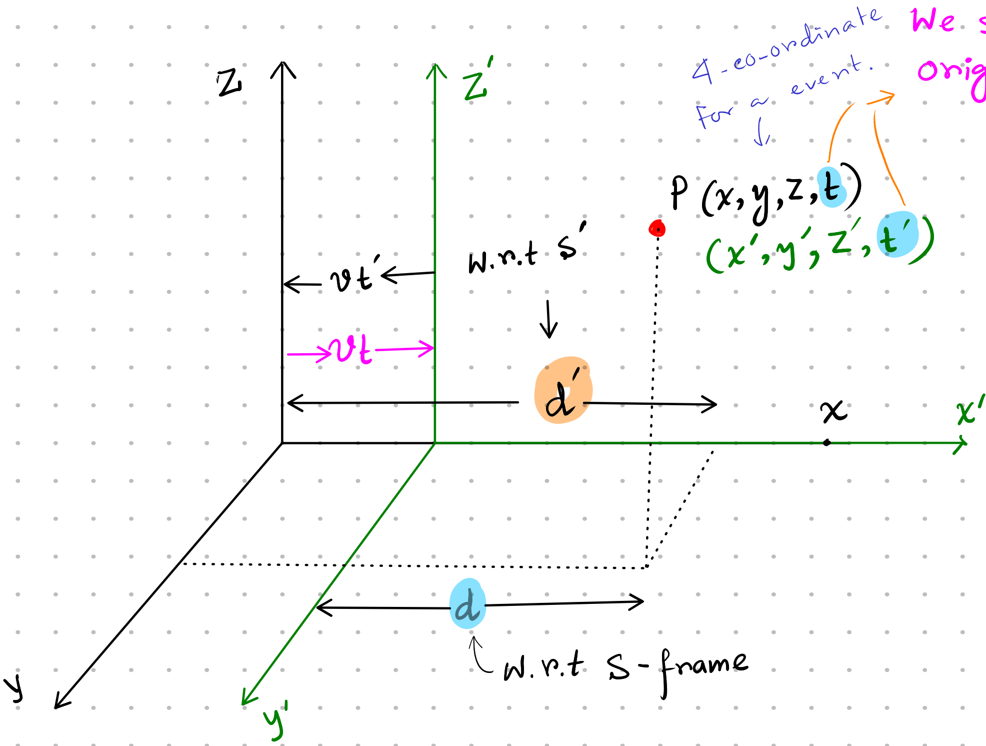

We started clock when two origin are overlapped.

- Classically,
- Intuition says that time will not differ in both frame.
  - $t = t'$
- $d = x'$

No absolute frame is there, so if we take S-frame to rest and consider the transformation,

$$x = d + vt \qquad \text{--- } (1)$$

No absolute frame is there, so if we take S'-frame to rest and consider the transformation,

$$x' = d' - vt' \qquad \text{--- } (2)$$

- Now we are going to use length construction,

"Moving length contracted."

- $d = \dfrac{x'}{\gamma}$

Moving length

rest length

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Plug this value in eq (1): $x = d + vt$

$$\Rightarrow \boxed{x' = \gamma(x - vt)} \qquad \text{--- } (3)$$

- Similarly, if we consider S' frame to be in rest,

Moving $\quad d' = \frac{1}{\gamma} x \quad$ rest length $\qquad \gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

Here we use eq(2): $\quad x' = d' - vt'$

$$\therefore \boxed{x = \gamma(x' + vt')} \quad \text{— (4)}$$

* Relativity of Simultaneity: Two events may not simultaneous in two frame of refarence.

$$\Rightarrow \quad t \neq t'$$

- Let's find t:

$$x = \gamma(x' + vt') \qquad\qquad x' = \gamma(x - vt)$$

$$\Rightarrow \quad x = \gamma(\gamma x - \gamma vt + vt')$$

$$\Rightarrow \quad \gamma v t' = (1-\gamma^2)x + \gamma^2 vt$$

$$\Rightarrow \quad t' = \gamma\left(t - \left(\frac{\gamma^2 - 1}{\gamma^2 v}\right)x\right)$$

$$\therefore \boxed{t' = \gamma\left(t - \frac{v}{c^2}x\right)} \quad \text{— (5)}$$

$$\frac{\gamma^2 - 1}{\gamma^2} = 1 - \frac{1}{\gamma^2} = 1 - 1 + \frac{v^2}{c^2} = v^2/c^2$$

Similarly, we can find t's expression,

$$\boxed{t = \gamma\left(t' + \frac{v}{c^2}x'\right)} \quad \text{— (6)}$$

Define: $\quad x_0 \equiv ct \qquad x_1 \equiv x \qquad \beta = \frac{v}{c}$

$\qquad\qquad x_0' \equiv ct' \qquad x_1' \equiv x'$

- $x_0' = \gamma(x_0 - \beta x_1) \qquad x_0 = \gamma(x_0' + \beta x_1')$

$\quad x_1' = \gamma(x_1 - \beta x_0) \qquad x_0' = \gamma(x_1' + \beta x_0)$

$$\underset{x'_M}{\begin{bmatrix} x_0' \\ x_1' \end{bmatrix}} = \underset{M}{\begin{bmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{bmatrix}} \underset{x_M}{\begin{bmatrix} x_0 \\ x_1 \end{bmatrix}} \qquad M = \begin{bmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{bmatrix}$$

The Matrix:

$$M = \begin{bmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{bmatrix}$$

- **Inverse:**

$$M^{-1} = \frac{1}{|M|} \mathrm{Adj}(M)$$

$$\gamma^2 = 1-\beta^2$$

- $$|M| = \gamma^2 - (\beta\gamma)^2 = (1-\beta^2)\gamma^2 = \frac{1-v^2/c^2}{1-v^2/c^2} = 1$$

$$\mathrm{adj}(M) = \begin{bmatrix} \gamma & +\beta\gamma \\ +\beta\gamma & \gamma \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{bmatrix}$$