ASSIGNMENT 5

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Download all python codes from

```
\begin{lstlisting}
https://github.com/Sowmyabandi99/Assignment5/
    blob/main/Ass5/assignment5.py
```

Latex-tikz codes from

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https://github.com/Sowmyabandi99/Assignment5/
    blob/main/Ass5/main.tex
```

1 Question No 2.51

Find the intervals in which the function

$$f(x) = x^2 - 4x + 6 \tag{1.0.1}$$

is

1) increasing
2) decreasing

2 SOLUTION

Given equation can be written as

$$y = x^2 - 4x + 6 \tag{2.0.1}$$

$$\implies x^2 - 4x - y + 6 = 0 \tag{2.0.2}$$

Comparing it with standard equation,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.3}$$

$$\therefore a = 1, b = 0, c = 0, d = -2, e = -\frac{1}{2}, f = 6 \tag{2.0.4}$$

General equation of parabola is

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.5}$$

$$\implies \mathbf{x}^T\begin{pmatrix} a & b \\ b & c \end{pmatrix}\mathbf{x} + 2\begin{pmatrix} d \\ e \end{pmatrix}\mathbf{x} + f = 0 \tag{2.0.6}$$

Here,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 2 \\ \frac{-1}{2} \end{pmatrix}, f = 6 \tag{2.0.7}$$

Using eigen decomposition on $\mathbf{V}$,

$$\mathbf{V} = \mathbf{P}\mathbf{D}\mathbf{P}^T \tag{2.0.8}$$

$$where, \mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.9}$$

$$\mathbf{P} = \begin{pmatrix} \mathbf{p_1} & \mathbf{p_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.10}$$

The vertex of parabola $\mathbf{c}$ is given by

$$\begin{pmatrix} \mathbf{u}^T + \eta\mathbf{p_1}^T \\ \mathbf{V} \end{pmatrix}\mathbf{c} = \begin{pmatrix} -f \\ \eta\mathbf{p_1} - \mathbf{u} \end{pmatrix} \tag{2.0.11}$$

$$where, \eta = \mathbf{u}^T\mathbf{p_1} = \frac{-1}{2} \tag{2.0.12}$$

Substituting values from (2.0.7),(2.0.10) and (2.0.12) in (2.0.11)

$$\begin{pmatrix} -2 & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{c} = \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} \tag{2.0.13}$$

Removing last row and representing (2.0.13) as augmented matrix and then converting the matrix to echelon form,

$$\begin{pmatrix} -2 & -1 & -6 \\ 1 & 0 & 2 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{-2}} \begin{pmatrix} 1 & \frac{1}{2} & 2 \\ 1 & 0 & 2 \end{pmatrix} \tag{2.0.14}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & \frac{1}{2} & 3 \\ 0 & \frac{-1}{2} & -1 \end{pmatrix} \tag{2.0.15}$$

$$\xleftrightarrow{R_2 \leftarrow (-2R_2)} \begin{pmatrix} 1 & \frac{1}{2} & 3 \\ 0 & 1 & 2 \end{pmatrix} \tag{2.0.16}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{R_2}{2}} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{pmatrix} \tag{2.0.17}$$

From (2.0.17) it can be observed that,

$$\mathbf{c} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{2.0.18}$$

1) f is increasing in interval (2,∞)
2) f is decreasing in interval (-∞,2)

Plot of Parabola-

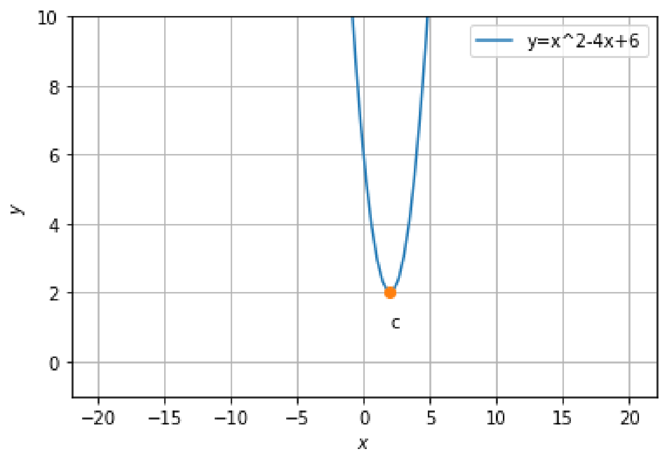

Fig. 2.1: Parabola