Summer 2025

ENERGY CONVERSION-I
EEE 221
LECTURE-03

Transformers

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Chapter 2 Transformers

- Types and construction of transformers
- The ideal transformer
- Theory of operation of real single-phase transformers
- Equivalent circuit of a transformer
- Transformer voltage regulation and efficiency
- Transformer taps and voltage regulation
- The autotransformer
- Three-phase transformer
- Instrument transformers

Measure the equivalent circuit parameters

- There are two types of measurements used for determination the equivalent circuit parameters
- *Open circuit test* – used to measure excitation branch
- *Short circuit test* – used to measure series branch

Open circuit test

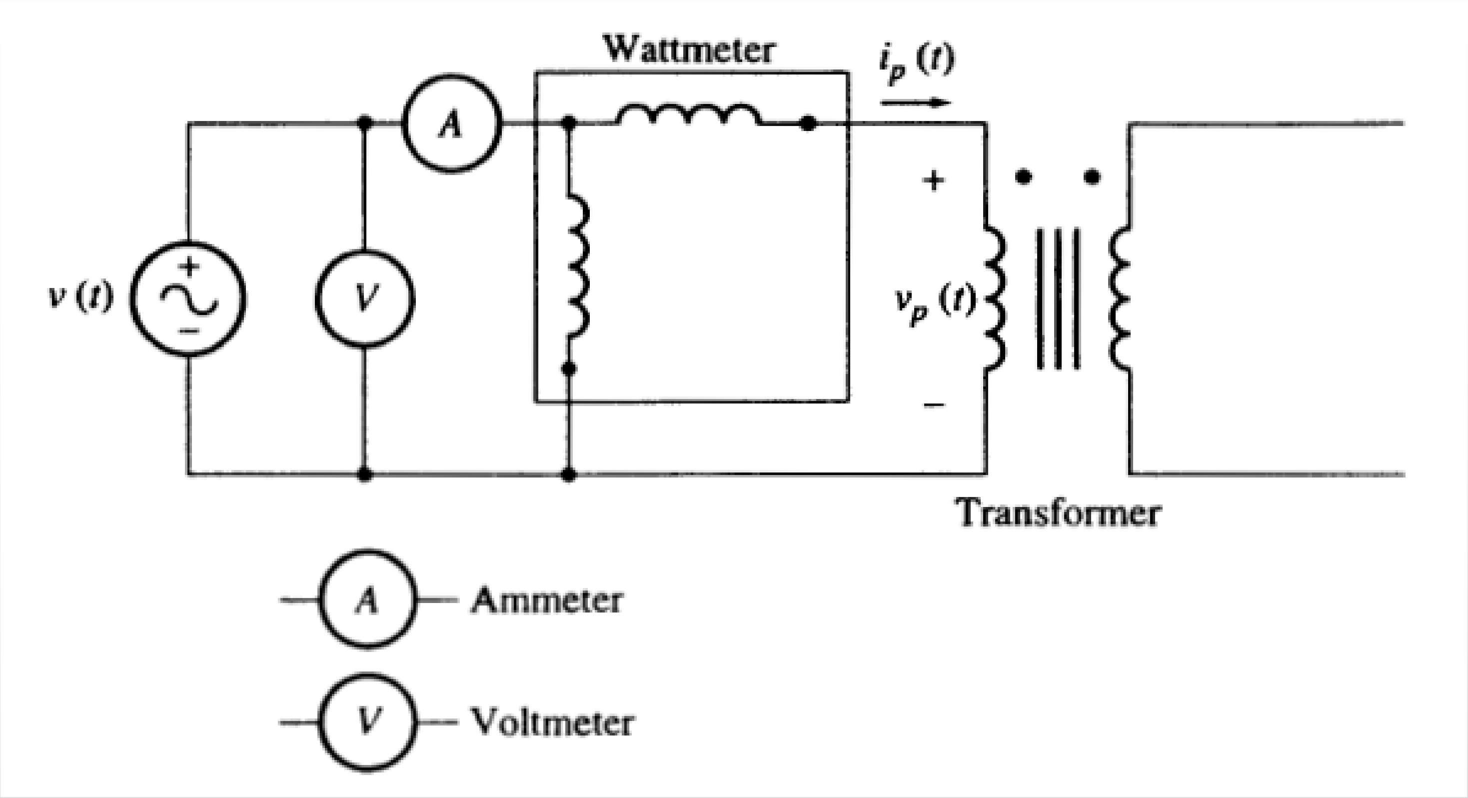

Figure 2-19

Open circuit test / Page-90

- Under the open circuit condition, all the input current flows through the excitation branch (Since the current is small, the test usually performs at high voltage side)

$$G_C = \frac{1}{R_C} \qquad (2\text{-}40)$$

$$B_M = \frac{1}{X_M} \qquad (2\text{-}41)$$

$$Y_E = G_C - jB_M = \frac{1}{R_C} - j\frac{1}{X_M} \qquad (2\text{-}43)$$

$$|Y_E| = \frac{I_{OC}}{V_{OC}} \qquad (2\text{-}44)$$

$$\text{PF} = \cos\theta = \frac{P_{OC}}{V_{OC}I_{OC}} \qquad (2\text{-}45)$$

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} \qquad (2\text{-}46)$$

$$Y_E = \frac{I_{OC}}{V_{OC}}\angle -\theta = \frac{I_{OC}}{V_{OC}}\angle -\cos^{-1}\text{PF} \qquad (2\text{-}47)$$

Short circuit test

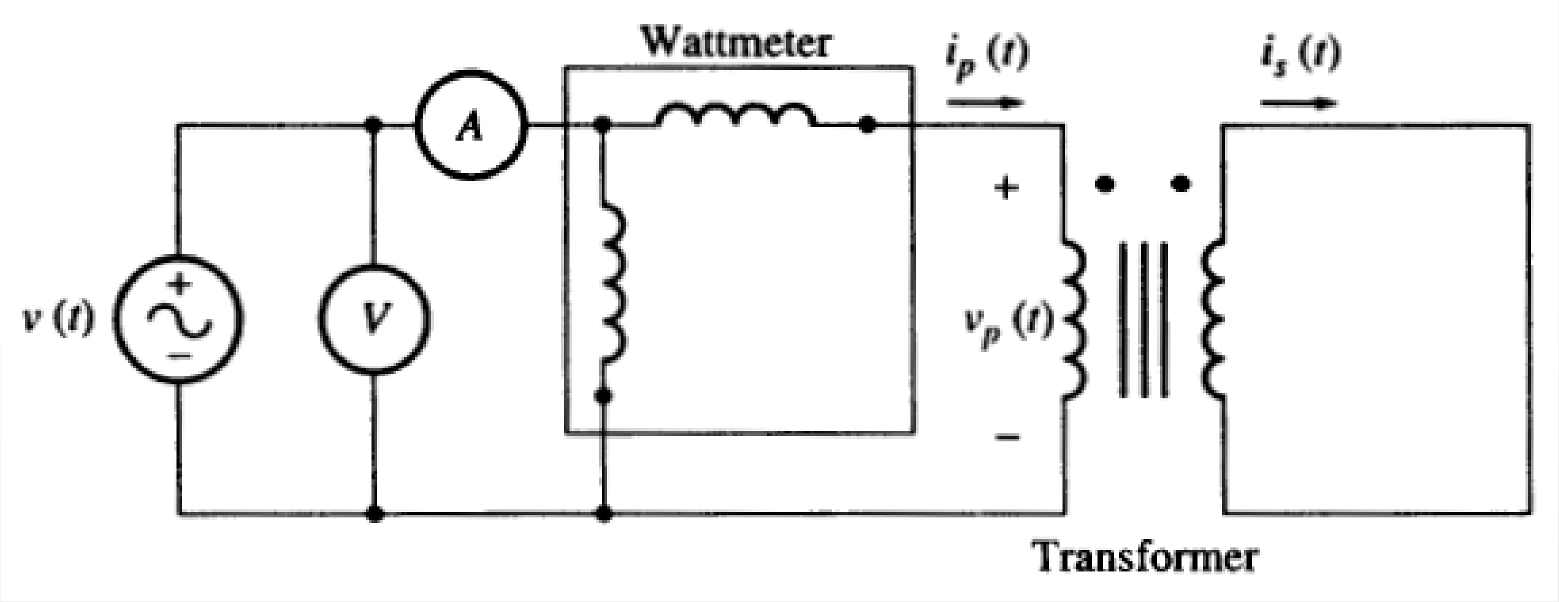

Figure 2-20

Short circuit test

- At secondary side short circuit condition, the input voltage must be a very low value to prevent input large short circuit current (usually performs at low voltage side)

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}} \qquad (2\text{-}48)$$

$$Z_{SE} = \frac{V_{SC}\angle 0^\circ}{I_{SC}\angle -\theta^\circ} = \frac{V_{SC}}{I_{SC}}\angle \theta^\circ \qquad (2\text{-}51)$$

$$PF = \cos\theta = \frac{P_{SC}}{V_{SC}I_{SC}} \qquad (2\text{-}49)$$

$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} \qquad (2\text{-}50)$$

$$\begin{aligned} Z_{SE} &= R_{eq} + jX_{eq} \\ &= (R_P + a^2R_S) + j(X_P + a^2X_S) \end{aligned} \qquad (2\text{-}52)$$

Open and Short circuit Test Summary

- Open Circuit Test (**Core loss**)

  H.V side open

  Measured in L.V side

  Components **Rc** and **Xm** (Ref. to low volt.)

- Short Circuit Test (**Cu loss**)

  L.V side Short Ckt. (Rated current flow)

  Measured in H.V side (for low current)

  Series Elements **Req** & **Xeq** (Ref. to High volt)

- In the math for final equivalent circuit, all the elements must be referred to the same side.

Example 2–2. The equivalent circuit impedances of a 20-kVA, 8000/240 V, 60-Hz transformer are to be determined. The open-circuit test was performed on the secondary side of the transformer (to reduce the maximum voltage to be measured) and the short-circuit test were performed on the primary side of the transformer (to reduce the maximum current to be measured). The following data were taken:

Open-circuit test (on secondary)	Short-circuit test (on primary)
$V_{OC} = 240$ V	$V_{SC} = 489$ V
$I_{OC} = 7.133$ A	$I_{SC} = 2.5$ A
Poc = 400 W	$P_{SC} = 240$ W

Find the impedances of the approximate equivalent circuit referred to the primary side, and sketch that circuit.

Solution

The turns ratio of this transformer is $a = 8000/240 = 33.3333$. The power factor during the *open-circuit* test is

$$PF = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}} \qquad (2\text{–}45)$$

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Solution:

$$PF = \cos\theta = \frac{P_{oc}}{V_{oc}\, I_{oc}}$$

$$\cos\theta = \frac{400\,W}{(240\,V)(7.133\,A)}.$$

$$\cos\theta = 0.234 \text{ Lagging}.$$

$$\therefore \theta = \cos^{-1}(0.234)$$

$$= 76.5^\circ$$

The excitation admittance is given by:

$$Y_E = \frac{I_{oc}}{V_{oc}} \angle -\theta$$

$$= \frac{I_{oc}}{V_{oc}} \angle -76.5^\circ$$

$$= \frac{7.133\,A}{240\,V} \angle -76.5^\circ$$

$$= 0.0297 \angle -76.5^\circ \ S$$

$$\therefore Y_E = 0.00693 - j\,0.02888$$

$$= \frac{1}{R_C} - j\frac{1}{X_M}$$

Therefore, the values of the excitation branch *referred to the low-voltage (secondary) side* are

$$R_C = \frac{1}{0.00693} = 144\ \Omega$$

$$X_M = \frac{1}{0.02888} = 34.63\ \Omega$$

The power factor during the *short-circuit* test is

$$\text{PF} = \cos\theta = \frac{P_{SC}}{V_{SC}I_{SC}} \qquad (2\text{–}49)$$

$$\text{PF} = \cos\theta = \frac{240\ \text{W}}{(489\ \text{V})(2.5\ \text{A})} = 0.196\ \text{lagging}$$

$$\therefore\ \theta = \cos^{-1}(0.196)$$

$$= 78.7^\circ$$

$\therefore$ The series impedance is given by;

$$Z_{SE} = \frac{V_{sc}}{I_{sc}} \angle\theta$$

$$= \frac{489\ \text{V}}{2.5\ \text{A}} \angle 78.7^\circ$$

$$= 195.6\angle 78.7^\circ = 38.4 + j192\ \Omega$$

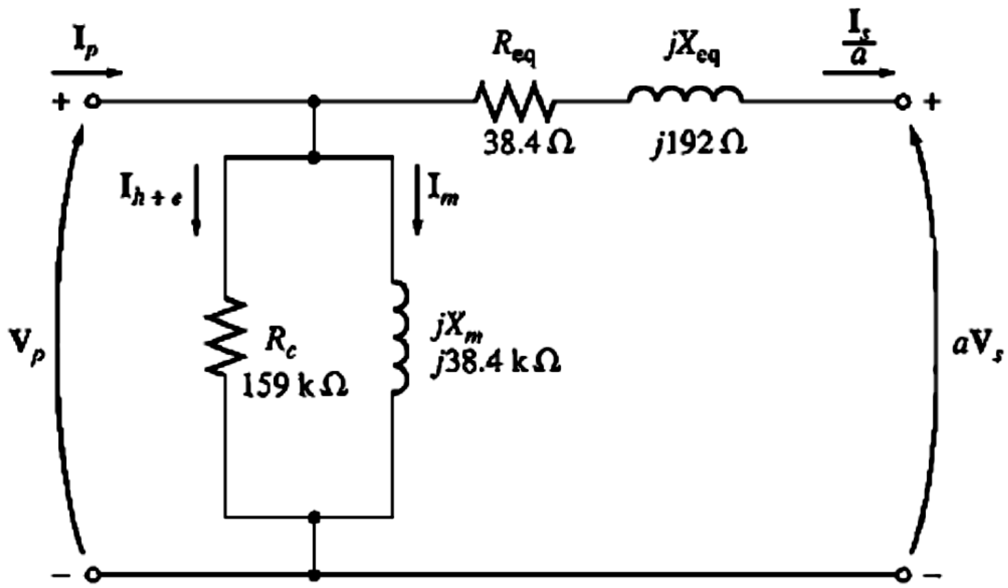

FIGURE 2–21
The equivalent circuit of Example 2–2.

Therefore, the equivalent resistance and reactance *referred to the high-voltage (primary) side* are

$$R_{eq} = 38.4\ \Omega \qquad X_{eq} = 192\ \Omega$$

The resulting simplified equivalent circuit referred to the high-voltage (primary) side can be found by converting the excitation branch values to the high-voltage side.

$$R_{C,P} = a^2 R_{C,S} = (33.333)^2\,(144\ \Omega) = 159\ \text{k}\Omega$$

$$X_{M,P} = a^2\, X_{M,S} = (33.333)^2\,(34.63\ \Omega) = 38.4\ \text{k}\Omega$$

The resulting equivalent circuit is shown in Figure 2–21.

Transformer voltage regulation and efficiency

Because a real transformer has series impedances within it, the output voltage of a transformer varies with the load even if the input voltage remains constant. To conveniently compare transformers in this respect, it is customary to define a quantity called *voltage regulation* (VR). *Full-load voltage regulation* is a quantity that compares the output voltage of the transformer at no load with the output voltage at full load. It is defined by the equation

$$\boxed{\mathrm{VR} = \frac{V_{S,\mathrm{nl}} - V_{S,\mathrm{fl}}}{V_{S,\mathrm{fl}}} \times 100\%} \tag{2–61}$$

Since at no load, $V_S = V_P/a$, the voltage regulation can also be expressed as

$$\boxed{\mathrm{VR} = \frac{V_P/a - V_{S,\mathrm{fl}}}{V_{S,\mathrm{fl}}} \times 100\%} \tag{2–62}$$