

ASSIGNMENT4

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Download all python codes from

<https://github.com/Sowmyabandi99/Assignment4/blob/main/Assignment4/assignment4.py>

and download all latex-tikz codes from

<https://github.com/Sowmyabandi99/Assignment4/blob/main/Assignment4/main.tex>

1 QUESTION No 2.40

Find the equation of the plane passing through the intersection of the planes

$$(2 \ 2 \ -3)\mathbf{x} = 7 \quad (1.0.1)$$

$$(2 \ 5 \ 3)\mathbf{x} = 9 \quad (1.0.2)$$

and the point $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

2 SOLUTION

The intersection of the planes is obtained by row reducing the augmented matrix as

$$\begin{pmatrix} 2 & 2 & -3 & 7 \\ 2 & 5 & 3 & 9 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & 1 & -\frac{3}{2} & \frac{7}{2} \\ 2 & 5 & 3 & 9 \end{pmatrix} \quad (2.0.1)$$

$$\xrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & 1 & -\frac{3}{2} & \frac{7}{2} \\ 0 & 3 & 6 & 2 \end{pmatrix} \quad (2.0.2)$$

$$\xrightarrow{R_2 \leftarrow \frac{R_2}{3}} \begin{pmatrix} 1 & 1 & -\frac{3}{2} & \frac{7}{2} \\ 0 & 1 & 2 & \frac{2}{3} \end{pmatrix} \quad (2.0.3)$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & -\frac{7}{2} & \frac{17}{6} \\ 0 & 1 & 2 & \frac{2}{3} \end{pmatrix} \quad (2.0.4)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} \frac{17}{6} \\ \frac{2}{3} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} \frac{7}{2} \\ -2 \\ 1 \end{pmatrix} \quad (2.0.5)$$

Thus, $\begin{pmatrix} \frac{17}{6} \\ \frac{2}{3} \\ 0 \end{pmatrix}$ is another point on the plane. The normal vector to the plane is then obtained as

$$\left(\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} \frac{17}{6} \\ \frac{2}{3} \\ 0 \end{pmatrix} \right) \begin{pmatrix} \frac{7}{2} \\ -2 \\ 1 \end{pmatrix} \quad (2.0.6)$$

which can be obtained by row reducing the matrix

$$\begin{pmatrix} \frac{7}{2} & -2 & 1 \\ \frac{-5}{6} & \frac{1}{3} & 3 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{2}{7}R_1} \begin{pmatrix} 1 & -\frac{4}{7} & \frac{2}{7} \\ \frac{-5}{6} & \frac{1}{3} & 3 \end{pmatrix} \quad (2.0.7)$$

$$\xrightarrow{R_2 \leftarrow R_2 + \frac{5}{6}R_1} \begin{pmatrix} 1 & -\frac{4}{7} & \frac{2}{7} \\ 0 & \frac{7}{7} & \frac{68}{21} \end{pmatrix} \quad (2.0.8)$$

$$\xrightarrow{R_1 \leftarrow R_1 - 4R_2} \begin{pmatrix} 1 & 0 & -\frac{38}{21} \\ 0 & 1 & \frac{68}{21} \end{pmatrix} \quad (2.0.9)$$

$$\Rightarrow \mathbf{n} = \begin{pmatrix} \frac{38}{3} \\ \frac{68}{3} \\ 1 \end{pmatrix} = \begin{pmatrix} 38 \\ 68 \\ 3 \end{pmatrix} \quad (2.0.10)$$

Since the plane passing through $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, using (2.0.10)

$$(38 \ 68 \ 3) \left(\mathbf{x} - \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \right) = 0 \quad (2.0.11)$$

$$\Rightarrow (38 \ 68 \ 3)\mathbf{x} = 153 \quad (2.0.12)$$

Alternatively, the plane passing through the intersection of (1.0.1) and (1.0.2) has the form

$$(2 \ 2 \ -3)\mathbf{x} + \lambda(2 \ 5 \ 3)\mathbf{x} = 7 + 9\lambda \quad (2.0.13)$$

substituting $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ in the above,

$$(2 \ 2 \ -3) \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda (2 \ 5 \ 3) \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 7 + 9\lambda \quad (2.0.14)$$

$$\Rightarrow -3 + 18\lambda = 7 + 9\lambda \quad (2.0.15)$$

$$\Rightarrow \lambda = \frac{10}{9} \quad (2.0.16)$$

substituting value of λ in (2.0.14) yields the equation of the plane.

$$(38 \ 38 \ 3) \mathbf{x} = 153 \quad (2.0.17)$$

Plot of the plane

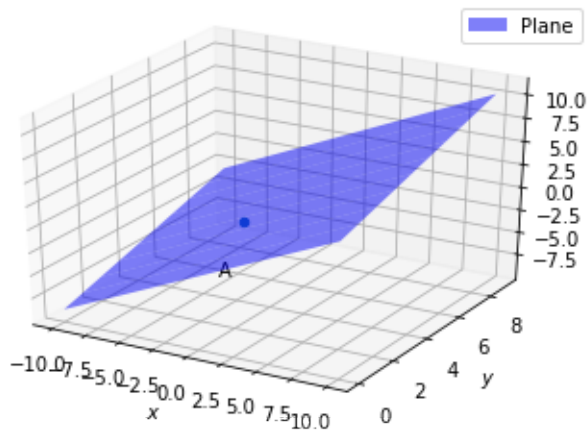


Fig. 2.1: Plot of the plane