ASSIGNMENT 7

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1 QUESTION No 2.29

Find the equation of the set of points $\mathbf{P}$ such that its distances from the points $\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ are equal.

2 SOLUTION

1) From the given information,

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \quad (2.0.1)$$

$$\implies \|\mathbf{P}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{P} \quad (2.0.2)$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T\mathbf{P} \quad (2.0.3)$$

$$\implies 2\mathbf{A}^T\mathbf{P} - 2\mathbf{B}^T\mathbf{P} = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.4)$$

2) Equation of plane is $\mathbf{n}^T\mathbf{P} = \mathbf{d}$
   where, $\mathbf{n}^T$ is the normal vector to the plane
   - From (2.0.4),

   $$\left(2\mathbf{A}^T - 2\mathbf{B}^T\right)\mathbf{P} = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.5)$$

   $\mathbf{P}$ is a plane and it is perpendicular bisector to $\mathbf{A}^T - \mathbf{B}^T$
   $\because \mathbf{P}$ is perpendicular to line joining $\mathbf{A}^T$ and $\mathbf{B}^T$
   - Midpoint of $\mathbf{A}^T$ and $\mathbf{B}^T$

   $$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (2.0.6)$$

   - Substitute in (2.0.4),

   $$\left(2\mathbf{A}^T - 2\mathbf{B}^T\right)\left(\frac{\mathbf{A} + \mathbf{B}}{2}\right) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.7)$$

$$\implies \frac{\mathbf{A}+\mathbf{B}}{2} \text{ satisfies (2.0.4)}$$

- $\therefore \mathbf{P}$ is the plane that is perpendicular bisector of the line joining the given points

3) Putting given values $\mathbf{A}$ and $\mathbf{B}$ in (2.0.4), we get

$$2\begin{pmatrix} 3 & 4 & -5 \end{pmatrix}\mathbf{P} - 2\begin{pmatrix} -2 & 1 & 4 \end{pmatrix}\mathbf{P} \quad (2.0.8)$$

$$= \left\|\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}\right\|^2 - \left\|\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}\right\|^2 \quad (2.0.9)$$

$$\implies \begin{pmatrix} 6 & 8 & -10 \end{pmatrix}\mathbf{P} + \begin{pmatrix} 4 & -2 & -8 \end{pmatrix}\mathbf{P} \quad (2.0.10)$$

$$= 50 - 21 \quad (2.0.11)$$

$$\implies \begin{pmatrix} 10 & 6 & -18 \end{pmatrix}\mathbf{P} = 29 \quad (2.0.12)$$

$\therefore$ The required equation is

$$\begin{pmatrix} 10 & 6 & -18 \end{pmatrix}\mathbf{P} = 29 \quad (2.0.13)$$
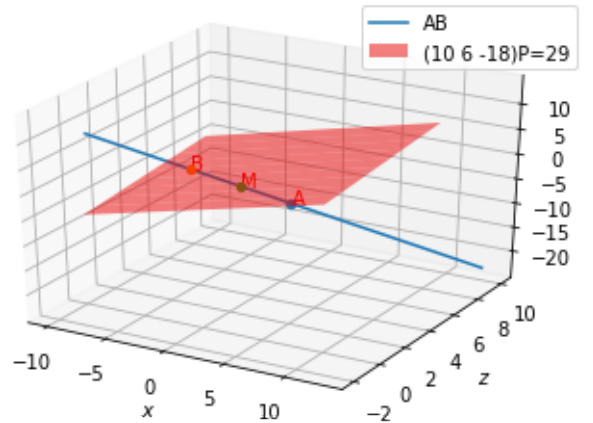
Plot of the equation whose distance from the points A and B are equal-

Fig. 2.1: Plot of the plane