Probability And Random Variables Assignment -I

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Q) A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independant events or not.

Solution :

We can say that two events A, B are independant if $P(A \cap B) = P(A).P(B)$ Here we are tossing a fair coin and rolling an unbiased die, so let us declare two random variables X for tossing a coin Y for rollling a die.

$$X = \begin{cases} 0, & \text{coin shows tail} \\ 1, & \text{coin shows heads} \end{cases}$$

$$Y = \begin{cases} 1, & \text{outcome of die is 1} \\ 2, & \text{outcome of die is 2} \\ 3, & \text{outcome of die is 3} \\ 4, & \text{outcome of die is 4} \\ 5, & \text{outcome of die is 5} \\ 6, & \text{outcome of die is 6} \end{cases}$$

parameter	value	description
$X = 1$	P(X=1)=$\frac{1}{2}$	occurence of head
$X = 0$	P(X=0) =$\frac{1}{2}$	occurence of tail
$Y = 1$	P(Y=1) =$\frac{1}{6}$	die shows 1
$Y = 2$	P(Y=2) =$\frac{1}{6}$	die shows 2
$Y = 3$	P(Y=3) =$\frac{1}{6}$	die shows 3
$Y = 4$	P(Y=4) =$\frac{1}{6}$	die shows 4
$Y = 5$	P(Y=5) =$\frac{1}{6}$	die shows 5
$Y = 6$	P(Y=6) =$\frac{1}{6}$	die shows 6

So, the total number of outcomes are (2).(6) = 12. Now, Favourable outcomes of A would be :

$$A : \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$$

H represents Head and T represents Tail
The required condition is X = 1, so Y can be 1,2,3,4,5,6

$$P(X = 1) = P(X = 1, Y = 1, 2, 3, 4, 5, 6) = \frac{6}{12} = \frac{1}{2}$$

Favourable outcomes of B would be :

$$B : \{(H, 3), (T, 3)\}$$

The required condition of is Y = 3, so X can be 0,1

$$P(Y = 3) = P(Y = 3, X = 0, 1) = \frac{2}{12} = \frac{1}{6}$$

The intersection of both the events would be :

$$= \{(H, 3)\} or (X = 1, Y = 3)$$

So,

$$P(X = 1, Y = 3) = \frac{1}{12}$$

And

$$P(X = 1).P(Y = 3) = \left(\frac{1}{2}\right).\left(\frac{1}{6}\right)$$

$$= \frac{1}{12}$$

So here we are getting

$$P(X = 1, Y = 3) = P(X = 1).P(Y = 3) = \frac{1}{12}$$

Therefore, we can say that A and B are independant events.