EEE 243 Signals and Systems

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Lecture 04: Classifications of systems

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Introduction

In this lecture, some of the basic classifications of systems will be briefly introduced and the most important properties of these systems are explained. Understanding these basic differences between systems and their properties will be a fundamental concept used in all signal and system courses.

Classification of Systems

Systems may be classified broadly in the following categories:

- Continuous vs. Discrete
- Analog vs. Digital
- Linear vs. Nonlinear
- Time-Invariant vs. Time-varying
- Causal vs. Noncausal
- Stable vs. Unstable
- Invertible vs. Noninvertible
- Instantaneous and Dynamic

Continuous vs. Discrete

A system in which the input signal and output signal both have continuous domains is said to be a continuous system. One in which the input signal and output signal both have discrete domains is said to be a discrete system (Figure 1).

However, it is possible to consider signals that belong to neither category, such as systems in which sampling of a continuous-time signal or reconstruction from a discrete-time signal takes place.

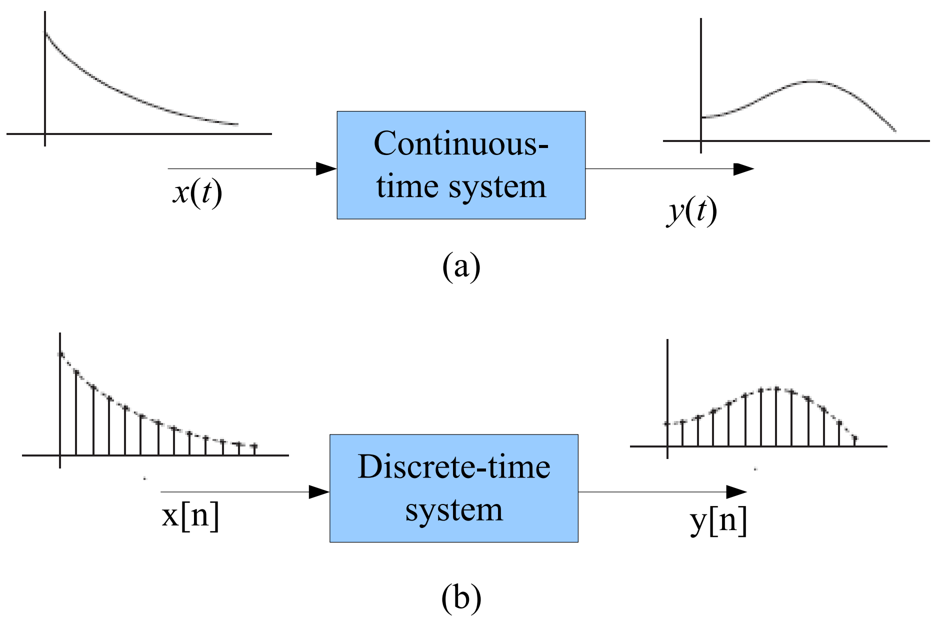
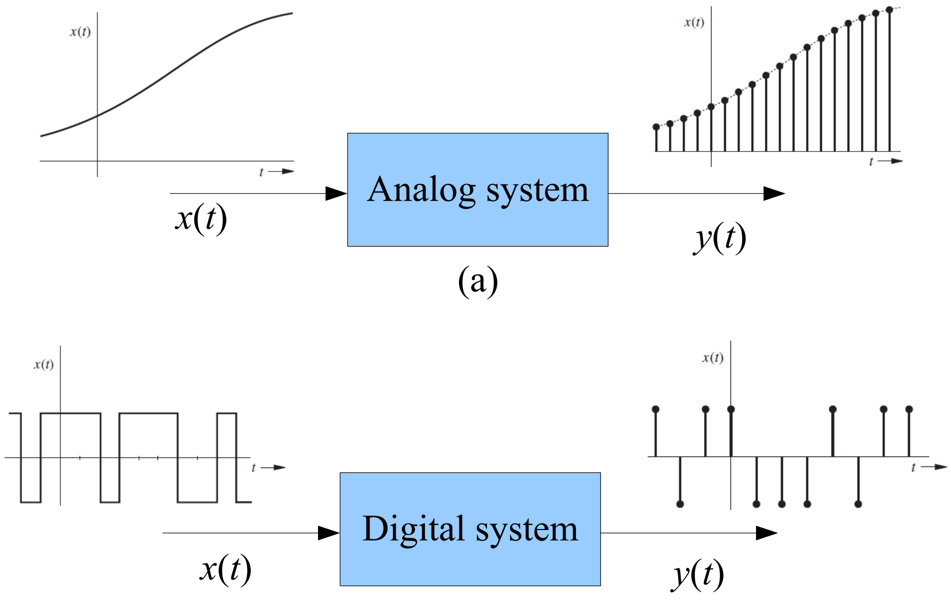


Figure 1. (a) continuous-time system (b) discrete-time system.

Analog vs. Digital

A system whose input and output signals are analog is an *analog system, and* a system whose input and output signals are digital is a *digital system*.



(a)

(b)

Figure 2. (a) an analog system and (b) a digital system, each performing the sampling operation on a continuous-time analog signal and a continuous-time digital signal, respectively.

Linear vs. Nonlinear

A linear system is any system that <u>obeys the properties of scaling and superposition (additivity)</u>.

A nonlinear system is any system that <u>does not have at least one</u> of these properties.

A system H obeys the <u>scaling</u> property when it obeys the following property (Figure 3)

$$H\left(k\,f\left(t\right)\right)=k\,H\left(f\left(t\right)\right)$$

Likewise, a system H obeys the superposition property if it demonstrates the following (Figure 4)

$$H\left(f_1\left(t\right)+f_2\left(t\right)\right)=H\left(f_1\left(t\right)\right)+H\left(f_2\left(t\right)\right)$$

A system for linearity can be verified in a single step by combining the above two steps for scaling and superposition properties, respectively as follows.

$$H\left(k_1 f_1\left(t\right)+k_2 f_2\left(t\right)\right)=k_1 H\left(f_1\left(t\right)\right)+k_2 H\left(f_2\left(t\right)\right)$$

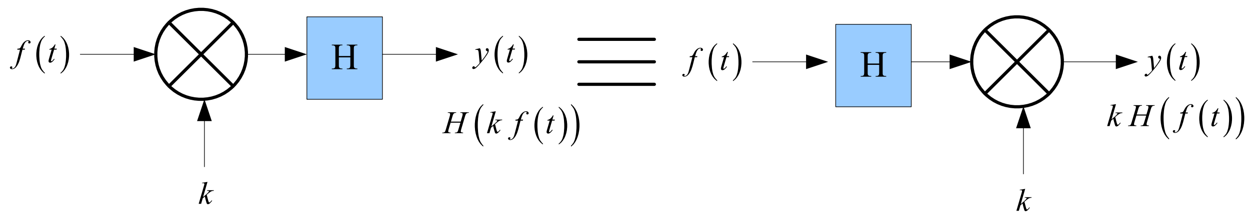


Figure 3. A block diagram demonstrating the scaling property of linearity.

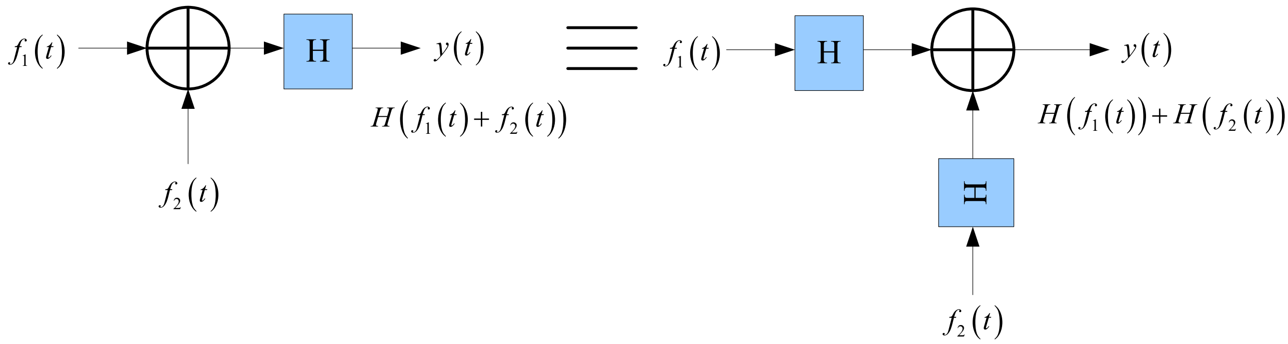


Figure 4. A block diagram demonstrating the superposition property of linearity.

Time-Invariant vs. Time-Varying

A system is said to be time-invariant if it commutes (showing commutative property, e.g. *ab*=*ba*) with the parameter shift operator defined by $S_T(f(t)) = f(t-T)$ for all *T*, i.e.,

$$HS_T = S_T H \text{ for all real } T.$$

Intuitively, any time shift of the input function will produce an output function identical in every way except that it is shifted by the same amount (Figure 5).

Any system that does not have this above property is said to be time-varying.

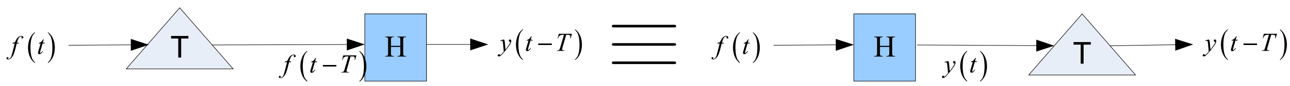


Figure 5. This block diagram shows the condition for time invariance. The output is the same whether the delay is put on the input or the output.

The time invariance property is expressed graphically in Figure 6. Since the input is delayed by *T* seconds, the output is the same as before but delayed by *T* (Figure 6b).

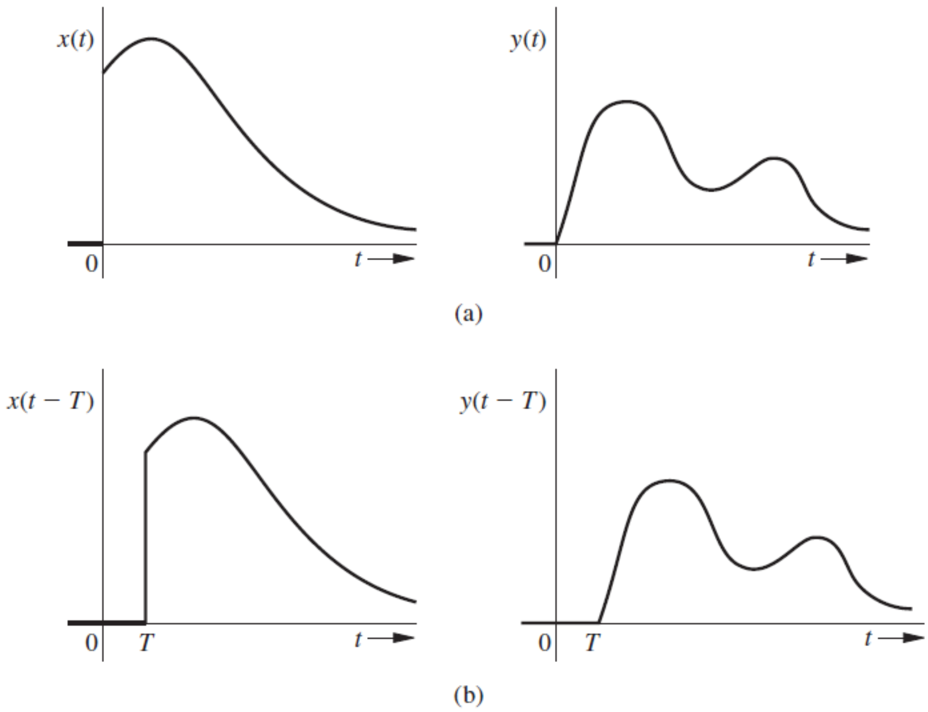


Figure 6. Time-invariance property.

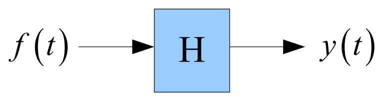
Causal vs. Noncausal

A *causal* system is one in which the output depends only on current or past inputs, but not future inputs (Figure 7).

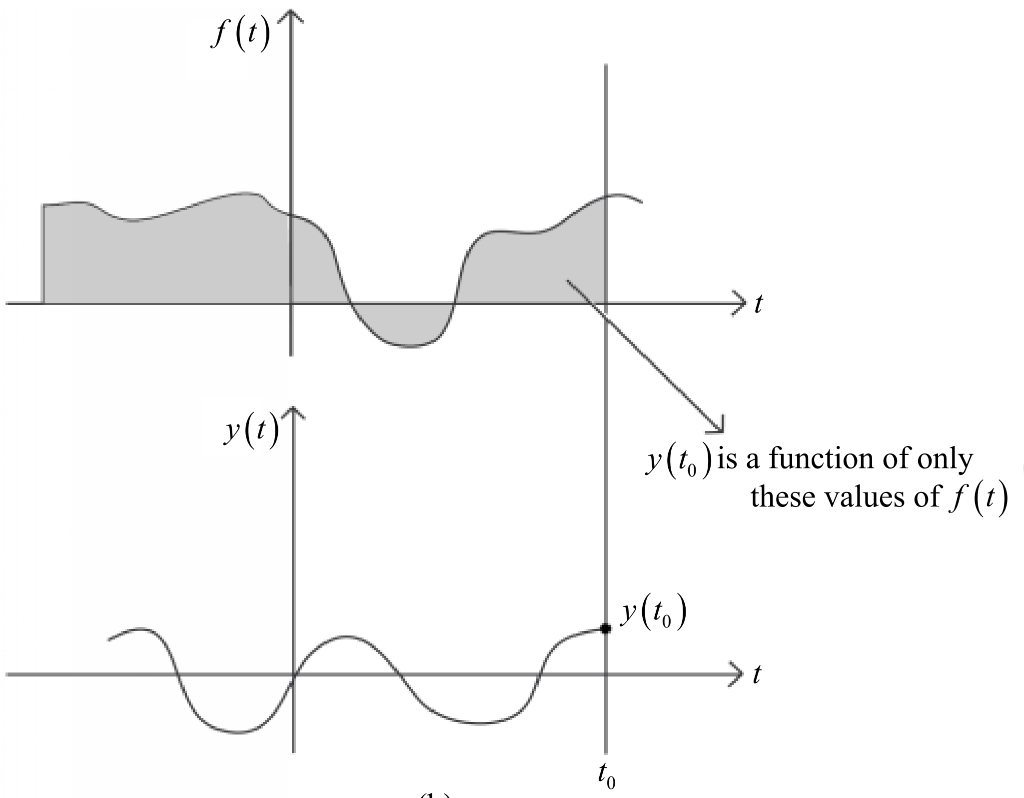
Similarly, an *anticausal* system is one in which the output depends only on current or future inputs, but not past inputs.

Finally, a *noncausal* system is one in which the output depends on both past and future inputs.

All "real-time" systems must be *causal* since they cannot have future inputs available to them. However, to understand the idea of future inputs, imagine that we wanted to do image processing. Then, the dependent variable might represent pixel positions to the left and right (the "future") of the current position on the image, and *we would not necessarily have a causal system*.



(a)



(b)

Figure 7. (a) For a typical system (a) to be causal, the output at time t_0, $y(t_0)$, can only depend on the portion of the input signal $f(t)$ before t_0.

Stable vs. Unstable

There are several definitions of stability, but the one that is used most frequently is be bounded input, bounded output (BIBO) stability.

In this context, a *stable system* is one in which the output is bounded if the input is also bounded. Similarly, an *unstable system* is one in which at least one bounded input produces an unbounded output.

Mathematically, a stable system must have the following property, where $f(t)$ is the input and $y(t)$ is the output. The output must satisfy the condition: $|y(t)| \le M_y < \infty$ for an input to the system that satisfies: $|f(t)| \le M_f < \infty$ where M_f and M_y both represent a set of finite positive numbers, and these relationships hold for all of t. Otherwise, the system is unstable.

Invertible vs. Noninvertible

A system S performs certain operation(s) on input signal(s). If we can obtain the input $x(t)$ back from the corresponding output $y(t)$ by some operation, the system S is said to be *invertible*.

The system that achieves the inverse operation [of obtaining $x(t)$ from $y(t)$] is the *inverse system* for S. For instance, if S is an ideal integrator, then its inverse system is an ideal differentiator. Consider a system S connected in tandem with its inverse S_i, as shown in Figure 8. The input $x(t)$ to this tandem system results in signal $y(t)$ at the output of S, and the signal $y(t)$, which now acts as an input to S_i, yields back the signal $x(t)$ at the output of S_i. Thus, S_i undoes the operation of S on $x(t)$, yielding back $x(t)$.

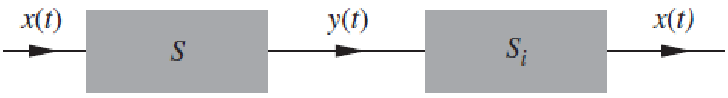


Figure 8. A cascade of a system with its inverse results in an identity system.

However, if it is impossible to obtain the input from the output, the system is said *noninvertible*. For example, a rectifier, specified by an equation $y(t) = |x(t)|$, is noninvertible because the rectification operation cannot be undone.

Inverse systems are very important in signal processing. In many applications, the signals are distorted during the processing, and it is necessary to undo the distortion. For instance, in the transmission of data over a communication channel, the signals are distorted owing to the nonideal frequency response and finite bandwidth of a channel. It is necessary to restore the signal as closely as possible to its original shape.

Instantaneous and Dynamic

A system is said to be *instantaneous (or memoryless)* if its output at any instant t depends, at most, on the strength of its input(s) at the same instant t, and not on any past or future values of the input(s). Otherwise, the system is said to be *dynamic* (or a system with memory).

In *resistive* networks, for example, any output of the network at some instant t depends only on the input at the instant t. In these systems, past history is irrelevant in determining the response. Such systems are said to be *instantaneous* or *memoryless* systems.

Contrary, networks containing *inductive and capacitive* elements generally have *infinite memory* because the response of such networks at any instant t is determined by their inputs over the entire past $(-\infty, t)$. However, a system whose response at t is completely determined by the input signals over the past T seconds [interval from $(t - T)$ to t] is a *finite-memory system* with a memory of T seconds.

References

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