1

ASSIGNMENT4

SOWMYA BANDI

Download all python codes from

https://github.com/Sowmyabandi99/Assignment4/blob/main/Assignment4/assignment4.py

and download all latex-tikz codes from

https://github.com/Sowmyabandi99/Assignment4/blob/main/Assignment4/main.tex

1 Question No 2.40

Find the equation of the plane passing through the intersection of the planes

$$(2 \ 2 \ -3)\mathbf{x} = 7$$
 (1.0.1)

$$\begin{pmatrix} 2 & 5 & 3 \end{pmatrix} \mathbf{x} = 9 \tag{1.0.2}$$

and the point $\begin{pmatrix} 2\\1\\3 \end{pmatrix}$

2 SOLUTION

The intersection of the planes is obtained by row reducing the augmented matrix as

$$\begin{pmatrix} 2 & 2 & -3 & 7 \\ 2 & 5 & 3 & 9 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & 1 & \frac{-3}{2} & \frac{7}{2} \\ 2 & 5 & 3 & 9 \end{pmatrix}$$
 (2.0.1)

$$\stackrel{R_2 \leftarrow R_2 - 2R_1}{\longleftrightarrow} \begin{pmatrix} 1 & 1 & \frac{-3}{2} & \frac{7}{2} \\ 0 & 3 & 6 & 2 \end{pmatrix} \qquad (2.0.2)$$

$$\stackrel{R_2 \leftarrow \frac{R_2}{3}}{\longleftrightarrow} \begin{pmatrix} 1 & 1 & \frac{-3}{2} & \frac{7}{2} \\ 0 & 1 & 2 & \frac{2}{3} \end{pmatrix} \qquad (2.0.3)$$

$$\stackrel{R_1 \leftarrow R_1 - R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & \frac{-7}{2} & \frac{17}{6} \\ 0 & 1 & 2 & \frac{2}{3} \end{pmatrix} \qquad (2.0.4)$$

$$\implies \mathbf{x} = \begin{pmatrix} \frac{17}{6} \\ \frac{2}{3} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} \frac{7}{2} \\ -2 \\ 1 \end{pmatrix} \qquad (2.0.5)$$

Thus, $\begin{pmatrix} \frac{17}{6} \\ \frac{2}{3} \\ 0 \end{pmatrix}$ is another point on the plane. The normal vector to the plane is then obtained as

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} \frac{17}{6} \\ \frac{2}{3} \\ 0 \end{pmatrix} \end{pmatrix} \begin{pmatrix} \frac{7}{2} \\ -2 \\ 1 \end{pmatrix}$$
(2.0.6)

which can be obtained by row reducing the matrix

$$\begin{pmatrix} \frac{7}{2} & -2 & 1\\ \frac{-5}{6} & \frac{1}{3} & 3 \end{pmatrix} \stackrel{R_1 \leftarrow \frac{2}{7}R_1}{\longleftrightarrow} \begin{pmatrix} 1 & \frac{-4}{7} & \frac{2}{7}\\ \frac{-5}{6} & \frac{1}{3} & 3 \end{pmatrix}$$
(2.0.7)

$$\stackrel{R_2 \leftarrow R_2 + \frac{5}{6}R_1}{\longleftrightarrow} \begin{pmatrix} 1 & \frac{-4}{7} & \frac{2}{7} \\ 0 & \frac{-1}{7} & \frac{68}{21} \end{pmatrix} \tag{2.0.8}$$

$$\stackrel{R_1 \leftarrow R_1 - 4R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & \frac{-38}{3} \\ 0 & \frac{-1}{7} & \frac{68}{21} \end{pmatrix}$$
 (2.0.9)

$$\implies \mathbf{n} = \begin{pmatrix} \frac{38}{3} \\ \frac{68}{3} \\ 1 \end{pmatrix} = \begin{pmatrix} 38 \\ 68 \\ 3 \end{pmatrix} \qquad (2.0.10)$$

Since the plane passing through $\begin{pmatrix} 2\\1\\3 \end{pmatrix}$, using (2.0.10)

$$(38 \ 68 \ 3) \left(\mathbf{x} - \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}\right) = 0$$
 (2.0.11)

$$\implies$$
 (38 68 3) $\mathbf{x} = 153$ (2.0.12)

PLOT OF THE PLANE:

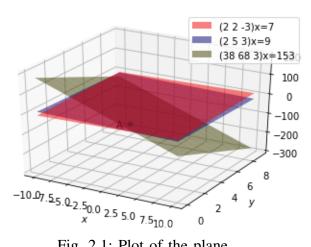


Fig. 2.1: Plot of the plane