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ENERGY CONVERSION-I
EEE 221
LECTURE-01

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Text Book: Electric Machinery Fundamentals

Stephen J. Chapman

5th Edition

Chapter 2

Transformers

Chapter 2 Transformers

- Types and construction of transformers
- The ideal transformer
- Theory of operation of real single-phase transformers
- Equivalent circuit of a transformer
- Transformer voltage regulation and efficiency
- Transformer taps and voltage regulation
- The autotransformer
- Three-phase transformer
- Instrument transformers

Transformers

A transformer is a device that converts one AC voltage to another AC voltage at the same frequency. It consists of one or more coil(s) of wire wrapped around a common ferromagnetic core. These coils are usually not connected electrically together. However, they are connected through the common magnetic flux confined to the core.

- The transformer winding connected to the power source is called the ***primary winding or input winding***
- The winding connected to the loads is called the ***secondary winding or output winding***
- ….***tertiary winding***

Why transformers are important to modern life

- The transformer ideally changes one ac voltage level to another voltage level without affecting the actual power supplied.
- The transformer can be used in distribution system for efficiency issues.
  - *The step-up transformer decreases the line current and decreases the power loss on power line.*
  - *The transmission/distribution system with transformer can keep high efficiency*

Types and construction of transformers

- **Core-form**: consists a simple rectangular laminated piece of steel with the transformer winding wrapped around two sides of the rectangle

- **Shell-form**: consists three legs laminated core with winding wrapped around the center leg

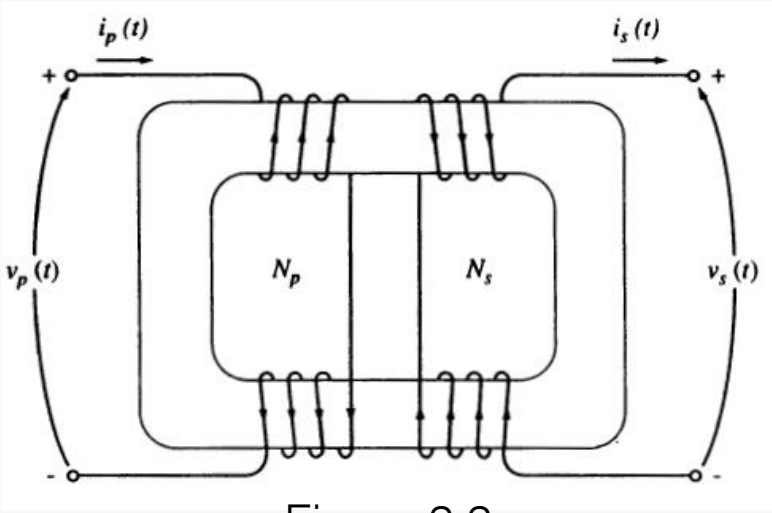


Figure 2.2

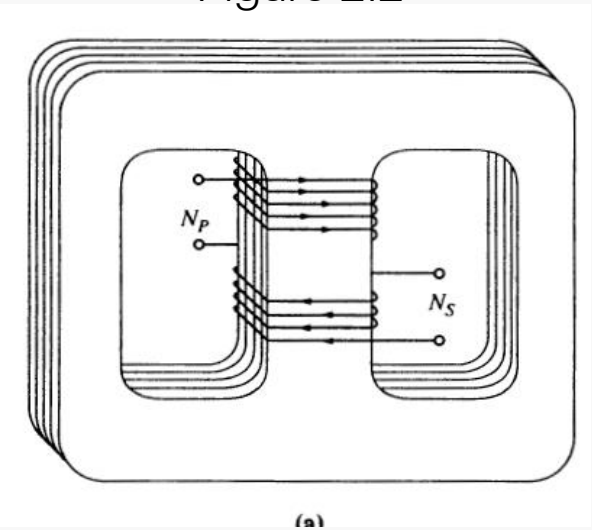


Figure 2.3

The ideal transformer characteristics

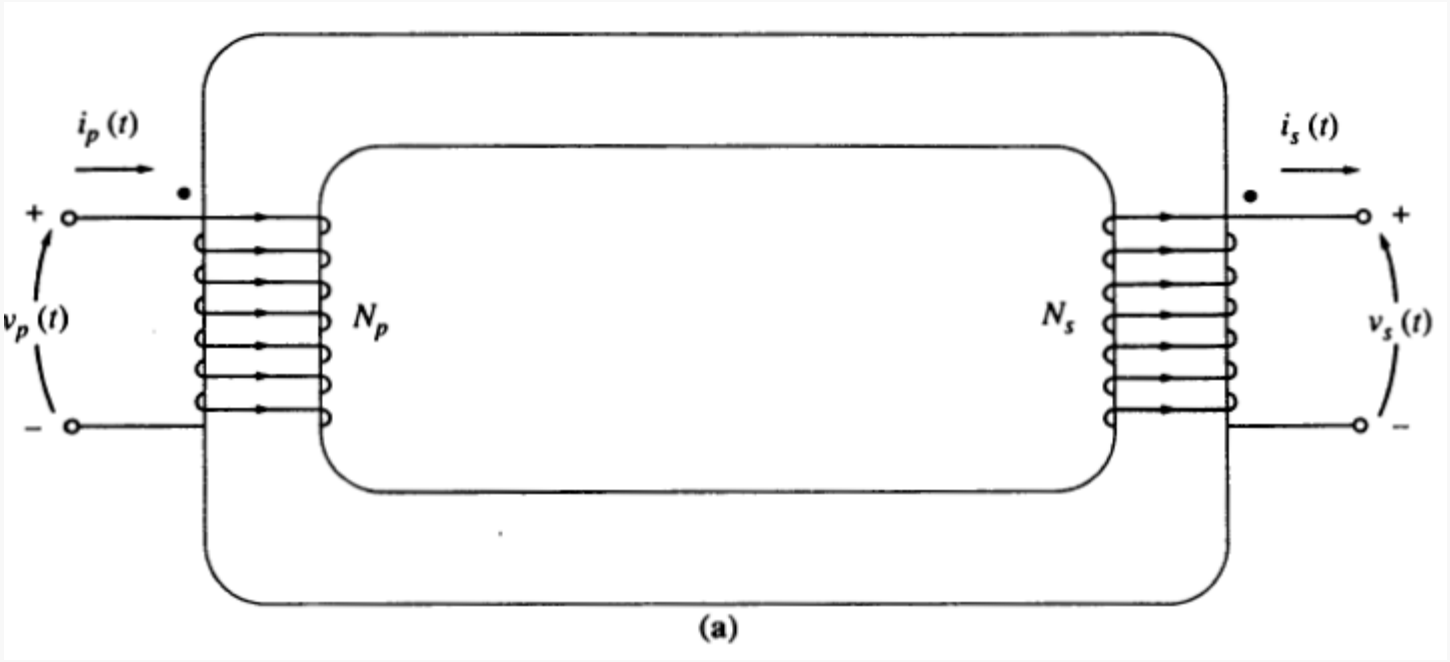


Figure 2.4 (a)

The ideal transformer characteristics

$$\frac{v_P(t)}{v_S(t)} = \frac{N_P}{N_S} = a \qquad (2.1)$$

- Where $a = N_p/N_s$ is the turns ratio (2.2)
- Energy balance relation

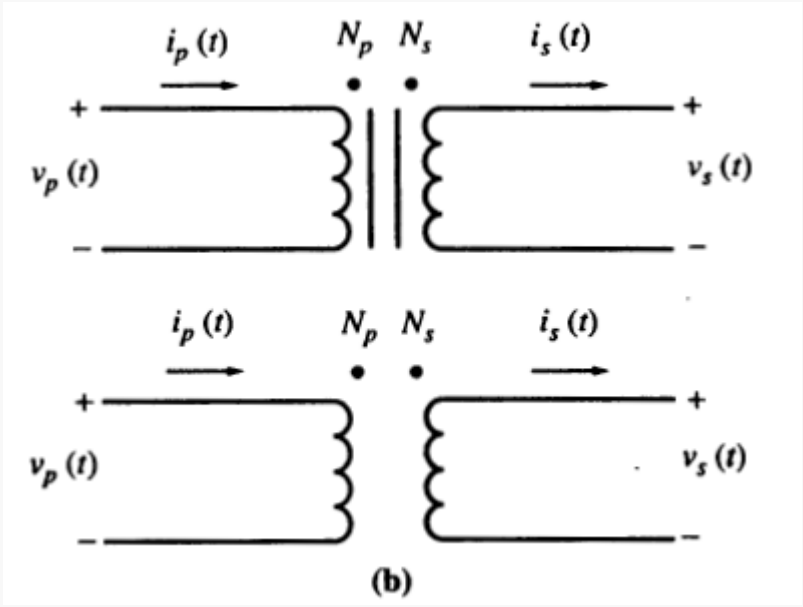
$$N_P i_P(t) = N_S i_S(t) \qquad \frac{i_P(t)}{i_S(t)} = \frac{1}{a}$$

2.3 (a) 2.3 (b)

- Phasor relation

$$\frac{\mathbf{V}_P}{\mathbf{V}_S} = a \quad (2.4) \qquad \frac{\mathbf{I}_P}{\mathbf{I}_S} = \frac{1}{a} \quad (2.5)$$

Figure 2.4 (b)



- The turns ratio ***a*** only effects the magnitude not the angle

Dot convention in ideal transformer

1. If the primary *voltage* is positive at the dotted end of the winding with respect to the undotted end, then the secondary voltage will be positive at the dotted end also. Voltage polarities are the same with respect to the dots on each side of the core.
2. If the primary *current* of the transformer flows *into* the dotted end of the primary winding, the secondary current will flow *out* of the dotted end of the secondary winding.

Power in an ideal transformer

The power supplied to the transformer by the primary circuit is given by the equation

$$P_{in} = V_P I_P \cos \theta_P \tag{2–6}$$

where θ_P is the angle between the primary voltage and the primary current. The power supplied by the transformer secondary circuit to its loads is given by the equation

$$P_{out} = V_S I_S \cos \theta_S \tag{2–7}$$

where θ_S is the angle between the secondary voltage and the secondary current. Since voltage and current angles are unaffected by an ideal transformer, $\theta_P - \theta_S = \theta$. The primary and secondary windings of an ideal transformer have the *same power factor.*

Power in an ideal transformer

$$P_{out} = V_S I_S \cos\theta \tag{2–8}$$

Applying the turns-ratio equations gives $V_S = V_P/a$ and $I_S = aI_P$, so

$$P_{out} = \frac{V_P}{a}(aI_P)\cos\theta$$

$$\boxed{P_{out} = V_P I_P \cos\theta = P_{in}} \tag{2–9}$$

Thus, *the output power of an ideal transformer is equal to its input power.*

The same relationship applies to reactive power Q and apparent power S:

$$\boxed{Q_{in} = V_P I_P \sin\theta = V_S I_S \sin\theta = Q_{out}} \tag{2–10}$$

and

$$\boxed{S_{in} = V_P I_P = V_S I_S = S_{out}} \tag{2–11}$$

Impedance transformation through a transformer

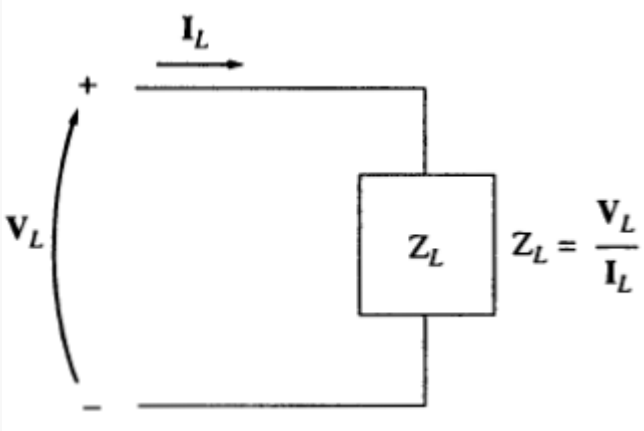
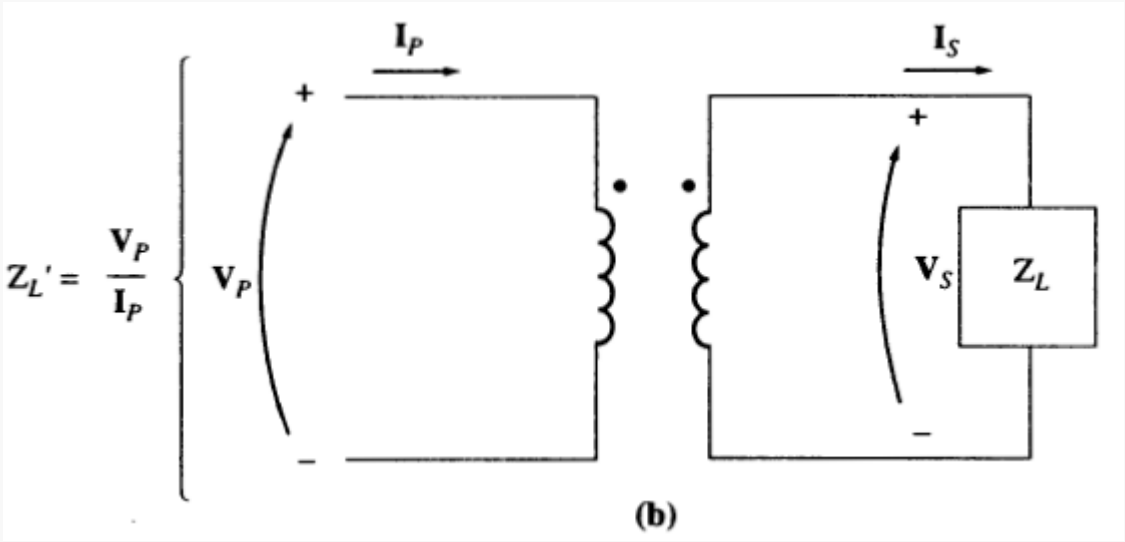


Figure 2.5

$$V_P = aV_S \qquad I_P = \frac{I_S}{a}$$

$$Z'_L = \frac{V_P}{I_P} = \frac{aV_S}{I_S/a} = a^2 \frac{V_S}{I_S}$$

$$\boxed{Z'_L = a^2 Z_L}$$



Analysis of circuits containing ideal transformers

- All the current and voltage are all referred to one side (primary side)
- Note the dot convention for current direction
- Impedance transformation

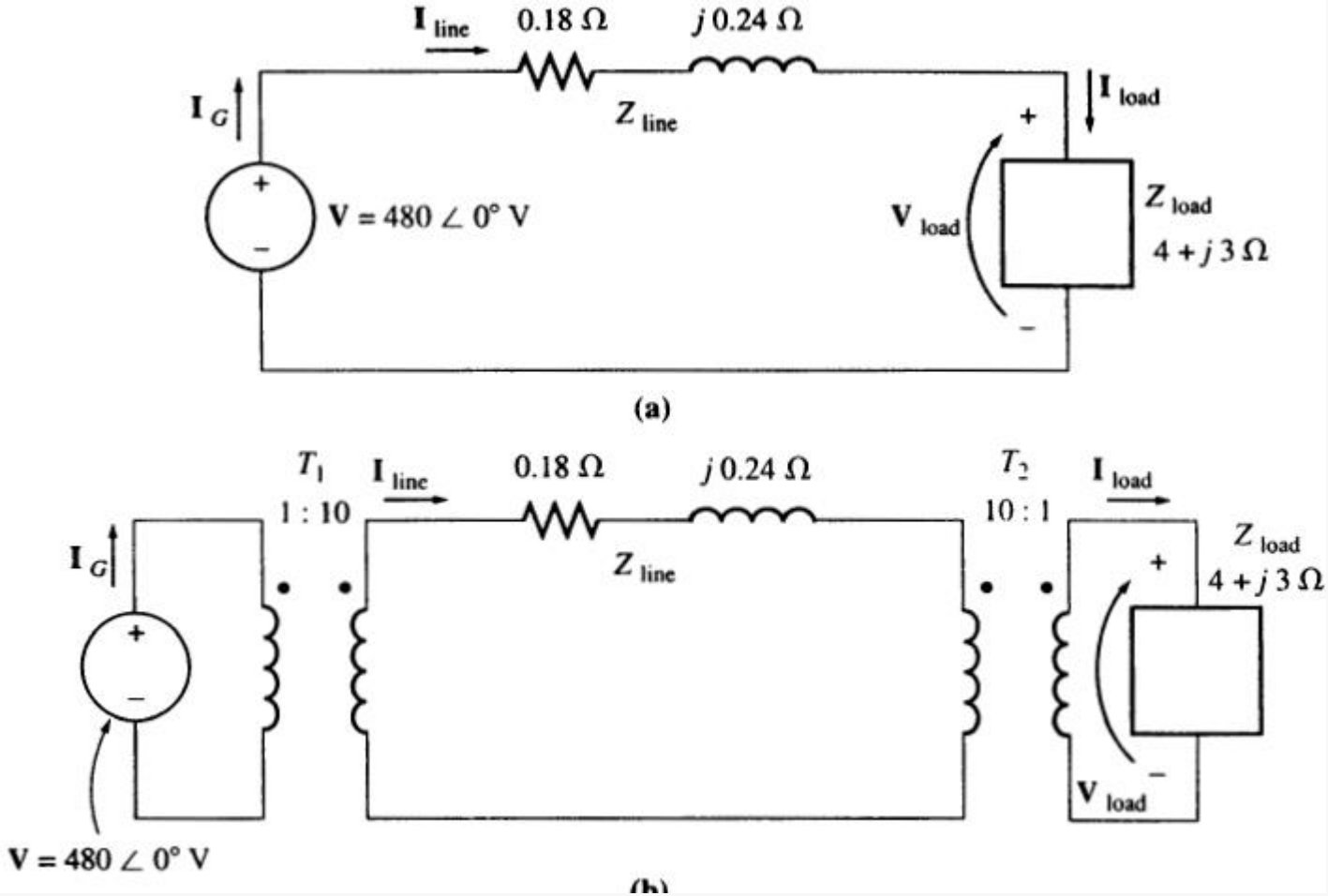
Example 2-1

Example 2–1. A single-phase power system consists of a 480-V 60-Hz generator supplying a load $Z_{load} = 4 + j3\ \Omega$ through a transmission line of impedance $Z_{line} = 0.18 + j0.24\ \Omega$. Answer the following questions about this system.

(a) If the power system is exactly as described above (Figure 2–6a), what will the voltage at the load be? What will the transmission line losses be?

(b) Suppose a 1:10 step-up transformer is placed at the generator end of the transmission line and a 10:1 step-down transformer is placed at the load end of the line (Figure 2–6b). What will the load voltage be now? What will the transmission line losses be now?

Example 2-1


I line
0.18 Ω
j 0.24 Ω
I G
Z line
I load
+
V = 480 ∠ 0° V
V load
Z load
4 + j 3 Ω
–
(a)
T1
1 : 10
I line
0.18 Ω
j 0.24 Ω
T2
10 : 1
I load
I G
Z line
Z load
4 + j 3 Ω
V load
V = 480 ∠ 0° V
(b)

Solution

(*a*) Figure 2–6a shows the power system without transformers. Here $\mathbf{I}_G = \mathbf{I}_{line} = \mathbf{I}_{load}$. The line current in this system is given by

$$\mathbf{I}_{line} = \frac{\mathbf{V}}{Z_{line} + Z_{load}}$$

$$= \frac{480\angle 0° \text{ V}}{(0.18\ \Omega + j0.24\ \Omega) + (4\ \Omega + j3\ \Omega)}$$

$$= \frac{480\angle 0°}{4.18 + j3.24} = \frac{480\angle 0°}{5.29\angle 37.8°}$$

$$= 90.8\angle -37.8° \text{ A}$$

Therefore the load voltage is

$$\mathbf{V}_{load} = \mathbf{I}_{line} Z_{load}$$

$$= (90.8\angle -37.8° \text{ A})(4\ \Omega + j3\ \Omega)$$

$$= (90.8\angle -37.8° \text{ A})(5\angle 36.9°\ \Omega)$$

$$= 454\angle -0.9° \text{ V}$$

and the line losses are

$$P_{loss} = (I_{line})^2 R_{line}$$

$$= (90.8 \text{ A})^2 (0.18\ \Omega) = 1484 \text{ W}$$

(*b*) Figure 2–6b shows the power system with the transformers. To analyze this system, it is necessary to convert it to a common voltage level. This is done in two steps:

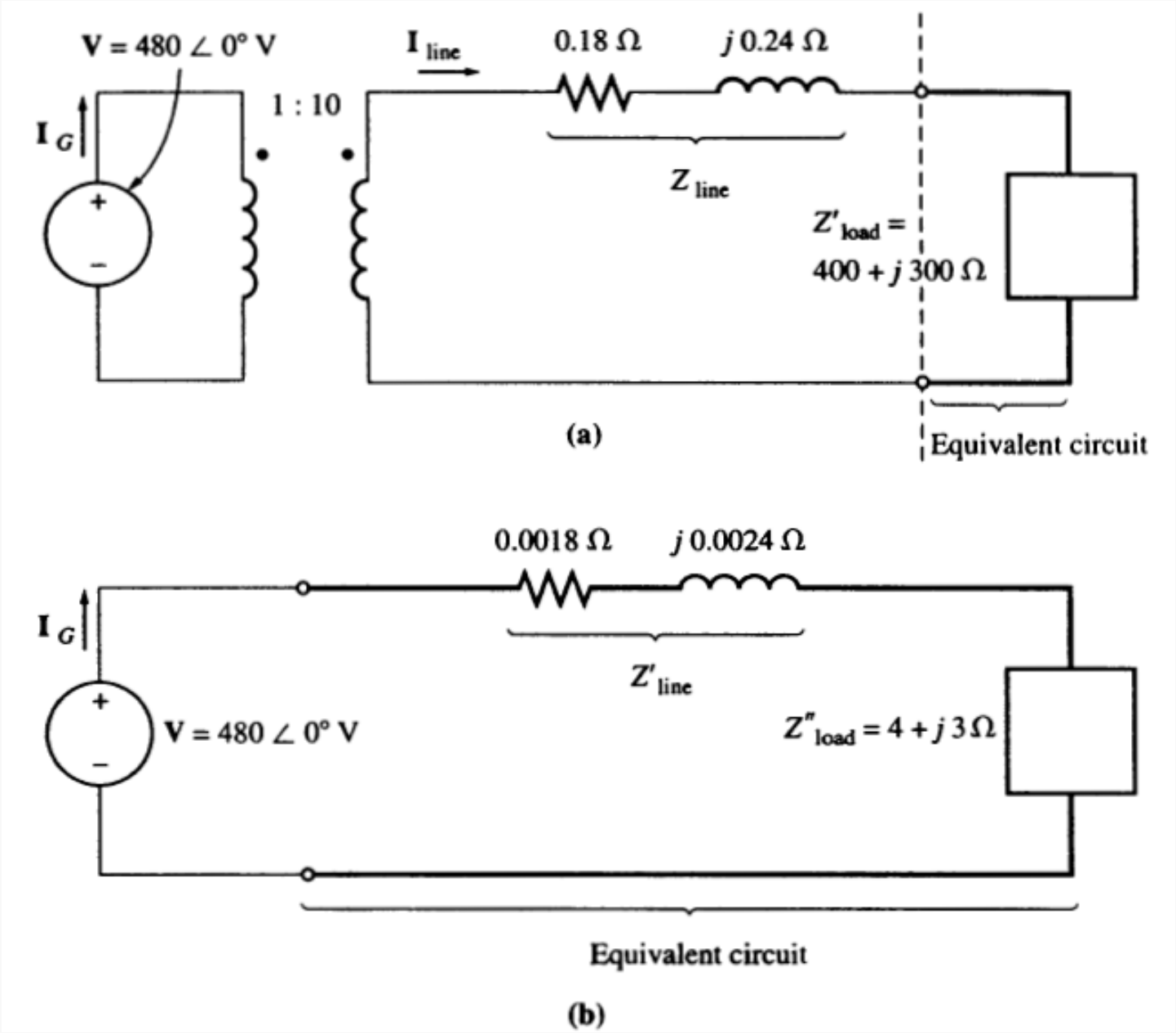
1. Eliminate transformer T_2 by referring the load over to the transmission line's voltage level.
2. Eliminate transformer T_1 by referring the transmission line's elements and the equivalent load at the transmission line's voltage over to the source side.

The value of the load's impedance when reflected to the transmission system's voltage is

$$\begin{aligned} Z'_{load} &= a^2 Z_{load} \\ &= \left(\frac{10}{1}\right)^2 (4\ \Omega + j3\ \Omega) \\ &= 400\ \Omega + j300\ \Omega \end{aligned}$$

The total impedance at the transmission line level is now

$$\begin{aligned} Z_{eq} &= Z_{line} + Z'_{load} \\ &= 400.18 + j300.24\ \Omega = 500.3\angle 36.88°\ \Omega \end{aligned}$$


V = 480 ∠ 0° V
I line
0.18 Ω
j 0.24 Ω
I G
1 : 10
Z line
Z'load =
400 + j 300 Ω
(a)
Equivalent circuit
0.0018 Ω
j 0.0024 Ω
I G
Z'line
V = 480 ∠ 0° V
Z"load = 4 + j 3 Ω
Equivalent circuit
(b)

This equivalent circuit is shown in Figure 2–7a. The total impedance at the transmission line level ($Z_{line} + Z'_{load}$) is now reflected across T_1 to the source's voltage level:

$$\begin{aligned} Z'_{eq} &= a^2 Z_{eq} \\ &= a^2(Z_{line} + Z'_{load}) \\ &= \left(\frac{1}{10}\right)^2 (0.18\ \Omega + j0.24\ \Omega + 400\ \Omega + j300\ \Omega) \\ &= (0.0018\ \Omega + j0.0024\ \Omega + 4\ \Omega + j3\ \Omega) \\ &= 5.003 \angle 36.88°\ \Omega \end{aligned}$$

Notice that $Z''_{load} = 4 + j3\ \Omega$ and $Z'_{line} = 0.0018 + j0.0024\ \Omega$. The resulting equivalent circuit is shown in Figure 2–7*b*. The generator's current is

$$I_G = \frac{480 \angle 0°\ V}{5.003 \angle 36.88°\ \Omega} = 95.94 \angle -36.88°\ A$$

Knowing the current I_G, we can now work back and find I_{line} and I_{load}. Working back through T_1, we get

$$N_{P1}\mathbf{I}_G = N_{S1}\mathbf{I}_{\text{line}}$$

$$\mathbf{I}_{\text{line}} = \frac{N_{P1}}{N_{S1}}\mathbf{I}_G$$

$$= \frac{1}{10}(95.94 \angle -36.88° \text{ A}) = 9.594 \angle -36.88° \text{ A}$$

Working back through T_2 gives

$$N_{P2}\mathbf{I}_{\text{line}} = N_{S2}\mathbf{I}_{\text{load}}$$

$$\mathbf{I}_{\text{load}} = \frac{N_{P2}}{N_{S2}}\mathbf{I}_{\text{line}}$$

$$= \frac{10}{1}(9.594 \angle -36.88° \text{ A}) = 95.94 \angle -36.88° \text{ A}$$

It is now possible to answer the questions originally asked. The load voltage is given by

$$\mathbf{V}_{\text{load}} = \mathbf{I}_{\text{load}} Z_{\text{load}}$$

$$= (95.94 \angle -36.88° \text{ A})(5 \angle 36.87° \ \Omega)$$

$$= 479.7 \angle -0.01° \text{ V}$$

and the line losses are given by

$$P_{\text{loss}} = (I_{\text{line}})^2 R_{\text{line}}$$

$$= (9.594 \text{ A})^2 (0.18 \ \Omega) = 16.7 \text{ W}$$

– *Thank You*