EEE 243 Signals and Systems

2022

Lecture 01 & 02: Classifications and Operations of Signals and Elementary signals

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School of Engineering
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Inspiring Excellence

Introduction

Signals

In the analysis of a communication system, we define a signal as a *single-valued function* of time that conveys information. For every time instant, there is a unique function value. This value can be either a real number, in which case we have a real-valued signal or a complex number, in which case we have a complex-valued signal.

A signal is typically written as x(t). The notation x(t) can actually be interpreted in two ways: (i) as the signal value at a *particular time* instant t, (ii) as a *function defined over all time t.*

Examples include a telephone or a television signal.

Systems

A system is a mapping (transformation) of the input signal, denoted by x(t), into the output signal, denoted by y(t). Let f denote this mapping. Then, we can write y(t) = f(x(t)). In other words, a system is *any (physical) device* that produces an output signal in response to an input signal.

Signals may be processed further by *systems,* which may modify them or extract additional information from them. Thus, a system is an entity that *processes* a set of signals (*inputs*) to yield another set of signals (*outputs).*

Classification of Signals

There are several classes of signals, we can classify signals in several ways as follows.

(a) According to the **predictability of their behavior**, signals can be *random* or *deterministic*. While a deterministic signal can be represented by a formula or a table of values, random signals can only be approached probabilistically.

(b) According to the **variation of their time variable and their amplitude**, signals can be either *continuous-time* or *discrete-time*, *analog* or *discrete* amplitude, or *digital*. This classification relates to the way signals are either processed, stored, or both.

(c) According to their **energy content**, signals can be characterized as *finite-* or *infinite-energy* signals.

(d) According to whether the **signals exhibit repetitive behavior or not** as *periodic* or *aperiodic* signals.

(e) According to the **symmetry with respect to the time origin**, signals can be *even* or *odd*.

(f) According to the **dimension of their support**, signals can be of *finite* or of *infinite* support. Support can be understood as the time interval of the signal outside of which the signal is always zero.

We discuss in detail what follows

Continuous-Time vs. Discrete-Time Signals

This classification is determined by whether or not *the time axis* is discrete (countable) or continuous (Figure 1).

A continuous-time signal will contain a value for all real numbers along the time axis. In contrast to this, a discrete-time signal is specified at discrete values of time t, often created by sampling a continuous signal that will only have values at equally spaced intervals along the time axis.

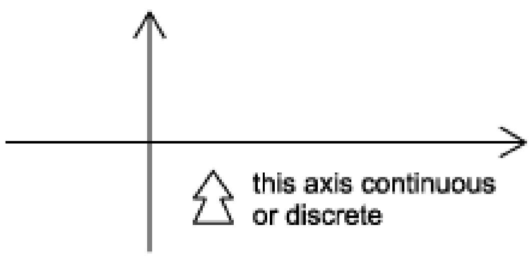


Figure 1.

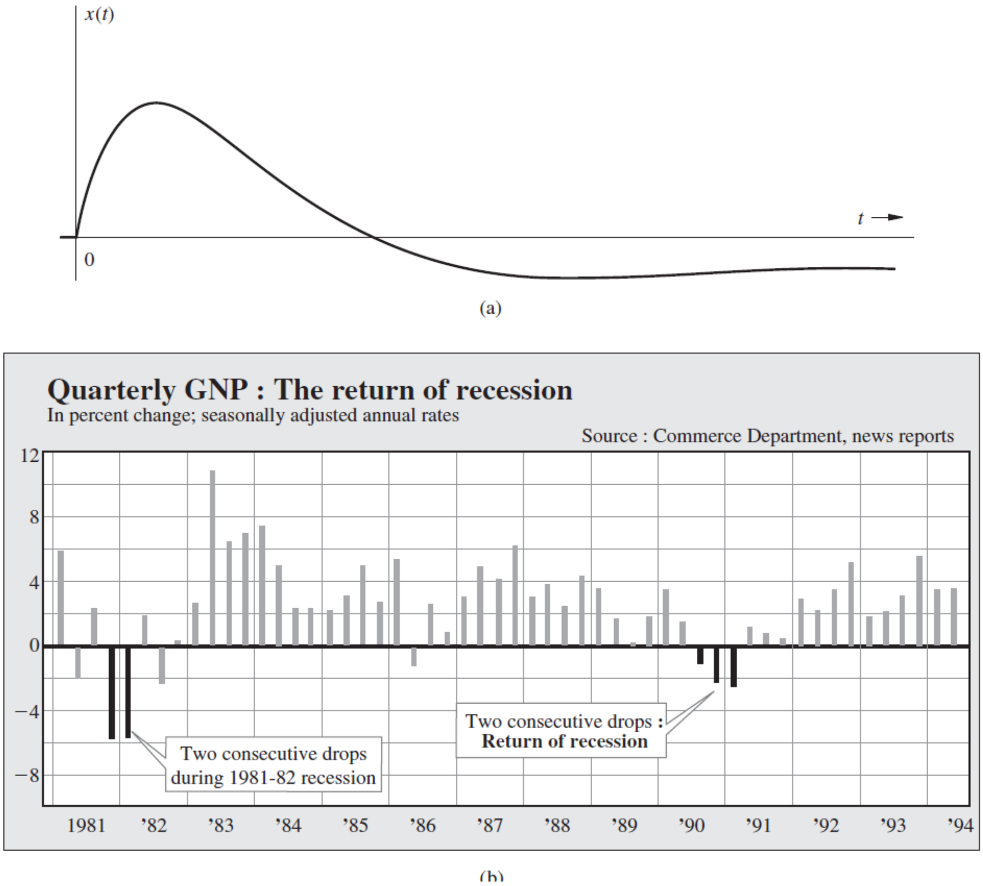


Figure 2. (a) Continuous-time and (b) discrete-time signals.

Analog and Digital Signals

This classification is determined by whether or not *the amplitude axis* is discrete (countable) or continuous (Figure 3).

A signal whose amplitude can take on any value in a continuous range is an *analog signal*. This means that an analog signal amplitude can take on an infinite number of values.

A *digital signal,* on the other hand, is one whose amplitude can take on only a *finite* number of values. Signals associated with a digital computer are digital because they take on only two values (binary signals). Note amplitudes can take on M values is an M-ary signal of which binary ($M = 2$) is a special case.

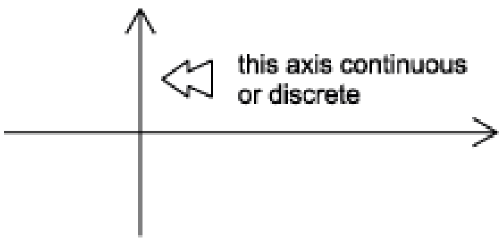


Figure 3.

The concept of continuous-time is often confused with that of analog. The two are not the same. The same is true of the concepts of discrete-time and digital.

The terms *continuous-time* and *discrete-time* qualify the nature of a signal along the time (horizontal) axis (Figure 1). The terms *analog* and *digital,* on the other hand, qualify the nature of the signal amplitude (vertical axis) (Figure 3).

Figure 4 shows examples of signals of various types.

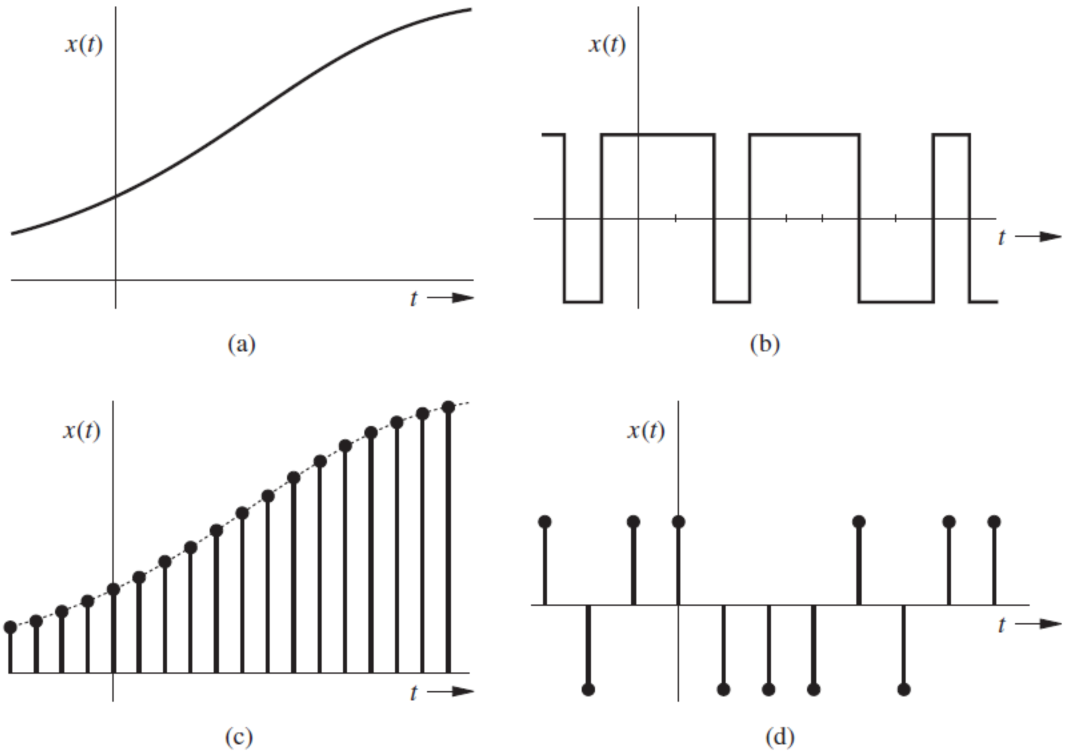


Figure 4. Examples of signals: (a) analog, continuous-time, (b) digital, continuous-time, (c) analog, discrete-time, and (d) digital, discrete-time.

Homework 01: "*analog is not necessarily continuous-time and digital need not be discrete-time*." Justify the statement.

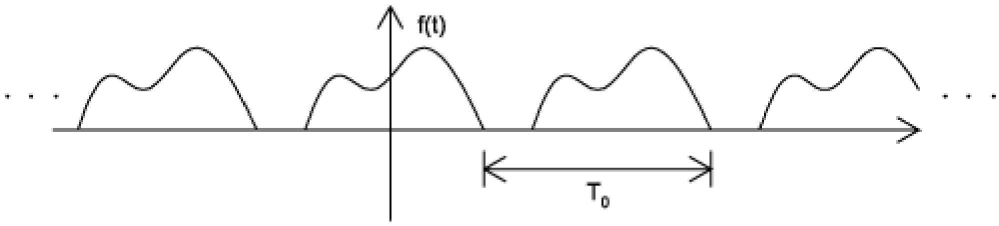
Periodic vs. Aperiodic

Periodic signals repeat with some period T_0, while aperiodic, or nonperiodic, signals do not (Figure 5).

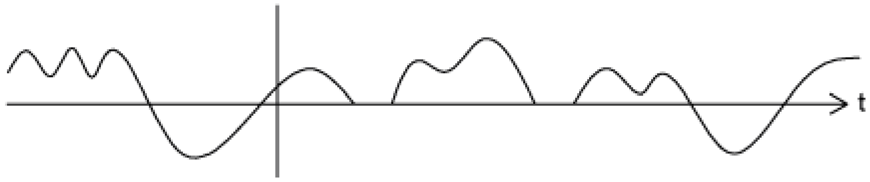
A signal $f(t)$ is said to be *periodic* if for some positive constant T_0

$$f(t) = f(t + T_0) \qquad \text{for all } t \qquad (1)$$

The smallest value of T_0 that satisfies the periodicity condition of (1) is the fundamental period of $f(t)$.



(a)



(b)

Figure 5: (a) A periodic signal with period T_0 (b) An aperiodic signal

Energy and Power Signals

A signal with finite energy is an *energy signal,* and a signal with finite and nonzero power is a *power signal*. Signals in Figures 6(a) and 6(b) are examples of energy and power signals, respectively.

A necessary condition for the energy to be finite is that the signal amplitude $\rightarrow 0$ as $|t| \rightarrow \propto$ (Figure 6(a)). Otherwise, the integral in (2) measuring the *signal energy* will not converge.

$$E_x = \int_{-\propto}^{\propto} x^2(t)\,dt \qquad (2)$$

When the amplitude of $x(t)$ does not $\rightarrow 0$ as $|t| \rightarrow \propto$ (Figure 6(b)), the signal energy is infinite. A more meaningful measure of the signal size in such a case would be the time average of the energy, if it exists. This measure is called the *power* of the signal. For a signal $x(t)$, we define its power P_x as

$$P_x = \lim_{T \rightarrow \propto} \frac{1}{T} \int_{-(T/2)}^{(T/2)} x^2(t)\,dt \qquad (3)$$

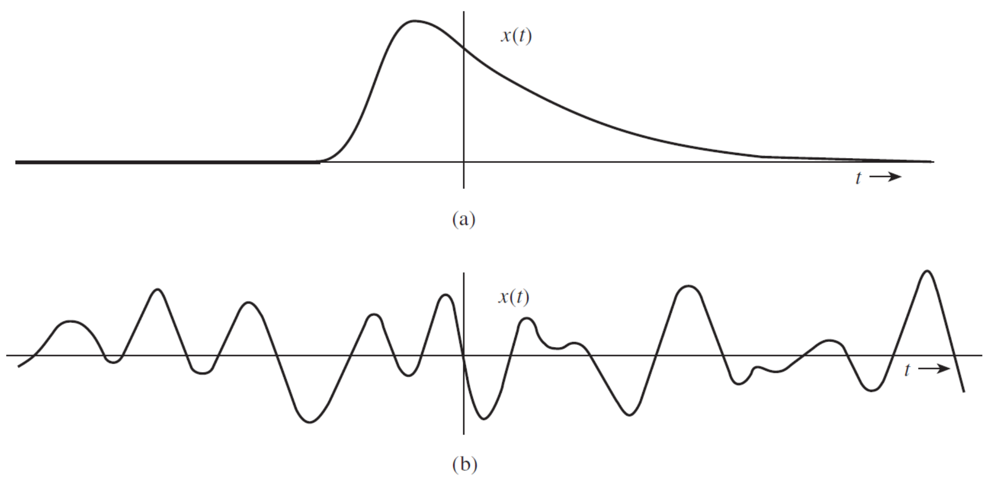


Figure 6. Examples of signals: (a) a signal with finite energy and (b) a signal with finite power.

Observe that power is the time average of energy. Since the averaging is over an infinitely large interval, a signal with finite energy has zero power, and a signal with *finite power has infinite energy*. Therefore, a signal cannot both be an energy signal and a power signal. If it is one, it cannot be the other.

Generally, the mean of an entity averaged over a large time interval approaching infinity exists if the entity either is periodic or has a statistical regularity. If such a condition is not satisfied, the average may not exist. For instance, a ramp signal $x(t) = t$ increases indefinitely as $|t| \to \infty$, and *neither the energy nor the power exists* for this signal. However, the unit step function, which is not periodic nor has statistical regularity, does have a finite power.

When $x(t)$ is periodic (Figure 7), $|x(t)|^2$ is also periodic. Hence, the power of $x(t)$ can be computed Using (3) by averaging $|x(t)|^2$ over one period.

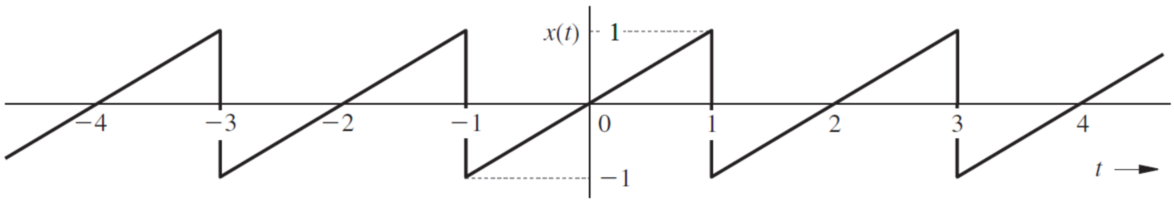


Figure 7.

Note: All practical signals have finite energies and are therefore energy signals. A power signal must necessarily have infinite duration; otherwise its power, which is its energy averaged over an infinitely large interval, will not approach a (nonzero) limit. Clearly, it is impossible to generate a true power signal in practice because such a signal has infinite duration and infinite energy.

Finite vs. Infinite Length

Signals can be characterized as to whether they have a finite or infinite length set of values. Most finite length signals are used when dealing with discrete-time signals or a given sequence of values.

Mathematically speaking, $f(t)$ is a finite-length signal if it is nonzero over a finite interval

$$t_1 < f(t) < t_2$$

where $t_1 > -\propto$ and $t_2 < \propto$. An example can be seen in Figure 8. Similarly, an infinite-length signal, f (t), is defined as nonzero over all real numbers.

$$-\propto \leq f(t) \leq \propto$$

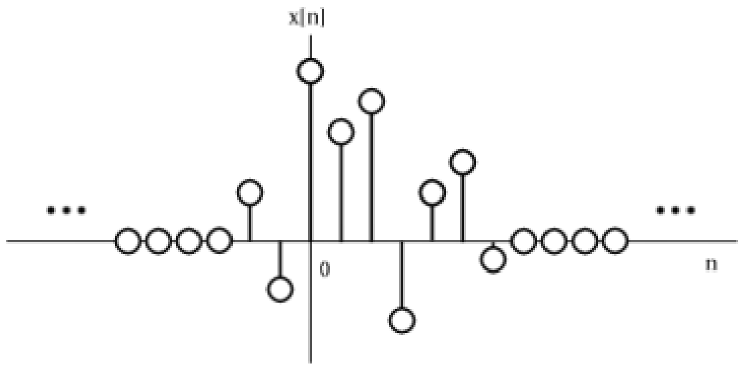
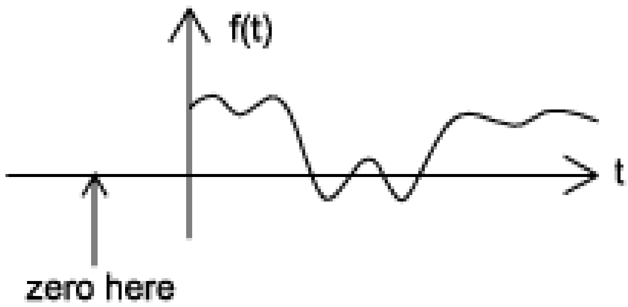


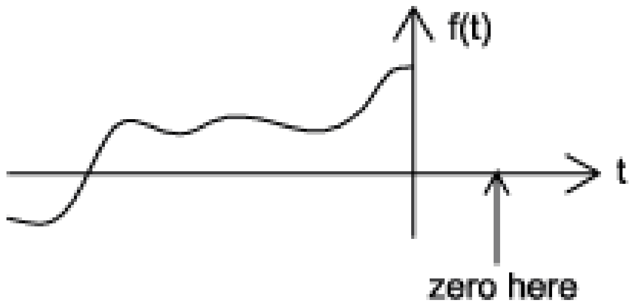
Figure 8. Finite-Length Signal. Note that it only has nonzero values on a set, finite interval.

Causal vs. Anticausal vs. Noncausal

Causal signals are signals that are zero for all *negative* time, while anticausal are signals that are zero for all *positive* time. Noncausal signals are signals that have nonzero values *in both positive and negative* time (Figure 9).



(a)



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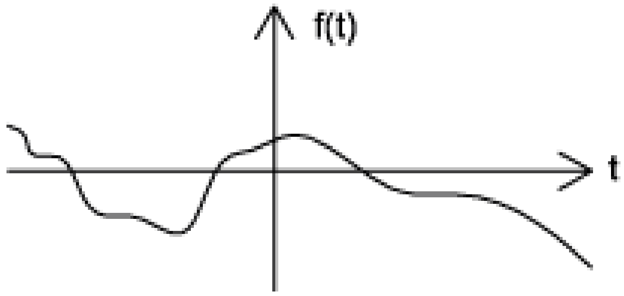


(c)

Figure 9. (a) A causal signal (b) An anticausal signal (c) A noncausal signal

Even vs. Odd

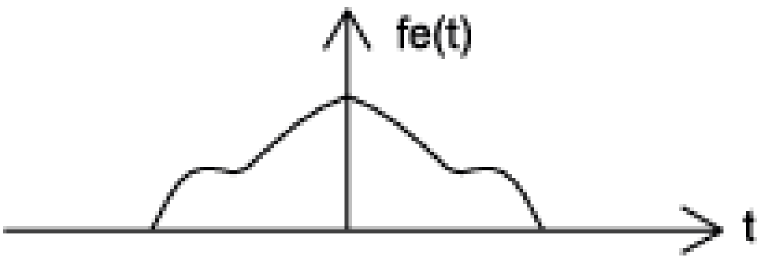
An even signal is any signal *f* such that (Figure 10(a)).

$$f(t) = f(-t).$$

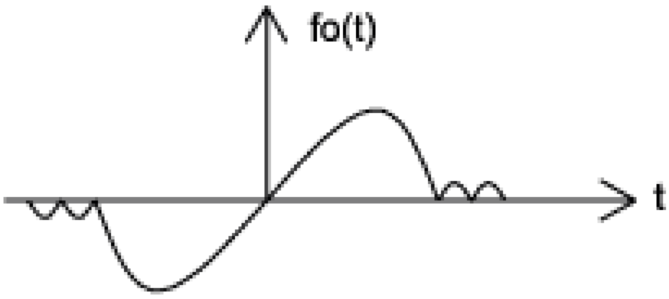
Even signals can be easily spotted as they are *symmetric* around the *vertical* axis.

An odd signal, on the other hand, is a signal *f* such that (Figure 10(b)).

$$f(t) = -f(-t)$$



(a)



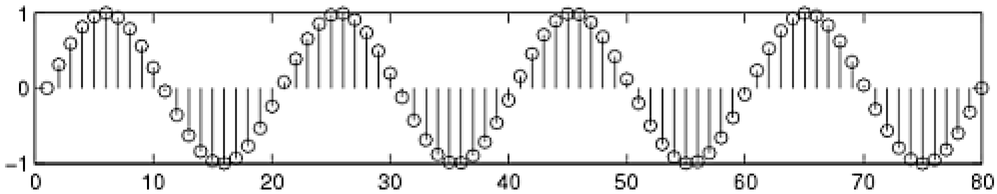
(b)

Figure 10. (a) An even signal (b) An odd signal.

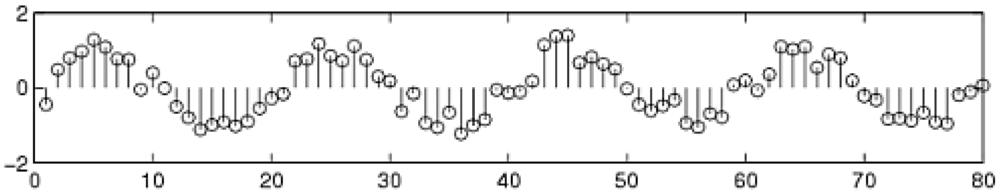
Deterministic vs. Random

A deterministic signal is a signal in which each value of the signal is fixed and can be determined by a mathematical expression, rule, or table. Because of this the future values of the signal can be calculated from past values with complete confidence. (Figure 11(a))

On the other hand, a random signal has a lot of uncertainty about its behavior. The future values of a random signal cannot be accurately predicted and can usually only be guessed based on the averages of sets of signals (Figure 11(b)).



(a)



(b)

Figure 11. (a) Deterministic Signal (b) Random Signal.

Signal Operations

Three useful signal operations called shifting, scaling, and inversion, are discussed. Since the independent variable in our signal description is time, these operations are discussed as *time shifting, time scaling,* and *time reversal* (inversion). However, this discussion is valid for functions having independent variables other than time (e.g., frequency or distance).

Time Shifting

Consider a signal $x(t)$ (Figure 12 (a)) and the same signal delayed by T seconds (Figure 12 (b)), which we shall denote by $\phi(t)$. Whatever happens in $x(t)$ at some instant t also happens in $\phi(t)$ T seconds later at the instant $t + T$. Therefore,

$$\phi(t+T) = x(t) \text{ and}$$
$$\phi(t) = x(t-T)$$

So, to time-shift a signal by T, we replace t with $t - T$. Thus $x(t-T)$ represents $x(t)$ time-shifted by T seconds.

- If T is positive, the shift is to the right (*delay*), as in Figure 12(b).
- If T is negative, the shift is to the left (*advance*), as in Figure 12(c).

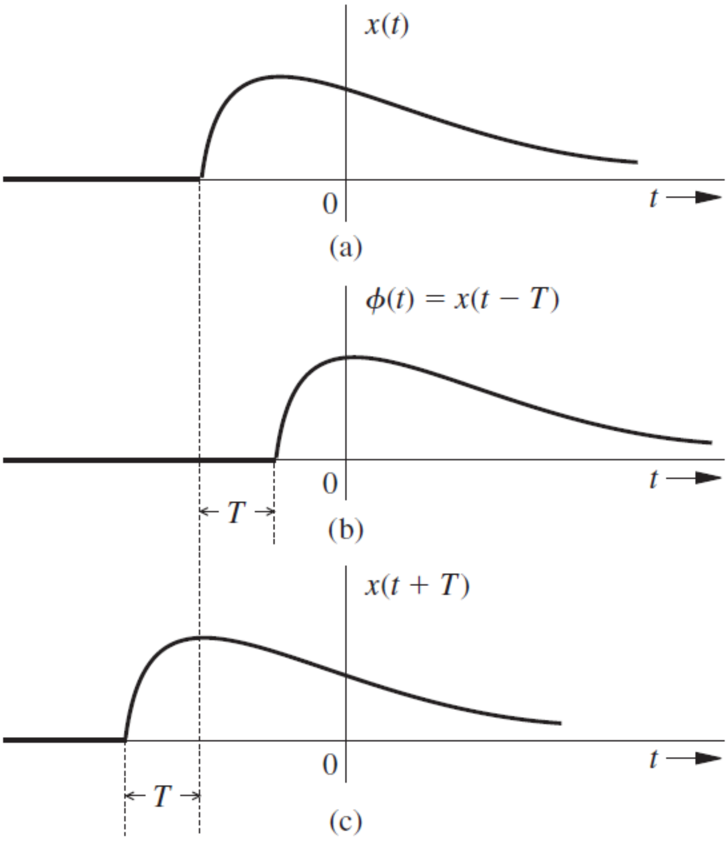


Figure 12. Time-shifting a signal.

Time Scaling

Time scaling compresses or dilates a signal by *multiplying the time variable by some quantity*. If that quantity is greater than one, the signal becomes narrower and the operation is called *compression*, while if the quantity is less than one, the signal becomes wider and is called *dilation*.

Consider the signal $x(t)$ of Figure 13 (a). The signal $\phi(t)$ in Figure 13(b) is $x(t)$ compressed in time by a factor of 2. Therefore, whatever happens in $x(t)$ at some instant t also happens to $\phi(t)$ at the instant $\frac{t}{2}$ so that

$$\varphi\left(\frac{t}{2}\right) = x(t)$$

$$\varphi(t) = x(2t)$$

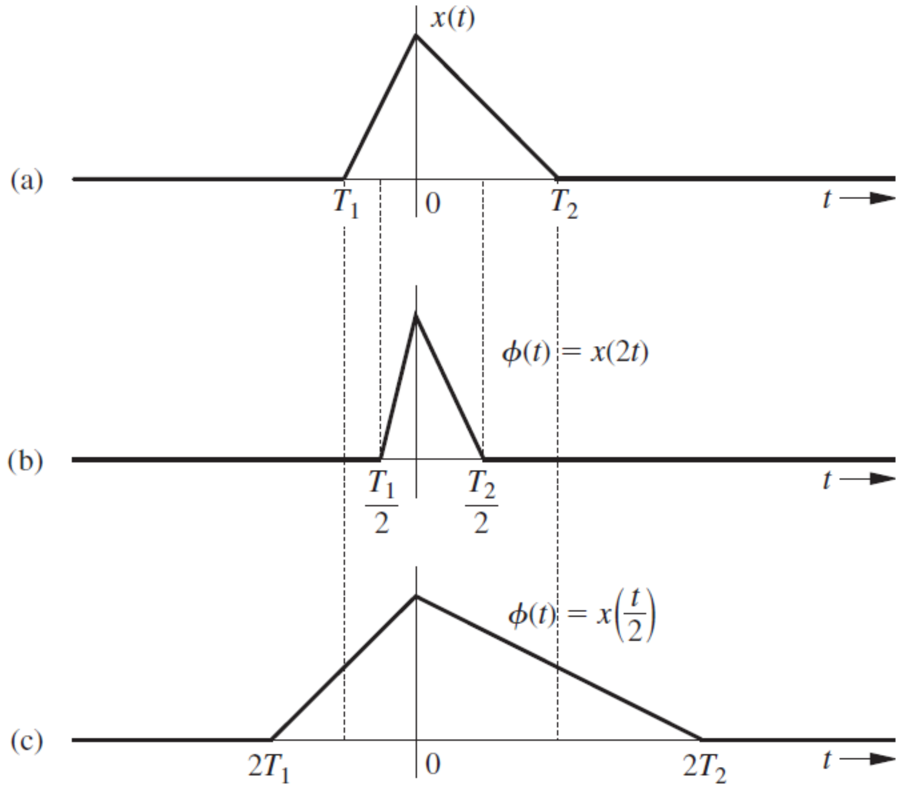


Figure 13**.** Time scaling a signal

If $x(t)$ were recorded on a tape and played back at twice the normal recording speed, we would obtain $x(t)$.

In general, if $x(t)$ is **compressed in time** by a factor a (**a > 1**), the resulting signal $\varphi(t)$ is given by

$$\varphi(t) = x(at)$$

Using a similar argument, we can show that $x(t)$ expanded (slowed down) in time by a factor a ($a > 1$) is given by

$$\varphi(t) = x\left(\frac{t}{a}\right)$$

In summary, to time-scale a signal by a factor a, we replace t with at. **If $a > 1$, the scaling results in compression, and if $a < 1$, the scaling results in expansion.**

Time Reversal

A natural question to consider when learning about time scaling is: What happens when the time variable is multiplied by a negative number? The answer to this is time reversal (Figure 14). This operation is the reversal of the time axis, or flipping the signal over the y-axis.

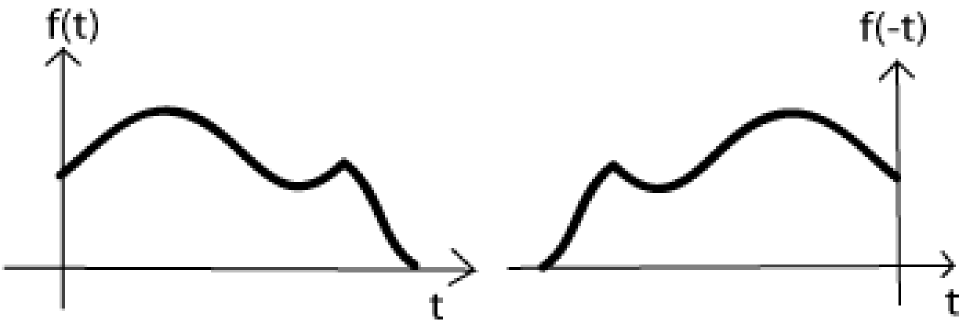


Figure 14. Reverse the time axis

Combined Operations

Certain complex operations require simultaneous use of more than one of the operations described above. The most general operation involving *all the three operations is* $x(at - b)$, which is realized in *two* possible sequences of operation:

1. **Time-shift** $x(t)$ by b to obtain $x(t - b)$. Now **time-scale** the shifted signal $x(t - b)$ by a (i.e., replace t with at) to obtain $x(at - b)$.

2. **Time-scale** $x(t)$ by a to obtain $x(at)$. Now **time-shift** $x(at)$ by b/a (i.e., replace t with $t - (b/a)$) to obtain $x[a(t - b/a)] = x(at - b)$.

In either case, if a is *negative*, time scaling involves **time reversal**.

Elementary signals

In this section, we define some elementary functions that will be used frequently *to represent more complicated signals*. Representing signals in terms of the elementary functions *simplifies the analysis and design* of linear systems.

Unit step function

The continuous-time unit step function $u(t)$ is defined as follows:

$$u(t)=\begin{cases}1, & t\geq 0\\ 0, & <0\end{cases}$$

In much of our discussion, the signals begin at $t = 0$ (*causal signals*). Such signals can be conveniently described in terms of unit step function *u(t)* shown in Figure 14 (a). More specifically, if we want a signal to start at $t = 0$ (so that it has a value of zero for $t < 0$), we need only multiply the signal by *u(t)*. For instance, the signal e^{-at} represents an everlasting exponential that starts at $t = -\infty$. The causal form of this exponential (Figure 14(b)) can be described as $e^{-at}u(t)$.

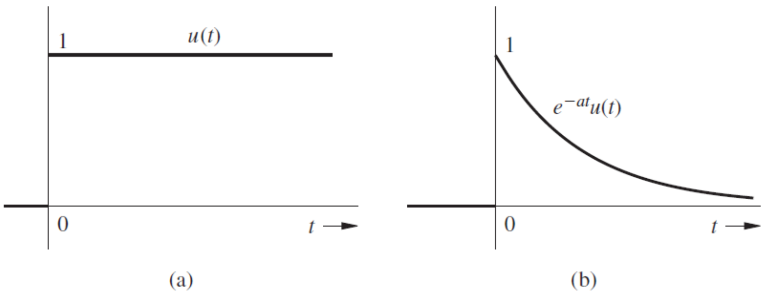


Figure 14 (a) Unit step function *u(t)*. (b) Exponential $e^{-at}u(t)$.

Ramp function

The continuous-time ramp function $r(t)$ is defined as follows:

$$r(t)=tu(t)=\begin{cases}t, & t\geq 0\\ 0, & <0\end{cases}$$

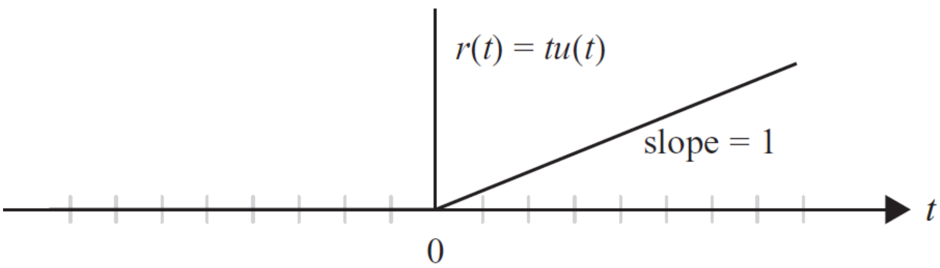


Figure 15. The ramp function.

Note that the ramp function can be obtained by integrating the unit-step function.

$$r(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$$

Signum Function

The *signum function* (or *sign*) function, denoted by sgn(t), is defined as follows:

$$\operatorname{sgn}(t) = \begin{cases} 1 & t > 1 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

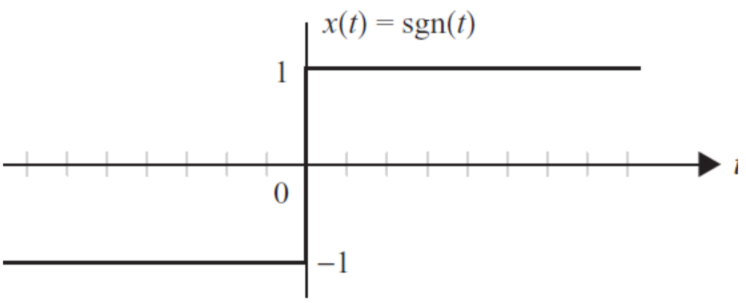


Figure17. The signum function.

The Sampling Function

A function frequently encountered in spectral analysis is the sampling function, defined as

$$\mathrm{Sa}(x) = \frac{\sin x}{x}$$

As the denominator is an increasing function of x and the numerator is bounded, i.e., $|\sin x| \le 1$, sampling function is a damped sine wave having its peak at x=0 and zero-crossing at x= $x = \pm n\Pi$.

The Sinc function

Another closely related function is the **Sinc function**, defined as

$$\operatorname{sinc}(x) = \frac{\sin \Pi x}{\Pi x} = Sa(\Pi x),$$

As shown in Figure 19. It can be found that sinc function is a compressed of version of Sa(x) where the compression factor is Π .

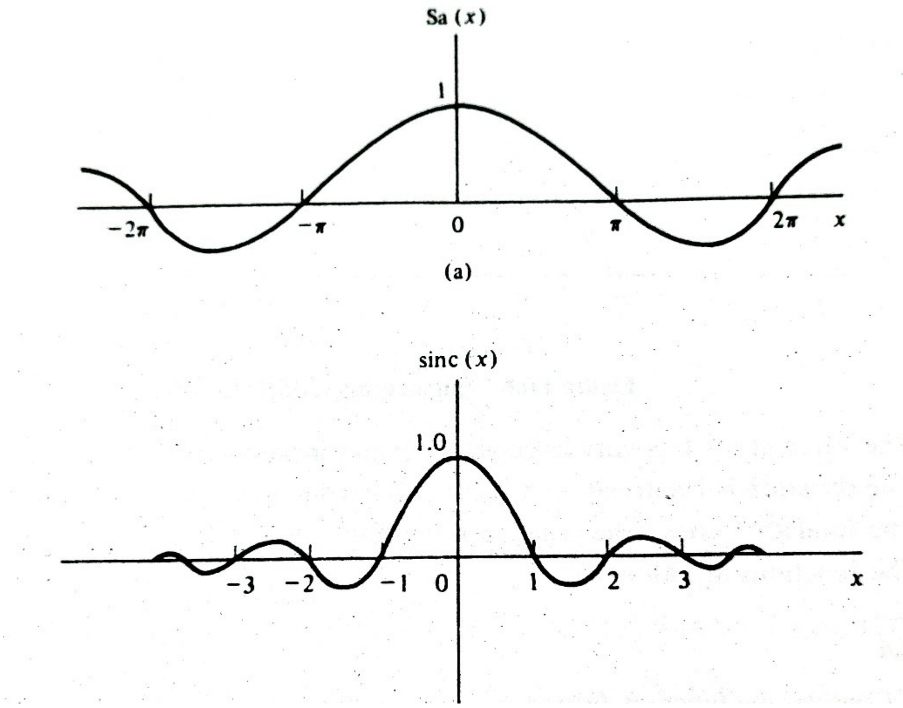


Figure 19. (a) Sampling function (b) Sinc function.

The Unit Impulse Function

The *unit impulse* function $\delta(t)$, also known as the *Dirac delta* function or simply the *delta* function, is defined in terms of two properties as follows:

$$(1)\quad \delta(t)=0, \quad t \neq 0$$

$$(2)\quad \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

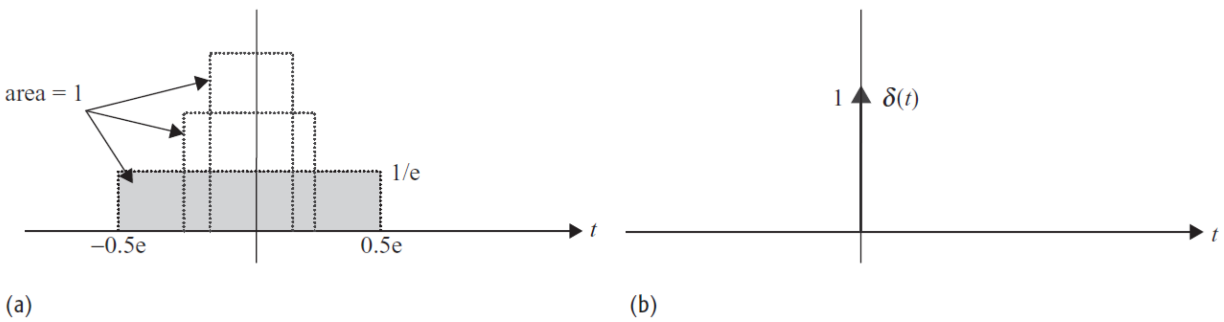


Figure 20. Impulse function $\delta(t)$. (a) Generating the impulse function from a rectangular pulse. (b) Notation used to represent an impulse function.

Direct visualization of a unit impulse function in the continuous time (CT) domain is difficult. One way to visualize a CT impulse function is to let it evolve from a rectangular function. Consider a tall narrow rectangle with width ε and height 1/ε, as shown in Figure 20 (a), such that the area enclosed by the rectangular function equals one. Next, we decrease the width and increase the height at the same rate such that the resulting rectangular functions have areas = 1. As the width $\varepsilon \rightarrow 0$, the rectangular function converges to the CT impulse function $\delta(t)$ with an infinite amplitude at $t = 0$. However, the area enclosed by CT impulse function is finite and equals one. Many physical phenomenon such as point sources, point charges, voltage or current sources (acting for a short duration) can be modelled as delta function.

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