# Problem 4 : The Caterer Problem

- A caterer knows that, in connection with the meals he has arranged to serve during the next n days, he will need $r_{j \geq 0}$ fresh napkins on the jthday,j=1,2,…..,n.Laundering normally takes p days; i.e., a soiled napkin sent for laundering immediately after use on the jth day is returned in time to be used again on the (j+p)th day. However, the laundry also has a higher-cost service which returns the napkins in q<p days (p and q are integers).

- If the caterer has no clean napkins on hand, he will meet his immediate needs by purchasing napkins at a cents each. Laundering costs b and c cents per napkin for the normal and high-cost service respectively. How does he arrange matters to meet his needs and minimize his outlay for the n days?

- Let $x_j$ represent the napkins purchased for use on the jth day; the remaining requirements, if any, are supplied by laundered napkins. Of the $r_j$ napkins which have been used on that day plus any other soiled napkins on hand,

- let $y_j$ be the number sent for laundering under normal service and $z_j$ the number under the rapid service.

- Finally, since soiled napkins need not be sent to the laundry immediately after use, let $s_j$ be the stock of soiled napkins on hand after $y_j+z_j$ have been shipped to the laundry: they will be available for laundering on the next day, together with those from the $r_{j+1}$ used on that day.

Consequently,

$$y_j + z_j + s_{j-1} = r_j \ (j=1,2,\ldots,n).$$

- That is, the napkins used on the jth day are equal to the number sent to the laundry plus the change in the stock of soiled napkins.

- The returns from the laundry are equal to the amounts shipped for normal service p days earlier plus the amounts shipped for rapid service q days earlier. Together with purchases, these provide for the needs on the same day. This gives

$$x_j + y_{j-p} + z_{j-q} = r_j (j = 1, 2, \ldots n)$$

- The total cost to be minimized, subject to these constraints, is

$$\sum_j (ax_j + by_j + cz_j),$$

Where

$$a > b, c > b : x_j, y_j, z_j, s_j \geq 0$$