Assignment 10

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1 QUESTION No. 2.26

A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below:

Machines			
Types of toys	I	II	III
A	12	18	6
B	6	0	9

TABLE 1.1: Toys table

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs 7.50 and that on each toy of type B is Rs 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

2 SOLUTION

Let the number of toys of type A be x and the number of toys of type B be y such that

$$x \geq 0 \qquad (2.0.1)$$
$$y \geq 0 \qquad (2.0.2)$$

According to the question,

$$12x + 6y \leq 360 \qquad (2.0.3)$$
$$\implies 2x + y \leq 60 \qquad (2.0.4)$$

and,

$$18x + 0y \leq 360 \qquad (2.0.5)$$
$$\implies x \leq 20 \qquad (2.0.6)$$

and,

$$6x + 9y \leq 360 \qquad (2.0.7)$$
$$\implies 2x + 3y \leq 120 \qquad (2.0.8)$$

$\therefore$ Our problem is

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 7.5 & 5 \end{pmatrix} \mathbf{x} \qquad (2.0.9)$$

$$s.t. \quad \begin{pmatrix} 2 & 1 \\ 1 & 0 \\ 2 & 3 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 60 \\ 20 \\ 120 \end{pmatrix} \qquad (2.0.10)$$

Lagrangian function is given by

$$\begin{aligned}
L(\mathbf{x}, \lambda) &= \begin{pmatrix} 7.5 & 5 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} - 60 \right] \right. \\
&+ \left[\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 20 \right] + \left[\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} - 120 \right] \\
&+ \left. \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \lambda
\end{aligned} \qquad (2.0.11)$$

where,

$$\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} \qquad (2.0.12)$$

Now,

$$\nabla L(\mathbf{x}, \lambda) = \begin{pmatrix} 7.5 + \begin{pmatrix} 2 & 1 & 2 & -1 & 0 \end{pmatrix} \lambda \\ 5 + \begin{pmatrix} 1 & 0 & 3 & 0 & -1 \end{pmatrix} \lambda \\ \begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} - 60 \\ \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 20 \\ \begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} - 120 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \qquad (2.0.13)$$

∴ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 2 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 & -1 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -7.5 \\ -5 \\ 60 \\ 20 \\ 120 \\ 0 \\ 0 \end{pmatrix}$$

$$(2.0.14)$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 3 \\ 2 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -7.5 \\ -5 \\ 60 \\ 120 \end{pmatrix} \qquad (2.0.15)$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 3 \\ 2 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -7.5 \\ -5 \\ 60 \\ 120 \end{pmatrix} \qquad (2.0.16)$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{3}{4} & \frac{-1}{4} \\ 0 & 0 & \frac{-1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{-1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} -7.5 \\ -5 \\ 60 \\ 120 \end{pmatrix} \qquad (2.0.17)$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 15 \\ 30 \\ -3.12 \\ -0.62 \end{pmatrix} \qquad (2.0.18)$$

$\because \lambda = \begin{pmatrix} -3.12 \\ -0.62 \end{pmatrix} > \mathbf{0}$

∴ Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 15 \\ 30 \end{pmatrix} \qquad (2.0.19)$$

$$Z = \begin{pmatrix} 7.5 & 5 \end{pmatrix} \mathbf{x} \qquad (2.0.20)$$

$$= \begin{pmatrix} 7.4 & 5 \end{pmatrix} \begin{pmatrix} 15 \\ 30 \end{pmatrix} \qquad (2.0.21)$$

$$= 262.5 \qquad (2.0.22)$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 14.99999998 \\ 90.99999996 \end{pmatrix} \qquad (2.0.23)$$

$$Z = 262.49999967 \qquad (2.0.24)$$

Hence, the manufacturer should manufacture $x = 15$ toys of type A and $y = 30$ toys of type
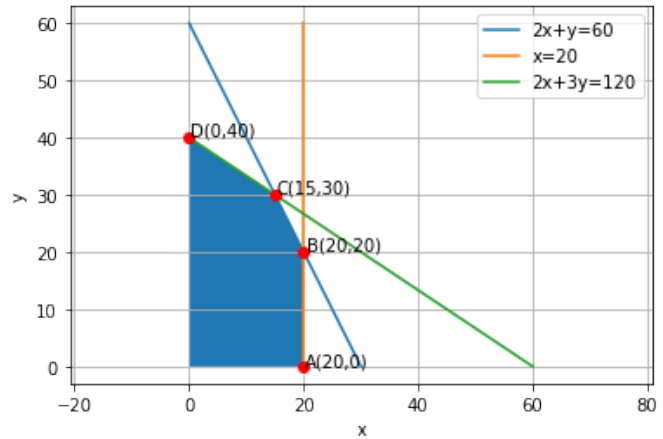
B in a day to get maximum profit $Z = 262.5$.

Fig. 2.1: Diet Problem