**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: probability that the service manager will not meet his demand will be 100-73.4=26.6 0r 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: Probability that the employee will be greater than age of 44=100-84.1345=15.86%

So the probability of number of employees between 38.44 years of age = pr(X<44)-

0.5=84.1345-0.5 = 34.1345%

Therefore the statement that more employees at the processing center are older than 44 between 38 is TRUE

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: So number of employees with probability 0.912 of them being under age

30=0.0912\*400=36.48 (36)

Therefore the statement B of the question is also TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: As we know that if *X* ~ *N*(μ, σ2) and Y ~ *N*(μ2, σ22) are two independent Random variables that X + Y ~ N(μ1 + μ2, σ12  + σ22 ) and X - Y ~ N(μ1 - μ2, σ12  + σ22)

Similarly if Z = ax + by, Where X and Y are as defined above I.e z is linear combination of X and Y then Z ~ N(aμ1 + bμ2 a2 σ12  + b2 σ22)

Therefore in the question

2X1 ~ N(2μ,4 σ2) and

X1 + X2 ~ N(μ + μ, σ2  + σ2 ) ~ N (2μ , 2σ2)

2X1 - (X1 + X2) = N(4μ , 6C2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Z\*σ + μ = X

Z(-0.005)\*20+100=(2.57)\*20+100 = 151.4

Z(0.005)\*20+100 =(-2.57)\*20+100 = 48.6

Ans: D.48.5,151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans:

Mean profit is RS 540 Million

Std deviation is RS 225 Million

Range is RS

State norms interval (0.95,540,225)

Range is RS 99.0081034, 980.991896

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:

Formula X=X= μ + Zσ; where in from z table, 5 percentile =-1.645

X = 540(-1.645)\*225

X = 169.875

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

Probability od division 1 making a loss p(X<0)

Stats.norm. cdf(0,5,3)

0.0477903

Probability od division 2 making a loss p(X<0)

Stats.norm. cdf(0,7,4)

0.0400591