ASSIGNMENT4

SOWMYA BANDI

Download all python codes from

https://github.com/Sowmyabandi99/Assignment4/blob/main/Assignment4/assignment4.py

and download all latex-tikz codes from

https://github.com/Sowmyabandi99/Assignment4/blob/main/Assignment4/main.tex

1 Question No 2.40

Find the equation of the plane passing through the intersection of the planes

$$\begin{pmatrix}2 & 2 & -3\end{pmatrix}\mathbf{x} = 7 \tag{1.0.1}$$

$$\begin{pmatrix}2 & 5 & 3\end{pmatrix}\mathbf{x} = 9 \tag{1.0.2}$$

and the point $\begin{pmatrix}2\\1\\3\end{pmatrix}$

2 SOLUTION

The intersection of the planes is obtained by row reducing the augmented matrix as

$$\begin{pmatrix}2 & 2 & -3 & 7\\2 & 5 & 3 & 9\end{pmatrix}\xleftrightarrow{R_1\leftarrow\frac{R_1}{2}}\begin{pmatrix}1 & 1 & \frac{-3}{2} & \frac{7}{2}\\2 & 5 & 3 & 9\end{pmatrix} \tag{2.0.1}$$

$$\xleftrightarrow{R_2\leftarrow R_2-2R_1}\begin{pmatrix}1 & 1 & \frac{-3}{2} & \frac{7}{2}\\0 & 3 & 6 & 2\end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow{R_2\leftarrow\frac{R_2}{3}}\begin{pmatrix}1 & 1 & \frac{-3}{2} & \frac{7}{2}\\0 & 1 & 2 & \frac{2}{3}\end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_1\leftarrow R_1-R_2}\begin{pmatrix}1 & 0 & \frac{-7}{2} & \frac{17}{6}\\0 & 1 & 2 & \frac{2}{3}\end{pmatrix} \tag{2.0.4}$$

$$\implies \mathbf{x} = \begin{pmatrix}\frac{17}{6}\\\frac{2}{3}\\0\end{pmatrix} + \lambda\begin{pmatrix}\frac{7}{2}\\-2\\1\end{pmatrix} \tag{2.0.5}$$

Thus, $\begin{pmatrix}\frac{17}{6}\\\frac{2}{3}\\0\end{pmatrix}$ is another point on the plane.The normal vector to the plane is then obtained as

$$\left(\begin{pmatrix}2\\1\\3\end{pmatrix} - \begin{pmatrix}\frac{17}{6}\\\frac{2}{3}\\0\end{pmatrix}\right)\begin{pmatrix}\frac{7}{2}\\-2\\1\end{pmatrix} \tag{2.0.6}$$

which can be obtained by row reducing the matrix

$$\begin{pmatrix}\frac{7}{2} & -2 & 1\\\frac{-5}{6} & \frac{1}{3} & 3\end{pmatrix}\xleftrightarrow{R_1\leftarrow\ \frac{2}{7}R_1}\begin{pmatrix}1 & \frac{-4}{7} & \frac{2}{7}\\\frac{-5}{6} & \frac{1}{3} & 3\end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_2\leftarrow R_2+\frac{5}{6}R_1}\begin{pmatrix}1 & \frac{-4}{7} & \frac{2}{7}\\0 & \frac{-1}{7} & \frac{68}{21}\end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow{R_1\leftarrow R_1-4R_2}\begin{pmatrix}1 & 0 & \frac{-38}{3}\\0 & \frac{-1}{7} & \frac{68}{21}\end{pmatrix} \tag{2.0.9}$$

$$\implies \mathbf{n} = \begin{pmatrix}\frac{38}{3}\\\frac{68}{3}\\1\end{pmatrix} = \begin{pmatrix}38\\68\\3\end{pmatrix} \tag{2.0.10}$$

Since the plane passing through $\begin{pmatrix}2\\1\\3\end{pmatrix}$, using (2.0.10)

$$\begin{pmatrix}38 & 68 & 3\end{pmatrix}\left(\mathbf{x} - \begin{pmatrix}2\\1\\3\end{pmatrix}\right) = 0 \tag{2.0.11}$$

$$\implies \begin{pmatrix}38 & 68 & 3\end{pmatrix}\mathbf{x} = 153 \tag{2.0.12}$$

PLOT OF THE PLANE:

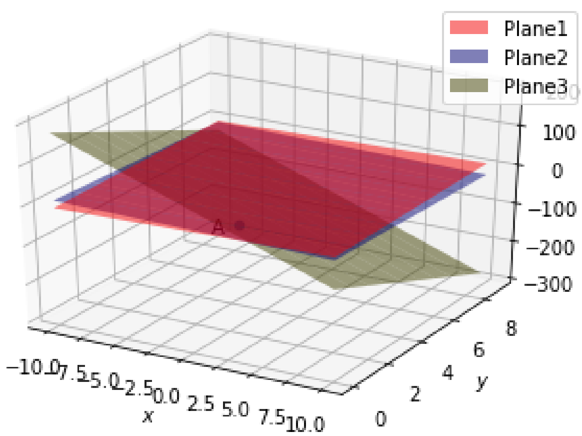

Fig. 2.1: Plot of the plane