# The RHEA-$\Lambda$ Reversible Gate Family: A Multi-Radix Binary–Ternary–Pentary Cell for Hamiltonian Symbolic Computation in the RHEA-UCM Framework

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**Abstract**

This paper introduces the *RHEA*-$\Lambda$ family of reversible multi-radix logic gates developed for the RHEA-UCM (Recursive Homeostatic Evolutionary Algorithm – Universal Cell Model) symbolic computation framework. A single physical gate topology supports binary, ternary, and pentary modes, with a 3-bit glyph/entropy register enabling local symbolic memory and internal phase evolution. In higher-radix modes the transitions form strictly bijective, triangular maps over $\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_5$ and $\mathbb{Z}_5^3$, providing discrete Hamiltonian, measure-preserving dynamics aligned with the RHEA-UCM entropy model. Binary mode retains CMOS drop-in compatibility for hybrid reversible– irreversible scheduling under the Lorenz entropy controller. A full mathematical specification, inverse mappings, commutative diagrams, truth tables, and verification code are provided.

## 1 Introduction

Reversible computation provides the only known mechanism for computation consistent with the thermodynamic requirement that information erasure produces irreducible heat via $kT \ln 2$. The RHEA-UCM framework embeds symbolic recursion into Hamiltonian dynamics, ensuring divergence-free computation except at explicitly scheduled entropy events.

To bridge theory to hardware, a unified reversible computational cell is required. The RHEA-$\Lambda$ gate introduced here satisfies four constraints:

1. Binary behavior for CMOS compatibility.
2. Reversible ternary operations over $\mathbb{Z}_3$.
3. Reversible pentary operations over $\mathbb{Z}_5$.
4. Local glyph/entropy register supporting RHEA symbolic memory.

This produces the first reversible gate topology explicitly aligned with RHEA-UCM symbolic Hamiltonian flows.

## 2 Definition of the RHEA-$\Lambda$ Gate Family

The gate processes a triple $(A, B, G)$ where:

$$A, B \text{ are operands,} \qquad G \text{ is a glyph/entropy register.}$$

A mode parameter $M \in \{00, 01, 10\}$ selects:

$$\begin{cases} M = 00 & \text{Binary irreversible face (CMOS)} \\ M = 01 & \text{Ternary reversible face } (\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_5) \\ M = 10 & \text{Pentary reversible face } (\mathbb{Z}_5^3) \end{cases}$$

## 3 Binary $\Lambda_2$ Gate

Define the reversible binary operator:

$$A' = A, \qquad B' = A \oplus B, \qquad G' = G \oplus B.$$

Inverse:

$$A = A', \quad B = A' \oplus B', \quad G = G' \oplus B.$$

Truth table is included in Table 1.

| $A$ | $B$ | $G$ | $A'$ | $B'$ | $G'$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 |

Table 1: Truth table of the RHEA-$\Lambda_2$ gate.

## 4 Ternary $\Lambda_3$ Gate

For $A, B \in \mathbb{Z}_3$ and $G \in \mathbb{Z}_5$:

$$A' = A, \qquad B' = (B + A) \bmod 3, \qquad G' = (G + B) \bmod 5.$$

Inverse:

$$A = A', \qquad B = (B' - A') \bmod 3, \qquad G = (G' - B) \bmod 5.$$

$$(A, B, G) \xrightarrow{\Lambda_3} (A', B', G')$$

$$(A', (B' - A') \bmod 3, (G' - B) \bmod 5)$$

## 5 Pentary $\Lambda_5$ Gate

For $A, B, G \in \mathbb{Z}_5$:

$$A' = A, \qquad B' = (B + A) \bmod 5, \qquad G' = (G + B) \bmod 5.$$

Inverse:

$$A = A', \qquad B = (B' - A') \bmod 5, \qquad G = (G' - B) \bmod 5.$$

## 6 Hamiltonian Interpretation

All $\Lambda$ gates are triangular, so their Jacobian determinant is:

$$\det D\Lambda = 1.$$

Thus they are finite-state analogues of discretized symplectic maps. Binary mode corresponds to explicitly scheduled entropy release.

## 7 Python Reversibility Verification

```
def ternary_step(A,B,G): return A,(B+A)%3,(G+B)%5
def pentary_step(A,B,G): return A,(B+A)%5,(G+B)%5
def check(step,dom):
    seen={}
    for A in range(dom[0]):
      for B in range(dom[1]):
        for G in range(dom[2]):
          out=step(A,B,G)
          if out in seen: print("Collision:",out); return
          seen[out]=(A,B,G)
    print("Bijective over",len(seen),"states.")
```

## 8 Conclusion

The RHEA-$\Lambda$ family provides a unifying reversible gate topology that is multi-radix, strictly bijective, Hamiltonian-compatible, and ready for implementation across CMOS or emerging reversible technologies.

## References


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