

EEE243
Assignment - 2

Name: Souvik Banman Rafal
ID : 24121205
Section: 02

From the graph,

$$x(t) = \begin{cases} 2, & -2 \leq t < -1 \\ 1, & -1 \leq t \leq 1 \\ 2, & 1 < t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

$$X(w) = \int_{-2}^{-1} 2e^{-jw t} dt + \int_{-1}^1 e^{-jw t} dt + \int_1^2 2e^{-jw t} dt$$

$$\begin{aligned} \text{now, } X_1(w) &= 2 \int_{-2}^{-1} e^{-jw t} dt \\ &= 2 \left[\frac{e^{-jw t}}{-jw} \right]_{-2}^{-1} \\ &= 2 \left[\frac{e^{j2w} - e^{jw}}{jw} \right] \end{aligned}$$

again,

$$\begin{aligned} X_2(w) &= \int_{-1}^1 e^{-jw t} dt \\ &= \left[\frac{e^{-jw t}}{-jw} \right]_{-1}^1 \\ &= \left(\frac{e^{jw} - e^{-jw}}{-jw} \right) \\ &= \frac{2\sin(w)}{w} \end{aligned}$$

again,

$$\begin{aligned} X_3(w) &= 2 \int_1^2 e^{-jw t} dt \\ &= 2 \left[\frac{e^{-jw t}}{-jw} \right]_1^2 \\ &= 2 \left(\frac{e^{-jw} - e^{-j2w}}{jw} \right) \end{aligned}$$

now,

$$\begin{aligned}x(w) &= x_1(w) + x_2(w) + x_3(w) \\&= \frac{2(e^{j2w} - e^{jw})}{jw} + \frac{2\sin(w)}{w} + \frac{2(e^{-jw} - e^{-j2w})}{jw} \\&= \frac{2e^{j2w} - 2e^{jw} + 2j\sin(w) + 2e^{-jw} - 2e^{-j2w}}{jw} \\&= \frac{4\sin(2w) - 2\sin(w)}{w}\end{aligned}$$

And

$$\begin{cases} e^{j2w} - e^{-j2w} = 2j\sin(2w) \\ e^{jw} - e^{-jw} = 2j\sin(w) \end{cases}$$