08/02/2023

Assignment Probability

1) Define Experiment, Sample Space, outcome & Event.

Ans:

Experiment: It is a mechanism that produces a definite outcome that cannot be predicted with certainty.

(or)

It is a mechanism that produces a definite outcome that predicts with uncertainty.

Sample Space: It is a collection or a set of all possible outcomes of a random experiment.

outcome: An outcome is a possible result of an experiment or trial.
Each possible outcome of a particular experiment is unique.

Event: An event is a subset of sample space.
These are the favourable outcomes of the experiment.

2) What is probability and explain different types of probability?

Ans:

Probability: It is a measure of the likelihood of an event occurs.

It ranges between 0 and 1.

0 indicates that an event is impossible

1 indicates that an event is certain.

Types of Probabilities:

There are mainly 4 types of probabilities.

1) Theoretical Probability
2) Experimental Probability
3) Axiomatic Probability
4) Subjective Probability

Theoretical Probability: It is based on the possible chances of something happening.

It is based on what is expected to happen in an experiment without conducting it.

$$\text{Theoretical Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total No. of outcomes}}$$

Experimental Probability : It is a probabity that is determined based on a series of experiments.
∴ It is based on the data which is obtained after an experiment is carried out.

$$\text{Experimental Probabity} = \frac{\text{No. of times an event occurs}}{\text{Total No. of experiments that are conducted}}$$

Axiomatic Probability : It is a set of rules or axioms are set, which applies to all types. In this, the chances of occurance and non-occurance of the events can be quantified.
It is the likelihood of an event/outcome occurring based on the occurance of a previous event/outcome.

Subjective Probability : It is based on a person's own personal reasoning & judgment. It is the probability that the outcome a person is expecting will actually occur.

3) In loan defaulters older people make up only 1.4%. Now the probability that someone defaults on a loan is 0.184. Find the probability of default on loan knowing that he is an old person. older people make up only 0.8%.

Sol:

Given data:

$P(old \mid Yes) = 1.4\% = 0.014$

$P(Yes) = 0.184$

$P(old) = 0.8\% = 0.008$

$P(Yes/old) = ?$

According to Baye's Theorem

$$P(A|B) = \frac{P(A)}{P(B)} \cdot P(B|A)$$

$$\Rightarrow P(Yes|old) = \frac{P(Yes)}{P(old)} \cdot P(old|Yes)$$

$$= \frac{0.184}{0.008} * 0.014 = 0.322$$

$$\therefore \boxed{P(Yes|old) = 0.322}$$

4) Define Bayes' theorem & Write the formulae.

Ans:

Definition : Bayes' theorem governs the likelihood that one event is based on the occurance of some other events. It depends upon the concepts of conditional probability.
It gives the probability of some events depending on some conditions related to the event.

Formula of Bayes' Theorem :

$$P(A|B) = \frac{P(A)}{P(B)} \cdot P(B|A) \quad \text{where } P(B) \neq 0$$

Where,

$P(A)$ → Probability of occurance of event A.

$P(B)$ → Probability of occurance of event B.

$P(A|B)$ → Probability of occurance of event A given B.

$P(B|A)$ → Probability of occurance of event B given A.

5) It may have learned that the word "free" appears in 30% of the mails marked as spam, i.e., P(free|spam) = 0.30. Assuming 1% of non-spam mail includes the word "free" & 50% of all mails received by the user are spam, find the probability that a mail is spam if the word "free" appears in it.

Sol:

Given Data:

$P(free|Spam) = 0.30$

$P(free|non\text{-}spam) = 1\% = 0.01$

$P(Spam) = 50\% = 0.5$

$\therefore P(non\text{-}spam) = 50\% = 0.5$

$P(Spam|free) = ?$

1st Method:

$$P(Spam|free) = \frac{P(Spam) * P(free|spam)}{P(free|spam) * P(spam) + P(free|non\text{-}spam) * P(non\text{-}spam)}$$

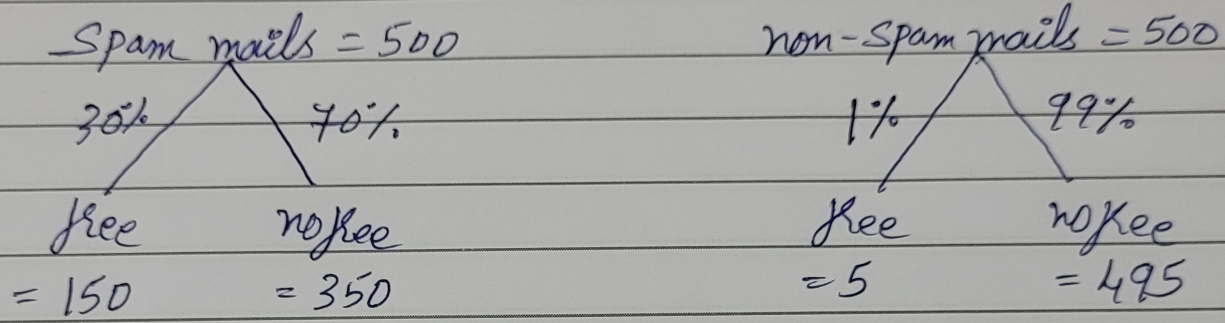
$$= \frac{0.5 * 0.3}{(0.3 * 0.5) + (0.01 * 0.5)} = \frac{0.15}{0.15 + 0.005}$$

$\therefore \boxed{P(Spam|free) = 0.96\#\#}$

2nd Method:

$$P(\text{Spam}|\text{free}) = \frac{P(\text{Spam})}{P(\text{free})} \cdot P(\text{free}|\text{spam})$$

Let, Total mails = 1000



$\therefore$ Total free mails = 150 + 5 = 155

$$P(\text{free}) = \frac{155}{1000} = 0.155$$

$$P(\text{Spam}|\text{free}) = \frac{0.5}{0.155} * 0.3$$

$$= \frac{0.15}{0.155} = 0.9677$$

$\therefore$ $\boxed{P(\text{Spam}|\text{free}) = 0.9677}$