Delivery Problem

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Outline

Problem Statement

Brute Force Search

Nearest Neighbor

Branch and Bound

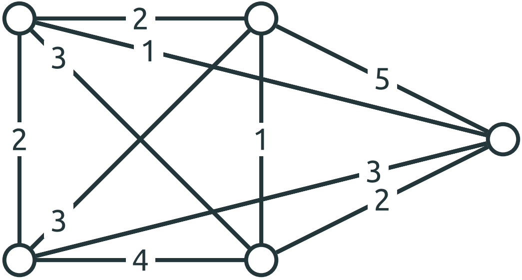
Dynamic Programming

Approximation Algorithm

Local Search

Traveling Salesman Problem

Given a complete weighted graph, find a cycle (or a path) of minimum total weight (length) visiting each node exactly once



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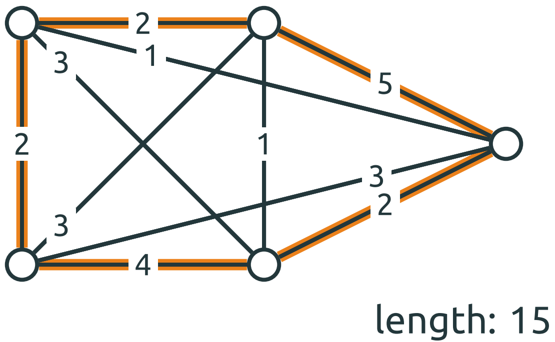

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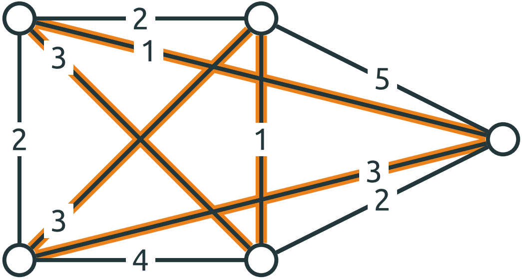

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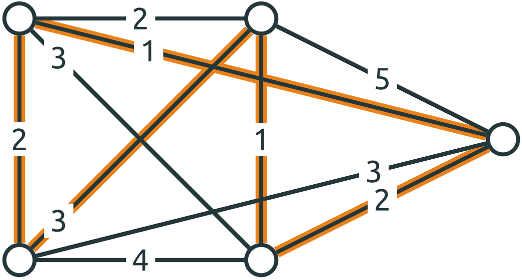

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- No polynomial time algorithms known
- Goal of this project: develop efficient programs for solving TSP problem

Delivering Goods

Need to visit several points. What is the optimal order of visiting them?

Traveling


Victoria
Seattle
WASHINGTON
MONTANA
NORTH DAKOTA
MINNESOTA
Québec City
NEW BRUNSWICK
Portland
Minneapolis
Ottawa
Montreal
MAINE
WISCONSIN
SOUTH DAKOTA
OREGON
IDAHO
MICHIGAN
Toronto
VERMONT
NOVA SC
WYOMING
NEW HAMPSHIRE
NEW YORK
Detroit
Chicago
MASSACHUSETTS
IOWA
NEBRASKA
CT
RI
New York
PENNSYLVANIA
ILLINOIS
INDIANA
OHIO
Philadelphia
Denver
United States
NEVADA
NJ
Kansas City
Indianapolis
MARYLAND
UTAH
DE
Sacramento
COLORADO
KANSAS
MISSOURI
WEST VIRGINIA
San Francisco
KENTUCKY
VIRGINIA
San Jose
CALIFORNIA
Las Vegas
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NORTH CAROLINA
Charlotte
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ARIZONA
NEW MEXICO
Atlanta
SOUTH CAROLINA
Phoenix
MISSISSIPPI
San Diego
Dallas
ALABAMA
GEORGIA
Bermuda
Tucson
El Paso
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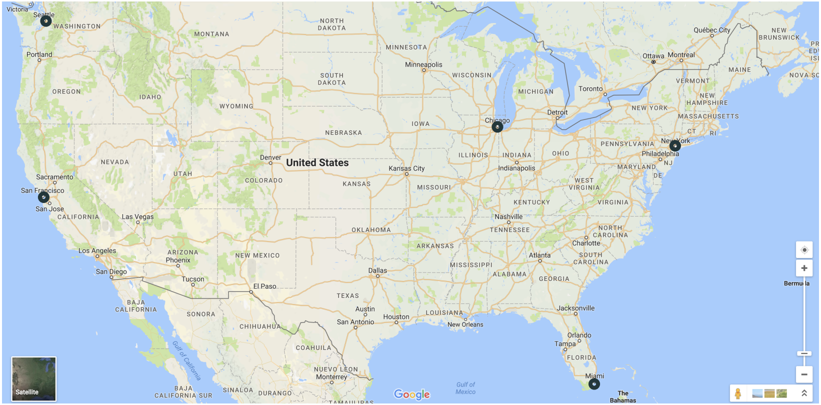
Traveling



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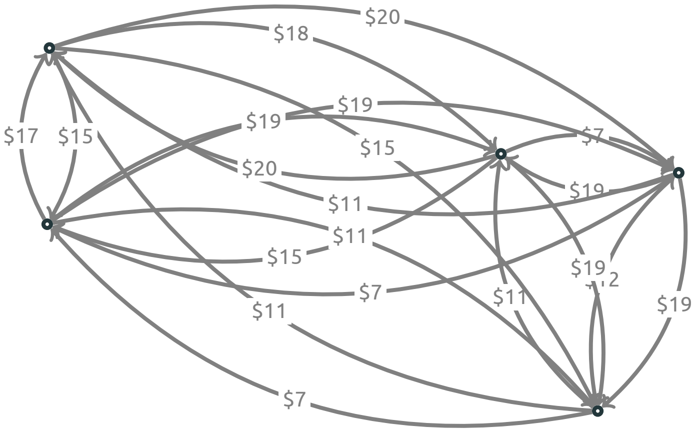
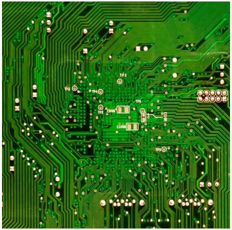


Drilling a Circuit Board



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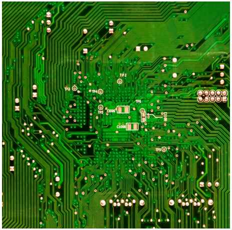

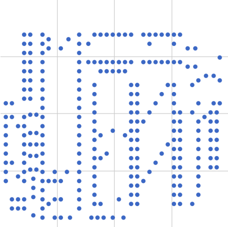

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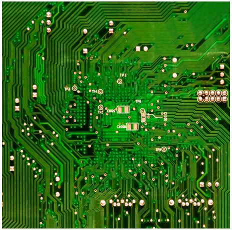

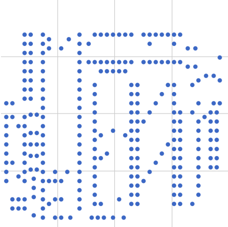

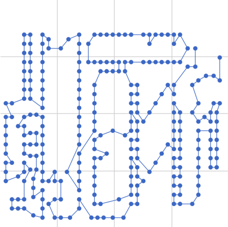

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Euclidean TSP

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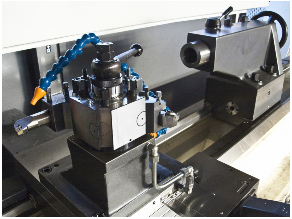
- Weights are symmetric: $d(p_i, p_j) = d(p_j, p_i)$
- Weights satisfy the triangle inequality: $d(p_i, p_j) \leq d(p_i, p_k) + d(p_k, p_j)$

Processing Components

There are n mechanical components to be processed on a complex machine. After processing the i-th component, it takes t_{ij} units of time to reconfigure the machine so that it is able to process the j-th component. What is the minimum processing cost?



Shortest Common Superstring

- The shortest common superstring problem (SCS): given a set $\{s_1, \ldots, s_n\}$ of n strings find a shortest string containing each s_i as a substring

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- Practical applications: data storage, data compression, genome assembly
- At the first look, it is not at all clear how this problem is related to TSP

SCS: Example

- Consider the following instance:

  ABE, DFA, DAB, CBD, ECA, ACB

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- To get a superstring, just concatenate them:

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- But the strings ECA and ACB have a non-empty overlap. One can get a shorter superstring by overlapping them:

  ECACB

SCS: Permutation Problem

ABE DFA DAB ECA CBD ACB

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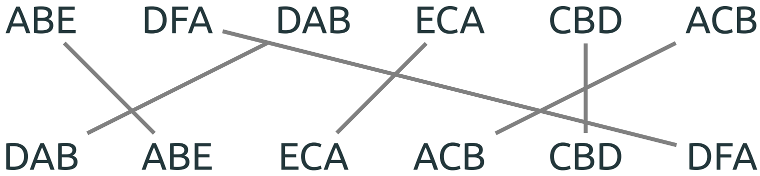

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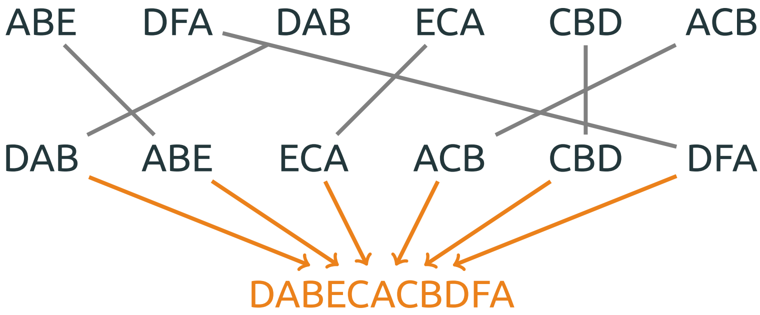

Overlap Graph: SCS$\rightarrow$MAX-ATSP

ABE DFA DAB CBD ECA ACB

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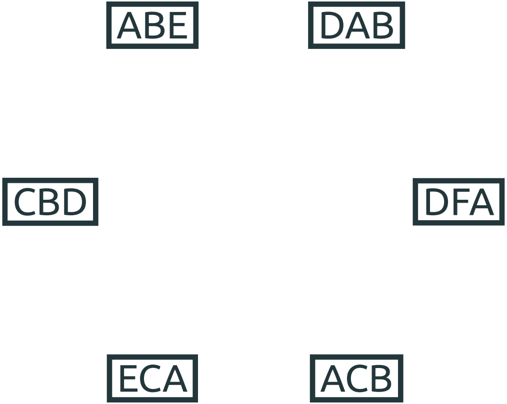

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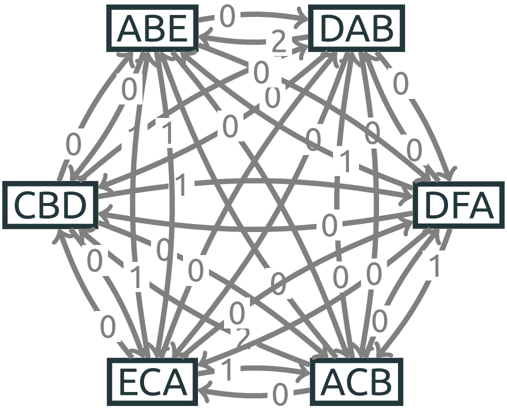

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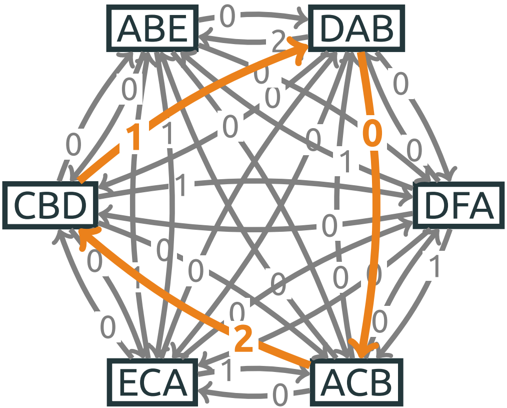

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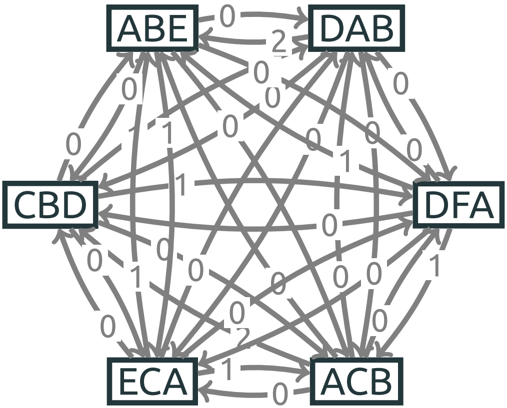

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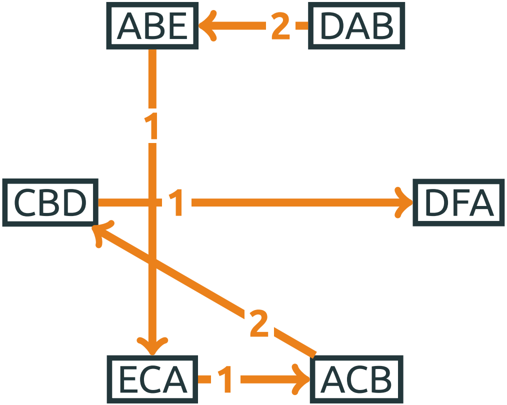

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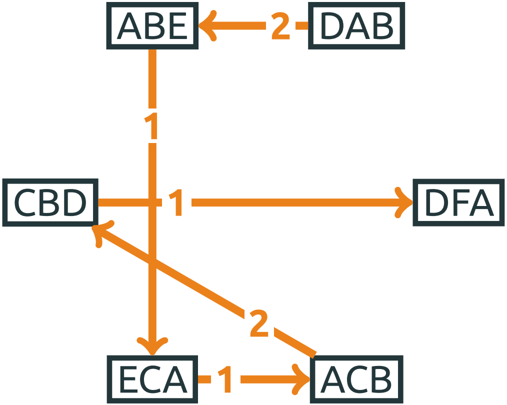

DABECACBDFA

Outline

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- Finding the best permutation is easy: simply iterate through all of them and select the best one

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- But the number of permutations of n objects is $n!$

$n!$: Growth Rate

n	$n!$
5	120
8	40320
10	3628800
13	6227020800
20	2432902008176640000
30	265252859812191058636308480000000

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Random Permutation

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- What if we just generate a random permutation?
- The length of a random permutation may be much worse than the minimum length, even for Euclidean TSP

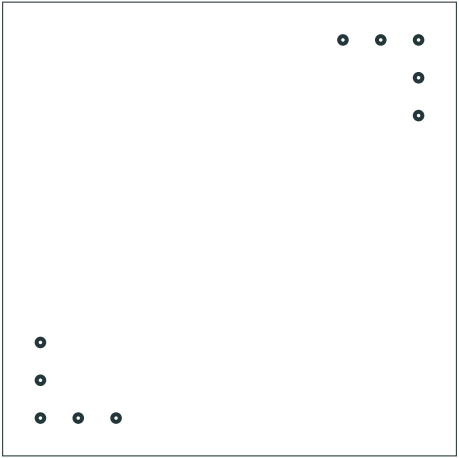
Expected Length

Lemma

For a complete directed graph G, the expected length of a random permutation is

$$\frac{1}{n-1} \cdot \sum_{u,v \in V(G)} w(u, v)$$

Bad Case



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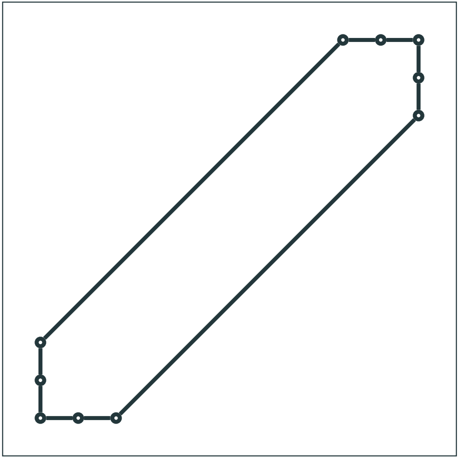

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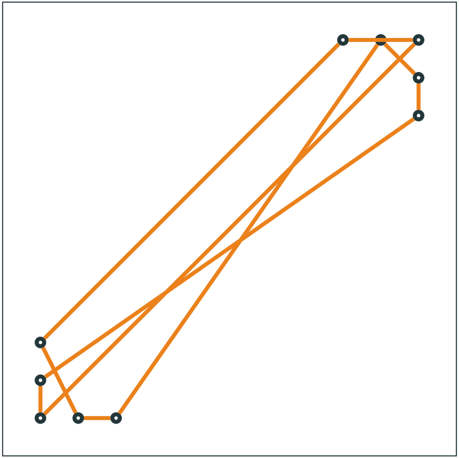

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- What about going to the nearest yet unvisited node at every iteration?
- Efficient, works reasonably well in practice
- For general graphs, may produce a cycle that is much worse than an optimal one
- For Euclidean instances, the resulting cycle may be about $\log n$ times worse than an optimal one

Nearest Neighbors: Bad Case

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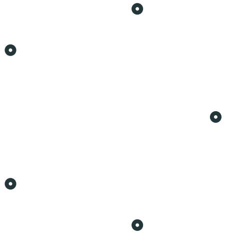

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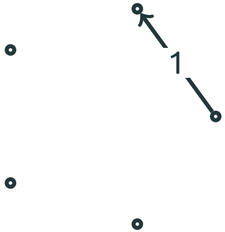

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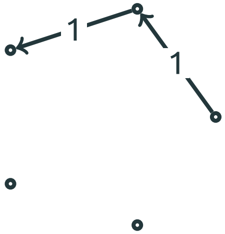

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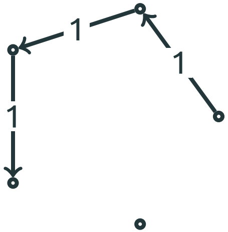

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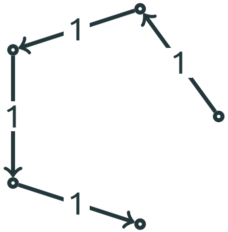

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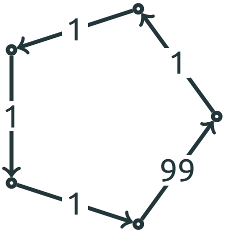
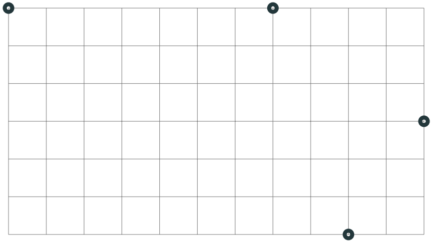


Suboptimal Solution for Euclidean TSP



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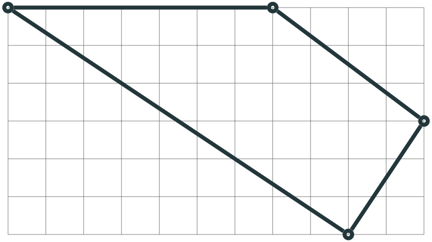

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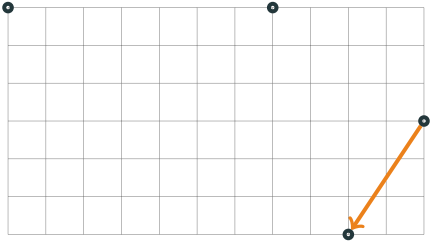

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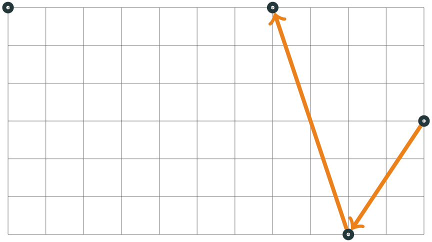

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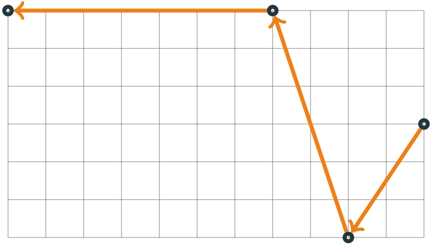

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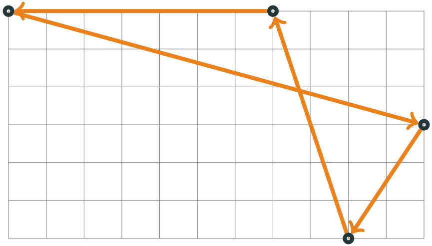

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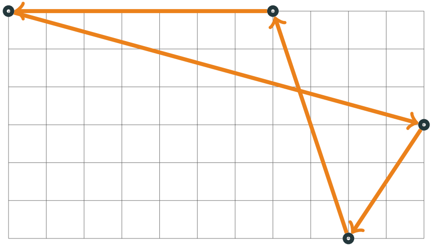

$\text{OPT} \approx 26.42$

$\text{NN} \approx 28.33$

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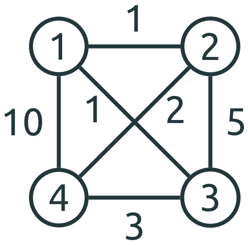
- At every iteration try to extend (recursively) the current path by every yet unvisited node
- But don’t continue extending the path, if it is already clear that it cannot be extended to an optimal cycle

Example: Brute Force Search



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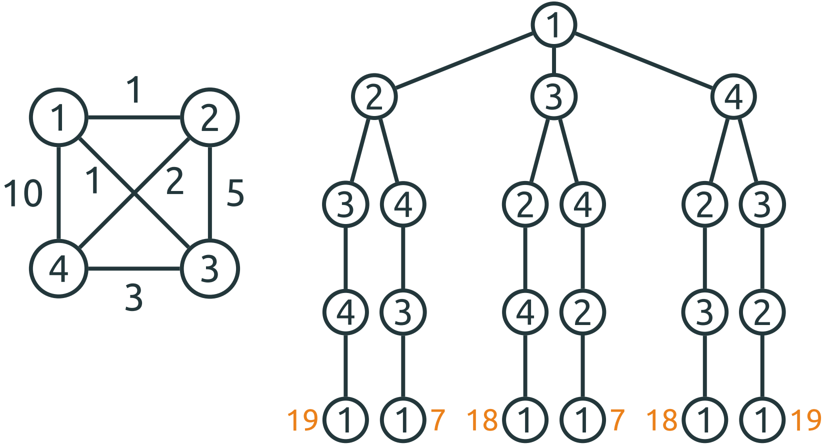
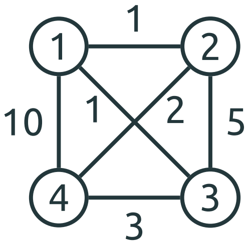
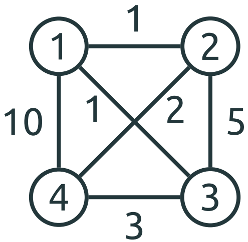
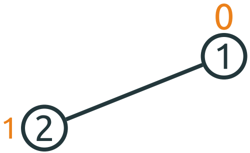


Example: Pruned Search



Example: Pruned Search





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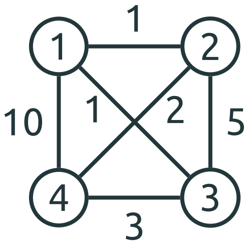

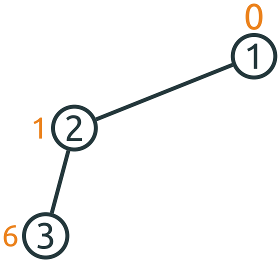

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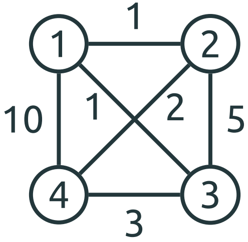

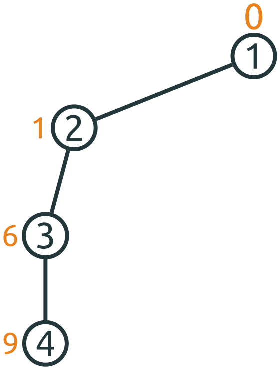

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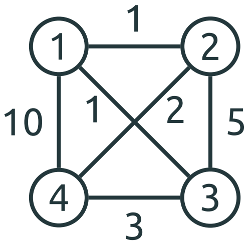

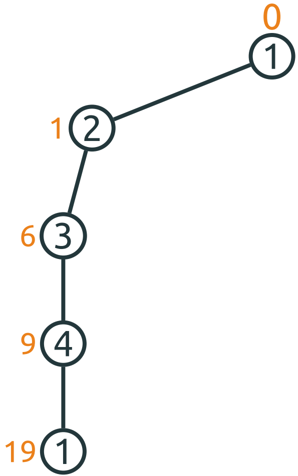

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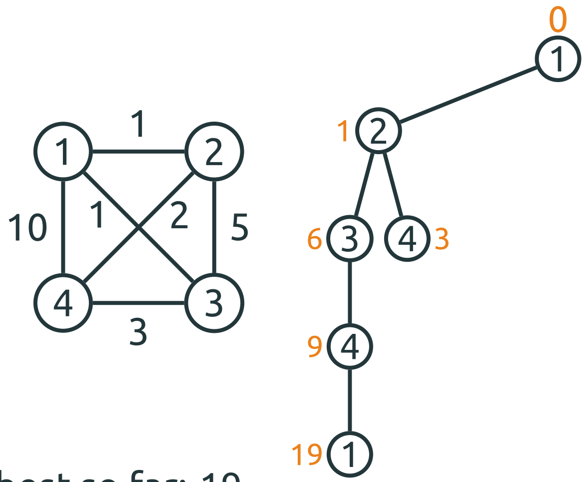

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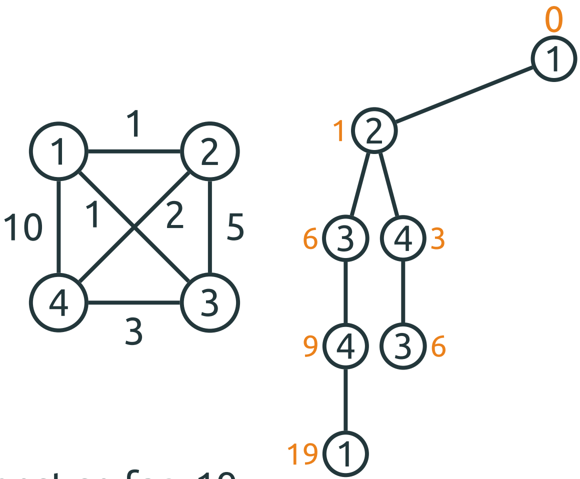

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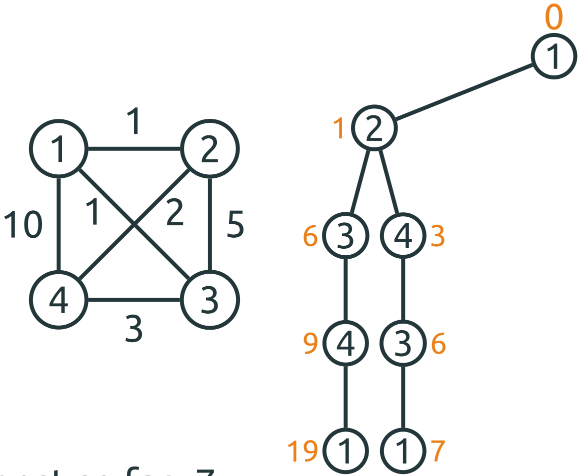

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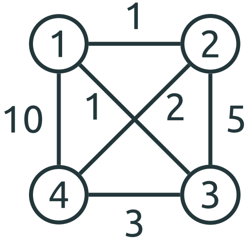

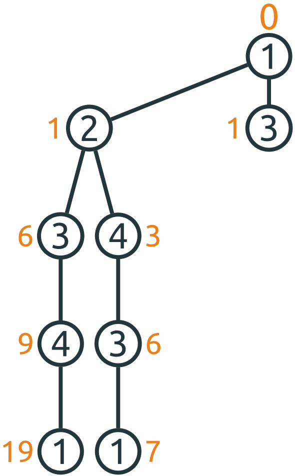

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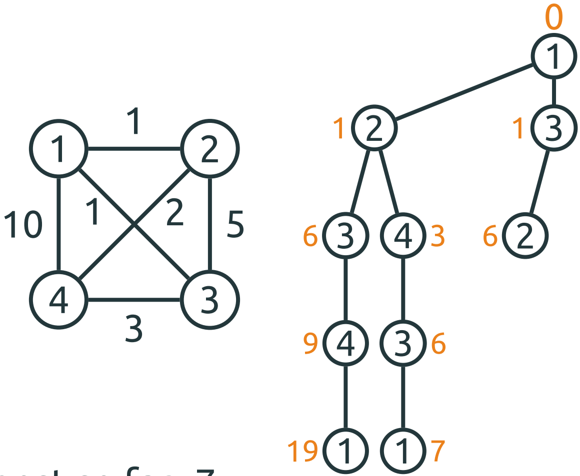

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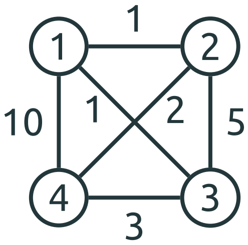

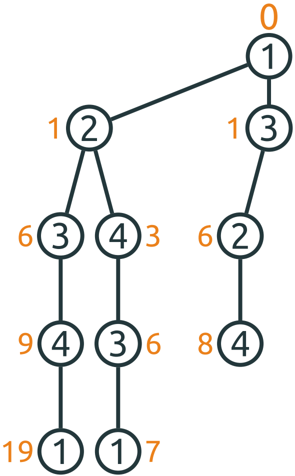

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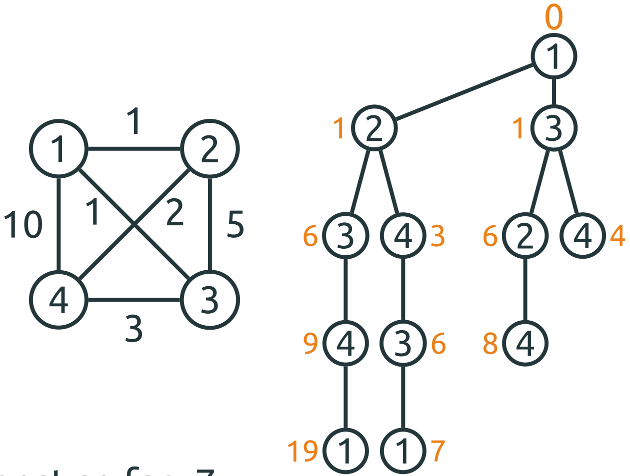

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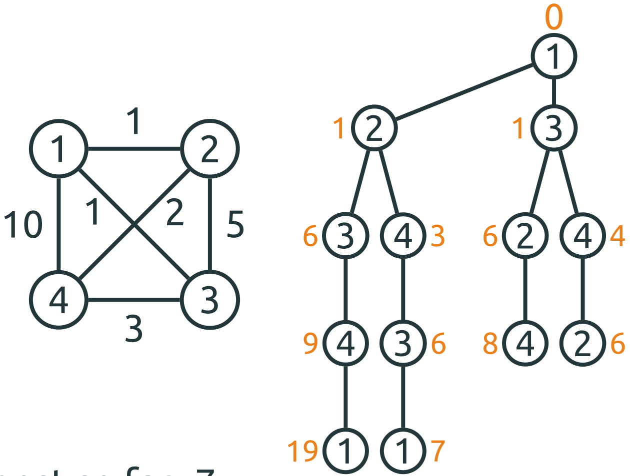

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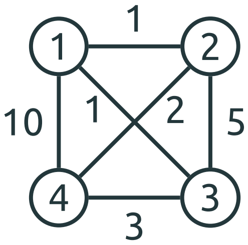

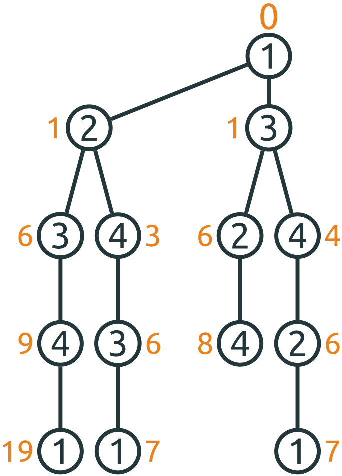

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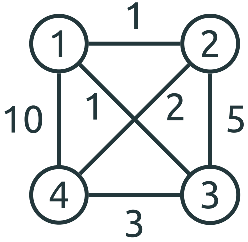

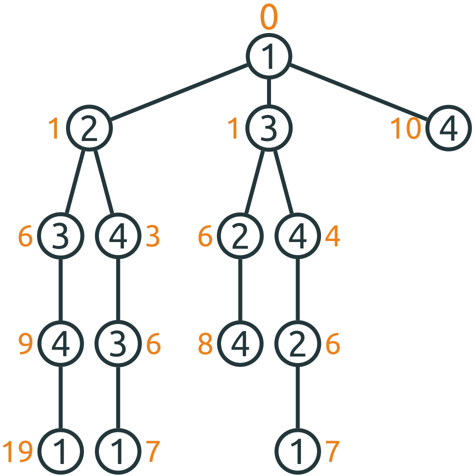

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Lower Bound

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- Modern TSP-solvers use smarter lower bounds to solve instances with thousands of vertices

Example: Lower Bounds (Still Simple)

The length of an optimal TSP cycle is at least

- $\frac{1}{2}\sum_{v \in V}$(two min length edges adj to v)

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The length of an optimal TSP cycle is at least

- $\frac{1}{2} \sum_{v \in V}$(two min length edges adj to v)
- the length of a minimum spanning tree (by taking out any edge of a TSP cycle, one gets a spanning tree)

Branch and Bound: Summary

- Main two heuristics of branch and bound:

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  - Bound: lower bounding the length of a path
- Finds an optimal solution
- The running time depends on the heuristics used as well as on the instance itself
- Used by state-of-the-art TSP-solvers that can handle instances with thousands of nodes!

Outline

Dynamic Programming

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- Rough idea: express a solution for a problem through solutions for smaller subproblems
- Solve subproblems one by one. Store solutions to subproblems in a table to avoid recomputing the same thing again

Subproblems

- For a subset of nodes $S \subseteq \{0, \ldots, n-1\}$ containing the node 0 and a node $i \in S$, let $C(i, S)$ be the length of the shortest path that starts at 0, ends at i, and visits all nodes from S exactly once

Subproblems

- For a subset of nodes $S \subseteq \{0, \ldots, n-1\}$ containing the node 0 and a node $i \in S$, let $C(i, S)$ be the length of the shortest path that starts at 0, ends at i, and visits all nodes from S exactly once
- $C(0, \{0\}) = 0$ and $C(0, S) = +\infty$ when $|S| > 1$

Recurrence Relation

- Consider the second-to-last node j on the required shortest path from 0 to i visiting all nodes from S

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- The subpath from 0 to j is the shortest one visiting all vertices from $S - \{i\}$ exactly once
- Hence $C(i, S) = \min\{C(j, S - \{i\}) + w(j, i)\}$, where the minimum is over all $j \in S$ such that $j \neq i$

Implementation Remark

- How to iterate through all subsets of $\{0, \ldots, n-1\}$?

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- How to iterate through all subsets of $\{0, \ldots, n-1\}$?
- There is a natural one-to-one correspondence between integers in the range from 0 to $2^n - 1$ and subsets of $\{0, \ldots, n-1\}$:

$$k \leftrightarrow \{i \colon \text{ } i\text{-th bit of } k \text{ is } 1\}$$

Example

k	$\text{bin}(k)$	$\{i\colon\ i\text{-th bit of } k \text{ is } 1\}$
0	`000`	$\emptyset$
1	`001`	{0}
2	`010`	{1}
3	`011`	{0,1}
4	`100`	{2}
5	`101`	{0,2}
6	`110`	{1,2}
7	`111`	{0,1,2}

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- If k corresponds to S, how to find out the integer corresponding to $S - \{j\}$ (for $j \in S$)?

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- In `C/C++`, `Java`, `Python`:
  `k ^ (1 << j)`

Code

```
def dp(G):
  n = G.number_of_nodes()
  T = [[float("inf")] * (1 << n) for _ in range(n)]
  T[0][1] = 0
  for s in range(1 << n):
    if sum(((s >> j) & 1) for j in range(n)) <= 1 or not (s & 1):
      continue

    for i in range(1, n):
      if not ((s >> i) & 1):
        continue
      for j in range(n):
        if j == i or not ((s >> j) & 1):
          continue

        T[i][s] = min(T[i][s],
                      T[j][s ^ (1 << i)] + G[i][j]['weight'])

  return min(T[i][(1 << n) - 1] + G[0][i]['weight']
             for i in range(1, n))
```

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- Better than $n!$, but still too slow (already for $n = 20$)

Outline

Problem Statement

Brute Force Search

Nearest Neighbor

Branch and Bound

Dynamic Programming

Approximation Algorithm

Local Search

Approximation

- Let’s focus on the metric version of TSP: $w(u, v) = w(v, u)$ and $w(u, v) \leq w(u, z) + w(z, v)$ (in particular, Euclidean TSP is metric)
- We will design a 2-approximation algorithm: it quickly finds a cycle that is at most twice longer than an optimal one

Minimum Spanning Trees

Lemma

Let G be an undirected graph with non-negative edge weights. Then $\mathrm{MST}(G) \leq \mathrm{TSP}(G)$.

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Proof

By removing any edge from an optimum TSP cycle one gets a spanning tree of G. $\square$

Algorithm

- $T \leftarrow$ minimum spanning tree of G

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- $D \leftarrow T$ with each edge doubled

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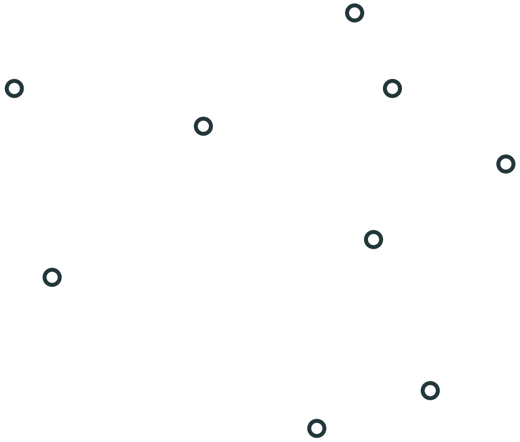
- $T \leftarrow$ minimum spanning tree of G
- $D \leftarrow T$ with each edge doubled
- find an Eulerian cycle C in D
- return a cycle that visits the nodes in the order of their first appearance in C

Example



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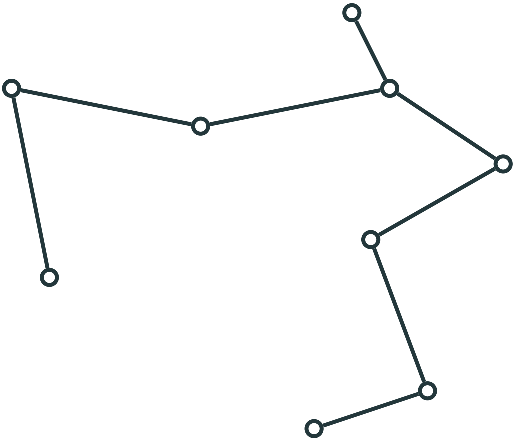

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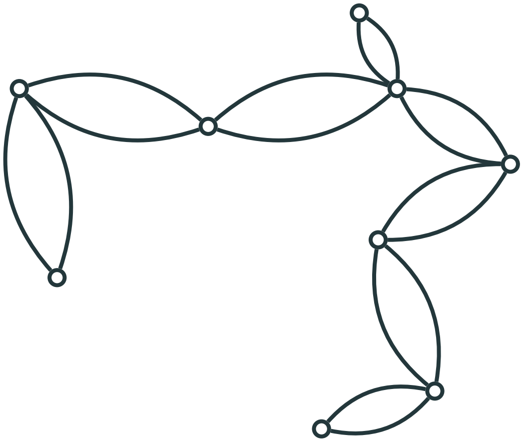

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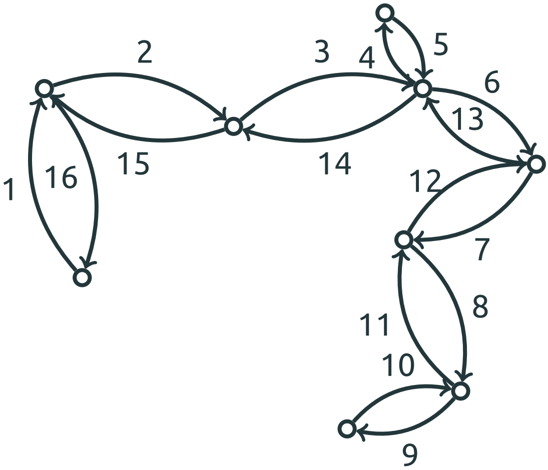

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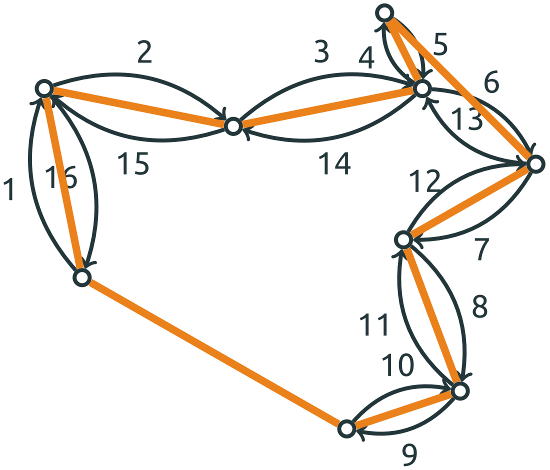

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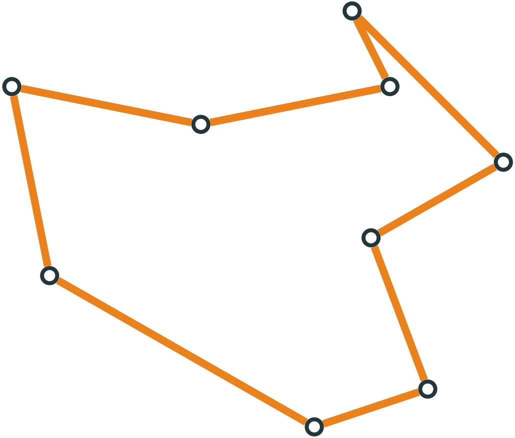

Provable Guarantee

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The algorithm is 2-approximate.

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Proof

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- Bypasses can only decrease the total length. $\square$

Final Remarks

- The currently best known approximation algorithm for metric TSP is Christofides' algorithm that achieves a factor of 1.5

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- If P $\neq$ NP, then there is no α-approximation algorithm for the general version of TSP for any constant α

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Properties

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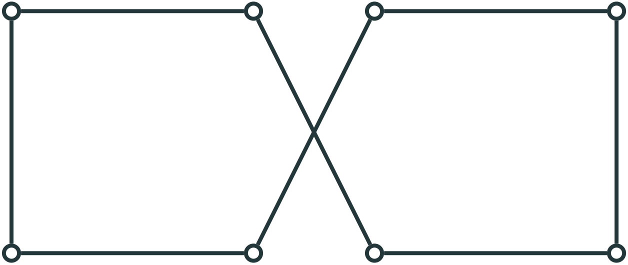
- The larger is d, the better is the resulting solution and the higher is the running time

Example

Changing two edges in a suboptimal solution:



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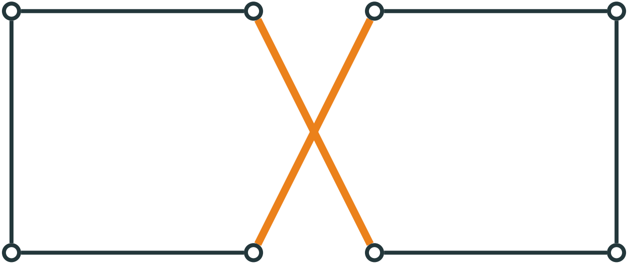

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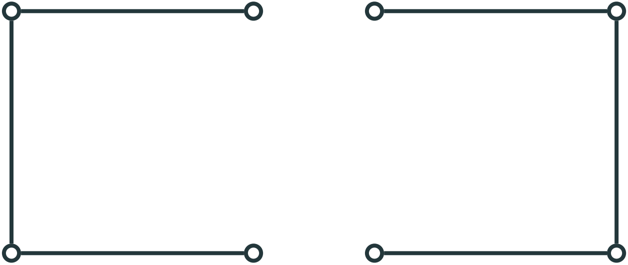

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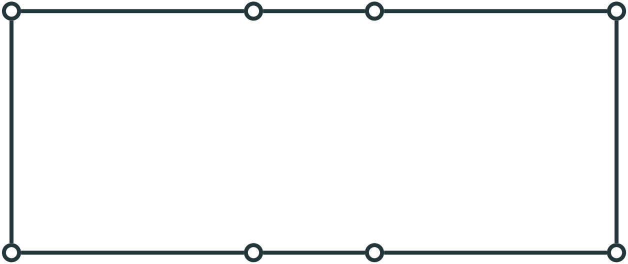

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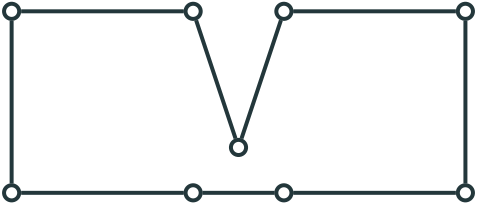

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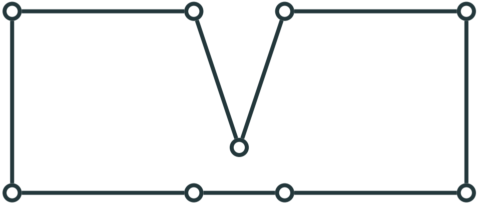

Need to allow changing three edges to improve this solution

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- But works well in practice

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- Approximation algorithms: nearest neighbors, MST-based, local search