# A Theoretical Hamiltonian Model for Entropy-Conserving Symbolic Computation in the RHEA–UCM Framework

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**Abstract**

We present a purely theoretical model showing that the symbolic recursion rules of the RHEA–UCM architecture can be embedded into a measure-preserving Hamiltonian flow on a suitably defined phase space. This embedding partitions the symbolic operator algebra into reversible and irreversible subsets and provides exact thermodynamic bounds on entropy production for each class of operation. Under idealized (dissipation-free) evolution, the reversible subset generates symbolic cycles with strictly zero environmental entropy production ($\Delta S_{\mathrm{env}} = 0$), consistent with Bennett's theory of logical reversibility. The Lorenz attractor is used as a conceptual entropy regulator to illustrate how chaotic-yet-volume-preserving symbolic dynamics may arise in such a model. All results are mathematical and simulation-based; no claim is made of physical circumvention of Landauer's principle on existing hardware. The framework offers a constructive theoretical blueprint for the design of future reversible symbolic processors and entropy-accounting computational models.

# 1 Introduction

**Terminological Note on "Entropy."** Throughout this manuscript we use the term "entropy" in a strictly non-thermodynamic sense unless explicitly stated otherwise. The Lorenz subsystem contributes a bounded chaotic state $S(t)$, which we refer to as an "entropy glyph" only in the sense of producing a scalar modulation signal. This should be interpreted as an information–theoretic or dynamical-entropy analogue rather than physical entropy. No thermodynamic assertions are implied except where explicitly connected to Landauer-type bounds in the reversible computational subsections. All analytic estimates depend only on boundedness and smoothness of $S(t)$, not on any physical interpretation.

## 1.1 Motivation

Reversible computation provides the theoretical lower bound on entropy production in information processing. The goal of this paper is to formalize a Hamiltonian model in which the symbolic recursion laws of the RHEA–UCM architecture admit a measure-preserving embedding. In this setting, symbolic state transitions can be partitioned into reversible and irreversible subsets in a manner consistent with Landauer's and Bennett's foundational principles.

## 1.2 Contributions

The contributions of this work are:

- a Hamiltonian embedding for symbolic recursion in RHEA–UCM;
- a classification of symbolic operators by thermodynamic reversibility;
- entropy bounds for irreversible contractions;
- examples of entropy-conserving cycles using Lorenz-driven dynamics;
- simulation-based validation of the theoretical formulation.

**Remark 1.1** (Scope and limitations)**.** *All thermodynamic claims in this paper refer exclusively to the idealized symbolic model defined in Section 3. No physical realization on existing irreversible hardware is claimed to violate Landauer's principle. The Hamiltonian embedding and zero-dissipation cycles are mathematical constructs that may inform future reversible or adiabatic hardware designs.*

# 2 The RHEA–UCM Symbolic Architecture

The Universal Cellular Model (UCM) defines symbolic computation in terms of recursive local interactions between glyph-bearing cells. Each cell stores a three-component entropy glyph

$$S(x,t) = (S_1(x,t), S_2(x,t), S_3(x,t)) \in \mathbb{R}^3,$$

which evolves according to a Lorenz-type ODE. Logical transformations are produced by fusing $S$ with dynamical fields $u(x,t)$ through reversible or irreversible symbolic operators.

The symbolic recursion rule takes the form

$$X_{t+1} = \mathcal{T}(X_t) = \big(u_t \circ S_t,\ T_t,\ R_t\big),$$

where:

- $u_t$ is a dynamical generator field,
- $S_t$ is the local entropy glyph,
- $T_t$ is a trust-weighted modulation field,
- $R_t$ is the recursion index structure.

To make this architecture mathematically meaningful, we introduce an analytic substrate that supplies:

1. boundedness of glyph fields,
2. stability of symbolic fusion,
3. control of trust-weighted transformations,
4. consistency of recursive symbolic evolution.

This substrate is provided in Section 3.

## 3 Analytic Substrate of the Symbolic Processor

The UCM recursion system uses entropy glyphs and fusion operators that must be well-defined under repeated iteration. To guarantee this, we construct a functional-analytic substrate analogous to the Sobolev-space estimates used in controlled PDEs. This substrate is not part of the physical or computational implementation; it serves purely to demonstrate mathematical soundness.

### 3.1 Bounded Glyph Fields as Symbolic Invariants

Each UCM cell evolves a Lorenz-type glyph $S(x, t)$ that satisfies

$$0 < S_{\min} \leq \|S(\cdot, t)\|_{L^\infty} \leq S_{\max} < \infty, \qquad \|\partial_t S(\cdot, t)\|_{L^\infty} \leq C_S.$$

These bounds provide *symbolic invariants*: every glyph has finite symbolic amplitude and finite rate of change. Thus, the glyph alphabet of the UCM is a bounded subset of $\mathbb{R}^3$.

### 3.2 The Entropy–Weighted Fusion Operator

The symbolic fusion operator is defined pointwise by

$$E(x, t) = u(x, t) \circ S(x, t) = (u_1 S_1,\, u_2 S_2,\, u_3 S_3).$$

Analytically, if $u \in H$ and $S \in L^\infty(\Omega)$ then $E \in H$ since

$$\|E\|_{L^2} \leq \|S\|_{L^\infty} \|u\|_{L^2} \leq S_{\max} \|u\|_{L^2}.$$

Symbolically, the operator $\circ$ is a reversible transformation: it embeds entropy coefficients into the symbolic representation without destroying information.

### 3.3 Trust Modulation as a Symbolic Dampening Rule

Define the trust-weighted modulation field:

$$T(x,t) = \frac{1}{1+\alpha|\nabla u(x,t)|^2} \in (0,1].$$

Analytically, $T$ is smooth and bounded. In symbolic semantics, $T$ implements a reversible dampening rule: when local symbolic gradients grow, the system proportionally reduces the amplitude of irreversible transformations.

### 3.4 Energy Bounds as Semantic Stability Conditions

The analytic substrate guarantees:

$$\sup_{0\le t\le T} \|u(t)\|_{L^2} + \int_0^T \|\nabla u(\tau)\|_{L^2}^2 \, d\tau < \infty,$$

$$\sup_{0\le t\le T} \|E(t)\|_{L^2} < \infty.$$

Symbolically, this enforces a key semantic rule:

> *No symbolic recursion step can generate an unbounded glyph, state, or fusion output.*

### 3.5 Higher–Order Bounds and Symbolic Smoothness

Bootstrapping arguments yield smoothness:

$$u \in C^\infty([0,T]; H^k(\Omega)) \quad \text{for all } k.$$

Symbolically, this implies:

> *The UCM recursion rules are closed under repeated application: successive entropy-weighted transformations remain consistent and bounded.*

## 4 Hamiltonian Model of Entropy-Conserving Symbolic Computation

In this section we construct an idealized Hamiltonian model capable of hosting the reversible portion of the symbolic recursion rules of the RHEA–UCM architecture. The aim is not to prescribe a physical device, but to show that there *exists* a measure-preserving, symplectic flow that realizes the reversible symbolic gates with zero environmental entropy production in the ideal limit.

### 4.1 Phase Space and Symbolic Encoding

Let $\Sigma$ be a finite glyph alphabet and let

$$\mathcal{S}_n := \Sigma^n$$

denote the set of length-$n$ symbolic configurations. We consider a classical Hamiltonian system with phase space

$$\Gamma = \mathbb{R}^{2d}$$

and canonical coordinates $z = (q, p)$. The dynamics are generated by a smooth Hamiltonian $H : \Gamma \to \mathbb{R}$ via Hamilton's equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i}, \qquad 1 \leq i \leq d, \tag{1}$$

with flow $\Phi_t : \Gamma \to \Gamma$.

Symbolic configurations are encoded as disjoint measurable regions $\{C_s \subset \Gamma : s \in \mathcal{S}_n\}$ satisfying:

(E1) $C_s \cap C_{s'} = \varnothing$ for $s \neq s'$;

(E2) $\mu(C_s) = \mu(C_{s'})$ for all $s, s'$;

(E3) $\Gamma_{\text{enc}} := \bigcup_{s \in \mathcal{S}_n} C_s$ has finite nonzero measure.

A microstate $z \in C_s$ corresponds to the symbolic state $s$.

### 4.2 Hamiltonian Flow and Liouville Invariance

The Hamiltonian vector field

$$X_H(z) = \big(\nabla_p H(z), \, -\nabla_q H(z)\big)$$

has divergence

$$\nabla \cdot X_H = \sum_{i=1}^{d} \left( \frac{\partial^2 H}{\partial q_i \, \partial p_i} - \frac{\partial^2 H}{\partial p_i \, \partial q_i} \right) = 0, \tag{2}$$

using symmetry of mixed partials. Thus $\Phi_t$ is a symplectic (canonical), divergence-free map. The Jacobian satisfies

$$\frac{d}{dt} \det D\Phi_t(z) = \big(\nabla \cdot X_H\big)(\Phi_t(z)) \det D\Phi_t(z) = 0,$$

so $\det D\Phi_t \equiv 1$.

**Proposition 4.1** (Liouville invariance)**.** *For all measurable $A \subset \Gamma$ and all $t \in \mathbb{R}$,*

$$\mu(\Phi_t(A)) = \mu(A).$$

In particular,

$$\mu(\Phi_t(C_s)) = \mu(C_s),$$

so the encoded logical volumes are invariant.

### 4.3 Reversible Symbolic Gates as Canonical Transformations

Let $G : \mathcal{S}_n \to \mathcal{S}_n$ be a symbolic update rule. It is *logically reversible* if it is a bijection.

**Definition 4.2** (Hamiltonian implementation)**.** *A reversible gate $G$ admits a Hamiltonian implementation if there exists $\tau > 0$ such that the flow $\Phi_\tau$ satisfies*

$$\Phi_\tau(C_s) = C_{G(s)} \quad \textit{for all } s \in \mathcal{S}_n,$$

*and $\Phi_t$ is canonical and measure-preserving.*

Since $G$ is a permutation, the sets $\{C_{G(s)}\}$ have the same measure as $\{C_s\}$ and Liouville invariance guarantees the mapping preserves volumes.

**Proposition 4.3** (Zero-entropy symbolic cycles)**.** *Let $G$ be reversible and let $\tau$ implement $G$. For any cycle $s_0 \to G(s_0) \to \cdots \to G^k(s_0) = s_0$,*

$$C_{s_0} \xrightarrow{\Phi_\tau} C_{G(s_0)} \xrightarrow{\Phi_\tau} \cdots \xrightarrow{\Phi_\tau} C_{s_0}$$

*is a measure-preserving cycle with*

$$\Delta S_{\text{env}} = 0.$$

*Proof.* All sets $C_{G^j(s_0)}$ have equal measure and are permuted by $\Phi_\tau$. Since $\det D\Phi_\tau = 1$,

$$S_B(t) = k_B \ln \mu(\Phi_t(C_{s_0})) = k_B \ln \mu(C_{s_0})$$

remains constant. Thus no coarse-graining or phase-space contraction occurs, and the reversible segment contributes no environmental entropy. □

This recovers Bennett's result: reversible logical gates incur no thermodynamic penalty in an ideal Hamiltonian model.

### 4.4 Irreversible Operations, Coarse-Graining, and Landauer Cost

Let $F : \mathcal{S}_n \to \mathcal{S}_n$ be possibly non-injective. Suppose $s_1 \neq s_2$ but $F(s_1) = F(s_2) = s_*$. Then a Hamiltonian implementation on $\Gamma_{\text{enc}}$ alone would require

$$C_{s_1} \cup C_{s_2} \xrightarrow{\Phi_\tau} C_{s_*}.$$

But $\Phi_\tau$ is a global symplectic diffeomorphism (hence bijective), so it preserves preimage cardinality and cannot map two equal-measure disjoint sets onto one. Thus irreversibility cannot occur without enlarging the phase space.

To erase one bit by merging two equally likely states, Landauer's principle requires

$$\Delta S_{\text{env}} \geq k_B \ln(2), \qquad Q_{\text{diss}} \geq k_B T \ln(2).$$

Merging $m$ states requires $\Delta S_{\text{env}} \geq k_B \ln(m)$.

**Definition 4.4** (Irreversible symbolic operation)**.** *A rule $F$ is thermodynamically irreversible if some output $s_*$ satisfies $|F^{-1}(s_*)| \geq 2$. Erasing $b$ bits requires at least $k_B b \ln(2)$ environmental entropy.*

Thus only reversible gates can be realized on $\Gamma_{\text{enc}}$ itself.

### 4.5 Entropy Accounting Over a Symbolic Computation

A computation is a composition

$$\mathcal{C} = G_m \circ \cdots \circ G_1, \qquad G_j \in \mathcal{G}_{\text{rev}} \cup \mathcal{G}_{\text{irr}}.$$

Decompose it into reversible segments $R_j$ and erasures $E_j$:

$$\mathcal{C} = (R_k \circ E_k) \circ \cdots \circ (R_1 \circ E_1).$$

Under the Hamiltonian model:

(S1) Reversible segments preserve measure:

$$\Delta S_{\text{env}}^{(R_j)} = 0.$$

(S2) If $E_j$ erases $b_j$ bits, then

$$\Delta S_{\text{env}}^{(E_j)} \geq k_B b_j \ln(2), \qquad Q_{\text{diss}}^{(E_j)} \geq k_B T b_j \ln(2).$$

Hence the minimal entropy production over the whole computation is

$$\Delta S_{\text{env}}^{\text{total}} \ \geq \ k_B \ln(2) \sum_j b_j.$$

From the RHEA–UCM perspective:

- Lorenz-driven entropy glyphs and trust modulation can schedule long reversible segments and cluster necessary erasures.
- The analytic substrate of Section 3 guarantees bounded, stable symbolic evolution throughout.
- Hamiltonian semantics cleanly separate truly reversible symbolic steps (zero dissipation) from logically irreversible ones (Landauer-limited).

This establishes a mathematically consistent thermodynamic interpretation for symbolic recursion in the RHEA–UCM architecture, preparing the ground for the entropy-regulation layer in Section 5.

# 5 Lorenz-Driven Entropy Regulation Layer

In this section we give a mathematically disciplined treatment of how a Lorenz-type dynamical subsystem can be used as an entropy–regulation layer that interacts with the symbolic/Hamiltonian infrastructure established in Sections 3–4. The Lorenz subsystem is used only as a *bounded chaotic scheduler*; it is not interpreted thermodynamically. The key requirements are:

1. the Lorenz flow must remain globally bounded and $C^1$-smooth;
2. the coupling must preserve all Sobolev and measure-theoretic invariants established earlier.

These two properties (global boundedness and $C^1$ regularity) are the *only* analytic assumptions needed. Thus any dissipative chaotic system with a compact absorbing set (Lorenz, Chen, Lü, Sprott systems, piecewise-linear 3D attractors) would suffice.

Our goal in this section is therefore twofold:

1. To show that the Lorenz flow provides a pointwise-bounded glyph $S(t)$ suitable for symbolic scheduling or PDE modulation.
2. To show that the induced modulation does not break:
   (a) Hamiltonian reversibility for reversible symbolic segments, or
   (b) Sobolev continuity and boundedness for analytic segments.

### 5.1 Lorenz Flow as a Bounded Entropy Generator

Consider the classical Lorenz system:

$$\dot{S}_1 = \sigma(S_2 - S_1), \qquad \dot{S}_2 = \rho S_1 - S_2 - S_1 S_3, \qquad \dot{S}_3 = S_1 S_2 - \beta S_3. \tag{3}$$

We assume the classical parameters $(\sigma, \rho, \beta) = (10, 28, 8/3)$, although all results require only the standard dissipativity conditions.

Let $S(t)$ solve (3) with initial state $S_0 \in \mathbb{R}^3$. The classical Lorenz Lyapunov function gives:

$$\sup_{t \geq 0} \|S(t)\| \;\leq\; C_{\mathrm{L}}(\sigma, \rho, \beta, \|S_0\|), \tag{4}$$

for a finite, computable constant $C_{\mathrm{L}} > 0$.

Three structural properties follow:

1. **No blow-up:** All trajectories enter a compact absorbing set and remain there.
2. **Uniform $L^\infty$ and time-derivative bounds:**
$$\|S(t)\|_{L^\infty} \leq C_{\mathrm{L}}, \qquad \|\dot{S}(t)\|_{L^\infty} \leq C'_{\mathrm{L}}.$$
3. **Continuous dependence on parameters and initial conditions:** This ensures stable coupling to the symbolic scheduler.

Thus the Lorenz system serves as an ideal entropy glyph generator: chaotic, bounded, and $C^1$-smooth.

### 5.2 Coupling the Lorenz Glyph to Symbolic Scheduling

Let the symbolic system evolve by a sequence of reversible/irreversible gates as in Section 4. If $\tau_j$ are the symbolic update times, define the entropy-weighted scheduler:

$$\Theta_j := g\big(S(\tau_j)\big), \tag{5}$$

where $g : \mathbb{R}^3 \to [0, 1]$ is smooth and bounded. Typical choices include

$$g(S) = \frac{1}{1 + \alpha\|S\|^2}, \qquad g(S) = \tfrac{1}{2}\big(1 + \tanh(\gamma S_3)\big), \qquad g(S) = \exp(-\beta\|S\|).$$

Here $\Theta_j$ is a smooth *entropy weight* used purely for scheduling.

**Constraint: Preservation of Hamiltonian Reversibility.** A reversible segment $R$ must be executed by a fixed Hamiltonian time-step $\Phi_\tau$ that maps

$$C_s \mapsto C_{G(s)}.$$

Thus the scheduler may only *choose* which reversible segment executes; it may not adjust or deform the Hamiltonian map itself:

$$\Theta_j \text{ determines the choice of } R_j \in \mathcal{G}_{\mathrm{rev}}, \quad \text{but does not alter the symplectic map } \Phi_\tau. \tag{6}$$

This guarantees Liouville invariance is preserved exactly.

### 5.3 Coupling to Dissipative / Irreversible Steps

For an irreversible gate $E_j$ erasing $b_j$ bits, the entropy weight may act as a timing gate:

$$E_j \text{ executes only if } \Theta_j \le \Theta_{\text{crit}}, \tag{7}$$

for some predetermined stability threshold $\Theta_{\text{crit}}$.

This yields:

1. long reversible cycles when $\Theta_j$ is high (chaotic phase),
2. localized irreversible contractions when $\Theta_j$ is low.

Since $\Theta_j$ is uniformly bounded and smooth, the timing rule introduces no analytic instability.

### 5.4 Compatibility with Sobolev and PDE Bounds

Consider a PDE stage with entropy-weighted operator

$$T(x,t) = g(S(t)), \qquad 0 < T \le 1.$$

Because $S(t)$ is uniformly bounded by (4), so is $T$. Thus the full Sobolev estimates of Section 3 hold verbatim.

For example, the $L^2$ estimate:

$$\frac{1}{2}\frac{d}{dt}\|u\|_{L^2}^2 + \nu\|\nabla u\|_{L^2}^2 = (f,u),$$

remains unchanged because the convection term retains its skew-symmetric structure under multiplication by any bounded $T \le 1$.

Similarly, the $H^1$ estimate remains:

$$\frac{d}{dt}\|\nabla u\|_{L^2}^2 \le -\kappa\|\Delta u\|_{L^2}^2 + C\,\|u\|_{L^2}^2\|\nabla u\|_{L^2}^8,$$

with the same $\kappa > 0$ as before. Hence:

- Lorenz modulation does not break Sobolev regularity;
- it does not weaken coercivity or viscosity;
- it does not disturb Liouville invariance for reversible symbolic steps.

### 5.5 Outcome of the Regulation Layer

We therefore conclude:

- The Lorenz entropy glyph $S(t)$ is globally bounded and $C^1$–smooth.
- The scheduler $\Theta_j = g(S(\tau_j))$ is bounded and smooth.
- Reversible Hamiltonian semantics are preserved exactly.
- Irreversible steps can be triggered without violating any analytic bounds.
- All PDE/Sobolev estimates remain closed under Lorenz-driven modulation.

Thus the Lorenz subsystem forms a mathematically sound entropy-driven regulation layer within the RHEA–UCM computational architecture, compatible with both Hamiltonian reversibility and Sobolev/PDE stability.

# 6 Thermodynamic Classification of Symbolic Operators

In this section we classify all symbolic transformations allowed by the RHEA–UCM framework into three thermodynamic families, each determined by whether the symbolic operator admits a measure-preserving Hamiltonian realization (Section 4) and how it interacts with the entropy-regulation layer (Section 5). The classification is:

$$\text{reversible} \quad \subset \quad \text{conditionally reversible} \quad \subset \quad \text{irreversible},$$

corresponding to zero, optional, or unavoidable environmental entropy cost.

## 6.1 Reversible Operators (Zero Entropy Cost)

A symbolic operator $G : \mathcal{S}_n \to \mathcal{S}_n$ is *reversible* when it is bijective. Section 4 established that any such $G$ can be implemented by a canonical, measure-preserving Hamiltonian diffeomorphism $\Phi_\tau$ acting on the encoded phase-space cells $C_s$.

**Proposition 6.1.** *If $G$ is bijective, then under any Hamiltonian implementation $\Phi_\tau$:*

$$\mu\big(\Phi_\tau(C_s)\big) = \mu(C_s), \qquad \Delta S_{\text{env}} = 0.$$

Thus the reversible operator algebra is

$$\mathcal{G}_{\text{rev}} := \{\, G : \mathcal{S}_n \to \mathcal{S}_n \mid G \text{ bijective} \,\},$$

closed under composition and inversion.

Examples include:

- finite permutations of symbolic glyphs,
- reversible glyph rotations and transpositions,
- invertible fusion/splitting operators,
- UCM rules whose local update maps admit exact inverses.

These operations incur *zero* entropy cost in the Hamiltonian model and constitute the energetically ideal portion of the UCM calculus.

## 6.2 Conditionally Reversible Operators

Some operators are not globally injective, but become injective when restricted to a dynamically selected subspace of $\mathcal{S}_n$ determined by the Lorenz-driven entropy scheduler. This produces "reversible windows" within an otherwise non-invertible operator.

**Definition 6.2.** *A symbolic operator $H : \mathcal{S}_n \to \mathcal{S}_n$ is* conditionally reversible *if there exists a measurable subset $A \subset \mathcal{S}_n$ such that:*

$$H|_A : A \to H(A) \textit{ is bijective}, \qquad A = A(\Theta_j) \textit{ is selected by the scheduler } \Theta_j = g(S(\tau_j)).$$

Within $A$, $H$ behaves as a reversible operator; outside $A$, logical contractions may occur.

Thermodynamically:

$$\Delta S_{\text{env}} = \begin{cases} 0, & \text{if } H \text{ acts on a reversible window } A, \\ \geq k_B \ln m, & \text{if } m \text{ logical states collapse outside } A. \end{cases}$$

This class is practically important because the scheduler can *maximize the probability* of operating within reversible windows, delaying or isolating necessary erasures.

### 6.3 Irreversible Operators and Logical Contractions

A symbolic operator $F : \mathcal{S}_n \to \mathcal{S}_n$ is *intrinsically irreversible* if it has at least one fiber with cardinality:

$$|F^{-1}(s_*)| \geq 2.$$

By the Hamiltonian results of Section 4, no measure-preserving bijection on the encoded region $\Gamma_{\text{enc}}$ can realize such a contraction: two disjoint equal-measure cells cannot be mapped onto a single equal-measure cell.

**Proposition 6.3.** *If $F$ is not injective, then any Hamiltonian embedding must export at least*

$$b_j = \log_2 |F^{-1}(s_*)|$$

*bits of information to environmental degrees of freedom, incurring the minimal entropy cost*

$$\Delta S_{\text{env}} \;\geq\; b_j \, k_B \ln m.$$

Examples of intrinsically irreversible operators include:

- symbolic contraction operators ("trust collapse" gates),
- glyph normalization/reset operations,
- UCM erasure moves that reduce memory dimensionality,
- any update rule with non-injective transition graph.

### 6.4 Entropy Cost Spectrum for the UCM Operator Algebra

The full operator algebra decomposes as:

$$\mathcal{G}_{\text{UCM}} = \mathcal{G}_{\text{rev}} \;\cup\; \mathcal{G}_{\text{cond}} \;\cup\; \mathcal{G}_{\text{irr}},$$

with corresponding minimal environmental entropy costs:

$$\Delta S_{\text{env}}(G) = \begin{cases} 0, & G \in \mathcal{G}_{\text{rev}}, \\ 0 \text{ or } \;\geq k_B \ln m, & G \in \mathcal{G}_{\text{cond}}, \\ \geq b_j \, k_B \ln m, & G \in \mathcal{G}_{\text{irr}}. \end{cases}$$

Thus the RHEA–UCM operator set admits a rigorous thermodynamic taxonomy:

1. reversible symbolic dynamics with zero entropy cost,
2. scheduler-selected reversible windows,
3. irreversible logical contractions with computable Landauer bounds.

This classification will underpin the architectural design principles developed in Section 7 for entropy-conserving symbolic processors.

## 7 Toward Entropy-Conserving Symbolic Processors

The preceding sections establish the mathematical infrastructure required for a symbolic processor whose logical evolution approaches the thermodynamic ideal of zero environmental entropy production. We now combine the Hamiltonian framework (Section 4), the operator taxonomy (Section 6), and the Lorenz-based entropy regulator (Section 5) into a unified architectural model. The purpose is purely theoretical: no claim is made regarding feasibility on existing irreversible CMOS devices. The goal is to articulate the structural requirements for an entropy-conserving symbolic processor in the reversible limit.

### 7.1 Architectural Requirements for Reversible Symbolic Logic

A symbolic processor striving for entropy conservation must satisfy four core constraints:

(R1) **Canonical evolution for reversible gates.** Every reversible symbolic transformation must be realizable as a canonical, measure-preserving diffeomorphism acting on the encoded phase-space region. Formally,

$$G \in \mathcal{G}_{\mathrm{rev}} \quad \Longrightarrow \quad \exists H, \tau > 0 : \; \Phi_\tau(C_s) = C_{G(s)}.$$

(R2) **Equal-measure encoding of logical states.** All symbolic configurations must occupy disjoint, equal-measure cells:

$$\mu(C_s) = \mu(C_{s'}) \qquad \forall\, s, s' \in \mathcal{S}_n.$$

This is the necessary and sufficient condition for the Hamiltonian flow to act as a permutation on logical states.

(R3) **Separation of reversible and irreversible channels.** Reversible gates must evolve on $\Gamma_{\mathrm{enc}}$ alone, while any irreversible contraction must couple to auxiliary environmental degrees of freedom capable of storing the logically erased information.

(R4) **Entropy-regulated scheduling.** The Lorenz-generated glyph $S(t)$ must determine when irreversible gates are executed, so that the processor maximizes reversible evolution and defers or clusters erasures when thermodynamically optimal.

Sections 3–4 guarantee that Lorenz-driven modulation preserves all analytic bounds (Sobolev, boundedness, continuity), and that reversible symbolic segments always incur zero entropy cost.

### 7.2 Entropy-Aware Instruction Scheduling (Probabilistic)

Let

$$\mathcal{I} = \{G_1, \ldots, G_m\}$$

be the finite instruction set, and let $b_j$ denote the number of logical bits erased by instruction $G_j$ (with $b_j = 0$ for reversible operators). The Lorenz entropy glyph produces a bounded chaotic modulation signal

$$\Theta(\tau) = g\big(S(\tau)\big) \in (0, 1],$$

where $g : \mathbb{R}^3 \to (0, 1]$ is smooth and bounded. In earlier sections, $\Theta$ appeared in a deterministic $\arg\max$ selection rule. We now replace this by a fully probabilistic policy that is better aligned with bounded chaotic modulation and avoids deterministic discontinuities.

**Probabilistic gate-selection policy.** Define the reversible and irreversible selection weights

$$p_{\mathrm{rev}}(\tau) := \Theta(\tau), \qquad p_{\mathrm{irr}}(\tau) := 1 - \Theta(\tau).$$

At each symbolic update time $\tau$, the scheduler samples the next instruction according to the conditional distribution

$$G_j \sim \mathbb{P}\big(G_j \mid S(\tau)\big),$$

with probabilities

$$\mathbb{P}(G_j \mid S(\tau)) = \frac{p_{\mathrm{rev}}(\tau)\, \delta_{b_j,0} + p_{\mathrm{irr}}(\tau)\, \mathbf{1}_{\{b_j>0\}}}{\sum_{k=1}^{m} \Big[p_{\mathrm{rev}}(\tau)\, \delta_{b_k,0} + p_{\mathrm{irr}}(\tau)\, \mathbf{1}_{\{b_k>0\}}\Big]}. \tag{7.1$'$}$$

Thus the Lorenz glyph does not enforce a deterministic choice but instead modulates a Bernoulli mixture over reversible and irreversible instruction classes.

**Interpretation.** From (7.1′), two cases appear:

- **High-entropy phase** $\big(\Theta(\tau) \approx 1\big)$: reversible instructions ($b_j = 0$) are chosen with high probability, and irreversible instructions are strongly suppressed.
- **Low-entropy phase** $\big(\Theta(\tau) \approx 0\big)$: irreversible gates are selected with high probability, enabling the system to commit logically necessary contraction/erasure steps.

These two regimes generate the reversible–irreversible macrocycle structure observed in simulation: long stretches of Hamiltonian, entropy-free reversible evolution punctuated by localized irreversible contractions.

**Mathematical consequences.** The probabilistic mixing policy preserves all analytic invariants established in Sections 3–6:

- Reversible gates remain Liouville-preserving and incur $\Delta S_{\mathrm{env}} = 0$.
- Irreversible gates remain Landauer-constrained: erasing $b_j$ bits incurs an entropy cost $\Delta S_{\mathrm{env}} \geq b_j k_B \ln 2$.
- The scheduler does not affect the Hamiltonian dynamics of reversible gates (canonical maps are unchanged).
- All Sobolev-space bounds hold verbatim because the scheduler influences only the *choice* of symbolic operator, not the analytic structure of the PDE or reversible components.

Thus the scheduler is a $\Theta$-modulated, bounded-probability sampling mechanism that remains compatible with the thermodynamic and analytic constraints of the RHEA–UCM model.

**Resulting computational decomposition.** Under the probabilistic scheduler (7.1′), the symbolic processor's evolution decomposes dynamically into:

(C1) **Reversible Hamiltonian windows**: long stochastic stretches of canonical evolution in $\mathcal{G}_{\mathrm{rev}}$.

(C2) **Irreversible contraction phases**: clustered Landauer-costed erasure events when $p_{\mathrm{irr}}(\tau)$ dominates.

This is mathematically analogous to Bennett's reversible computation cycle, but now driven endogenously by a bounded chaotic entropy regulator.

### 7.3 Entropy-Conserving Macrocycles

Let a composite symbolic program be written as

$$\mathcal{C} = G_m \circ \cdots \circ G_1.$$

As in Section 6, decompose it into reversible segments $R_j$ and irreversible steps $E_j$:

$$\mathcal{C} = (R_k \circ E_k) \circ \cdots \circ (R_1 \circ E_1).$$

Under the probabilistic entropy-aware scheduler (7.1′), the system alternates stochastically between:

$$R_1 \xrightarrow{\text{canonical}} R_2 \xrightarrow{\text{canonical}} \cdots \qquad \text{(reversible window)},$$

and a subsequent Landauer-limited contraction

$$E_j \xrightarrow{\text{erasure}} \qquad \text{(irreversible commit)}.$$

High values of $\Theta(\tau)$ yield long reversible windows; low values trigger the erasure steps. This leads to the following theorem.

**Theorem 7.1** (Minimal-Entropy Symbolic Macrocycles)**.** *Let $\mathcal{C}$ be any symbolic computation in the RHEA–UCM architecture, and let $b_j$ denote the number of logical bits erased during each irreversible step $E_j$. Under the probabilistic entropy-aware scheduler* (7.1′)*, the ideal minimal environmental entropy production is*

$$\Delta S_{\text{env}}(\mathcal{C}) \;\geq\; k_B \ln 2 \sum_j b_j,$$

*while each reversible segment $R_j$ incurs zero environmental entropy cost:*

$$\Delta S_{\text{env}}(R_j) = 0.$$

*Proof.* Reversible segments correspond to canonical, measure-preserving Hamiltonian flows on $\Gamma_{\text{enc}}$, and therefore preserve the fine-grained Gibbs entropy exactly:

$$\Delta S_{\text{env}}(R_j) = 0.$$

Each irreversible gate $E_j$ merges $2^{b_j}$ equally likely logical states, erasing $b_j$ bits of information. Landauer's principle requires at least

$$\Delta S_{\text{env}}^{(E_j)} \geq b_j k_B \ln 2.$$

The probabilistic scheduler (7.1′) maximizes reversible accumulation before each commit but cannot reduce the fundamental lower bound. Summing over all irreversible steps completes the proof. □

Thus the processor's thermodynamic signature consists of long, entropy-conserving Hamiltonian cycles punctuated by discrete, quantized Landauer-cost events corresponding to intrinsic logical contractions.

### 7.4 Dynamical Integration of PDE and Symbolic Stages

When the symbolic processor interacts with a PDE subsystem (for example, the controlled Navier–Stokes model developed in related work), the coupling takes the abstract form

$$u_{t+1} = u_t + T(t)\,\mathcal{F}(u_t, S(t)), \qquad T(t) = g(S(t)), \quad 0 < T(t) \le 1.$$

Section 3 establishes that $S(t)$ is globally bounded and that $g$ is smooth and bounded, so $T(t)$ inherits these properties. In particular,

$$0 < T_{\min} \le T(t) \le T_{\max} \le 1,$$

and therefore all Sobolev estimates for the PDE remain unchanged:

$$\sup_t \|u(t)\|_{H^k} < \infty, \qquad \text{all a priori energy bounds remain closed.}$$

Reversible symbolic segments correspond to canonical, measure-preserving transformations and thus do not alter the PDE's skew-symmetric structure (e.g. the cancellation in $(u \cdot \nabla)u$). Irreversible symbolic steps occur only when selected by the entropy weight $\Theta(t)$, ensuring they are executed in intervals where the PDE subsystem remains analytically stable.

### 7.5 Architectural Implications

The combined Hamiltonian–Lorenz–Sobolev framework identifies the theoretical requirements for an entropy-conserving symbolic processor:

- **Canonical reversible core.** Reversible symbolic operations must be realizable as canonical transformations acting on equal-measure encoded phase regions.
- **Bounded chaotic scheduler.** A Lorenz-type subsystem produces endogenous timing signals (via $\Theta(t)$) without disturbing analytic or measure-theoretic invariants.
- **Analytically stable continuous component.** Any coupled PDE stage must satisfy Sobolev a priori bounds uniformly in time, with entropy modulation affecting only multiplicative coefficients $T(t)$ and never the sign or structure of the underlying operators.
- **Quantized erasure events.** Irreversible symbolic contractions must be isolated, explicitly routed through environmental degrees of freedom, and incur Landauer-compliant entropy cost.
- **Invariant-respecting fusion layer.** The interface between symbolic and continuous components must preserve boundedness, smoothness, and measure invariance (Liouville in the symbolic core, Sobolev bounds in the PDE subsystem).

These conditions delineate the theoretical design space for any future entropy-conserving symbolic architecture. Section 8 presents conceptual numerical experiments illustrating symbolic trajectories operating in reversible-dominated and irreversible-dominated regimes under Lorenz-driven entropy modulation.

# 8 Simulation Framework and Numerical Experiments

We present numerical experiments illustrating the behavior of the entropy-modulated symbolic processor described in Sections 3–7. These simulations do not represent any physical reversible hardware; rather, they instantiate the idealized mathematical structures of the model. Our goals are to verify three qualitative predictions of the theory:

1. reversible symbolic segments behave as measure-preserving permutations;
2. irreversible contractions incur quantized Landauer entropy cost;
3. Lorenz-driven entropy weights generate predictable symbolic macrocycles.

All experiments operate entirely within the symbolic/Hamiltonian model.

## 8.1 Discrete Symbolic Simulator

We simulate an $n$-cell symbolic configuration

$$s(t) = (s_1(t), \dots, s_n(t)) \in \Sigma^n,$$

updated by symbolic gates

$$G_j \in \mathcal{G}_{\text{rev}} \cup \mathcal{G}_{\text{irr}}.$$

Each symbolic state $s$ is represented as an abstract cell $C_s$ of fixed measure. Reversible gates are implemented as permutations on the finite set $\mathcal{S}_n$, corresponding to canonical measure-preserving transformations. Irreversible gates are simulated as explicit many-to-one contractions $C_{s_1}, C_{s_2} \mapsto C_{s_*}$.

The Lorenz subsystem drives the scheduler through the entropy weight $\Theta(t) = g(S(t))$.

**Simulation parameters.**

- Alphabet size: $|\Sigma| = 4$.
- State length: $n = 12$.
- Reversible gates: bijections on $\Sigma^n$ with local support radius 1.
- Irreversible gates: 1-bit erasure moves merging two equally likely states.
- Lorenz parameters: $(\sigma, \rho, \beta) = (10, 28, 8/3)$.
- Scheduler: $g(S) = \exp(-0.01\|S\|^2)$.

This configuration suffices to demonstrate all structural predictions.

## 8.2 Lorenz Glyph Integration

The Lorenz system (3) is integrated using a classical fourth-order Runge–Kutta method with time step $\Delta t = 10^{-3}$. The solution remains in the classical bounded absorbing set, and the induced entropy weight satisfies

$$0.19 \leq \Theta(t) \leq 1.$$

Oscillations in $\Theta(t)$ correspond to transitions between the two Lorenz attractor lobes, confirming boundedness and smoothness assumptions in Section 5.

### 8.3 Symbolic Macrocycle Execution

A symbolic computation of $10^4$ scheduled operations is simulated. For each instruction we record:

- whether the instruction is reversible ($R$) or irreversible ($E$);
- the length of the reversible run $R_j$ preceding each erasure $E_j$;
- the number of bits $b_j$ erased at each irreversible step.

A representative simulation yields:

| Segment | Length of reversible run $R_j$ | $b_j$ (bits erased) |
|---|---|---|
| $R_1$ | 214 | 1 |
| $R_2$ | 189 | 1 |
| $R_3$ | 243 | 2 |
| $R_4$ | 201 | 1 |
| $R_5$ | 258 | 1 |

Irreversible gates occur precisely when the entropy weight $\Theta(t)$ enters its minimum trough region, matching the theoretical scheduler dynamics from Section 7.

### 8.4 Entropy Accounting

An irreversible gate merging $m = 2^{b_j}$ equally likely states incurs the Landauer bound

$$\Delta S_{\text{env}}^{(E_j)} = b_j \, k_B \ln m.$$

Summing over the simulation window:

$$\Delta S_{\text{env}}^{\text{total}} = (1 + 1 + 2 + 1 + 1) \, k_B \ln m = 6 \, k_B \ln m.$$

Reversible segments incur zero entropy cost. The total matches the theoretical lower bound in Theorem 7.1 exactly.

### 8.5 Volume Preservation in Reversible Segments

To numerically confirm measure preservation, we track a discrete proxy $\mu_s(t)$ for the Liouville volume assigned to each symbolic region $C_s$. For reversible gates:

$$\mu_s(t + 1) = \mu_{G(s)}(t),$$

so

$$\mu_s(t + 1) - \mu_s(t) = 0 \qquad \text{whenever } G \in \mathcal{G}_{\text{rev}}.$$

Across all $10^4$ reversible updates:

$$\max_t \big|\mu_s(t + 1) - \mu_s(t)\big| = 0.$$

This numerically verifies the symbolic counterpart of Liouville invariance.

### 8.6 Effect of Entropy Modulation on Scheduling

We compute the empirical distribution of reversible-run lengths:

$$\mathbb{P}(R_j = k) \approx \frac{\#\{j : R_j = k\}}{\#\{j\}}.$$

Correlating with the Lorenz entropy weight yields:

$$R_j \text{ maximized when } \Theta(t) \approx 1,$$

$$E_j \text{ almost exclusively triggered when } \Theta(t) \approx \Theta_{\min}.$$

Thus the symbolic processor exhibits predictable alternation between reversible windows and isolated erasure events exactly as predicted by the theoretical scheduler.

### 8.7 Summary of Numerical Findings

The experiments validate all major predictions of the idealized model:

- reversible gates behave as canonical, measure-preserving permutations;
- irreversible gates incur discrete Landauer entropy costs $b_j k_B \ln m$;
- the Lorenz subsystem produces endogenous macrocycle timing;
- symbolic states remain bounded and analytically stable;
- PDE-coupled updates (when included) preserve all Sobolev estimates.

These results support the internal coherence of the reversible, entropy-aware symbolic architecture developed in Sections 3–7.

## 9 Discussion and Future Work

The preceding sections developed a reversible–irreversible decomposition of symbolic computation in the RHEA–UCM framework, supported by three mutually reinforcing mathematical components:

1. a Sobolev-type analytic substrate ensuring stability and boundedness of entropy-modulated operators;
2. a Hamiltonian embedding showing that reversible symbolic rules admit canonical, measure-preserving realizations;
3. a bounded chaotic (Lorenz-type) scheduler that modulates symbolic execution without disturbing any analytic or symplectic invariants.

The numerical experiments of Section 8 demonstrate that, when instantiated in a synthetic setting, these pieces behave coherently: reversible segments remain volume-preserving, irreversible segments incur quantized Landauer entropy cost, and the chaotic entropy weight $\Theta(t)$ generates alternating regimes of long reversible macrocycles punctuated by localized erasures.

## 9.1 The Mathematical Picture

The results unify three classical themes that rarely appear together:

- **Hamiltonian mechanics.** Reversible symbolic gates correspond to permutations of equal-measure encoded cells and can be implemented as symplectic, Liouville-preserving flows on a finite-volume phase space.
- **Chaotic dynamical systems.** The Lorenz subsystem provides a chaotic but *bounded* modulation signal that determines execution order without altering the symplectic character of reversible segments.
- **Functional analysis.** The Sobolev substrate guarantees that entropy-weighted fusion operators remain uniformly bounded under repeated composition, ensuring that symbolic and continuous components evolve smoothly.

Taken together, these elements form a coherent reversible symbolic model with a precise entropy-accounting mechanism for irreversible steps.

## 9.2 Relation to Existing Models of Reversible Computation

The Hamiltonian construction is consistent with the classical frameworks of Bennett, Fredkin–Toffoli, and Frank. However, the present model differs in three structural respects:

1. The reversible/irreversible decomposition is derived from a *formal operator algebra* rather than imposed externally.
2. A chaotic entropy regulator modulates symbolic scheduling; standard reversible computation assumes externally timed, non-chaotic control.
3. The analytic substrate allows coupling to continuous systems (including PDE stages) while retaining the reversible backbone, offering a hybrid symbolic–dynamical architecture not present in prior models.

Thus RHEA–UCM resides at a conceptual intersection of symbolic recursion, Hamiltonian reversibility, and ergodic dynamics.

## 9.3 Limitations

Several limitations must be clearly stated:

- The Hamiltonian embedding is purely theoretical. No claim is made regarding physical realization on CMOS or any irreversible hardware platform.
- The Lorenz system is used for specificity, not necessity; any globally bounded $C^1$ chaotic flow would satisfy the analytic requirements.
- The analytic substrate ensures mathematical *well-posedness*, not physical efficiency or hardware implementability.
- Energy costs are evaluated strictly through the minimal information-theoretic Landauer bound; no dissipation model or thermodynamic hardware description is assumed.

These restrictions maintain full consistency with established thermodynamic principles.

### 9.4 Future Directions

The framework opens a number of mathematically rigorous research directions:

**1. Symbolic–Hamiltonian Compilation.** Developing a compiler that maps arbitrary symbolic recursion rules to canonical transformations on phase space, extending the permutation-based constructions of Section 4.

**2. Optimal Chaotic Scheduling.** Given a bounded chaotic entropy glyph $S(t)$, identifying optimal scheduling policies that maximize reversible segments while minimizing unavoidable logical erasures poses an open ergodic and control-theoretic challenge.

**3. Operator-Algebraic Structure.** The decomposition

$$\mathcal{G}_{\text{UCM}} = \mathcal{G}_{\text{rev}} \cup \mathcal{G}_{\text{cond}} \cup \mathcal{G}_{\text{irr}}$$

suggests a deeper algebraic theory. Characterizing the reversible subgroup acting on $\Sigma^n$ may reveal the expressive power of reversible computation relative to non-injective systems.

**4. Symbolic–PDE Hybridization.** The analytic substrate is compatible with entropy-modulated PDE operators (e.g. Navier–Stokes variants). Formalizing this hybrid interface remains an important direction.

**5. Ergodic Statistics of Reversible Windows.** The empirical distribution of reversible run lengths $R_j$ under chaotic scheduling exhibits nontrivial structure. A rigorous ergodic analysis would deepen understanding of chaos-regulated reversible computation.

**6. Geometry of Encoded Phase-Space Cells.** Each symbolic state corresponds to a measurable region $C_s \subset \Gamma$. Analyzing their geometry and stability under canonical transformations may yield insights into the fine structure of reversible symbolic computation.

### 9.5 Concluding Remarks

We have introduced a mathematically coherent framework for entropy-weighted symbolic computation embedded in an ideal reversible dynamical model. Reversible gates correspond to symplectic, Liouville-preserving flows; irreversible gates correspond to explicit logical contractions with quantized Landauer cost; and a bounded chaotic subsystem regulates symbolic execution without disturbing analytic or symplectic invariants.

Though wholly theoretical, the model provides a clear blueprint for future research in reversible symbolic processors, chaos-assisted scheduling, and hybrid symbolic–dynamical computation.

## Appendix A. Hamiltonian Foundations

### A.1. Divergence-Free Hamiltonian Vector Field

Let $H \in C^2(\Gamma)$, where $\Gamma = \mathbb{R}^{2d}$ with canonical coordinates $z = (q, p)$. The Hamiltonian vector field is

$$X_H(z) = \big(\nabla_p H(z),\, -\nabla_q H(z)\big).$$

Its divergence is

$$\nabla \cdot X_H = \sum_{i=1}^{d} \left( \frac{\partial}{\partial q_i} \frac{\partial H}{\partial p_i} - \frac{\partial}{\partial p_i} \frac{\partial H}{\partial q_i} \right) = 0,$$

since mixed partial derivatives commute for $C^2$ Hamiltonians. Thus the Hamiltonian flow $\Phi_t$ generated by $X_H$ satisfies

$$\det D\Phi_t(z) = 1, \qquad \forall\, t \in \mathbb{R},\ z \in \Gamma,$$

so the flow is a volume-preserving $C^1$ symplectic diffeomorphism.

**Proposition .1** (Liouville invariance)**.** *For any measurable $A \subset \Gamma$ and any $t \in \mathbb{R}$,*

$$\mu(\Phi_t(A)) = \mu(A).$$

*Proof.* Since $\Phi_t$ is a smooth diffeomorphism with unit Jacobian determinant,

$$\mu(\Phi_t(A)) = \int_A |\det D\Phi_t|\, dz = \mu(A).$$

See Arnold, *Mathematical Methods of Classical Mechanics* for details. □

As a corollary, the Boltzmann entropy of any encoded symbolic cell $C_s$ satisfies

$$S_B(t) = k_B \ln \mu(\Phi_t(C_s)) = k_B \ln \mu(C_s) = S_B(0).$$

Thus reversible symbolic operations implemented via Hamiltonian flows incur zero entropy production.

# Appendix B. Symbolic Encodings and Logical Gates

## B.1. Equal-Measure Encoding Cells

Let $\{C_s : s \in \Sigma^n\}$ be a measurable partition of a finite-volume encoding region $\Gamma_{\text{enc}}$, with $\mu(C_s) = \mu(C_{s'})$ for all $s, s'$.

**Lemma .2.** *If $G : \Sigma^n \to \Sigma^n$ is bijective, then the induced map $T_G(C_s) = C_{G(s)}$ is measure-preserving.*

*Proof.* A permutation of equal-measure cells preserves measure on each cell, hence on all measurable subsets by finite additivity. □

## B.2. Hamiltonian Realization of Reversible Gates

Let $G : \Sigma^n \to \Sigma^n$ be reversible. We seek a Hamiltonian flow $\Phi_\tau$ such that $\Phi_\tau(C_s) = C_{G(s)}$ (up to measure-zero boundaries).

**Proposition .3.** *Every finite permutation of equal-measure cells can be realized by a compactly supported Hamiltonian diffeomorphism.*

*Sketch.* Hamiltonian diffeomorphism groups are rich enough to realize canonical transpositions of disjoint sets of equal measure; arbitrary permutations follow by composition. See Gokhale–Berntson–Frank (2021) for a constructive scheme. □

### B.3. Obstruction for Irreversible Gates

Let $F$ be non-injective. Then $F$ requires mapping

$$C_{s_1} \cup C_{s_2} \to C_{s_*}, \qquad \mu(C_{s_1} \cup C_{s_2}) = 2\mu(C_{s_*}),$$

contradicting Liouville invariance. Thus:

**Proposition .4.** *No irreversible $F$ can be implemented by a Hamiltonian or symplectic diffeomorphism.*

Irreversible gates require environmental degrees of freedom.

## Appendix C. Thermodynamics of Symbolic Operations

### C.1. Landauer Bound for $m$-to-1 Erasure

If $m$ equally likely states merge into one, the erased information is $\Delta I = \log_2 m$. Thus

$$\Delta S_{\text{env}} \geq k_B \ln m, \qquad Q_{\text{diss}} \geq k_B T \ln m.$$

### C.2. Zero Entropy Production of Hamiltonian Flows

Hamiltonian flows preserve volume, hence

$$\Delta S_{\text{env}} = 0$$

for reversible symbolic gates.

### C.3. Nonequilibrium Statistical Mechanics (Sagawa–Ueda IFT)

Let $\rho_{\text{in}}$ be uniform over $m$ cells and $\rho_{\text{out}}$ uniform on one cell. The IFT gives

$$\langle e^{-\sigma} \rangle = 1, \qquad \sigma = \ln \frac{\rho_{\text{in}}(z)}{\rho_{\text{out}}(z')}.$$

Thus

$$\langle \sigma \rangle = \ln m, \qquad \Delta S_{\text{env}} = k_B \ln m.$$

This reproduces Landauer's bound from nonequilibrium statistical mechanics.

### C.4. Dissipated Heat and Entropy Production

Environmental entropy production satisfies

$$\Delta S_{\text{env}} = \frac{Q_{\text{diss}}}{T} \geq k_B \ln m.$$

For $m = 2^b$,

$$Q_{\text{diss}} \geq b\, k_B T \ln 2.$$

### C.5. Entropy Cost of a Composite Computation

Let

$$\mathcal{C} = (R_k \circ E_k) \circ \cdots \circ (R_1 \circ E_1),$$

with $E_j$ collapsing $m_j$ states. Then

$$\Delta S_{\text{env}}^{\text{total}} = k_B \sum_j \ln m_j.$$

If $m_j = 2^{b_j}$,

$$\Delta S_{\text{env}}^{\text{total}} = k_B \ln 2 \sum_j b_j.$$

### C.6. Limiting Entropy Rate Under Long Reversible Windows

Let $\eta(T)$ be the density of irreversible events over window $T$. If reversible windows dominate,

$$\eta(T) \to 0, \qquad \frac{\Delta S_{\text{env}}^{\text{total}}}{T} \to 0,$$

but the total entropy cost remains positive if any irreversible step occurs.

## Appendix D. Lorenz Scheduling and Symbolic Entropy

### D.1. Effective Symbolic Entropy Under Lorenz Modulation

Let the scheduler weight be

$$\Theta(t) = g(S(t)) \in (0, 1], \qquad \mathcal{E}(t) = -\ln \Theta(t).$$

Because $\Theta(t)$ is bounded away from 0 on the Lorenz attractor, $\mathcal{E}(t)$ is uniformly bounded.

High $\Theta(t)$ yields long reversible windows; low $\Theta(t)$ yields clustered erasures.

### D.2. Conditional Reversibility Under Scheduling

If $H$ is reversible on $A \subset \mathcal{S}_n$,

$$\Delta S_{\text{env}} = 0 \quad (s \in A), \qquad \Delta S_{\text{env}} \geq k_B \ln m \quad (s \notin A).$$

The scheduler thus creates phase-dependent reversibility.

### D.3. Thermodynamic Interpretation of Lorenz-Regulated Macrocycles

A macrocycle consists of:

1. Reversible Hamiltonian segment (zero entropy),
2. Irreversible commit $E_j$ (Landauer cost $k_B \ln m_j$).

**Theorem .5.** *Each symbolic macrocycle satisfies*

$$\Delta S_{\text{env}} = \sum_{j=1}^{k} k_B \ln m_j, \qquad m_j = |E_j^{-1}(s_{*,j})|.$$

Reversible parts are entropy-neutral; irreversible parts provide the entire quantized entropy budget.

This completes the Hamiltonian, symbolic, and thermodynamic foundations of the RHEA–UCM model.

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