Floating-point number parsing with perfect accuracy at a gigabyte per second

Daniel Lemire

professor, Université du Québec (TÉLUQ)

Montreal 🇨🇦

blog: https://lemire.me

twitter: @lemire

GitHub: https://github.com/lemire/

❗ work with Michael Eisel, Ivan Smirnov, Nigel Tao, R. Oudompheng, Carl Verret and others!

How fast is your disk?

PCIe 4 disks: 5 GB/s reading speed (sequential)

Fact

Single-core processes are often CPU bound

How fast can you ingest data?


```
{ "type": "FeatureCollection",
  "features": [
[[[-65.613616999999977,43.420273000000009],
[-65.619720000000029,43.418052999999986],
[-65.625,43.421379000000059],
[-65.636123999999882,43.449714999999969],
[-65.633056999999951,43.474709000000132],
[-65.611389000000031,43.513054000000068],
[-65.605835000000013,43.516105999999979],
[-65.598343,43.515830999999935],
[-65.566101000000003,43.508331000000055],
...
```

How fast can you parse numbers?

```
std::stringstream in(mystring);
while(in >> x) {
   sum += x;
}
return sum;
```

50 MB/s (Linux, GCC -O3)

Source: https://lemire.me/blog/2019/10/26/how-expensive-is-it-to-parse-numbers-from-a-string-in-c/

Some arithmetic

5 GB/s divided by 50 MB/s is 100.

Got 100 CPU cores?

Want to cause climate change all on your own?

How to go faster?

- Fewer instructions (simpler code)
- Fewer branches

How fast can you go?

AMD Rome (Zen 2). GNU GCC 10, -O3.

function	bandwidth	instructions	ins/cycle
strtod (GCC 10)	200 MB/s	1100	3
ours	1.1 GB/s	280	4.2

17-digit mantissa, random in [0,1].

Floats are easy

- Standard in Java, Go, Python, Swift, JavaScript…
- IEEE standard well supported on all recent systems
- 64-bit floats can represent all integers up to 2^53 exactly.

Floats are hard

```
> 0.1 + 0.2 == 0.3
false
```

Generic rules regarding "exact" IEEE support

- Always round to nearest floating-point number (*,+,/)
- Resolve ties by rounding to nearest with an even decimal mantissa/significand.

Benefits

- Predictable outcomes.
- Debuggability.
- Cross-language compatibility (same results).

Challenges

- Machine A writes float x to string
- Machine B reads string gets float x'
- Machine C reads string gets float x''

Do you have $x = x'$ and $x = x''$?

What is the problem?

Need to go from

$$w \times 10^q$$

(e.g., 123e5)

to

$$m \times 2^p$$

Example

$$0.1 \rightarrow 7205759403792793 \times 2^{-56}$$

0.1000000000000000055

$$0.2 \rightarrow 7205759403792794 \times 2^{-55}$$

0.2000000000000000111

$$0.3 \rightarrow 5404319552844595 \times 2^{-54}$$

0.299999999999999988897769753

Problems

Start with 32323232132321321111e124.

Lookup 10^{124} as a float (not exact)

Convert 32323232132321321111 to a float (not exact)

Compute $(10^{124}) \times (32323232132321321111)$

Approximation $\times$ Approximation = Even worse approximation!

Insight

You can always represent floats exactly (binary64) using at most 17 digits.

Never to this:

3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170679

WHAT THE NUMBER OF DIGITS IN YOUR COORDINATES MEANS

LAT/LON PRECISION	MEANING
28°N, 80°W	YOU'RE PROBABLY DOING SOMETHING SPACE-RELATED
28.5°N, 80.6°W	YOU'RE POINTING OUT A SPECIFIC CITY
28.52°N, 80.68°W	YOU'RE POINTING OUT A NEIGHBORHOOD
28.523°N, 80.683°W	YOU'RE POINTING OUT A SPECIFIC SUBURBAN CUL-DE-SAC
28.5234°N, 80.6830°W	YOU'RE POINTING TO A PARTICULAR CORNER OF A HOUSE
28.52345°N, 80.68309°W	YOU'RE POINTING TO A SPECIFIC PERSON IN A ROOM, BUT SINCE YOU DIDN'T INCLUDE DATUM INFORMATION, WE CAN'T TELL WHO
28.5234571°N, 80.6830941°W	YOU'RE POINTING TO WALDO ON A PAGE
28.523457182°N, 80.683094159°W	"HEY, CHECK OUT THIS SPECIFIC SAND GRAIN!"
28.523457182818284°N, 80.683094159265358°W	EITHER YOU'RE HANDING OUT RAW FLOATING POINT VARIABLES, OR YOU'VE BUILT A DATABASE TO TRACK INDIVIDUAL ATOMS. IN EITHER CASE, PLEASE STOP.

credit: xkcd

We have 64-bit processors

So we can express all positive floats as `12345678901234567E+/-123`.

Or $w \times 10^q$

where mantissa $w < 10^{17}$

But 10^{17} fits in a 64-bit word!

Factorization

$10 = 5 \times 2$

Overall algorithm

- Parse decimal mantissa to a 64-bit word!
- Precompute 5^q for all powers with up to 128-bit accuracy.
- Multiply!
- Figure out right power of two

Tricks:

- Deal with "subnormals"
- Handle excessively large numbers (infinity)
- Round-to-nearest, tie to even

Check whether we have 8 consecutive digits

```
bool is_made_of_eight_digits_fast(const char *chars) {
  uint64_t val;
  memcpy(&val, chars, 8);
  return (((val & 0xF0F0F0F0F0F0F0F0) |
           (((val + 0x0606060606060606) & 0xF0F0F0F0F0F0F0F0) >> 4))
           == 0x3333333333333333);
}
```

(Works with ASCII, harder if input is UTF-16 as in Java/C#)

Then construct the corresponding integer

Using only three multiplications (instead of 7):

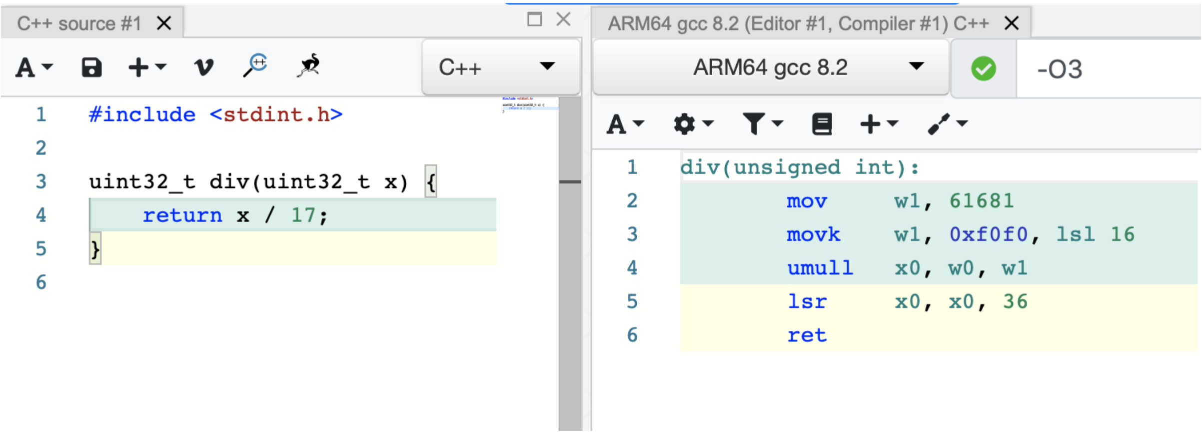
```
uint32_t parse_eight_digits_unrolled(const char *chars) {
  uint64_t val;
  memcpy(&val, chars, sizeof(uint64_t));
  val = (val & 0x0F0F0F0F0F0F0F0F) * 2561 >> 8;
  val = (val & 0x00FF00FF00FF00FF) * 6553601 >> 16;
  return (val & 0x0000FFFF0000FFFF) * 42949672960001 >> 32;
}
```

Positive powers

- Compute $w \times 5^q$ where 5^q is only approximate (128 bits)
- 99.99% of the time, you get provably accurate 55 bits

Negative powers

- Compilers replace division by constants with multiply and shift



credit: godbolt

Negative powers

- Precompute $2^b/5^q$ (reciprocal, 128-bit precision)
- 99.99% of the time, you get provably accurate results

What about tie to even?

- Need absolutely exact mantissa computation, to infinite precision.
- But only happens for small decimal powers ($q \in [-4, 23]$) where absolutely exact results are practical.

What if you have more than 19 digits?

- Truncate the mantissa to 19 digits, map to w.
- Do the work for $w \times 10^q$
- Do the work for $(w + 1) \times 10^q$
- When get same results, you are done. (99% of the time)

Overall

- With 64-bit mantissa.
- With 128-bit powers of five.
- Can do exact computation 99.99% of the time.
- Fast, cheap, accurate.

Full product?

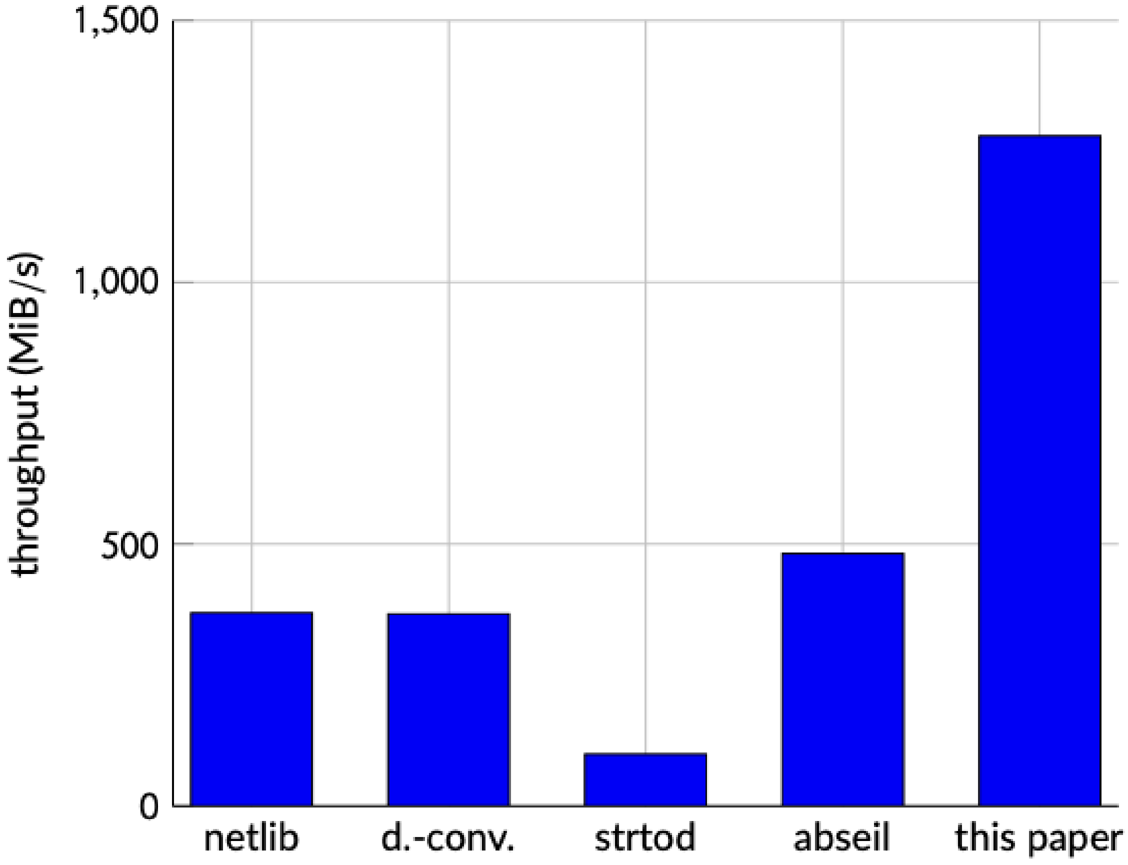
- 64-bit $\times$ 64-bit $\to$ 128-bit product
- GNU GCC: `__uint128_t`.
- Microsoft Visual Studio: `_umul128`
- ARM intrinsic: `__umulh`
- Go: `bits.Mul64`
- C#: `Math.BigMul`

Leading zeros

- How many consecutive leading zeros in 64-bit word?
- GNU GCC: `__builtin_clzll`
- Microsoft Visual Studio: `_BitScanReverse64`
- C++20: `std::countl_zero`
- Go: `bits.LeadingZeros64`
- C#: `BitOperations.LeadingZeroCount`

C/C++

- https://github.com/lemire/fast_float
  (used by Apache Arrow, Yandex ClickHouse) + Microsoft LightGBM



(a) canada

Go

- Algorithm adapted to Go's standard library (ParseFloat) by Nigel Tao and others
- Release notes (version 1.16): *ParseFloat (...) improving performance by up to a factor of 2.*
- Perfect rounding.
- Blog post by Tao: The Eisel-Lemire ParseNumberF64 Algorithm

Rust

function	speed
from_str (standard)	130 MB/s
lexical (popular lib.)	370 MB/s
fast-float	1200 MB/s

R

rcppfastfloat: https://github.com/eddelbuettel/rcppfastfloat

3x faster than standard library

C#

`FastFloat.ParseDouble` is 5x faster than standard library (`Double.Parse`)

https://github.com/CarlVerret/csFastFloat/

credit: Carl Verret and Egor Bogatov (Microsoft)

Further reading

- Daniel Lemire, Number Parsing at a Gigabyte per Second, https://arxiv.org/abs/2101.11408
- Blog: https://lemire.me/blog/