Summer 2025

ENERGY CONVERSION-I
EEE 221
LECTURE-02

Electric Machinery Fundamentals

Stephen J. Chapman

5th Edition

Chapter 2

Transformers

BRAC UNIVERSITY

Inspiring Excellence

Chapter 2 Transformers

- Types and construction of transformers
- The ideal transformer
- **Theory of operation of real single-phase transformers**
- **Equivalent circuit of a transformer**
- Transformer voltage regulation and efficiency
- Transformer taps and voltage regulation
- The autotransformer
- Three-phase transformer
- Instrument transformers

Theory of operation of real single-phase transformers – secondary side open

- Secondary side is open circuit
- Input voltage and current to measure hysteresis curve
- Flux is proportional to v_p and magneto motive force is proportional to i_p
- $i_p(t) = 0$ for ideal transformer

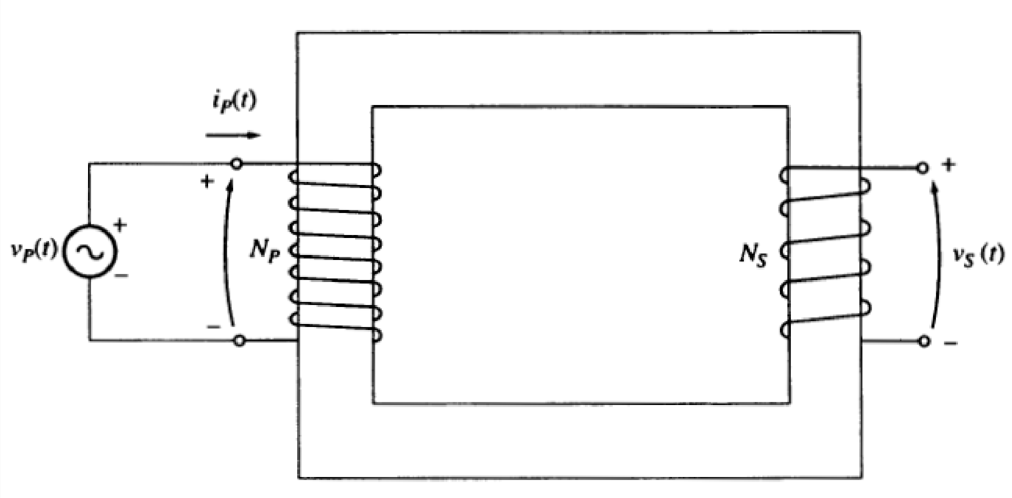


Figure 2-8

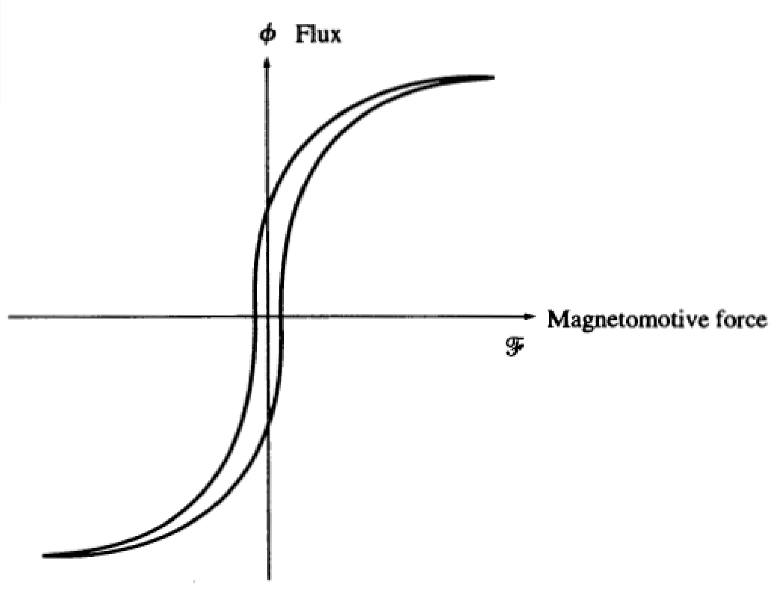


Figure 2-9

Induced voltage and flux linkage λ

1. The induced voltage $$e_{\text{ind}} = \frac{d\lambda}{dt}$$ 1-41
2. The flux linkage $$\lambda = \sum_{i=1}^{N} \phi_i$$ 1-42
3. Simplified by average flux $$\bar{\phi} = \frac{\lambda}{N}$$ 2-16
4. The final induced voltage $$e_{\text{ind}} = N\frac{d\bar{\phi}}{dt}$$ 2-17

Voltage relation between primary and secondary derived from Faraday's law

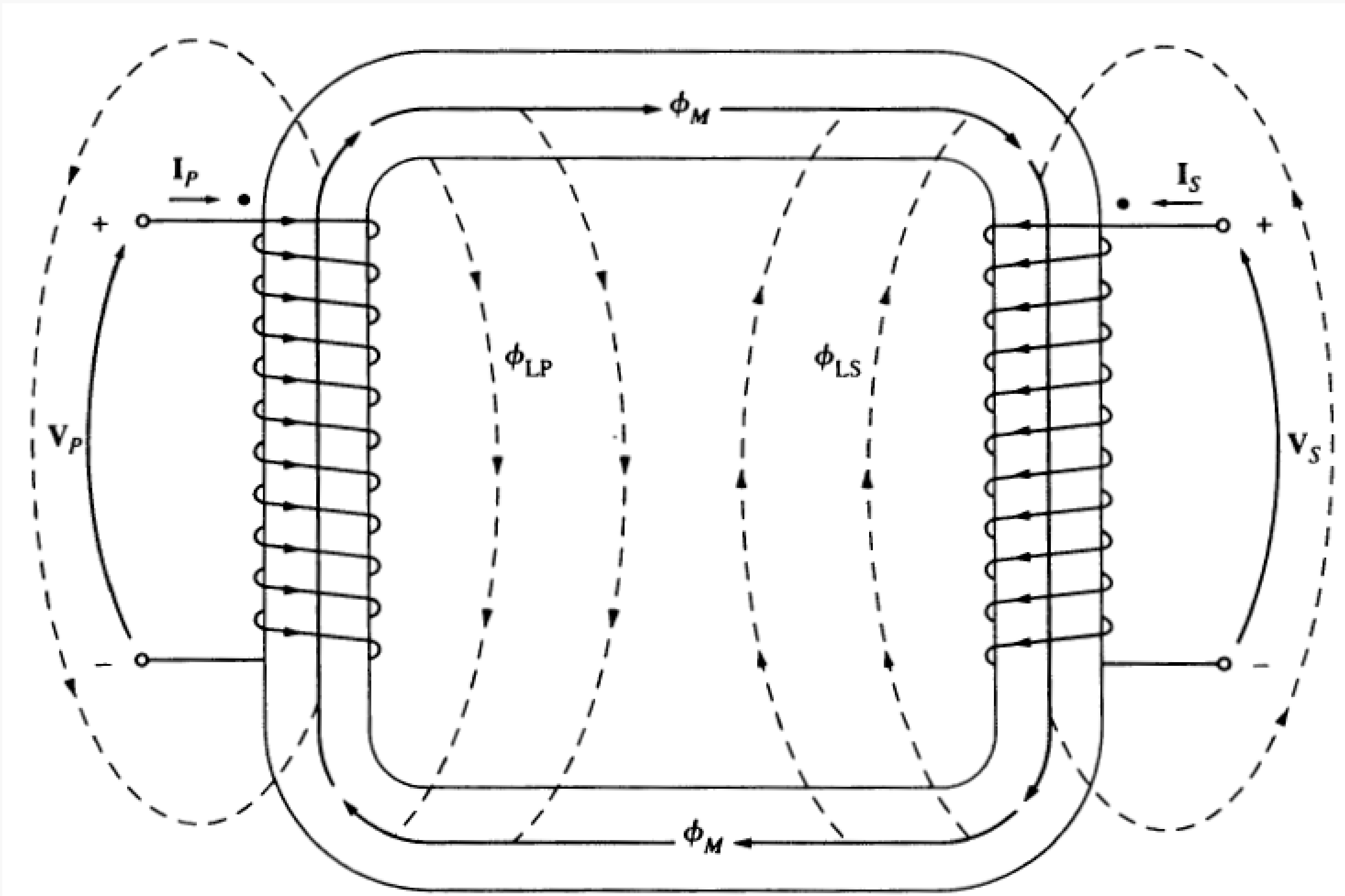


Figure 2-10: Mutual and leakage fluxes in a transformer core

Voltage relation

1. Induced voltage on each side $\bar{\phi} = \frac{1}{N_P} \int v_P(t)\, dt$ 2-18

2. Primary side flux

$$\boxed{\bar{\phi}_P = \phi_M + \phi_{LP}}$$

$\bar{\phi}_P$ = total average primary flux
ϕ_M = flux component linking both primary and secondary coils
ϕ_{LP} = primary leakage flux

2-19

3. Secondary side flux

$$\boxed{\bar{\phi}_S = \phi_M + \phi_{LS}}$$

$\bar{\phi}_S$ = total average secondary flux
ϕ_M = flux component linking both primary and secondary coils
ϕ_{LS} = secondary leakage flux

2-20

Voltage relation

1. Induced voltage on primary side

$$v_P(t) = N_P \frac{d\bar{\phi}_P}{dt}$$

$$= N_P \frac{d\phi_M}{dt} + N_P \frac{d\phi_{LP}}{dt} \quad \text{2-21}$$

$$v_P(t) = e_P(t) + e_{LP}(t) \quad \text{2-22}$$

2. Induced voltage on secondary side

$$v_S(t) = N_S \frac{d\bar{\phi}_S}{dt}$$

$$= N_S \frac{d\phi_M}{dt} + N_S \frac{d\phi_{LS}}{dt} \quad \text{2-23}$$

$$= e_S(t) + e_{LS}(t) \quad \text{2-24}$$

Induced voltage relation - Induced by mutual flux

The primary voltage *due to the mutual flux* is given by

$$e_P(t) = N_P \frac{d\phi_M}{dt} \qquad (2\text{–}25)$$

and the secondary voltage *due to the mutual flux* is given by

$$e_S(t) = N_S \frac{d\phi_M}{dt} \qquad (2\text{–}26)$$

Notice from these two relationships that

$$\frac{e_P(t)}{N_P} = \frac{d\phi_M}{dt} = \frac{e_S(t)}{N_S}$$

Therefore,

$$\boxed{\frac{e_P(t)}{e_S(t)} = \frac{N_P}{N_S} = a} \qquad (2\text{–}27)$$

Terminal voltage relation - Neglecting the leakage flux

This equation means that *the ratio of the primary voltage caused by the mutual flux to the secondary voltage caused by the mutual flux is equal to the turns ratio of the transformer*. Since in a well-designed transformer $\phi_M >> \phi_{LP}$ and $\phi_M >> \phi_{LS}$, the ratio of the total voltage on the primary of a transformer to the total voltage on the secondary of a transformer is approximately

$$\frac{v_P(t)}{v_S(t)} = \frac{N_P}{N_S} = a \tag{2–28}$$

The smaller the leakage fluxes of the transformer are, the closer the total transformer voltage ratio approximates that of the ideal transformer discussed in Section 2.3.

Magnetization current in real transformer

1. The magnetization current i_M is used to generate mutual flux ϕ_M
2. While secondary side is opened, the current measured at primary side contains two parts and is called the excitation current i_{ex}
   1. *Magnetization current i_M: to generate mutual flux*
   2. *Core loss current i_{h+e} : hysteresis and eddy currents*

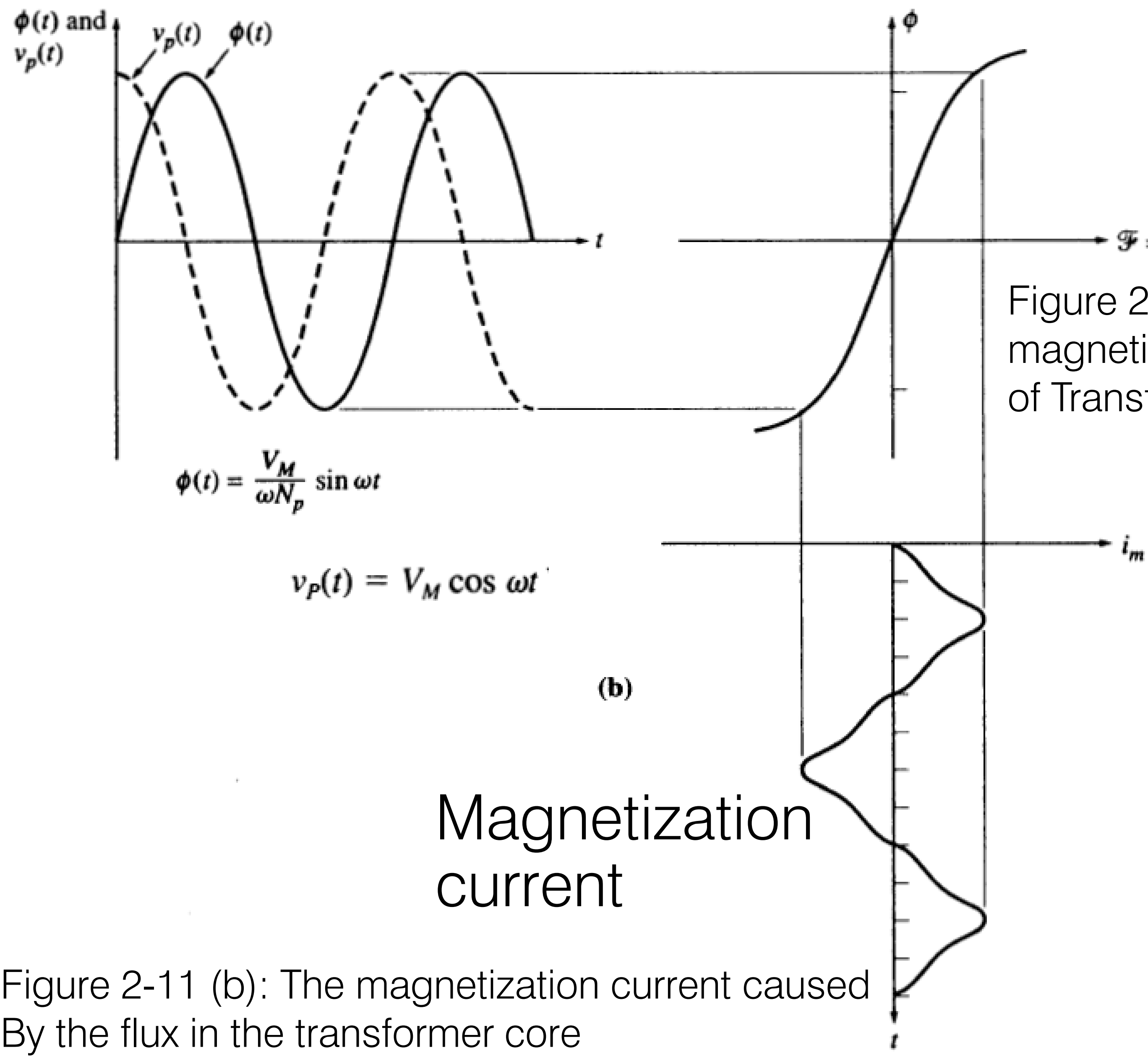


Figure 2-11 (a): The magnetization curve of Transformer core

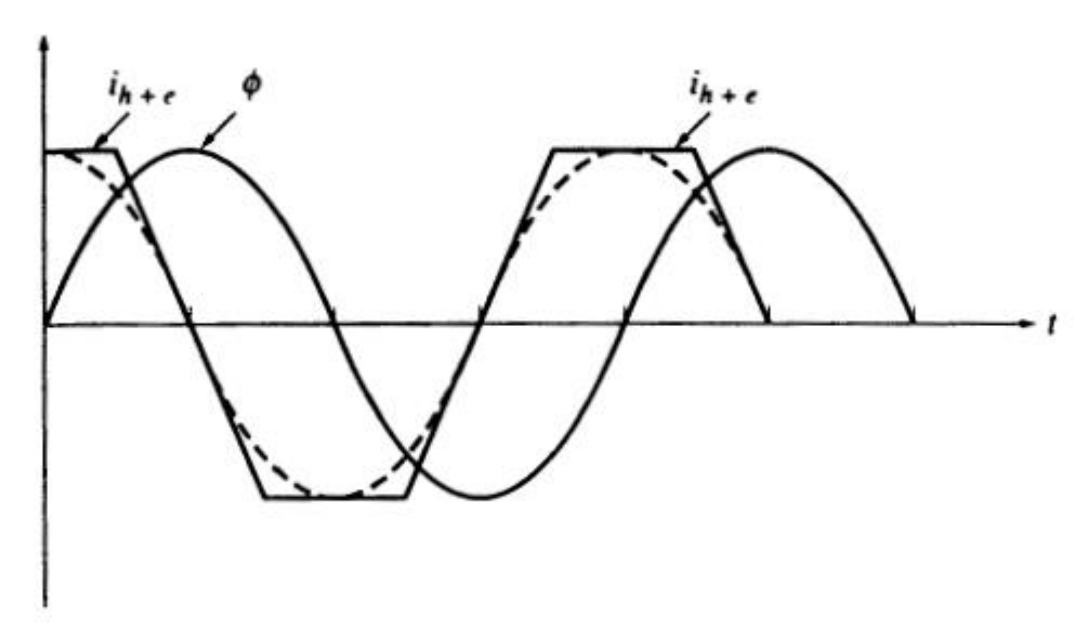
Magnetization current

Figure 2-11 (b): The magnetization current caused By the flux in the transformer core

Magnetization current

1. The magnetization current in the transformer is not sinusoidal. The higher-frequency components in the magnetization current are due to magnetic saturation in the transformer core.
2. Once the peak flux reaches the saturation point in the core, a small increase in peak flux requires a very large increase in the peak magnetization current.
3. The fundamental component of the magnetization current lags the voltage applied to the core by 90°.
4. The higher-frequency components in the magnetization current can be quite large compared to the fundamental component. In general, the further a transformer core is driven into saturation, the larger the harmonic components will become.

Core loss current



1. The core-loss current is nonlinear because of the nonlinear effects of hysteresis.
2. The fundamental component of the core-loss current is in phase with the voltage applied to the core.

Figure 2-12: The core loss current in a transformer

Excitation current i_{ex}

$$i_{ex} = i_m + i_{h+e}$$

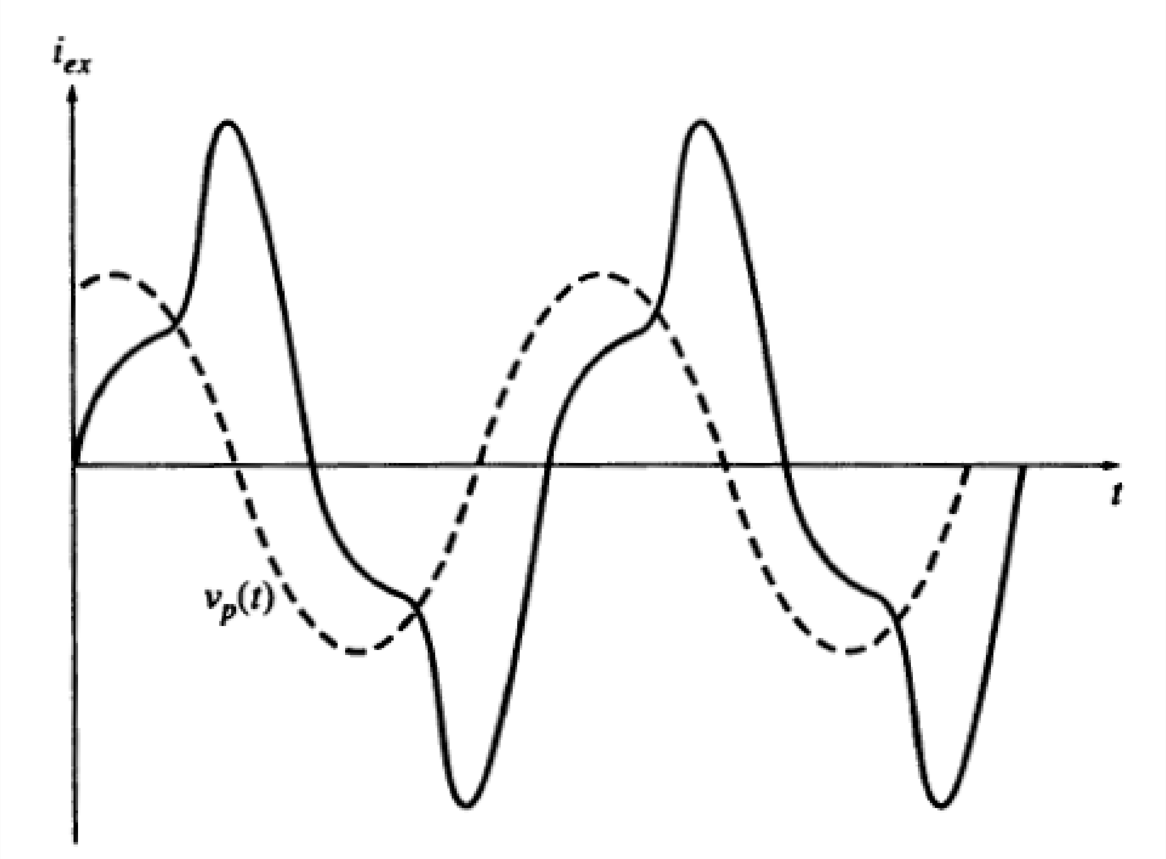


Figure 2-13 : The total excitation current in a Transformer

Current ratio on a transformer

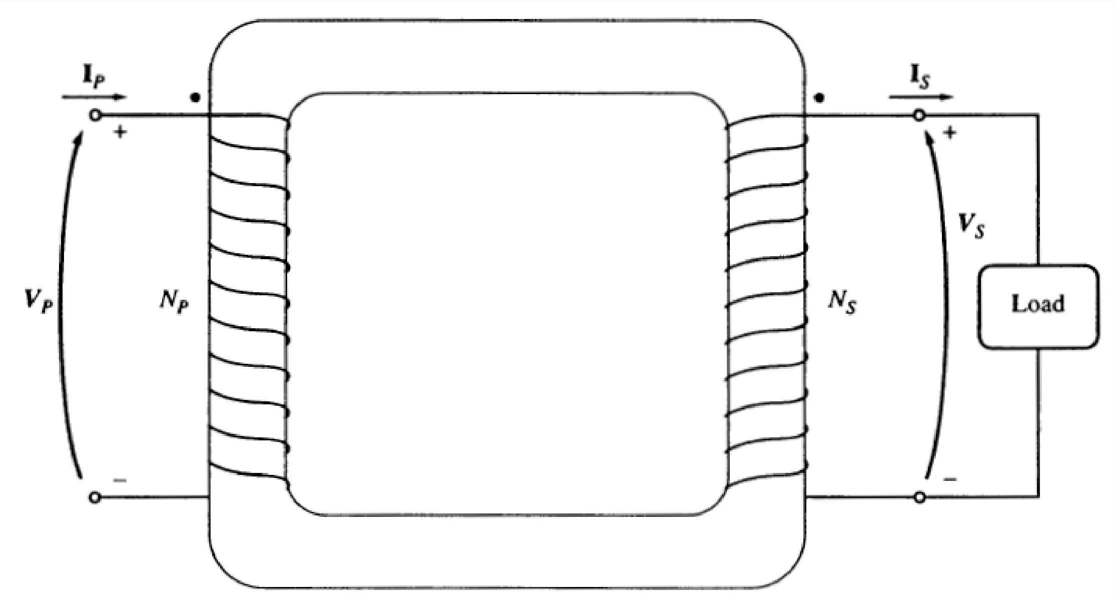


Figure 2-14 : A real Transformer with a load connected to Its secondary

Polarity of the magnetomotive force

1. Current flows into "Dot" will produce a positive magnetomotive force
2. Current flows out "Dot" will produce a negative magnetomotive force

The net magnetomotive force and magnetic circuit

In the situation shown in Figure 2–14, the primary current produces a positive magnetomotive force $\mathcal{F}_P = N_P i_P$, and the secondary current produces a negative magnetomotive force $\mathcal{F}_S = -N_S i_S$. Therefore, the net magnetomotive force on the core must be

$$\mathcal{F}_{\text{net}} = N_P i_P - N_S i_S \tag{2–31}$$

This net magnetomotive force must produce the net flux in the core, so the net magnetomotive force must be equal to

$$\boxed{\mathcal{F}_{\text{net}} = N_P i_P - N_S i_S = \phi \mathcal{R}} \tag{2–32}$$

Ideal transformer

$$\mathcal{F}_{net} = N_P i_P - N_S i_S \approx 0$$

$$\boxed{N_P i_P \approx N_S i_S}$$

$$\boxed{\frac{i_P}{i_S} \approx \frac{N_S}{N_P} = \frac{1}{a}}$$

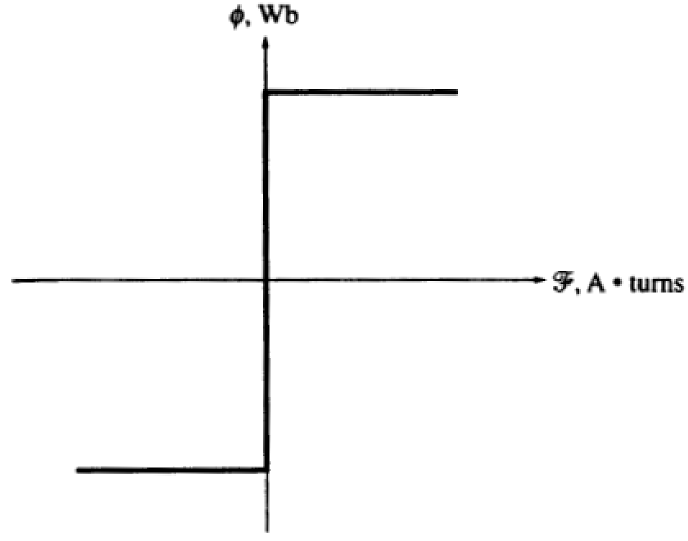


Figure 2-15 : The magnetization curve of an ideal transformer

The assumptions from real to ideal transformer

1. The core must have no hysteresis or eddy currents.
2. The magnetization curve must have the shape shown in Figure 2–15. Notice that for an unsaturated core the net magnetomotive force $\mathcal{F}_{net} = 0$, implying that $N_P i_P = N_S i_S$.
3. The leakage flux in the core must be zero, implying that all the flux in the core couples both windings.
4. The resistance of the transformer windings must be zero.

The equivalent circuit of a transformer – to model the non-ideal characteristics

1. *Copper (I^2R) losses*. Copper losses are the resistive heating losses *in the primary and secondary windings* of the transformer. They are proportional to the square of the current in the windings.
2. *Eddy current losses*. Eddy current losses are resistive heating losses *in the core* of the transformer. They are proportional to the square of the voltage applied to the transformer.
3. *Hysteresis losses*. Hysteresis losses are associated with the rearrangement of the magnetic domains in the core during each half-cycle, as explained in Chapter 1. They are a complex, nonlinear function of the voltage applied to the transformer.
4. *Leakage flux*. The fluxes ϕ_{LP} and ϕ_{LS} which escape the core and pass through only one of the transformer windings are leakage fluxes. These escaped fluxes produce a *self-inductance* in the primary and secondary coils, and the effects of this inductance must be accounted for.

Modeling the leakage flux by leakage inductance

As explained in Section 2.4, the leakage flux in the primary windings ϕ_{LP} produces a voltage ϕ_{LP} given by

$$e_{LP}(t) = N_P \frac{d\phi_{LP}}{dt} \tag{2–36a}$$

and the leakage flux in the secondary windings ϕ_{LS} produces a voltage e_{LS} given by

$$e_{LS}(t) = N_S \frac{d\phi_{LS}}{dt} \tag{2–36b}$$

$$\phi_{LP} = (\mathcal{P} N_P) i_P$$

$$\phi_{LS} = (\mathcal{P} N_S) i_S$$

where $\mathcal{P}$ = permeance of flux path

N_P = number of turns on primary coil

N_S = number of turns on secondary coil

Modeling the leakage flux by leakage inductance

Substitute Equations (2–37) into Equations (2–36). The result is

$$e_{LP}(t) = N_P \frac{d}{dt}(\mathcal{P} N_P) i_P = N_P^2 \mathcal{P} \frac{di_P}{dt}$$

$$e_{LS}(t) = N_S \frac{d}{dt}(\mathcal{P} N_S) i_S = N_S^2 \mathcal{P} \frac{di_S}{dt}$$

The constants in these equations can be lumped together. Then

$$\boxed{e_{LP}(t) = L_P \frac{di_P}{dt}}$$

$$\boxed{e_{LS}(t) = L_S \frac{di_S}{dt}}$$

Modeling excitation current and copper loss

- The hysteresis and eddy currents is in-phase with input voltage (modeled as a shunt resistor R_c)
- The magnetization current is lagging input voltage by 90 degrees (modeled as a shunt inductor X_m)
- The copper loss can be modeled as the series resistors R_p and R_s

The resulting equivalent circuit

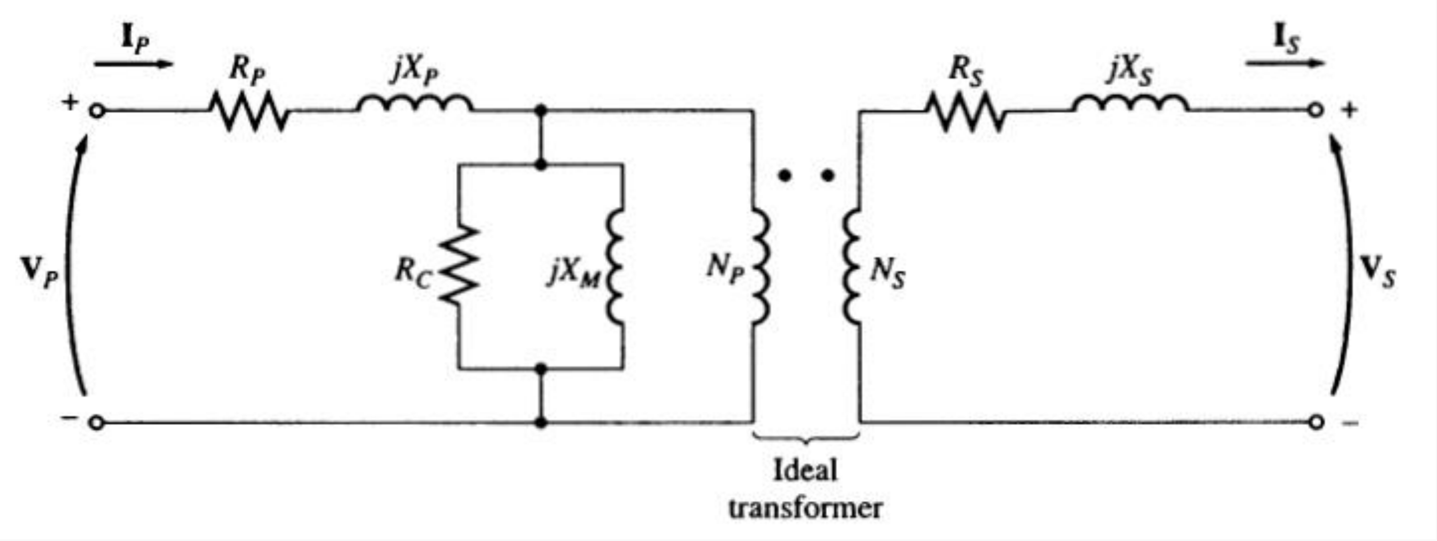


Figure 2-16 : The model of a real transformer

Exact Equivalent Circuit

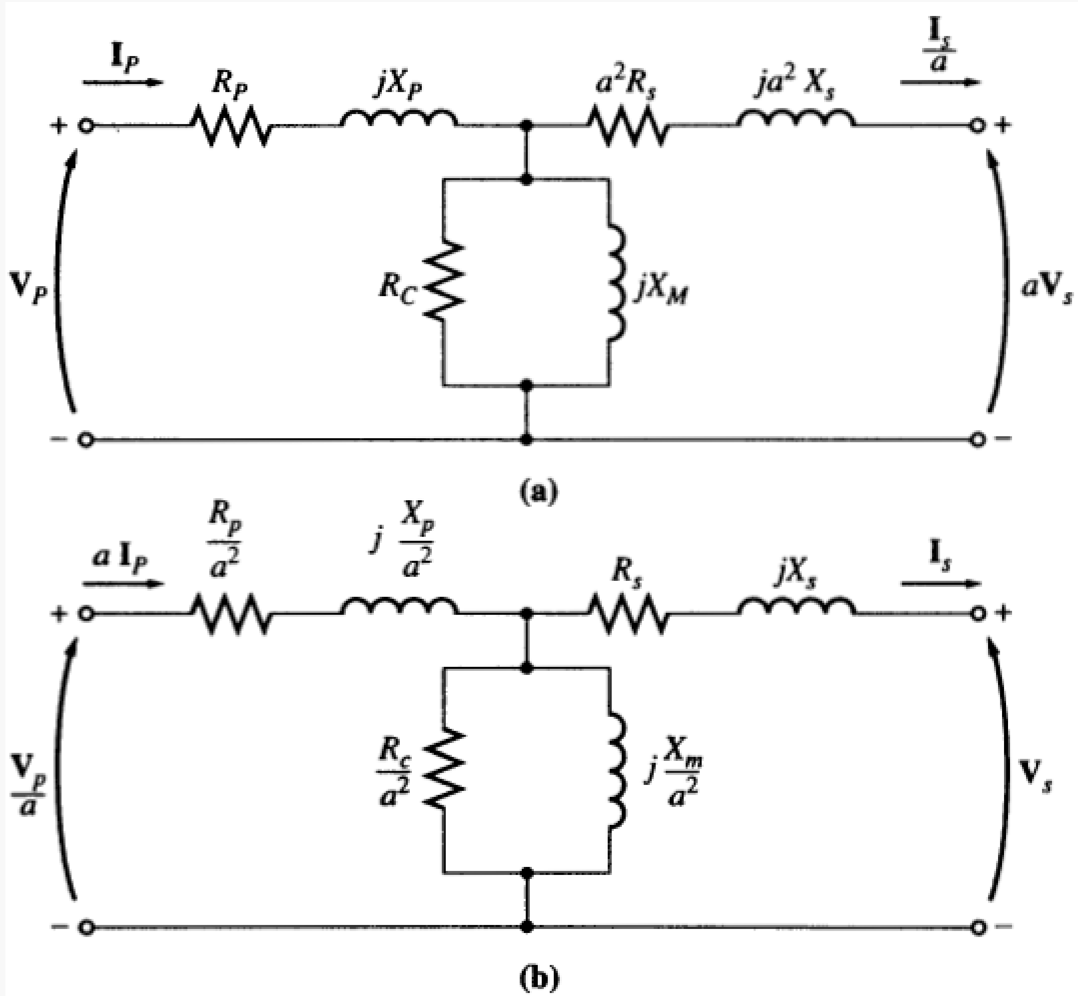


FIGURE 2- 17
(a) The transformer model referred to its primary voltage level. (b) The transformer model referred to it's secondary voltage level

Approximate equivalent circuit

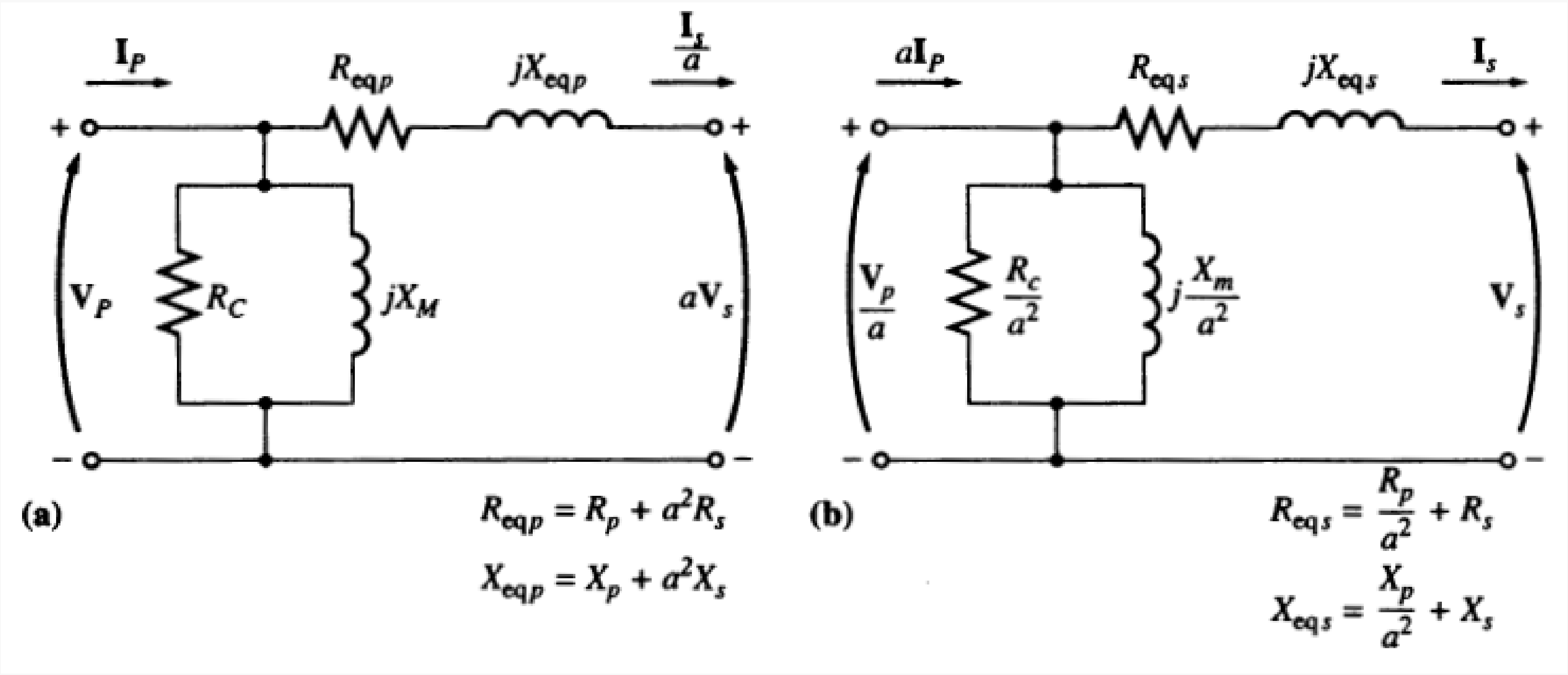


FIGURE 2-18
Approximate transformer models. (a) Referred to the primary side;
(b) referred to the secondary side;

Neglecting excitation current

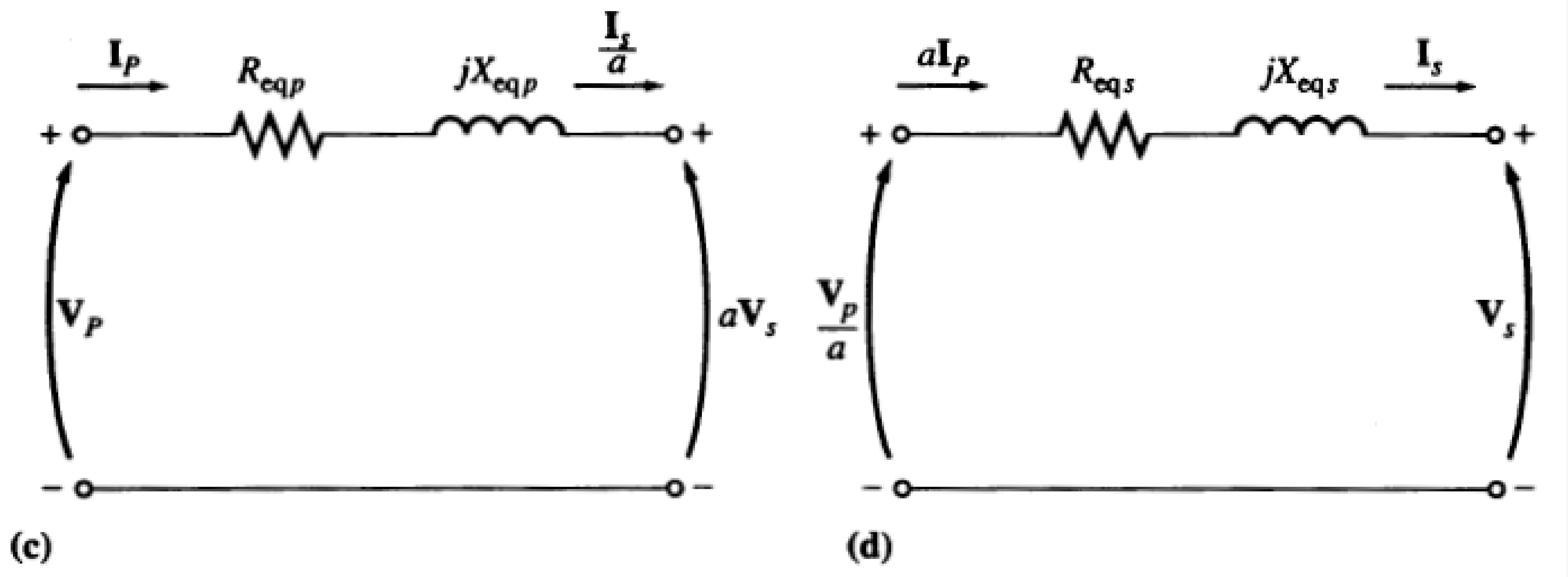


FIGURE 2-18
Approximate transformer models. (c) with no excitation branch, referred to the primary side; (d} with no excitation branch, referred to the secondary side.

Thank You