1. **The jeep Problem (optimization with constraints)** We shall illustrate a problem in which it is desired to minimize a function subject to constraints given as recursive relations.

If s_i is the amount of fuel stored at the ith storage point $(i = 0, 1, \dots, n)$, d_i is the distance between the (i-1)st and the ith storage points, and k_i is the number of round trips the vehicle makes between these two points, then

$$s_{i-1} = s_i + 2k_i d_{\cancel{1}\,i} + d_i \quad (i = 1, \dots, n)$$

Thus, the amount of fuel stored at (i-1)st point is equal to the amount stored at the ith point plus the amount consumed along the route in making k_i round trips and a single forward trip.

It is not difficult to see that a minimum use of fuel is made if for each trip the vehicle proceeds loaded at full capacity. For, in that case, a smaller number of trips would be made and, hence, less fuel is consumed in travel. Also, in order to have no fuel left at the end, it is necessary that the vehicle be loaded with 500 units of fuel at the 500-mile point. From these two facts, it follows that the ith storage point should be located so that the vehicle makes k_{i+1} round trips between this point and the (i+1)st storage point and one last forward trip, always fully loaded and ultimately leaving no fuel behind at the ith storage point. Working backward, the last statement is valid back to the first storage point but is not possible for the starting point since the position of that point is predetermined. Hence, the vehicle will make its last forward trip between the starting point and the first storage point with a load $c < 500$. Thus, we have:

$$\begin{aligned} s_i &= k_i(500 - 2d_i) + 500 - d_i \quad (i = 2, \dots, n), \\ s_1 &= k_1(500 - 2d_1) + c - d_1. \end{aligned}$$

It is desired to minimize s_0 given by

$$s_0 = s_1 + 2k_1 d_1 + d_1.$$

Now, using for s_1 the value previously given, one has

$$s_0 = 500k_1 + c.$$

Since the vehicle can travel the last 500 miles without need for stored fuel, in order to minimize s_0, it suffices to put $s_n = 500$ and to place the storage points along the first 500 miles of the route. Thus,

$$\sum_{i=1}^{n} d_i = 500.$$

It turns out that $k_i = 8 - i, d_i = 500/(17 - 2i),\ i = 1, \ldots, 7 \quad and \quad s_0 = 3836.45.$