Assignment 10

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Download all python codes from

https://github.com/Sowmyabandi99/Assignment10/blob/main/assignment10_2.py

and latex-tikz codes from

https://github.com/Sowmyabandi99/Assignment10/blob/main/main.tex

1 Question No. 2.26

A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below:

Machines			
Types of toys	I	II	III
A	12	18	6
B	6	0	9

TABLE 1.1: Toys table

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs 7.50 and that on each toy of type B is Rs 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

2 Solution

Let the number of toys of type A be x and the number of toys of type B be y such that

$$x \geq 0 \tag{2.0.1}$$
$$y \geq 0 \tag{2.0.2}$$

According to the question,

$$12x + 6y \leq 360 \tag{2.0.3}$$
$$\implies 2x + y \leq 60 \tag{2.0.4}$$

and,

$$18x + 0y \leq 360 \tag{2.0.5}$$
$$\implies x \leq 20 \tag{2.0.6}$$

and,

$$6x + 9y \leq 360 \tag{2.0.7}$$
$$\implies 2x + 3y \leq 120 \tag{2.0.8}$$

$\therefore$ Our problem is

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 7.5 & 5 \end{pmatrix} \mathbf{x} \tag{2.0.9}$$
$$s.t. \quad \begin{pmatrix} 2 & 1 \\ 1 & 0 \\ 2 & 3 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 60 \\ 20 \\ 120 \end{pmatrix} \tag{2.0.10}$$

Lagrangian function is given by

$$\begin{aligned} &L(\mathbf{x}, \boldsymbol{\lambda}) \\ &= \begin{pmatrix} 7.5 & 5 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} - 60 \right] \right. \\ &+ \left[\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 20 \right] + \left[\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} - 120 \right] \\ &+ \left. \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \boldsymbol{\lambda} \end{aligned} \tag{2.0.11}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} \tag{2.0.12}$$

Now,

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = \begin{pmatrix} 7.5 + \begin{pmatrix} 2 & 1 & 2 & -1 & 0 \end{pmatrix} \boldsymbol{\lambda} \\ 5 + \begin{pmatrix} 1 & 0 & 3 & 0 & -1 \end{pmatrix} \boldsymbol{\lambda} \\ \begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} - 60 \\ \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 20 \\ \begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} - 120 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{2.0.13}$$

$\therefore$ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 2 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 & -1 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -7.5 \\ -5 \\ 60 \\ 20 \\ 120 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.14}$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 3 \\ 2 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -7.5 \\ -5 \\ 60 \\ 120 \end{pmatrix} \tag{2.0.15}$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 3 \\ 2 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -7.5 \\ -5 \\ 60 \\ 120 \end{pmatrix} \tag{2.0.16}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{3}{4} & \frac{-1}{4} \\ 0 & 0 & \frac{-1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{-1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} -7.5 \\ -5 \\ 60 \\ 120 \end{pmatrix} \tag{2.0.17}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 15 \\ 30 \\ -3.12 \\ -0.62 \end{pmatrix} \tag{2.0.18}$$

$\because \lambda = \begin{pmatrix} -3.12 \\ -0.62 \end{pmatrix} \succ \mathbf{0}$

$\therefore$ Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 15 \\ 30 \end{pmatrix} \tag{2.0.19}$$

$$Z = \begin{pmatrix} 7.5 & 5 \end{pmatrix} \mathbf{x} \tag{2.0.20}$$

$$= \begin{pmatrix} 7.4 & 5 \end{pmatrix} \begin{pmatrix} 15 \\ 30 \end{pmatrix} \tag{2.0.21}$$

$$= 262.5 \tag{2.0.22}$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 14.99999998 \\ 29.99999996 \end{pmatrix} \tag{2.0.23}$$

$$Z = 262.49999967 \tag{2.0.24}$$

Hence, the manufacturer should manufacture $\boxed{x = 15}$ toys of type A and $\boxed{y = 30}$ toys of type B in a day to get maximum profit $\boxed{Z = 262.5}$.

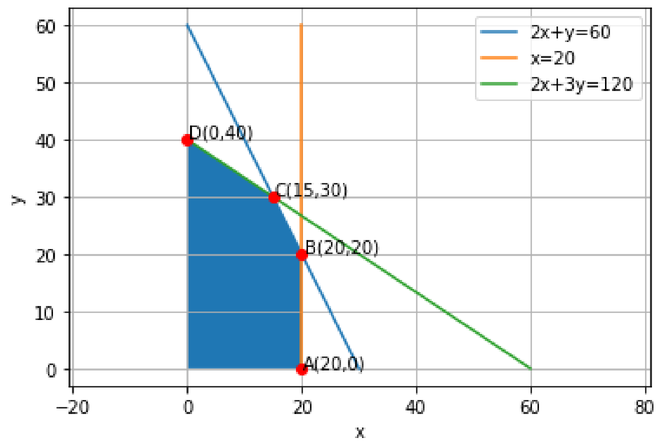

Fig. 2.1: Diet Problem