

ASSIGNMENT 5

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Download all python codes from

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\begin{lstlisting}
https://github.com/Sowmyabandi99/Assignment5/
blob/main/Ass5/assignment5.py
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Latex-tikz codes from

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https://github.com/Sowmyabandi99/Assignment5/
blob/main/Ass5/main.tex
```

1 QUESTION No 2.51

Find the intervals in which the function

$$f(x) = x^2 - 4x + 6 \quad (1.0.1)$$

is

- 1) increasing
- 2) decreasing

2 SOLUTION

Given equation can be written as

$$y = x^2 - 4x + 6 \quad (2.0.1)$$

$$\Rightarrow x^2 - 4x - y + 6 = 0 \quad (2.0.2)$$

Comparing it with standard equation,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.3)$$

$$\therefore a = 1, b = 0, c = 0, d = -2, e = -\frac{1}{2}, f = 6 \quad (2.0.4)$$

General equation of parabola is

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.5)$$

$$\Rightarrow \mathbf{x}^T \begin{pmatrix} a & b \\ b & c \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} d \\ e \end{pmatrix} \mathbf{x} + f = 0 \quad (2.0.6)$$

Here,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 2 \\ -\frac{1}{2} \end{pmatrix}, f = 6 \quad (2.0.7)$$

Using eigen decomposition on \mathbf{V} ,

$$\mathbf{V} = \mathbf{P} \mathbf{D} \mathbf{P}^T \quad (2.0.8)$$

$$\text{where, } \mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.9)$$

$$\mathbf{P} = (\mathbf{p}_1 \quad \mathbf{p}_2) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (2.0.10)$$

The vertex of parabola \mathbf{c} is given by

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (2.0.11)$$

$$\text{where, } \eta = \mathbf{u}^T \mathbf{p}_1 = \frac{-1}{2} \quad (2.0.12)$$

Substituting values from (2.0.7), (2.0.10) and (2.0.12) in (2.0.11)

$$\begin{pmatrix} -2 & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} \quad (2.0.13)$$

Removing last row and representing (2.0.13) as augmented matrix and then converting the matrix to echelon form,

$$\begin{pmatrix} -2 & -1 & -6 \\ 1 & 0 & 2 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{R_1}{-2}} \begin{pmatrix} 1 & \frac{1}{2} & 3 \\ 1 & 0 & 2 \end{pmatrix} \quad (2.0.14)$$

$$\xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & \frac{1}{2} & 3 \\ 0 & -\frac{1}{2} & -1 \end{pmatrix} \quad (2.0.15)$$

$$\xrightarrow{R_2 \leftarrow (-2R_2)} \begin{pmatrix} 1 & \frac{1}{2} & 3 \\ 0 & 1 & 2 \end{pmatrix} \quad (2.0.16)$$

$$\xrightarrow{R_1 \leftarrow R_1 - \frac{R_2}{2}} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{pmatrix} \quad (2.0.17)$$

From (2.0.17) it can be observed that,

$$\mathbf{c} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (2.0.18)$$

$\therefore a > 0$, the vertex of the parabola turns from decreasing to increasing.

1) f is increasing in interval $(2, \infty)$

2) f is decreasing in interval $(-\infty, 2)$

Plot of Parabola-

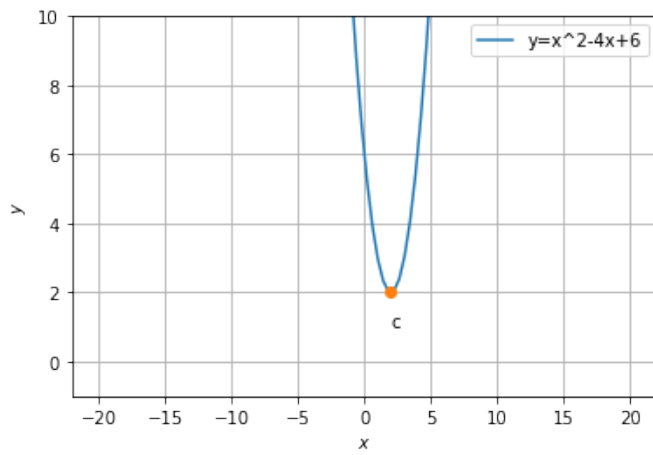


Fig. 2.1: Parabola