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**REPORT: “Universal Robots (UR10)”**

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## Requirements:

1. Use Matlab / Python for implementation.
2. Both FK and IK should be implemented as distinct files.
3. IK function should take into account singularities, workspace limits and possibility of multiple solutions.
4. Your code should contain file with example of the usage of FK and IK functions.
5. Your code should contain file with tests.
6. Report should be of following structure: Description of the robot.
  - Kinematic scheme with description of the parameters.
  - Step by step explanation of inverse kinematics solution.

# I. INTRODUCTION

Universal Robots is a Danish manufacturer of smaller flexible industrial collaborative robot arms. Universal Robots was founded in 2005 by the engineers Esben Østergaard, Kasper Støyer, and Kristian Kassow. During joint research at the Syddansk Universitet Odense, they came to the conclusion that the robotics market was dominated by heavy, expensive, and unwieldy robots. As a consequence they developed the idea to make robot technology accessible to small and medium-sized businesses.

There is a plenty part of this robots such as UR5, UR10, UR3...



## Universal Robot

## II. Forward Kinematic

Forward kinematics specifies the joint parameters and computes the configuration of the chain. For serial manipulators this is achieved by direct substitution of the joint parameters into the forward kinematics equations for the serial chain. For parallel manipulators substitution of the joint parameters into the kinematics equations requires solution of a set of polynomial constraints to determine the set of possible end-effector locations.

For UR10 we used 12 parameters of Denavit-Hartenberg:  
 $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, d, a, \text{ and } \alpha)$ .

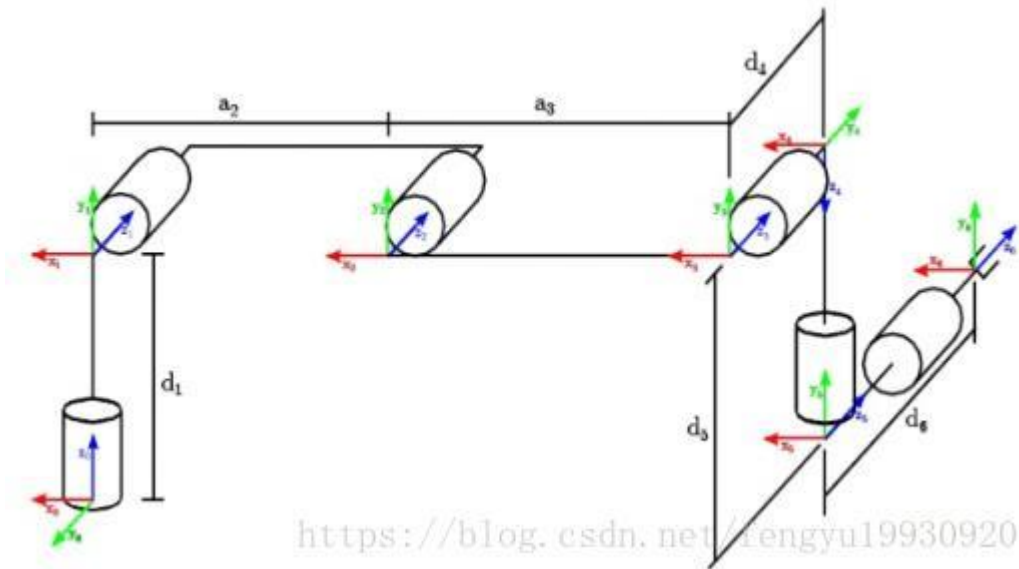
$$A_i = \text{Trans}_x(a_i) \text{Rot}_x(\alpha_i) \text{Trans}_z(d_i) \text{Rot}_z(\theta_i)$$

Where

$$\text{Trans}_x(a_i) = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Rot}_x(\alpha_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \alpha_i & -\sin \alpha_i & 0 \\ 0 & \sin \alpha_i & \cos \alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\text{Trans}_z(d_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Rot}_z(\theta_i) = \begin{bmatrix} \cos \theta_i & -\sin \theta_i & 0 & 0 \\ \sin \theta_i & \cos \theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Coordinate frames for UR10 arm. Joints rotate around the z-axes



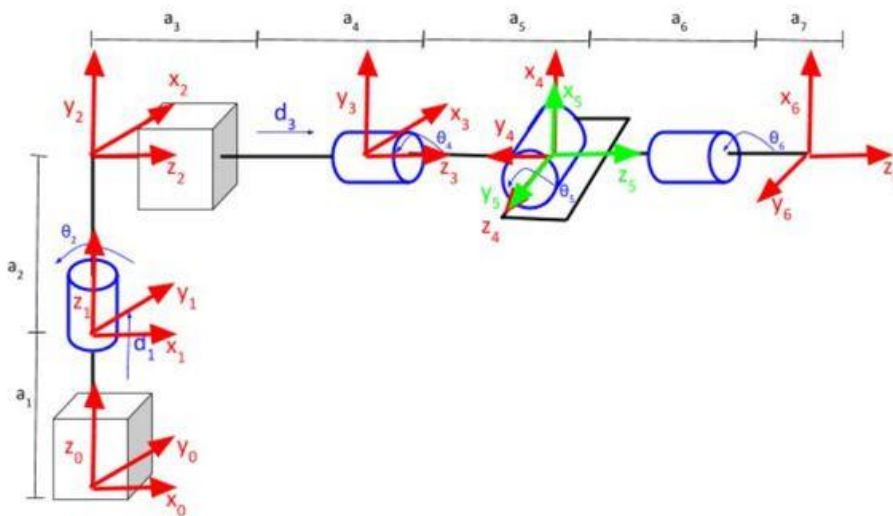
## Denavitt-Hartenberg Parameters

Link	$a_i$	$\alpha_i$	$d_{i0}$	$\theta_{i0}$
1	0	$\pi/2$	0.089	$\theta_1$
2	-0.425	0	0	$\theta_2$
3	-0.392	0	0	$\theta_3$
4	0	$\pi/2$	0.109	$\theta_4$
5	0	$-\pi/2$	0.094	$\theta_5$
6	0	0	0.082	$\theta_6$

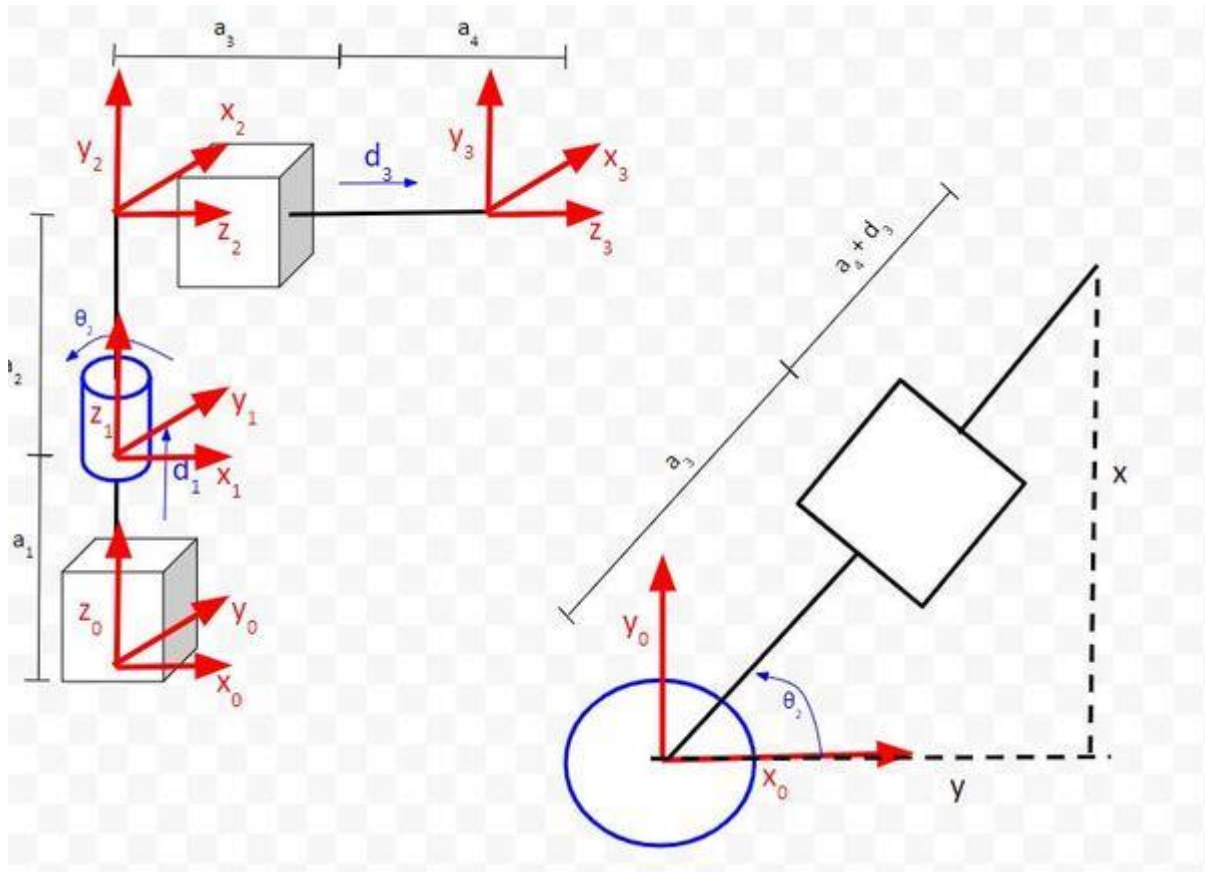
## III. Inverse Kinematic

Inverse kinematics specifies the end-effector location and computes the associated joint angles. For serial manipulators this requires solution of a set of polynomials obtained from the kinematics equations and yields multiple configurations for the chain. The case of a general 6R serial manipulator (a serial chain with six revolute joints) yields sixteen different inverse kinematics solutions, which are solutions of a 17degree polynomial. For parallel manipulators, the specification of the end-effector location simplifies the kinematics equations, which yields formulas for the joint parameters.

For universal robot (UR10), we need to study 6DOF of robot :



The first 3 joints:



we can see that we have two equations that come out of that.

- $\theta_2 = \tan^{-1}(y/x)$
  - $d_3 = \sqrt{x^2 + y^2} - a_3 - a_4$
- ❖ Calculate rot\_mat\_0\_3

$$R_3^0 = \begin{bmatrix} -\sin \theta_2 & 0 & \cos \theta_2 \\ \cos \theta_2 & 0 & \sin \theta_2 \\ 0 & 1 & 0 \end{bmatrix}$$

- ❖ Calculate the inverse of rot\_mat\_0\_3

$$R_6^0 = R_3^0 R_6^3$$

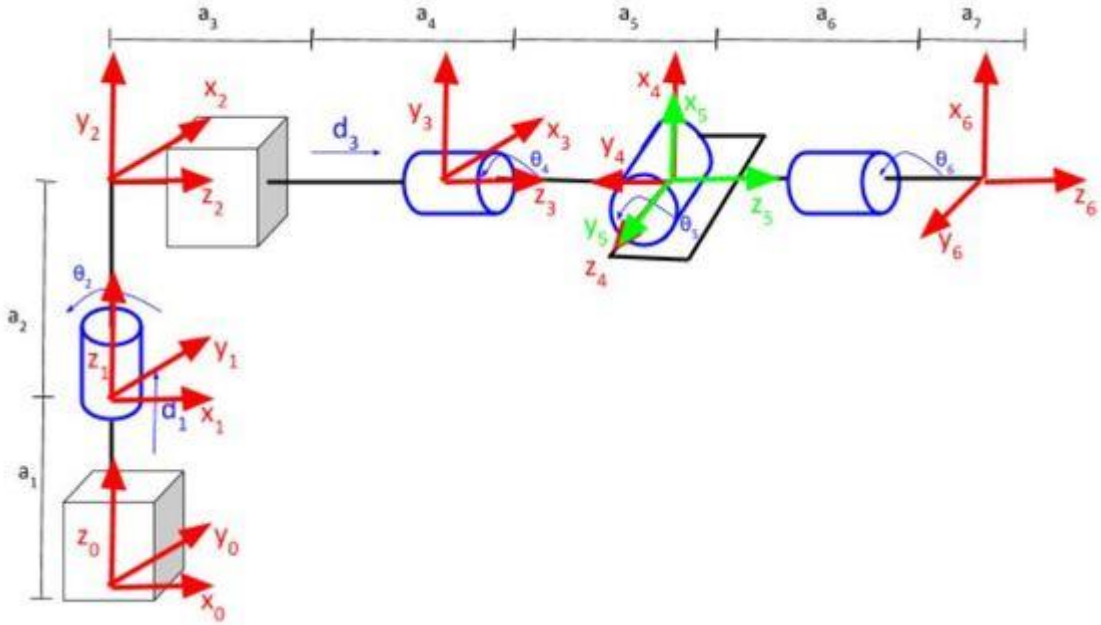
$$R_6^0 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(R_3^0)^{-1} R_6^0 = (R_3^0)^{-1} R_3^0 R_6^3$$

$$R_6^3 = (R_3^0)^{-1} R_6^0$$

❖ Calculate rot\_mat\_3\_6

To calculate the rotation of frame 6 relative to frame 3, we need to go back to the kinematic diagram we drew earlier.



Using either the rotation matrix method or Denavit-Hartenberg, here is the rotation matrix you get when you consider just the frames from 3 to 6.

$$R_6^3 = \begin{bmatrix} -\sin \theta_4 \cos \theta_5 \cos \theta_6 - \cos \theta_4 \sin \theta_6 & \sin \theta_4 \cos \theta_5 \sin \theta_6 - \cos \theta_4 \cos \theta_6 & -\sin \theta_4 \sin \theta_5 \\ \cos \theta_4 \cos \theta_5 \cos \theta_6 - \sin \theta_4 \sin \theta_6 & -\cos \theta_4 \cos \theta_5 \sin \theta_6 - \sin \theta_4 \cos \theta_6 & \cos \theta_4 \sin \theta_5 \\ -\sin \theta_5 \cos \theta_6 & \sin \theta_5 \sin \theta_6 & \cos \theta_5 \end{bmatrix}$$

We want to set a desired position and orientation (relative to the base frame) for the end effector of the universal robot (UR10) and then have the program calculate the servo angles necessary to move the end effector to that position and orientation.

## IV. Conclusion

What I've shown in this tutorial are two popular methods for calculating a universal robots (UR10) on Forward kinematic and Inverse kinematic. There are Denavit-Hartenberg Parameters and Inverse Kinematics Analytical Approach.

By using the method, we can learn clearly how to rotation and translation the robot and especially we can use python to solve the equation by using "Numpy" for known a position and orientation of robot.