Jeff Plaisance, Nathan Kurz, Daniel Lemire

Inverted Indexes

Inverted Index

- Like index in the back of a book
- words = terms, page numbers = doc ids
- Term list is sorted
- Doc list for each term is sorted

Standard Index

doc id	query	country	impressions	clicks
0	software	Canada	10	1
1	blank	Canada	10	0
2	sales	US	5	0
3	software	US	8	1
4	blank	US	10	1

Constructing an Inverted Index

	query		country		impression			clicks		
doc id	blank	sales	software	Canada	US	5	8	10	0	1
0			V	V				~		~
1	~			•				~	~	
2		~			•	•			•	
3			V		✓		/			/
4	•				✓			'		/

Constructing an Inverted Index

field	term	0	1	2	3	4
query	blank		~			✓
	sales			/		
	software	~			~	
country	Canada	~	~			
	US			/	~	✓
impressions	5			/		
	8				~	
	10	~	'			✓
clicks	0		'	•		
	1	~			'	/

Inverted Index

field	term	doc list
query	blank	1, 4
	sales	2
	software	0, 3
country	Canada	0, 1
	US	2, 3, 4
impressions	5	2
	8	3
	10	0, 1, 4
clicks	0	1, 2
	1	0, 3, 4

Inverted Indexes

Allow you to:

- Quickly find all documents containing a term
- Intersect several terms to perform boolean queries

Inverted Index Optimizations

- Compressed data structures
 - Better use of RAM and processor cache
 - Better use of memory bandwidth
 - Increased CPU usage and time
- Optimizations matter!

Delta / VByte Encoding

- Doc id lists are sorted
- Delta between a doc id and the previous doc id is sufficient
- Deltas are usually small integers

Delta Encoding

field	term	doc list
query	nursing	34, 86, 247, 301, 674, 714

Delta Encoding

field	term	doc list
query	nursing	34, 86, 247, 301, 674, 714
		34, 52, 161, 54, 373, 40

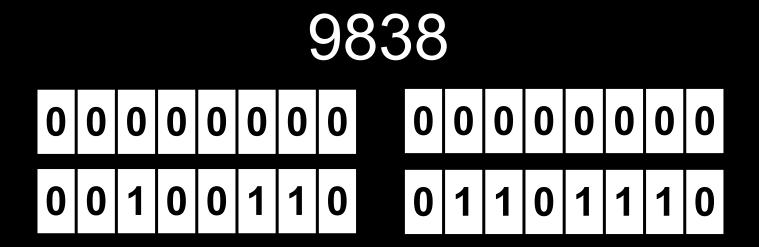
Small Integer Compression

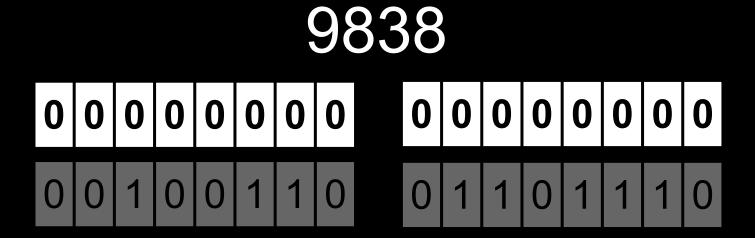
- Golomb/Rice
- VByte (or Varint)
- Binary Packing
- PForDelta

Small Integer Compression

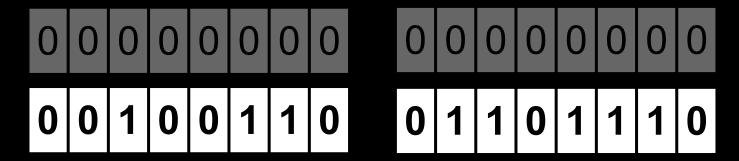
- Golomb/Rice
- VByte
- Bit Packing
- PForDelta

9838

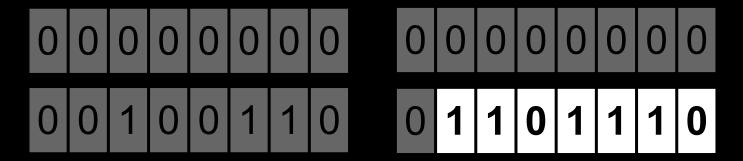




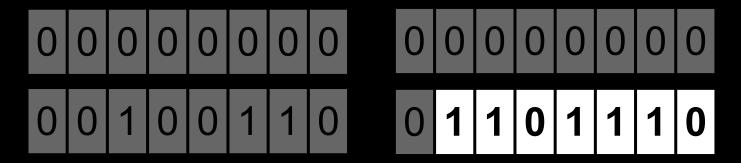




9838

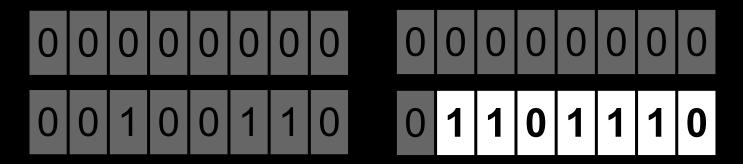


9838

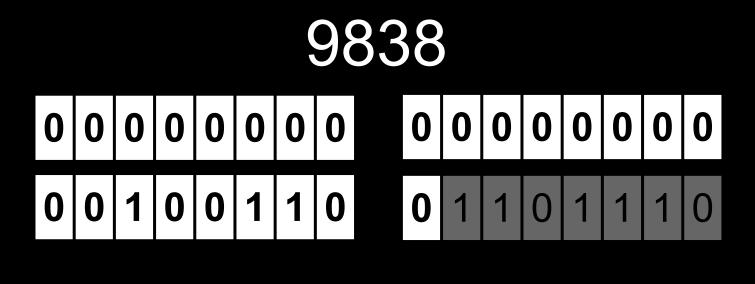


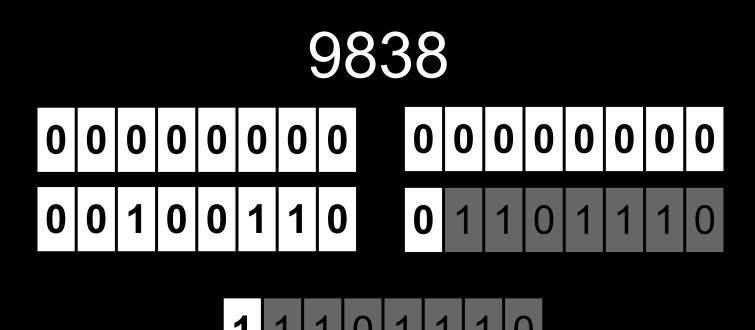
? 1 1 0 1 1 1 0

9838

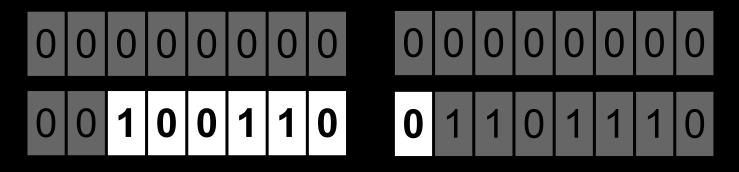


? 1 1 0 1 1 1 0



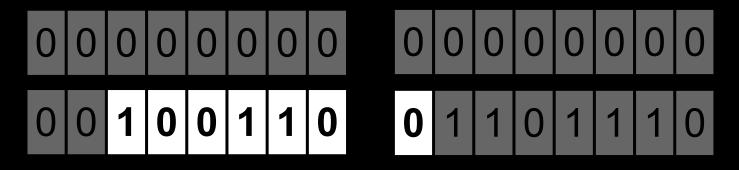


9838



1 1 1 0 1 1 0

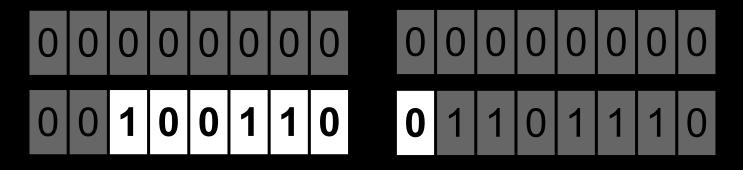




 1
 1
 1
 0
 1
 1
 1
 0

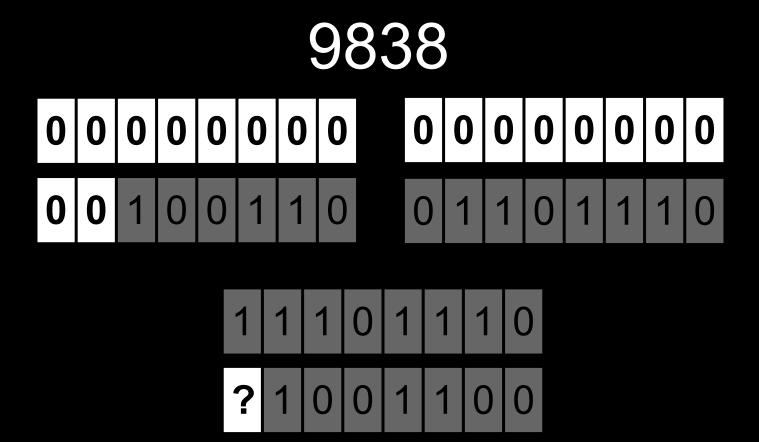
 ?
 1
 0
 0
 1
 1
 0
 0

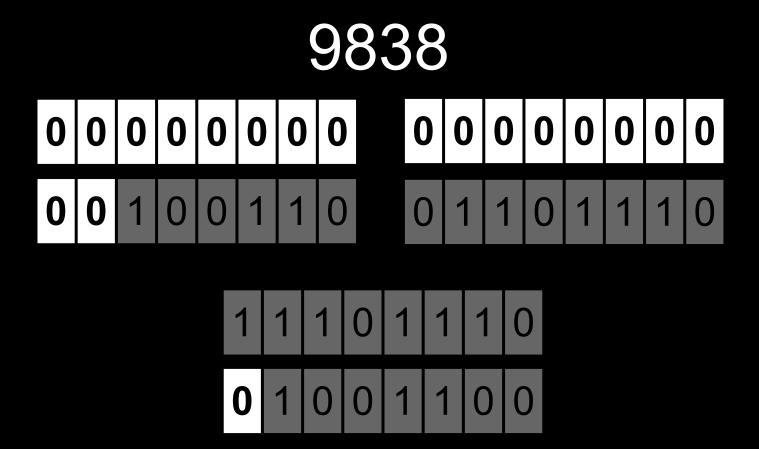
9838



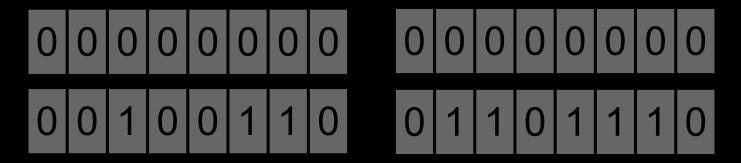
 1
 1
 1
 0
 1
 1
 1
 0

 ?
 1
 0
 0
 1
 1
 0
 0





9838



 1
 1
 1
 0
 1
 1
 1
 0

 0
 1
 0
 0
 1
 1
 0
 0

VByte

Pros:

- Compression
- Can fit more of index in RAM
- Higher information throughput per byte read from disk

VByte

Cons:

- Decodes one byte at a time
- Lots of branch mispredictions
- Not fast to decode
- Largest ints expand to 5 bytes

Optimized decoder implemented using x86_64 intrinsics

```
01001010 11001000 01110001 01001110
10011011 01101010 10110101 00010111
01110110 10001101 10110011 11000001
```

```
01001010 11001000 01110001 01001110
10011011 0110101 101011 1000001
01110110 10001101 10110011 11000001
```

pmovmskb: Extract top bit of each byte

```
01001010 11001000 01110001 01001110
10011011 0110101 101011 1000001
01110110 10001101 10110011 11000001
```

pmovmskb: Extract top bit of each byte

010010100111

010010100111

Pattern of leading bits determines:

- how many varints to decode
- sizes and offsets of varints
- length of longest varint in bytes
- number of bytes to consume

- how many varints to decode
- sizes and offsets of varints
- length of longest varint in bytes
- number of bytes to consume

- how many varints to decode
- sizes and offsets of varints
- length of longest varint in bytes
- number of bytes to consume

- how many varints to decode
- sizes and offsets of varints
- length of longest varint in bytes
- number of bytes to consume

0100<mark>10</mark>100111

- how many varints to decode
- sizes and offsets of varints
- length of longest varint in bytes
- number of bytes to consume

010010<mark>10</mark>0111

- how many varints to decode
- sizes and offsets of varints
- length of longest varint in bytes
- number of bytes to consume

- how many varints to decode
- sizes and offsets of varints
- length of longest varint in bytes
- number of bytes to consume

- how many varints to decode
- sizes and offsets of varints
- length of longest varint in bytes
- number of bytes to consume

- how many varints to decode
- sizes and offsets of varints
- length of longest varint in bytes
- number of bytes to consume

Decoding options for:

- sixteen 1 byte varints
- six 1-2 byte varints
- four 1-3 byte varints
- two 1-5 byte varints

Decoding options for:

- sixteen 1 byte varints special case
- six 1-2 byte varints 2⁶, 64 possibilities
- four 1-3 byte varints 3⁴, 81 possibilities
- two 1-5 byte varints 5², 25 possibilities

170 total possibilities

Data Distribution:

- Longer doc id lists are necessarily composed of smaller deltas
- Most deltas in real datasets (ClueWeb09, Indeed's internal datasets) fall into 1 byte case or 1-2 byte case

Most Significant Bit Decoding

- We separate most significant bit decoding from integer decoding
- Reduces duplicate most significant bit decoding work if we don't consume all 12 bytes
- Better instruction level parallelism

- If most significant bits of next 16 bytes are all 0, handle sixteen 1 byte ints case
- Otherwise lookup most significant bits of next 12 bytes in 4096 entry lookup table

Lookup table contains:

- Shuffle vector index from 0-169 representing which possibility we are decoding
- Number of bytes of input that will be consumed

Branch on shuffle vector index to determine which case we are decoding

- 0-63 six 1-2 byte ints
- 64-144 four 1-3 byte ints
- 145-169 two 1-5 byte ints

Six 1-2 Byte Ints

```
01001010 11001000 01110001 01001110
10011011 0110101 1011011 10000111
01110110 10001101 10110011 11000001
```

Decode 6 varints from 9 bytes

Expected Positions

Six 1-2 byte ints



Four 1-3 byte ints



Two 1-5 byte ints



Six 1-2 Byte Ints

```
      01001010
      11001000
      01110001
      01001110

      10011011
      0110101
      10110011
      0001011

      01110110
      10001101
      10110011
      11000001
```

Pad out 1 byte ints to 2 bytes

Six 1-2 Byte Ints

```
      01001010
      00000000
      11001000
      01110001

      01001110
      00000000
      10011011
      0110101

      10110101
      00000000
      01110110
      00000000
```

Pad out 1 byte ints to 2 bytes

Shuffle input

- Use index to lookup appropriate shuffle vector
- Shuffle input bytes to get them in the expected positions

```
for (i = 0; i < 16; i++) {
    if (mask[i] & 0x80) {
        dest[i] = 0;
    } else {
        dest[i] = src[mask[i] \& 0xF];
```

src

DE	DF	27	E3	7C	A9	60	55	1C	EA	45	56	A6	43	C9	48
	וט	21	LJ	/ 0	79	00	33	10		45	30	Λ0	40	Ca	40

mask

0 11 2 -1 12 -1 13 4 0 10 2 6 4 3 13

src

DF 27 E3 7C A9 60 55 1C EA 45 56 A6 43 C9	DE	7C A9	A9 60 5	55 1C	EA 45	56 A6	43 C9	48
---	----	-------	---------	-------	---------	-------	-------	----

mask

0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5	

src

DE	DF	27	E3	7C	A9	60	55	1C	EA	45	56	A6	43	C9	48

mask

src

DE DF 27 E3 7C A9 60 55 1C EA 45 56 A6 43 C9 48	DE	DF	27	E3	7C	A9	60	55	1C	EA	45	56	A6	43	C9	48
---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

mask

DE				

src

			1					1	1		1		1	1		
	5-								4.0	^	4 -			4.0		4.0
	DE	DF	27	E3	7C	A9	60	55	1C	EA	45	56	A6	43	C9	48
								-				-				
- 1																

mask

0 11 2 -1 12 -1 13 4 0 10 2	6 4	3 13	5
-----------------------------	-----	------	---

src

DE DF 27 E3 7C A9 60 55 1C EA	45 56 A6 43 C9 48	EA	1C	55	60	A9	7C	E3	27	∣ DF	DE	
-------------------------------	--------------------------	----	----	----	----	----	----	----	----	------	----	--

mask

0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5

DE								

src

DE DF 27 E3 7C A9 60 55 1C EA 45 56	A6 43 C9 48
-------------------------------------	-------------

mask

0 11 2 -1 12 -1 13 4 0 10 2	6 4	3 13	5
-----------------------------	-----	------	---

DE	56														
----	----	--	--	--	--	--	--	--	--	--	--	--	--	--	--

src

DE DF 27 E3 7C A9 60 55 1C EA 45 56 A6 43 C9	48
--	----

mask

0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5

DE	56														
----	----	--	--	--	--	--	--	--	--	--	--	--	--	--	--

src

DE DF 27 E3 7C A9 60 55 1C EA 45 56 A6 43 C9	48
--	----

mask

0	11	2 -1	12	-1	13	4	0	10	2	6	4	3	13	5	
---	----	------	----	----	----	---	---	----	---	---	---	---	----	---	--

DE	56														
----	----	--	--	--	--	--	--	--	--	--	--	--	--	--	--

src

DE DF 27 E3 7C A9 60 55 1C EA 45 56 A6 43 C9	48
--	----

mask

0	11	2 -1	12	-1	13	4	0	10	2	6	4	3	13	5	
---	----	------	----	----	----	---	---	----	---	---	---	---	----	---	--

DE	56	27													
----	----	----	--	--	--	--	--	--	--	--	--	--	--	--	--

src

DE	DF	27	E3	7C	A9	60	55	1C	EA	45	56	A6	43	C9	48

mask

0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5

DE	56	27													
----	----	----	--	--	--	--	--	--	--	--	--	--	--	--	--

src

DF 27 E3 7C A9 60 55 1C EA 45 56 A6 43 C9	DE	7C A9	A9 60 5	55 1C	EA 45	56 A6	43 C9	48
---	----	-------	---------	-------	---------	-------	-------	----

mask

0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5

DE	56	27	0											
----	----	----	---	--	--	--	--	--	--	--	--	--	--	--

src

DE	DF	27	E3	7C	A9	60	55	1C	EA	45	56	A6	43	C9	48
DL	וט	21	LJ	/ 0	79	00	33	10		45	30	Α0	40	Ca	40

mask

_		_			_		_	_		_		_	_		_
0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5

DE	56	27	0												
----	----	----	---	--	--	--	--	--	--	--	--	--	--	--	--

src

DE DF 27 E3 7C A9 60 55 1C EA 45 56 A6 43 C9
--

mask

0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5

DE	56	27	0												
----	----	----	---	--	--	--	--	--	--	--	--	--	--	--	--

src

		DE	DF	27	E3	7C	A9	60	55	1C	EA	45	56	A6	43	C9	48
--	--	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

mask

0 11 2 -1 12 -1 13 4 0 10 2 6 4 3 13 5
--

src

		1		1	ı	1	1		1						
DE	DF	27	E3	7C	A9	60	55	1C	EA	15	56	۸۵	42	C9	10
		21	⊢⊏ડ	70	A9	00	55	10	⊏A	45	00	A6	43	U9	48
															1

mask

		0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5
--	--	---	----	---	----	----	----	----	---	---	----	---	---	---	---	----	---

src

DE	DF	27	E3	7C	A9	60	55	1C	EA	45	56	A6	43	C9	48

mask

		0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5
--	--	---	----	---	----	----	----	----	---	---	----	---	---	---	---	----	---

DE	56 27 0	A6 0)			
----	---------	------	---	--	--	--

src

DE	DF	27	E3	7C	A9	60	55	1C	EA	45	56	A6	43	C9	48
DE	DF	21	ES	/ (A9	00	55	10	LA	43	50	AO	43	<u>C9</u>	40

mask

0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5

src

DE DF 27 E3 7C A9 60 55 1C EA 45 56 A6 43 C9
--

mask

0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5

DE	56	27	0	A6	0										
----	----	----	---	----	---	--	--	--	--	--	--	--	--	--	--

src

	DΕ	27 E3 7C A9	.9 60 55	1C EA	45 56	A6 43	C9 48
--	----	-------------------	----------	---------	-------	-------	-------

mask

0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5

DE	56	27	0	A6	0	43									
----	----	----	---	----	---	----	--	--	--	--	--	--	--	--	--

src

	DE	DF	27	E3	7C	A9	60	55	1C	EA	45	56	A6	43	C9	48
--	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

mask

0	11	2	-1	12	-1	13	4	0	10	2	6	4	3	13	5	
O				12	'	10	_		10	_		_ ~		10		

DF	56	27	0	A6	0	43	7C	DF	45	27	60	7C	F3	43	A9	
				7.0		10	, 0		10			'		40	/ (0	

Shuffle input

- Use index to lookup appropriate shuffle vector
- Shuffle input bytes to get them in the expected positions

```
      0
      1
      0
      1
      0
      1
      1
      0
      1
      1
      0
      1
      1
      0
      1
      1
      1
      0
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
```

Reverse bytes in 2 byte varints

*not actually necessary since x86 is little endian

```
      00000000
      01001010
      01110001
      11001000

      00000000
      01001110
      01101010
      10011011

      00010111
      10110101
      00000000
      01110110
```

Reverse bytes in 2 byte varints

^{*}not actually necessary since x86 is little endian

```
      00000000
      0
      1001010
      0
      11001000

      00000000
      0
      1001110
      0
      1101010
      10011011

      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
      0
```

Mask out leading purple 1's

```
      00000000
      01001010
      01110001
      01001000

      00000000
      01001110
      01101010
      0000000
      01110110

      00010111
      00110101
      00000000
      01110110
```

Mask out leading purple 1's

```
      00000000
      01001010
      01110001
      01001000

      00000000
      01001110
      01101010
      0000000
      01110110
```

Shift top bytes of each varint 1 bit right (mask/shift/or)

```
      00000000
      01001010
      00111000
      11001000

      00000000
      01001110
      00110101
      0000000
      01110110

      00001011
      10110101
      00000000
      01110110
```

Shift top bytes of each varint 1 bit right (mask/shift/or)

Done!

11101110 00011101 11110101 11101101 01111001 11111000 01101001 00100001 00001011 10110101 10111001 01110110

```
      11101110
      00011101
      11110101
      11101101

      01111001
      11111000
      01101001
      00100001

      00001011
      10110101
      10111001
      01110110
```

101101000110

```
      11101110
      00011101
      11110101
      11101101

      01111001
      11111000
      01101001
      00100001

      00001011
      10110101
      10111001
      01110110
```

101101000110

```
      11101110
      00011101
      11110101
      11101101

      01111001
      11111000
      01101001
      00100001

      00001011
      10110101
      10111001
      01110110
```

10<mark>110</mark>1000110

```
      11101110
      00011101
      11110101
      11101101

      01111001
      11111000
      01101001
      00100001

      00001011
      10110101
      10111001
      01110110
```

10110<mark>10</mark>00110

```
      11101110
      00011101
      11110101
      11101101

      01111001
      11111000
      01101001
      00100001

      00001011
      10110101
      10111001
      01110110
```

101101000110

```
      11101110
      00011101
      11110101
      11101101

      01111001
      11111000
      01101001
      00100001

      00001011
      10110101
      10111001
      01110110
```

101101000110

```
      11101110
      00011101
      11110101
      11101101

      01111001
      11111000
      01101001
      00100001

      00001011
      10110101
      10111001
      01110110
```

Decode 4 varints from 8 bytes

```
      11101110
      00011101
      11110101
      11101101

      01111001
      11111000
      01101001
      00100001

      00001011
      10110101
      10111001
      01110110
```

Pad ints to 4 bytes

```
      11101110
      00011101
      0000000
      0000000

      11110101
      111101101
      01111001
      0000000

      11111000
      01101001
      0000000
      0000000
      0000000

      00100001
      0000000
      0000000
      0000000
      0000000
```

Pad ints to 4 bytes

```
      0000000
      000011101
      11101110

      0000000
      01111001
      11101101
      11110101

      0000000
      0000000
      01101001
      11111000

      0000000
      0000000
      0000000
      00100001
```

Reverse bytes

*not actually necessary since x86 is little endian

```
      00000000
      00011101
      11101110

      00000000
      01111001
      11101101
      11110101

      00000000
      0000000
      01101001
      11111000

      00000000
      0000000
      0000000
      00100001
```

Clear top bit of each byte

```
      00000000
      00011101
      01101110

      00000000
      01111001
      01101101
      011110101

      00000000
      0000000
      01101001
      01111000

      00000000
      00000000
      0000000
      00100001
```

Clear top bit of each byte

```
      00000000
      000011101
      01101110

      00000000
      01111001
      01101101
      011110101

      00000000
      0000000
      01101001
      01111000

      00000000
      00000000
      0000000
      00100001
```

Shift 2nd least significant bytes over by 1 bit

```
      00000000
      00001110
      11101110

      00000000
      01111001
      00110110
      11110101

      00000000
      0000000
      00110100
      1111000

      00000000
      0000000
      0000000
      00100001
```

Shift 2nd least significant bytes over by 1 bit

```
      00000000
      000001110
      11101110

      00000000
      01111001
      00110110
      11110101

      00000000
      0000000
      00110100
      11111000

      00000000
      00000000
      0000000
      00100001
```

Shift 3rd least significant bytes over by 2 bits

```
      00000000
      000001110
      11101110

      00000000
      00001110
      111101110

      00000000
      00000000
      00110100
      11111000

      00000000
      00000000
      00000000
      00100001
```

Shift 3rd least significant bytes over by 2 bits

```
      00000000
      0000000
      00001110
      11101110

      00000000
      00011110
      01110110
      11110101

      00000000
      0000000
      00110100
      11111000

      0000000
      0000000
      0000000
      0010000
      00100001
```

Done!

```
      11101110
      10011101
      11110101
      11101101

      00000011
      11111000
      11101001
      10110101

      00001011
      10110101
      10111001
      01110110
```

111101110110

```
      11101110
      10011101
      11110101
      11101101

      00000011
      11111000
      11101001
      10110101

      00001011
      10110101
      10111001
      01110110
```

111101110110

```
      11101110
      10011101
      11110101
      11101101

      00000011
      11111000
      11101001
      10110101

      00001011
      10110101
      10111001
      01110110
```

1111<mark>01110</mark>110

```
      11101110
      10011101
      11110101
      11101101

      00000011
      11111000
      11101001
      10110101

      00001011
      10110101
      10111001
      01110110
```

1111<mark>0</mark>1110110

```
      11101110
      10011101
      11110101
      11101101

      00000011
      11111000
      11101001
      10100001

      00001011
      10110101
      10111001
      01110110
```

Decode 2 varints from 9 bytes

```
      11101110
      10011101
      11110101
      11101101

      00000011
      11111000
      11101001
      10110110

      00001011
      10110101
      10111001
      01110110
```

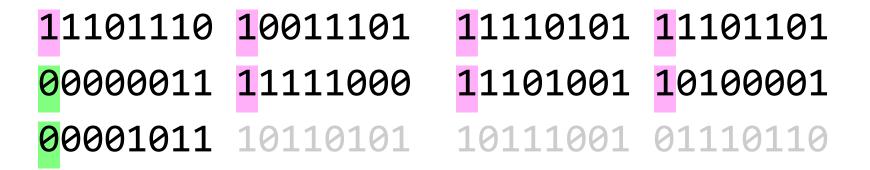
- Could handle the same way as other cases
- Would require 5 AND operations, 4 shift operations, and 4 OR operations

```
      11101110
      10011101
      11110101
      11101101

      00000011
      11111000
      11101001
      10110110

      00001011
      10110101
      10111001
      01110110
```

- We can simulate shifting by different amounts with multiplication
- Only needs 1 multiplication, 1 shift, 1 OR, 1 shuffle



Two 1-5 byte ints



Treat SIMD register as eight 16 bit registers, loading 1 byte into each. First byte doesn't need to be shifted.

11101110 10011101 00000011 11111000 00001011 10110101

11110101 11101101 11101001 10100001 10111001 01110110

0000000 0000000

```
      11101110
      10011101
      11110101
      11101101

      00000011
      11111000
      11101001
      10100001

      00001011
      10110101
      10111001
      01110110
```

 11101110
 10011101

 00000011
 11111000

 00001011
 10110101

11110101 11101101 11101001 10100001 10111001 01110110

11101110 00000000 0000000 00000000

11101110 10011101 00000011 11111000 00001011 10110101

 11110101
 11101101

 1110101
 10100001

 10111001
 01110110

11101110 00000000 00000000 <mark>11110101</mark>

 11101110
 10011101
 11110101
 11101101

 00000011
 11111000
 11101001
 10110101

 00001011
 101110101
 10111001
 01110110

11101110 00000000 000000000 <mark>11101101</mark> 00000000 11110101 00000000 10011101

11101110 10011101 11110101 11101101 00000011 11111000 11101001 10100001 00001011 10110101 10111001 01110110

```
      11101110
      00000011

      00000000
      11101101

      00000000
      11110101

      00000000
      10011101
```

Clear top bit of each byte

Clear top bit of each byte

```
01101110 00000011 * 16 (<< 4)
00000000 01101101 * 32 (<< 5)
00000000 01110101 * 64 (<< 6)
00000000 00011101 * 128 (<< 7)
```

Multiply to shift bits into place

```
      0
      1101110
      0
      00000011
      * 16 (<< 4)</td>

      0
      0
      0
      0
      1101101
      * 32 (<< 5)</td>

      0
      0
      0
      0
      1110101
      * 64 (<< 6)</td>

      0
      0
      0
      0
      111011
      * 128 (<< 7)</td>
```

Multiply to shift bits into place

```
11100000 00110000 * 16 (<< 4)
00001101 10100000 * 32 (<< 5)
00011101 01000000 * 64 (<< 6)
00001110 10000000 * 128 (<< 7)
```

Multiply to shift bits into place

 1110
 000
 0011
 000
 0000
 1101
 101
 0000

 000
 1101
 01
 0000
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00
 00

```
    1110
    000
    0011
    000
    0000
    1101
    101
    0000

    000
    1110
    01
    0000
    00
    000
    000
    000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    00000
    0000
    0000
    0000
    0000
    0000
    0000
    00000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    00000
    0000
    0000
    0000
    0000
    0000
    0000
    00000
    0000
    00000
    0000
    0000
    0000
    0000
    0000
```

Left shift everything by 8 bits

```
    1110
    0000
    0011
    0000
    0000
    1101
    101
    0000

    000
    1110
    01
    0000
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
```

Left shift everything by 8 bits

```
    0011
    0000
    101
    101
    0000
    000
    000
    1101

    01
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    00000
    0000
    0000
    0000
    0000
    000
```

```
    1110
    0000
    0011
    0000
    0000
    1101
    101
    0000

    000
    1101
    01
    0000
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
```

```
    0011
    0000
    101
    101
    0000
    000
    000
    1101

    01
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    0000
    00000
    0000
    0000
    0000
    0000
    000
```

```
    1110
    0000
    0011
    0000
    0000
    1101
    101
    0000

    000
    1101
    01
    0000
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
```

```
    0011
    0000
    101
    0000
    0000
    1101

    01
    0000
    0000
    0000
    0000
    0000
```

```
    1111
    0000
    0011
    0000
    0000
    1101
    101
    0000

    000
    1101
    01
    0000
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
```

```
<mark>0011</mark>0000 0000<mark>1101 101</mark>00000 000<mark>11101</mark>
<mark>01</mark>000000 00<mark>001110 1</mark>0000000 00000000
```

```
    1111
    0000
    0011
    0000
    0000
    1101
    101
    0000

    000
    1101
    01
    0000
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
    00
```

```
00110000 0000<mark>1101 101</mark>00000 000<mark>11101</mark>
01000000 00<mark>001110 1</mark>0000000 00000000
```

```
    1111
    0000
    0011
    1101
    101
    01101
    101
    00000

    000
    1110
    01
    00000
    00
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    <td
```

```
    00110000
    0000
    1101
    101
    00000
    000
    11101

    01
    000000
    000000
    000000
    0000000
```

```
    1111
    0000
    0011
    1101
    101
    01101
    101
    00000

    000
    1110
    01
    00000
    00
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    000
    <td
```

```
00110000 00001101 10100000 000<mark>11101</mark>

<mark>01</mark>000000 00<mark>001110 1</mark>0000000 00000000
```

```
    1111
    0000
    0011
    1101
    101
    01101
    101
    11101

    01
    01
    01
    000000
    00
    0001110
    10000000
```

```
00110000 00001101 10100000 000<mark>11101</mark>

<mark>01</mark>000000 00<mark>001110 1</mark>0000000 00000000
```

```
    1111
    0000
    0011
    1101
    101
    01101
    101
    11101

    01
    01
    01
    000000
    00
    0001110
    10000000
```

```
00110000 00001101 10100000 00011101
01000000 00<mark>001110 1</mark>0000000 00000000
```

```
    1111
    0000
    0011
    1101
    1010
    1101
    1011

    010
    1110
    0100
    1000
    1000
    1000
    1000
```

```
00110000 00001101 10100000 00011101
01000000 00<mark>001110 1</mark>0000000 00000000
```

```
    1111
    0000
    0011
    1101
    1010
    1101
    1011

    010
    1110
    0100
    1110
    1000
    1000
    1000
```

```
    1111
    0000
    00111101
    10101101
    10111101

    010
    11101
    01001110
    10001110
    10001110
    10000000
```

```
11110000 <mark>0011</mark>1101 10101101 101<mark>11101</mark>
01011101 01001110 10001110 10000000
```

```
      00110000
      00111101
      10101101
      1011101

      01011101
      01001110
      10001110
      10000000
```

```
      00110000
      00111101
      10101101
      101

      01011101
      01001110
      10001110
      10000000
```

```
      00110000
      00111101
      10101101
      10111101

      01011101
      01001110
      10001110
      10000000
```

```
      00110000
      00111101
      10101101
      10111101

      01011101
      01001110
      10000000
```

```
      00110000
      00111101
      10101101
      10111101

      01011101
      01001110
      10001110
      10000000
```

```
      00110000
      00111101
      10101101
      10111101

      01011101
      01001110
      10001110
      10000000
```

```
      00110000
      00111101
      10101101
      10111101

      01011101
      01001110
      10001110
      10000000
```

Extract result from every other byte

OR in low 7 bits of least significant byte

(remember that we stored it in most significant byte position originally)

```
      00110000
      00111101
      10101101
      10111101

      01011101
      01001110
      10001110
      11101110
```

OR in low 7 bits of least significant byte

(remember that we stored it in most significant byte position originally)

00111101 10111101 01001110 11101110

Final result!

00111101 10111101 01001110 11101110

Final result!

Checking my work against initial varint:

11101110 10011101 11110101 11101101 00000011

00111101 10111101 01001110 1<mark>1101110</mark>

Final result!

Checking my work against initial varint:

1<mark>1101110</mark> 10011101 11110101 11101101 00000011

00111101 10111101 01<mark>001110 1</mark>1101110

Final result!

Checking my work against initial varint:

```
11101110 1<mark>0011101</mark> 11110101 11101101
00000011
```

00111101 101<mark>11101 01</mark>001110 11101110

Final result!

Checking my work against initial varint:

11101110 10011101 1<mark>1110101</mark> 11101101 00000011

0011<mark>1101 101</mark>11101 01001110 11101110

Final result!

Checking my work against initial varint:

11101110 10011101 11110101 1<mark>1101101</mark> 0000011

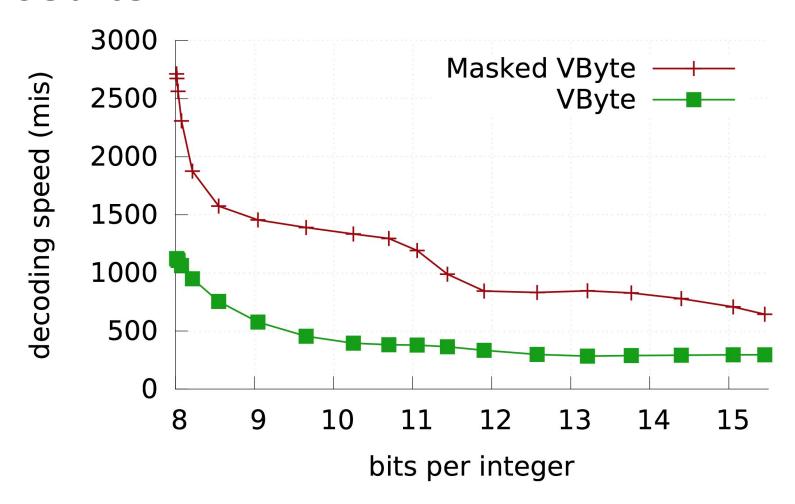
<mark>0011</mark>1101 10111101 01001110 11101110

Final result!

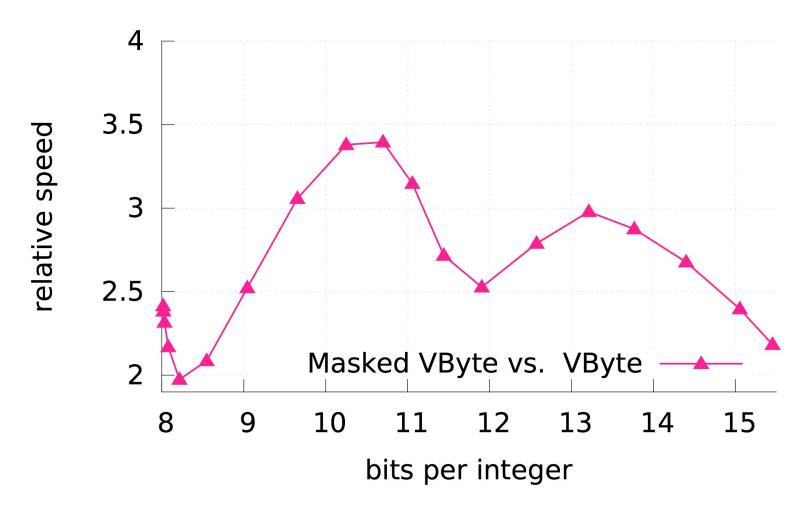
Checking my work against initial varint:

11101110 10011101 11110101 11101101 0000<mark>0011</mark>

Results



Results



Q&A