Solving Simple Arithmetic Word Problems with Schemas Precisely

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Problem

Solving simple arithmetic word problems presented in English with the help of schemas and generate the solution.

Examples

John has 5 apples. He ate 1. How many does he have now?

John has 5 apples. He got 3 apples from Mary. How many does he have now?

Motivation

- Solving word problems automatically and giving an interpretable solution benefits students in concept clarification

- It is an interesting natural language understanding problem where semantics of the language in a mathematical context has to be modelled.

- Online question answering systems would fail unless the exact word problem is in a web document somewhere

Related Work

Knowledge Based

- Precise
- Model human behaviour
- Interpretable
- Less focus on NLP
- Not comparable

Empirical Systems

- High recall
- Simple representation
- Not interpretable
- More focus on NLP
- Comparable

Need for Representation Layer

Sally has 3 apples. John has 4 apples. Kathy has 5 apples more than Sally. How many apples do they have altogether?

Equation
3 + 4 + (5 + 3)

Sally is 10 years older than Mary. In 6 years, she will be twice as old as Mary. How old are they now?

s - m = 10
s - 2m = 6

(thrice)

s - m = 10
s - 3m = 12

Schemas

- Schemas are templates that organize similar experiences and provide a mechanism to find the characteristic components of these experiences and reason with them.

- In this context, they describe what are the types of word problems and what are the equations associated with them.

Schemas

Schema	Example Word Problem
Combine	John has 5 apples. Mary has 3 apples. How many apples do they have altogether?
Compare	John has 5 apples. Mary has 2 apples less than John. How many apples does Mary have?
Change	John has 5 apples. He gave 1 apple to Mary. How many does he have now?

Schemas

Combine Schema

Slots expected - Superset, Subsets
Equation - $\sum |\text{subset}| = |\text{superset}|$

Compare Schema

Slots expected - large, small, difference
Equation - large - small = difference

Change Schema

Slots expected - initial, change, final
Equation - initial + change = final

Schema Identifier

Need for Schema Identifier :

- A simple keyword based identifier fails many times

- “John had 5 apples **altogether**. He **gave** 3 apples to Mary. How many apples does he have now?”

Schema Identifier

- Binary feature vector
- Trained on a multi-layer perceptron
- Resources - WordNet, ConceptNet, Stanford Core NLP Suite
- Features
  - Change in tense
  - Presence of multiple objects
  - Presence of multiple agents
  - Presence of schema specific keywords
  - Presence of units
  - ....

Representation of Schemas

- Represented in Prolog

- The various slots and the equations connecting the slots were coded in a declarative fashion

Representation of Schemas

Keyword Check:

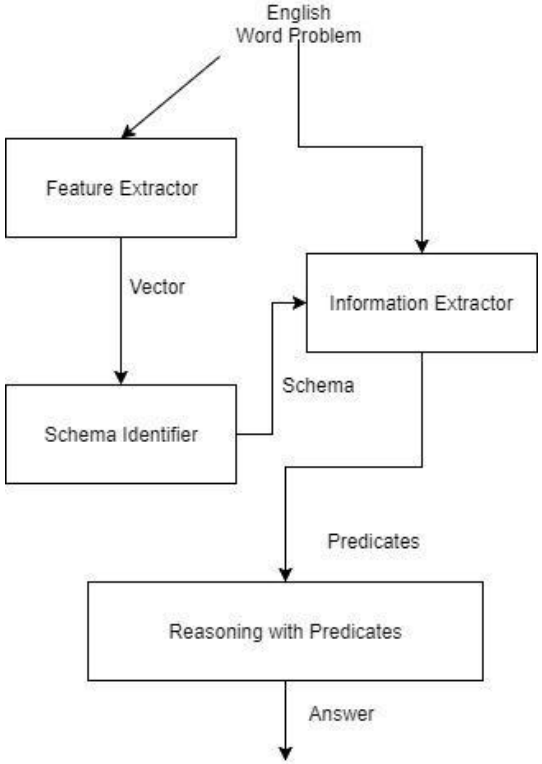
cType(C, comparePlus) :- keyword(C, taller).
cType(C, comparePlus) :- keyword(C, more).
cType(C, compareMinus) :- keyword(C, shorter).
cType(C, compareMinus) :- keyword(C, less).

Representation of Schemas

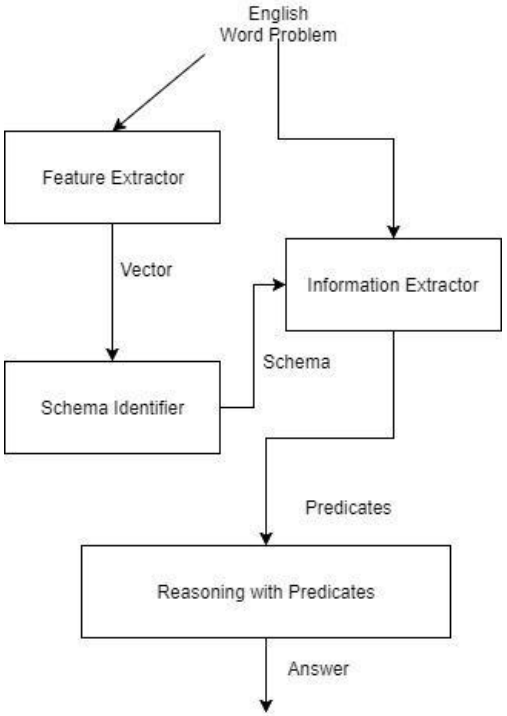
Rules:

cValue(C, Num) :- compare(C), entity(K1, X), type(X, Ent), entity(K2, Y), type(Y, Ent), X <> Y, value(X, V1), value(Y, V2), not(var(V1)), not(var(V2)), V1 > V2, Num is V1 - V2, !.

Architecture

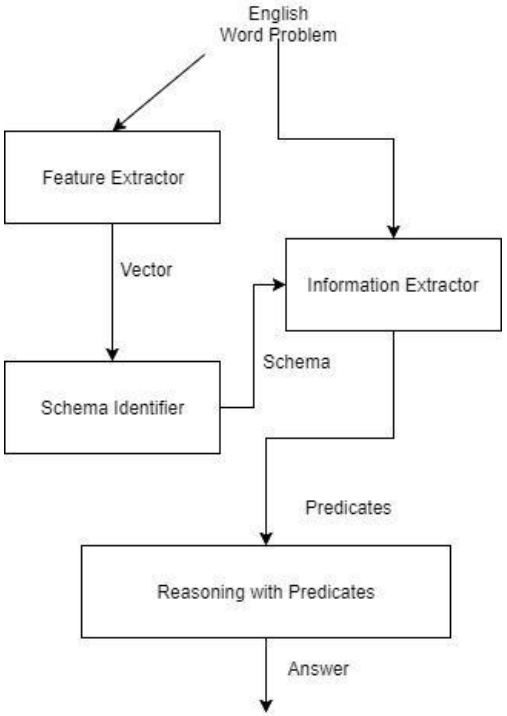


Complete Example



There are 5 apples in a basket. Ruth put 2 apples in a basket. How many apples are in the basket now?

Complete Example

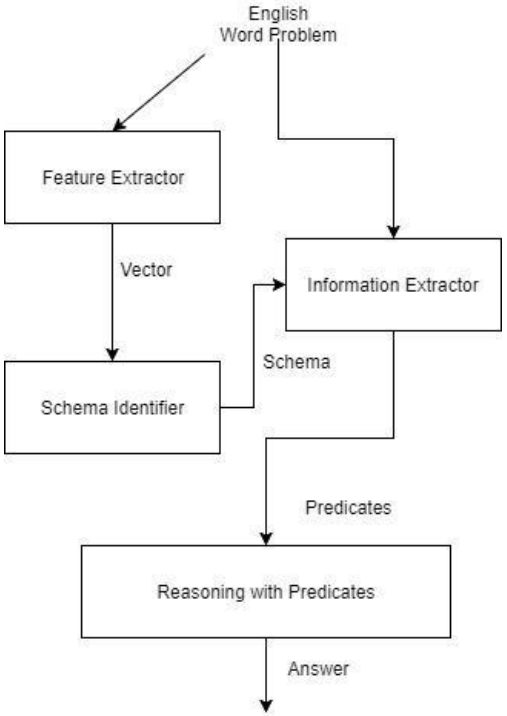


There are 5 apples in a basket. Ruth put 2 apples in a basket. How many apples are in the basket now?

There are 5 apples in a basket.

holdsAt(f1, 10).
entity(f1, ent1).
type(ent1, apple).
loc(f1, basket).
value(ent1, 5).

Complete Example

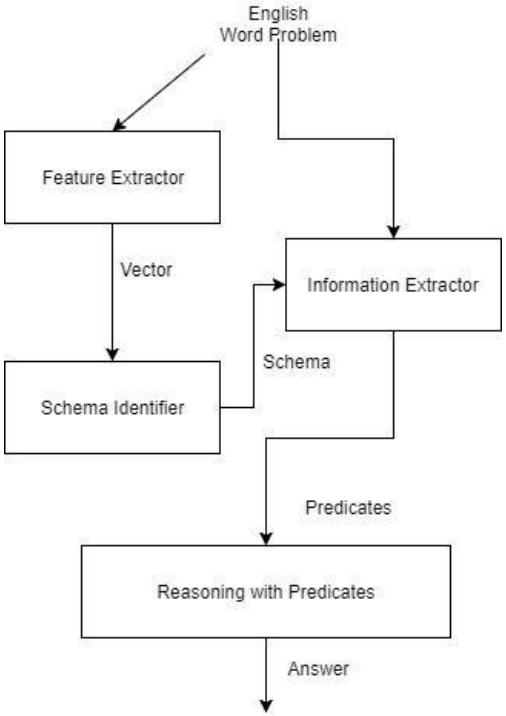


There are 5 apples in a basket. Ruth put 2 apples in the basket. How many apples are in the basket now?

Ruth put 2 apples in the basket.

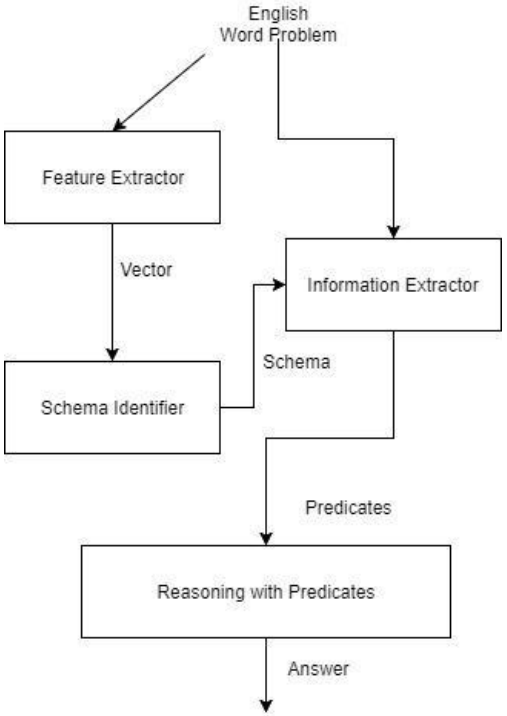
holdsAt(f2, 20).
entity(f2, ent2).
type(ent2, apple).
loc(f2, basket).
agent(f2, ruth).
value(ent1, 2).

Complete Example



Schema identifier result : CHANGE

Complete Example

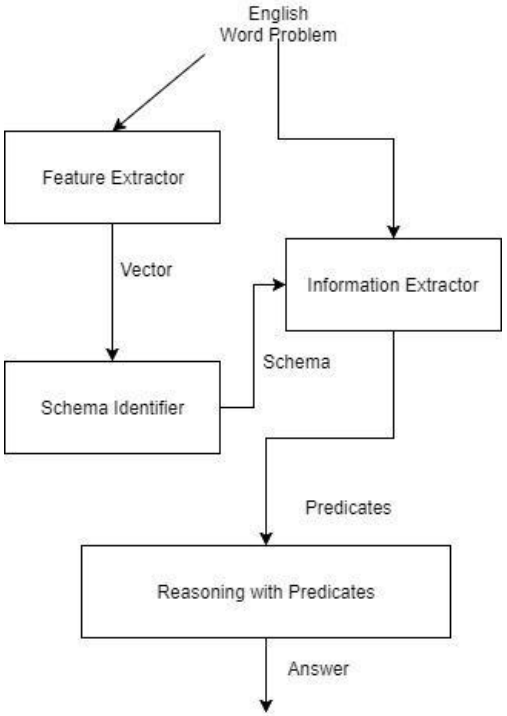


Schema identifier result : CHANGE

Schema specific predicates:

change(c).
agent(c, ruth).
value(c, 2).
keyword(c, put).
loc(c, basket).

Complete Example



Facts that can be asserted:

holdsAt(ques, 30)
loc(ques, basket).
entity(ques, ent3).
type(ent3, apple).
value(ent3, 7).

Query:
entity(ques, X), value(X, Ans)

Implementation Details

- Multi-layer perceptron in WEKA
- Prolog interfaced with Java using jpl
- Program developed in Java

Results

Precision

	DS1	DS2	DS3
Our System	98.3	97.7	92.3
Competitor 1[1]	94.0	77.1	81.0
Competitor 2[2]	96.3	80.0	90.1

[1] (Hosseini et. al., 2014)

[2] (S. Sundaram and Khemani, 2015)

Discussion

- Advantages
  - High interpretability
  - Amenable for explanation generation
  - Precise - no garbage values

- Disadvantages
  - High precision, low recall - drawback of knowledge based systems

Future Work

- Collect enough data to perform robust semantic parsing
- Run user-studies to ascertain actual use for students
- Extend to semantically richer domains

Thank You