Linear Algebra & Fourier Analysis

Assignment 01

Total - 75 Marks, Due date: July 21
(You need to answer all questions except the bonus, which will be added if you didn't obtain a full score)

Use this page as the cover page of your assignment.

1. (a) Solve the following system,

$$\begin{cases} x + 3z + 6t &= 0 \\ y + 2z + 5t &= 0 \end{cases}$$

If the right-hand side is zero, we call it a **homogeneous system** like above.

(b) Now, if we change the right-hand side as,

$$\begin{cases} x + 3z + 6t &= 9 \\ y + 2z + 5t &= 7 \end{cases}$$

Can you find the general solution?
Hint: You don't need to solve it explicitly. Think wisely.

(c) So, we solve two systems:

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 6 & 0 \\ 0 & 1 & 2 & 5 & 0 \end{array}\right] \quad \text{and} \quad \left[\begin{array}{cccc|c} 1 & 0 & 3 & 6 & 9 \\ 0 & 1 & 2 & 5 & 7 \end{array}\right]$$

We can consider them as $A\vec{x} = \vec{0}$ and $A\vec{x} = \vec{b}$, where $\vec{b} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$. How can you describe the relation between the solution set of $A\vec{x} = \vec{b}$ and the solution set of $A\vec{x} = \vec{0}$?

(5 Marks)

2. Find the reduced row echelon form of the augmented matrix for the following system:

$$\begin{aligned} 3x + 5y - 2z + 7t &= 4 \\ -6x + y + 3z - 2t &= -1 \\ 9x - 4y - 5z + 8t &= 7 \end{aligned}$$

Identify the pivot columns and determine whether any variables are free. If so, express the solution set in parametric form. (5 Marks)

3. Find the inverse of the following matrix using Gaussian Elimination:

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 2 & 5 & 7 \end{pmatrix}$$

Wait, let's have some fun. I know you apply the four row operation to find the inverse. Now, apply each of those four row operation to the identity matrix I_3 and label them as E_1, E_2, E_3 and E_4. And verify that, $E_4E_3E_2E_1A = I_3$. Can you describe why it happen? And how can you write the inverse matrix A^{-1} in terms of E_1, E_2, E_3 and E_4 matrices. (10 Marks)

4. (a) Find the values of k when the following systems will be consistent:

$$\left(\begin{array}{cc|c} 3 & -4 & k \\ -6 & 8 & 5 \end{array}\right), \left(\begin{array}{cc|c} k & 1 & -2 \\ 4 & -1 & 2 \end{array}\right)$$

(b) Find the value of λ and μ such that the following system of equations have (i) No solution, (ii) Many solution, and (iii) Unique solution (if possible):

$$\begin{cases} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= \mu \end{cases}, \qquad \begin{cases} x + y + z &= 6 \\ y + 2z &= 4 \\ (\lambda - 3)z &= \mu - 10 \end{cases}$$

(5 Marks)

5. Solve the following system of linear equations using Gaussian elimination:

$$\begin{aligned} x + 2y &= 2 \\ 2x + y + z &= 1 \\ 2x + z &= 2 \end{aligned}$$

(5 Marks)

6. Determine whether the following system of equations has a solution.

$$\begin{aligned} 3x + y + 3z &= 2 \\ x + 2z &= -3 \\ 2x + y + z &= 4 \end{aligned}$$

(5 Marks)

7. Solve the following system using Gaussian elimination:

$$\begin{aligned} w + x + y + 2z &= 1 \\ 2x + 2y + z &= 0 \\ 2w + 2y + z &= 1 \end{aligned}$$

(5 Marks)

8. Use row reduction to transform the matrix into RREF.

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

(5 Marks)

9. Solve the following system by Gauss-Jordan Elimination:

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &= 5 \\ 2x_1 - x_2 + 3x_3 &= 2 \\ x_1 + 3x_2 - x_3 &= 3 \end{aligned}$$

(5 Marks)

10. Find the inverse of the following matrix using elementary row operations:

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & -2 & 1 \end{bmatrix}$$

(5 Marks)

11. Is the vector b in the span of the vectors v_1, v_2 ?

$$\mathbf{b} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}, \mathbf{v}_1 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -5 \\ 6 \\ 1 \end{bmatrix}$$

(5 Marks)

12. Determine the values of a for which the following system has no solutions, exactly one solution, or infinitely many solutions

$$\begin{aligned} x + 2y - z &= 3 \\ 3x - y + 2z &= 3 \\ 4x + y + \left(a^2 - 8\right) z &= a + 3 \end{aligned} \tag{5}$$

(5 Marks)

13. Answer the following with True or False, and explain your answer.

- The vector $\mathbf{b} = (1, 2, 3)$ is in the span of the set of vectors

$$\left\{ \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ -5 \\ 0 \end{bmatrix} \right\}$$

- The solution set of the linear system whose augmented matrix is $\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{b} \end{bmatrix}$ is the same as the solution set of the vector equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{b}$.
- Suppose that the augmented matrix $\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & b \end{bmatrix}$ has an inconsistent row. Then either $\mathbf{b}$ can be written as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ or $\mathbf{b} \in \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.
- The span of the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ (at least one of which is nonzero) contains only the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ and the zero vector $\mathbf{0}$.

(5 Marks)

14. Answer in short.

- What is the largest possible number of pivots a 4×6 matrix can have? Why?
- What is the largest possible number of pivots a 6×4 matrix can have? Why?
- How many solutions does a consistent linear system of 3 equations and 4 unknowns have? Why?
- Suppose the coefficient matrix corresponding to a linear system is 4×6 and has 3 pivot columns. How many pivot columns does the augmented matrix have if the linear system is inconsistent?

(5 Marks)

Bonus Question:

1. (a) Let t be an arbitrary real number and let,

$$s_1 = -\frac{3}{2} - 2t$$
$$s_2 = \frac{3}{2} + t$$
$$s_3 = t$$

Is the list (s_1, s_2, s_3) a solution to the linear system for any choice of parameter t? If not, then for which value of t will it become?

$$x_1 + x_2 + x_3 = 0$$
$$x_1 + 3x_2 - x_3 = 0$$

(b) Suppose we have given the null space (remember I introduced it as nullifying vectors which appear inside our general solution) of a matrix which was in the reduced row echelon form:

$$N = \alpha \begin{pmatrix} 7 \\ 0 \\ 0 \\ 8 \\ 0 \\ 0 \\ 3 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 0 \\ 0 \\ 4 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 2 \\ 0 \\ 0 \\ 3 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Now, can you predict that linear system?
Hint: Can you identify the pivot columns? Look carefully at the nullifying vectors and what information they contain (like how they interact with the pivots).

(2+3 Marks)

Best of Luck!

MAT216

Assignment 1

Name: Souvik Barman Ratul

ID: 24121205

Section: 22

CamScanner

Answer to the Q No-1

a)
$$\begin{cases} x+3z+6t=0 \\ y+2z+5t=0 \end{cases}$$

Augmented matrix, $\left(\begin{array}{cccc|c} 1 & 0 & 3 & 6 & 0 \\ 0 & 1 & 2 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

let, $a, b \in \mathbb{R}$

$z=a,\ t=b,$

$x+3z+6t=0$

$\Rightarrow x=-3z-6t$

$\therefore x=-3a-6b$

$y+2z+5t=0$

$\Rightarrow y=-2z-5t$

$y=-2a-5b$

The parametric equation is,

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} -3a-6b \\ -2a-5b \\ a \\ b \end{pmatrix} = \begin{pmatrix} 0+(-3a-6b) \\ 0+(-2a-5b) \\ 0+(a+0b) \\ 0+(0+b) \end{pmatrix}$$

The general equation is,

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \\ 1 \\ 0 \end{pmatrix} a + \begin{pmatrix} -6 \\ -5 \\ 0 \\ 1 \end{pmatrix} b$$

Ans

CamScanner

b) $\begin{cases} x+3z+6t=9 \\ y+2z+5t=7 \end{cases}$

Augmented matrix, $\left(\begin{array}{cccc|c} 1 & 0 & 3 & 6 & 9 \\ 0 & 1 & 2 & 5 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

let,

$a, b \in \mathbb{R}$

$z=a,\ t=b$

now,

$x+3z+6t=9 \Rightarrow x=9-3z-6t \Rightarrow x=9-3a-6b$

$y+2z+5t=7 \Rightarrow y=7-2t-5t \Rightarrow y=7-2a-5b$

The parametric equation,

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 9-3a-6b \\ 7-2a-5b \\ a \\ b \end{pmatrix} = \begin{pmatrix} 9 & -3a & -6b \\ 7 & -2a & -5b \\ 0 & +a & +0b \\ 0 & +0a & +b \end{pmatrix}$$

The general equation,

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \\ 1 \\ 0 \end{pmatrix} a + \begin{pmatrix} -6 \\ -5 \\ 0 \\ 1 \end{pmatrix} b$$

Ans

c) Here,

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 6 & 0 \\ 0 & 1 & 2 & 5 & 0 \end{array}\right] \text{ and } \left[\begin{array}{cccc|c} 1 & 0 & 3 & 6 & 9 \\ 0 & 1 & 2 & 5 & 7 \end{array}\right]$$

we can consider them $A\vec{x}=\vec{0}$ and $A\vec{x}=\vec{b}$ where,

A : A known matrix of size 2×4

$\vec{x} \in \mathbb{R}^4$, $\vec{b}=\begin{bmatrix} 9 \\ 4 \end{bmatrix}$

The first solution is the set of homogeneous system

$$S_h = \{\vec{x} \in \mathbb{R}^4 : A\vec{x}=\vec{0}\}$$

The second solution is the set of non-homogeneous system

$$S_{nh} = \{\vec{x} \in \mathbb{R}^4 : A\vec{x}=\vec{b}\}$$

$\therefore$ The relation is $\Rightarrow S_{nh} = \begin{pmatrix} 9 \\ 7 \\ 0 \\ 0 \end{pmatrix} + S_h$

$\therefore S_{nh} = x_p + S_h$

Ans

Answer to the Q. NO-02

Here,

$$3x+5y-2z+7t=4$$
$$-6x+y+3z-2t=-1$$
$$9x-4y-5z+8t=7$$

Augmented matrix $\left(\begin{array}{cccc|c} 3 & 5 & -2 & 7 & 4 \\ -6 & 1 & 3 & 2 & -1 \\ 9 & -4 & -5 & 8 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$$\sim\left(\begin{array}{cccc|c} 1 & 5/3 & -2/3 & 7/3 & 4/3 \\ -6 & 1 & 3 & 2 & -1 \\ 9 & -4 & -5 & 8 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \left[R_1' = \frac{R_1}{3}\right]$$

$$\sim\left(\begin{array}{cccc|c} 1 & 5/3 & -2/3 & 7/3 & 4/3 \\ 0 & 11 & -1 & 12 & 7 \\ 0 & -19 & 1 & -13 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \left[\begin{array}{l} R_2' = R_2 + 6R_1' \\ R_3' = R_3 - 9R_1' \end{array}\right]$$

$$\sim\left(\begin{array}{cccc|c} 1 & 5/3 & -2/3 & 7/3 & 4/3 \\ 0 & 1 & -1/11 & 12/11 & 7/11 \\ 0 & 0 & -8/11 & 85/11 & 78/11 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \left[\begin{array}{l} R_2'' = \frac{R_2'}{11} \\ R_3'' = R_3'' + 19R_2 \end{array}\right]$$

$$\sim\left(\begin{array}{cccc|c} 1 & 0 & -17/33 & 17/33 & 3/11 \\ 0 & 1 & -1/11 & 12/11 & 7/11 \\ 0 & 0 & -8/11 & 85/11 & 78/11 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \left[R_1'' = R_1 - \frac{5}{3}R_2''\right]$$

$$\sim\left(\begin{array}{cccc|c} 1 & 0 & -17/33 & 17/33 & 3/11 \\ 0 & 1 & -1/11 & 12/11 & 7/11 \\ 0 & 0 & 1 & -85/8 & -39/4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \left[R_3'' = R_3'' \times \left(-\frac{11}{8}\right)\right]$$

The pivot columns are : 1(x), 2(y), 3(z) and free variable are in column 4 where variable t is free.

Let, $a \in \mathbb{R}$ and $a = t$

$$z - \frac{85}{8}a = -\frac{39}{4} \Rightarrow z = -\frac{39}{4} + \frac{85}{8}a.$$

$$y + \frac{1}{11}z + \frac{12}{11}a = \frac{7}{11} \Rightarrow y = \frac{7}{11} + \frac{1}{11}z - \frac{12}{12}a$$

$$\Rightarrow y = \frac{7}{11} + \frac{1}{11} \times \left(-\frac{39}{4} + \frac{85}{8}a\right) - \frac{12}{11}a.$$

$$y = -\frac{1}{4} - \frac{1}{8}a.$$

$$x = -\frac{17}{33}z + \frac{17}{33}a = \frac{3}{11}.$$

$$\Rightarrow x = \frac{17}{33}z + \frac{3}{11} - \frac{17}{33}a$$

$$\Rightarrow x = \frac{17}{33} \times \left(-\frac{39}{4} + \frac{85}{8}a\right) + \frac{3}{11} - \frac{17}{33}a$$

$$\Rightarrow x = -\frac{19}{4} + \frac{119}{24} a$$

The parametric equation

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} -\frac{19}{4} + \frac{119}{24} a \\ -\frac{1}{4} - \frac{1}{8} a \\ -39/4 + 85/8\, a. \\ 0 + a \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} -\frac{19}{4} \\ -\frac{1}{4} \\ -\frac{39}{4} \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{119}{24} \\ -1/8 \\ 85/8 \\ 1 \end{pmatrix} a$$

Ans

Answer to the Q. No-3

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 2 & 5 & 7 \end{bmatrix}$$

$$(A|I) = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 6 & 0 & 1 & 0 \\ 2 & 5 & 7 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0. \\ 0 & 1 & 1 & -2 & 0 & 1 \end{array}\right] \quad \begin{bmatrix} R_2' = R_2 - 2R_1 \\ R_3' = R_3 - 2R_1 \end{bmatrix}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \quad [R_3'' = R_3' - R_2']$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 3 & -3 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \quad [R_1' = R_1 - 3R_3'']$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 1 & -3 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \quad [R_1'' = R_1' - 2R_2'']$$

$$\therefore A^{-1} = \begin{bmatrix} 5 & 1 & -3 \\ -2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

now,

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_5 = \begin{bmatrix} 5 & 1 & -3 \\ -2 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

CamScanner

$AE_1 \rightarrow$ Partial row-reduced matrix.

$E_2E_1A \Rightarrow$ more reduced.

$\therefore E_5E_4E_3E_2E_1A = I_3$

$\Rightarrow (E_5E_4E_3E_2E_1) = A^{-1}$ $\left[A \cdot A^{-1} = I_3\right]$

$$\therefore A^{-1} = \begin{bmatrix} 5 & 1 & -3 \\ -2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Ans

Ans to the Q. No-5

$$\begin{cases} x+2y=2 \\ 2x+y+z=1 \\ 2x+z=2 \end{cases}$$

Augmented matrix $\left(\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 2 & 1 & 1 & 1 \\ 2 & 0 & 1 & 2 \end{array}\right)$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & -3 & 1 & -3 \\ 0 & -4 & 1 & -2 \end{array}\right) \quad \left[\begin{array}{l} R_2' = R_2 - 2R_1 \\ R_3' = R_3 - 2R_1 \end{array}\right]$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 1 & -2/3 & 1 \\ 0 & -4 & 1 & -2 \end{array}\right) \quad \left[R_2'' = (-1)\frac{R_2}{3}\right]$$

$$\sim\left(\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 1 & -\frac{1}{5} & 1 \\ 0 & 4 & -1 & 2 \end{array}\right) \quad \left[R_3'' = (-1)R_3''\right]$$

$$\sim\left(\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 1 & -1/3 & 1 \\ 0 & 0 & \frac{1}{3} & -2 \end{array}\right) \quad \left[R_3''' = R_3'' - 4R_2\right]$$

$$\frac{1}{3}z = -2 \Rightarrow \therefore z = -6$$

$$y - \frac{1}{3}z = 1,$$

$$\Rightarrow y + \frac{6}{3} = 1$$

$$\Rightarrow y + 2 = 1$$

$$\therefore y = -1$$

$$\therefore x + 2y = 2$$

$$x + 2(-1) = 2$$

$$x - 2 = 2$$

$$x = 4$$

$$\therefore \left.\begin{array}{l} x = 4 \\ y = -1 \\ z = -6 \end{array}\right\} \text{Ans}$$

Ans to the Q. NO-6

$$\begin{cases} 3x+y+3z=2 \\ x+2z=-3 \\ 2x+y+z=4 \end{cases}$$

Augmented matrix $\left[\begin{array}{ccc|c} 3 & 1 & 2 & 2 \\ 1 & 0 & 2 & -3 \\ 2 & 1 & 1 & 4 \end{array}\right]$

$$\sim \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 2/3 \\ 1 & 0 & 2 & -3 \\ 2 & 1 & 1 & 4 \end{array}\right] \quad \left[R_1' = \frac{R_1}{3}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 2/3 \\ 0 & -1/3 & 1 & -11/3 \\ 0 & 1/3 & -1 & \frac{8}{3} \end{array}\right] \quad \left[\begin{array}{l} R_2' = R_2 - R_1' \\ R_3' = R_3 - 2R_1' \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 2/3 \\ 0 & -1/3 & 1 & -11/3 \\ 0 & 0 & 0 & -1 \end{array}\right] \quad \left[R_3'' = R_3'' - R_2'\right]$$

The system have no solution. Because it is inconsistent.

Answer to the Q. No-7

$$\begin{cases} w+x+y+2z=1 \\ 2x+2y+z=0 \\ 2w+2y+z=1 \end{cases}$$

Augmented matrix $\left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 1 \\ 0 & 2 & 2 & 1 & 0 \\ 2 & 0 & 2 & 1 & 1 \end{array}\right)$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 1 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & -2 & 0 & -3 & 1 \end{array}\right] \quad [R_3' = R_3 - 2R_1]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1/2 & 0 \\ 0 & -2 & 0 & -3 & -1 \end{array}\right] \quad [R_2' = R_2/2]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1/2 & 0 \\ 0 & 0 & 2 & -2 & -1 \end{array}\right] \quad [R_3'' = R_3' + 2R_2']$$

let, $t \in \mathbb{R}$ and $z = t$.

$2y - 2z = -1 \Rightarrow 2y = -1 + 2z = -1 + 2t$

$\therefore y = -\frac{1}{2} + t$

$x + y + \frac{1}{2}z = 0 \Rightarrow x = -y - \frac{1}{2}z = -\left(-\frac{1}{2} + t\right) - \frac{1}{2}t$

$\Rightarrow x = \frac{1}{2} - t - \frac{1}{2}t$

$\Rightarrow x = -\frac{3}{2}t + \frac{1}{2}$

$\therefore x = \frac{1}{2} - \frac{3}{2}t$

$\therefore w + x + y + 2z = 1$

$\Rightarrow w = 1 - x - y - 2z$

$\Rightarrow w = 1 - \frac{1}{2} + \frac{3}{2}t + \frac{1}{2} - t - 2t$

$\therefore w = 1 - \frac{3}{2}t$

The parametic equation,

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 - t \cdot \frac{3}{2} \\ \frac{1}{2} - t \cdot \frac{3}{2} \\ -\frac{1}{2} + t \\ 0 + t \end{bmatrix}$$

The general equation,

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ -1/2 \\ 0 \end{bmatrix} + \begin{bmatrix} -3/2 \\ -3/2 \\ 1 \\ 1 \end{bmatrix}$$

Ans

Ans to the Q. No-8

Here,

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix} \quad [R_1 \leftrightarrow R_2]$$

$$\sim \begin{bmatrix} 1 & -\frac{7}{3} & \frac{8}{3} & -\frac{5}{3} & \frac{8}{3} & \frac{9}{3} \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix} \quad \left[R_1' = \frac{R_1}{3}\right]$$

$$\sim \begin{bmatrix} 1 & -\frac{7}{3} & \frac{8}{3} & -\frac{5}{3} & \frac{8}{3} & \frac{9}{3} \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 0 & -2 & 4 & -4 & -2 & 6 \end{bmatrix} \quad [R_3' = R_3 - 3R_1]$$

$$\sim \begin{bmatrix} 1 & -\frac{7}{3} & \frac{8}{3} & -\frac{5}{3} & \frac{8}{3} & \frac{9}{3} \\ 0 & 1 & -2 & 2 & \frac{4}{3} & -\frac{5}{3} \\ 0 & -2 & 4 & -4 & -2 & 6 \end{bmatrix} \quad \left[R_2' = \frac{R_2}{3}\right]$$

$$\sim\begin{bmatrix} 1 & 0 & -2 & 3 & \frac{52}{9} & -8/9 \\ 0 & 1 & -2 & 2 & 4/3 & -5/3 \\ 0 & 0 & 0 & 0 & 2/3 & 8/3 \end{bmatrix} \quad \begin{bmatrix} R_1' = R_1 + \frac{7}{3}R_2' \\ R_3'' = R_3 + 2R_2' \end{bmatrix}$$

$$\sim\begin{bmatrix} 1 & 0 & -2 & 3 & \frac{52}{9} & -\frac{8}{9} \\ 0 & 1 & -2 & 2 & \frac{4}{3} & -\frac{5}{3} \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix} \quad \left[R_3''' = R_3'' \times \frac{3}{2}\right]$$

Ans

Ans to the Q. NO - 9

$$\begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ 2x_1 - x_2 + 3x_3 = 2 \\ x_1 + 3x_2 - x_3 = 3 \end{cases}$$

Augmented matrix $\left(\begin{array}{ccc|c} 3 & 2 & 1 & 5 \\ 2 & -1 & 3 & 2 \\ 1 & 3 & -1 & 3 \end{array}\right)$

$$\sim\left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 5/3 \\ 2 & -1 & 3 & 2 \\ 1 & 3 & -1 & 3 \end{array}\right) \quad \left[R_1' = \frac{R_1}{3}\right]$$

$$\sim\left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 5/3 \\ 0 & -7/3 & 7/3 & -4/3 \\ 0 & 7/3 & -4/3 & 4/3 \end{array}\right) \quad \begin{bmatrix} R_2' = R_2 - 2R_1' \\ R_3' = R_3 - R_1' \end{bmatrix}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 5/3 \\ 0 & 1 & -1 & 4/7 \\ 0 & 7/3 & -4/3 & 4/3 \end{array}\right) \left[R_2'' = R_2' \times \left(-\frac{3}{7}\right)\right]$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 5/3 \\ 0 & 1 & -1 & 4/7 \\ 0 & 0 & 1 & 0 \end{array}\right) \left[R_3'' = R_3 - \frac{7}{3} R_2''\right]$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 5/3 \\ 0 & 1 & 0 & 4/7 \\ 0 & 0 & 1 & 0 \end{array}\right) \left[R_2'' = R_2'' + R_3''\right]$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2/3 & 0 & 5/3 \\ 0 & 1 & 0 & 4/7 \\ 0 & 0 & 1 & 0 \end{array}\right) \left[R_1'' = R_1' - \frac{1}{3} R_3''\right]$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 9/7 \\ 0 & 1 & 0 & 4/7 \\ 0 & 0 & 1 & 0 \end{array}\right) \left[R_1''' = R_1'' - \frac{2}{3} R_2'''\right]$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 9/7 \\ 0 & 1 & 0 & 4/7 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$x_3 = 0,\ x_2 = 4/7,\ x_1 = \frac{9}{7}$ Ans

Ans to the Q. No-10

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & -2 & 1 \end{bmatrix}$$

$$[A|I] = \left[\begin{array}{ccc|ccc} 2 & -1 & 4 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 3 & -2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1/2 & 2 & 1/2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 3 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \quad \left[R_1' = \frac{R_1}{2}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1/2 & 2 & 1/2 & 0 & 0 \\ 0 & 3/2 & -1 & -1/2 & 1 & 0 \\ 0 & -1/2 & -5 & -3/2 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{l} R_2' = R_2 - R_1' \\ R_3' = R_3 - 3R_1' \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1/2 & 2 & 1/2 & 0 & 0 \\ 0 & 1 & -2/3 & -1/3 & 2/3 & 0 \\ 0 & -1/2 & -5 & -3/2 & 0 & 1 \end{array}\right] \quad \left[R_2'' = R_2 \times \frac{2}{3}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1/2 & 2 & 1/2 & 0 & 0 \\ 0 & 1 & -2/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & -16/3 & -5/3 & 1/3 & 1 \end{array}\right] \quad \left[R_3' = R_3 + \frac{R_2}{2}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1/2 & 2 & 1/2 & 0 & 0 \\ 0 & 1 & -2/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & 1 & 9/16 & -1/16 & -\frac{3}{16} \end{array}\right] \quad \left[R_3'' = R_3 \times \left(-\frac{3}{1}\right.\right.$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1/2 & 2 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & -1/8 & 5/8 & -\frac{1}{8} \\ 0 & 0 & 1 & 5/16 & -\frac{1}{16} & -\frac{3}{16} \end{array}\right] \left[R_2''' = R_2'' + \left(\tfrac{2}{5}\right) R_3'''\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 7/16 & 5/16 & -1/16 \\ 0 & 1 & 0 & -1/8 & 5/8 & -1/8 \\ 0 & 0 & 1 & 5/16 & -1/16 & -3/16 \end{array}\right] \left[R_1'' = R_1' + \tfrac{1}{2} R_2'''\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3/16 & 7/16 & 5/24 \\ 0 & 1 & 0 & -1/5 & 5/8 & -1/8 \\ 0 & 0 & 1 & 5/16 & -1/16 & -3/16 \end{array}\right] \left[R_1''' = R_1'' - 2R_3'''\right]$$

$$A^{-1} = \begin{bmatrix} -3/16 & 7/16 & 5/24 \\ -1/8 & 5/8 & -1/8 \\ 5/16 & -1/16 & -3/16 \end{bmatrix} \text{ Ans}$$

Ans to the Q. No-11

Here, $b = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$, $V_1 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$, $V_2 = \begin{bmatrix} -5 \\ 6 \\ 1 \end{bmatrix}$

$$\Rightarrow \alpha \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} -5 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3\alpha \\ -2\alpha \\ \alpha \end{bmatrix} + \begin{bmatrix} -5\beta \\ 6\beta \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$$

CamScanner

$$\therefore \left.\begin{aligned} 3\alpha - 5\beta &= 0 \\ -2\alpha + 6\beta &= 4 \\ 1\alpha + 1\beta &= 4 \end{aligned}\right\} \left(\begin{array}{cc|c} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{array}\right)$$

$$\sim \left(\begin{array}{cc|c} 1 & -5/3 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{array}\right) \quad \left[R_1' = \frac{R_1}{3}\right]$$

$$\sim \left(\begin{array}{cc|c} 1 & -5/3 & 0 \\ 0 & 8/3 & 4 \\ 0 & 8/3 & 4 \end{array}\right) \quad \left[\begin{array}{l} R_2' = R_2 + 2R_1' \\ R_3' = R_3 - R_1' \end{array}\right]$$

$$\sim \left(\begin{array}{cc|c} 1 & -5/3 & 0 \\ 0 & 1 & 2/3 \\ 0 & 8/3 & 4 \end{array}\right) \quad \left[R_2'' = R_2' \times \frac{3}{8}\right]$$

$$\sim \left(\begin{array}{cc|c} 1 & 0 & 5/2 \\ 0 & 1 & 3/2 \\ 0 & 8/3 & 4 \end{array}\right) \quad \left[R_1'' = R_1' + \frac{5}{3} R_2''\right]$$

$$\sim \left(\begin{array}{cc|c} 1 & 0 & 5/2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \end{array}\right) \quad \left[R_3' = R_3' - \frac{8}{3} R_2'\right],$$

$\beta = \frac{3}{2}$, $\alpha = 5/2$

If the value of there scalars $\alpha = 5/2$ $\beta = \frac{3}{2}$ the the vector b in in the span of v_1 and v_2 Ans

Ans to the Q. No-12

$$\left.\begin{aligned} x+2y-z&=3 \\ 3x-y+2z&=3 \\ 4x+y+(a^2-8)z&=a+3 \end{aligned}\right\} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 3 \\ 4 & 1 & (a^2-8) & (a+3) \end{array}\right]$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -6 \\ 0 & -7 & (a^2-4) & (a-9) \end{array}\right) \left[\begin{array}{l} R_2' = R_2 - 3R_1 \\ R_3' = R_3 - 4R_1 \end{array}\right]$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -6 \\ 0 & 0 & a^2-9 & (a-3) \end{array}\right) \left[R_3'' = R_3' - R_2'\right]$$

for no solution,

$$a^2-9=0 \Rightarrow a^2=9 \Rightarrow a=\pm 3.$$

$$a-3=0 \Rightarrow a \neq 3 \quad \text{Ans}$$

For many solution,

$$a^2-9=0 \Rightarrow a^2=\pm 3$$

$$a-3=0, \Rightarrow a=3 \quad \text{Ans}$$

For unique solution,

$$a^2-9 \neq 0$$

$$a \neq \pm 3 \quad \text{Ans}$$

Ans to the Q. NO-134

1] 4 is the largest possible number of pivots a 4x6 matrix can have. Because a matrix can have at most one pivot per row

2] 4 is the largest possible of pivots a 6x4 matrix can have. Because a pivot must be in a different column and there are only 4 column.

3] A consistent linear system of 3 equations and 4 unknowns have many solutions. Because there are more variable than equation and one free variable in exist

4] 4. Because the 3 pivots columns came from the coefficient matrix and 1 extra pivot must exist in the Augmented column to make the system inconsistent.

CamScanner

Ans to the Q. No-4

a) Here, $\left(\begin{array}{cc|c} 3 & -4 & K \\ -6 & 8 & 5 \end{array}\right)$

$\sim \left(\begin{array}{cc|c} 1 & -4/3 & K/3 \\ -6 & 8 & 5 \end{array}\right)$ $[R_1' = R_1/3]$

$\sim \left(\begin{array}{cc|c} 1 & -4/3 & K/3 \\ 0 & 0 & 5+2K \end{array}\right)$ $[R_2' = R_2 + 6R_1']$

It will be consistent when it's have many solution. For many solution, $5+2K=0$

$\therefore K = -\frac{5}{2}$ Ans

again, $\left(\begin{array}{cc|c} K & 1 & -2 \\ 4 & -1 & 2 \end{array}\right)$

$\sim \left(\begin{array}{cc|c} 1 & 1/K & -2/K \\ 4 & -1 & 2 \end{array}\right)$ $[R_1' = R_1/K]$

$\sim \left(\begin{array}{cc|c} 1 & 1/K & -2/K \\ 0 & (-1-4/K) & (2+\frac{2}{K}) \end{array}\right)$ $[R_2' = R_2 - 4R_1]$

It will be consistent when it's have many and unique solution.

for many solution,

$(-1-\frac{4}{K}) = 0 \Rightarrow -1 = \frac{4}{K} \therefore K = -4$

$(2+\frac{2}{K}) = 0 \Rightarrow \frac{2}{K} = -2 \therefore K = -1$ Ans

CamScanner

For unique solution,

$$\left(-1-\frac{4}{k}\right)\neq 0 \Rightarrow k\neq -4$$

$$\left(2+\frac{2}{k}\right)\neq 0 \Rightarrow k\neq 1$$ Ans

b) $\begin{cases} x+y+z=6 \\ x+2y+3z=10 \\ x+2y+\lambda z=\mu \end{cases}$ $\begin{cases} x+y+z=6 \\ y+2z=4 \\ (\lambda-3)z=\mu-10 \end{cases}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right)$$

$$\sim\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right) ; \quad \left[\begin{array}{l} R_2'=R_2-R_1 \\ R_3'=R_3-R_1 \end{array}\right]$$

$$\sim\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-16 \end{array}\right) \quad \left[R_3''=R_3'-R_2'\right]$$

For no solution,

$\lambda-3=0 \Rightarrow \lambda=3$

$\mu-16\neq 0 \Rightarrow \mu\neq 16$

For many solution,

$\lambda-3=0 \Rightarrow \lambda=3$

$\mu-16=0 \Rightarrow \mu=6$

For unique solution,

$\lambda-3\neq 0$

$\lambda\neq 3$ Ans

again, $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & (\lambda-3) & (\mu-10) \end{array}\right]$

For no solution,

$\lambda-3=0 \Rightarrow \lambda=3$

$\mu-10\neq 0 \Rightarrow \mu\neq 10$

For many solution,

$\lambda-3=0$, $\lambda=3$

$\mu-10=0$, $\mu=10$

For unique solution,

$\lambda-3\neq 0$

$\therefore \lambda\neq 3$ Ans

Answer to the Q. No-13

1] $\alpha\begin{bmatrix}-1\\3\\0\end{bmatrix}+\beta\begin{bmatrix}2\\-7\\0\end{bmatrix}+\gamma\begin{bmatrix}4\\-5\\0\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$

$\Rightarrow \begin{bmatrix}-\alpha+2\beta+4\gamma\\3\alpha-7\beta-5\gamma\\0\alpha-0\beta-0\gamma\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$

The statment is false.

The third component of all vectors in the set is 0. So it always give a third component of 0. But the third component of b is 3. So b is not in the span of the given vector.

2) Here,

$$x_1 v_1 + x_2 y_2 + x_3 y_3 = b.$$

Augmented matrix.

$$\left[\begin{array}{ccc|c} v_1 & v_2 & v_3 & b \end{array}\right]$$

The statement is true.

3) The statement is false

The system has no solution as the augmented matrix has an inconsistent row,

$$b \notin \text{Span}\{v_1, v_2, v_3\}$$

4) The statement is false.

$$\text{Span}\{v_1, v_2, v_3\} = \{x_1 v_1 + x_2 v_2 + x_3 v_3 \mid x_1, x_2, x_3 \in R\}$$

It includes, → The original vectors v_1, v_2, v_3

→ The zero vector.

→ Infinitely many other combinations,

So the span is much bigger than just the 3 vectors and 0.