ASSIGNMENT4

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Download all python codes from

https://github.com/Sowmyabandi99/Assignment3/tree/main/Assignment3/Assignment3

and download all latex-tikz codes from

https://github.com/CRAMYATULASI/ASSIGNMENT5/tree/main/ASSIGNMENT5

1 Question No 2.40

Find the equation of the plane passing through the intersection of the planes

$$\begin{pmatrix} 2 & 2 & -3 \end{pmatrix} \mathbf{x} = 7 \tag{1.0.1}$$

$$\begin{pmatrix} 2 & 5 & 3 \end{pmatrix} \mathbf{x} = 9 \tag{1.0.2}$$

and the point $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

2 Solution

The intersection of the planes is obtained by row reducing the augmented matrix as

$$\begin{pmatrix} 2 & 2 & -3 & 7 \\ 2 & 5 & 3 & 9 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & 1 & \frac{-3}{2} & \frac{7}{2} \\ 2 & 5 & 3 & 9 \end{pmatrix} \tag{2.0.1}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & 1 & \frac{-3}{2} & \frac{7}{2} \\ 0 & 3 & 6 & 2 \end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{3}} \begin{pmatrix} 1 & 1 & \frac{-3}{2} & \frac{7}{2} \\ 0 & 1 & 2 & \frac{2}{3} \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & \frac{-7}{2} & \frac{17}{6} \\ 0 & 1 & 2 & \frac{2}{3} \end{pmatrix} \tag{2.0.4}$$

$$\implies \mathbf{x} = \begin{pmatrix} \frac{17}{6} \\ \frac{2}{3} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} \frac{7}{2} \\ -2 \\ 1 \end{pmatrix} \tag{2.0.5}$$

Thus, $\begin{pmatrix} \frac{17}{6} \\ \frac{2}{3} \\ 0 \end{pmatrix}$ is another point on the plane.The normal vector to the plane is then obtained as

$$\left(\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} \frac{17}{6} \\ \frac{2}{3} \\ 0 \end{pmatrix} \right) \begin{pmatrix} \frac{7}{2} \\ -2 \\ 1 \end{pmatrix} \tag{2.0.6}$$

which can be obtained by row reducing the matrix

$$\begin{pmatrix} \frac{7}{2} & -2 & 1 \\ \frac{-5}{6} & \frac{1}{3} & 3 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{2}{7} R_1} \begin{pmatrix} 1 & \frac{-4}{7} & \frac{2}{7} \\ \frac{-5}{6} & \frac{1}{3} & 3 \end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 + \frac{5}{6} R_1} \begin{pmatrix} 1 & \frac{-4}{7} & \frac{2}{7} \\ 0 & \frac{-1}{7} & \frac{68}{21} \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 4R_2} \begin{pmatrix} 1 & 0 & \frac{-38}{3} \\ 0 & \frac{-1}{7} & \frac{68}{21} \end{pmatrix} \tag{2.0.9}$$

$$\implies \mathbf{n} = \begin{pmatrix} \frac{38}{3} \\ \frac{68}{3} \\ 1 \end{pmatrix} = \begin{pmatrix} 38 \\ 68 \\ 3 \end{pmatrix} \tag{2.0.10}$$

Since the plane passing through $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, using (2.0.10)

$$\begin{pmatrix} 38 & 68 & 3 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \right) = 0 \tag{2.0.11}$$

$$\implies \begin{pmatrix} 38 & 68 & 3 \end{pmatrix} \mathbf{x} = 153 \tag{2.0.12}$$

Alternatively,the plane passing through the intersection of (1.0.1) and (1.0.2) has the form

$$\begin{pmatrix} 2 & 2 & -3 \end{pmatrix} \mathbf{x} + \lambda \begin{pmatrix} 2 & 5 & 3 \end{pmatrix} \mathbf{x} = 7 + 9\lambda \tag{2.0.13}$$

substituting $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ in the above,

$$\begin{pmatrix} 2 & 2 & -3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 & 5 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 7 + 9\lambda \tag{2.0.14}$$

$$\implies -3 + 18\lambda = 7 + 9\lambda \tag{2.0.15}$$

$$\implies \lambda = \frac{10}{9} \tag{2.0.16}$$

substituting value of λ in (2.0.14) yields the equation of the plane.

$$\begin{pmatrix} 38 & 38 & 3 \end{pmatrix} \mathbf{x} = 153 \tag{2.0.17}$$

Plot of the plane

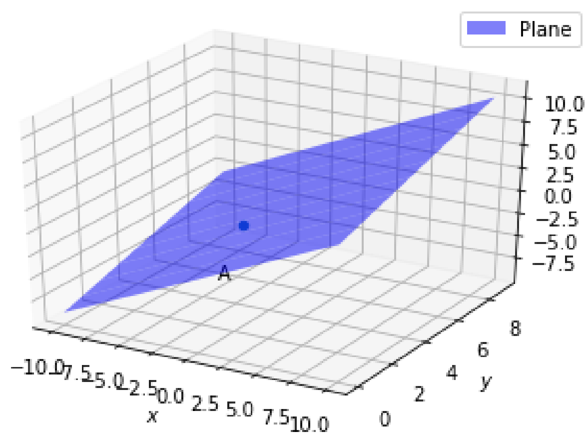

Fig. 2.1: Plot of the plane