^2}{2v^2\cos^2\theta}$

To find R, put $y = 0$

$\Rightarrow x = 0$ and, $x = \dfrac{v^2 \sin(2\theta)}{g}$

$$A = \int_0^R y\, dx$$

$$= \int_0^R \left(\tan\theta \cdot x - \frac{g}{2v^2\cos^2\theta}\, x^2\right) dx$$

$$= \tan\theta \cdot \frac{R^2}{2} - \frac{g}{2v^2\cos^2\theta} \frac{R^3}{3}$$

$$= \frac{1}{2} \frac{\sin\theta}{\cos\theta} \left(\frac{v^4 \cdot 4\sin^2\theta\cos^2\theta}{g^2}\right) - \frac{g \cdot 8 \cdot v^6 \sin^3\theta\cos^3\theta}{6v^2\cos^2\theta\; g^3}$$

$$= \frac{v^4 \sin^3\theta\cos\theta}{g^2}\left(\frac{4}{2} - \frac{8}{6}\right) = \frac{2}{3} \frac{v^4\sin^3\theta\cos\theta}{g^2}$$

$$A = \frac{2}{3}\frac{v^4}{g^2}\sin^3\theta\cos\theta$$

To find the extrema $\boxed{\frac{dA}{d\theta} = 0}$

$$\frac{dA}{d\theta} = \frac{2v^4}{3g^2}\left(3\sin^2\theta\cos^2\theta - \sin^4\theta\right) = 0$$

$$\Rightarrow \sin^2\theta = 0 \quad \text{or,} \quad \tan\theta = \sqrt{3}$$

$$\theta = 0^\circ \qquad\qquad \theta = 60^\circ$$

$\uparrow$ Minima $\qquad\qquad$ $\uparrow$ Maxima.

$$A_{max} = \frac{2}{3}\frac{v^4}{g^2}\sin^3(60^\circ)\cos(60^\circ)$$

$$\boxed{A_{max} \propto v^4}$$