ASSIGNMENT 8

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Download all python codes from

https://github.com/Sowmyabandi99/Assignment8/blob/main/assignment8.py

and latex codes from

https://github.com/Sowmyabandi99/Assignment8/blob/main/main.tex

1 Question No 2.50

Balance the following chemical equation.

$$NaOH + H_2SO_4 \rightarrow Na_2SO_4 + H_2O \tag{1.0.1}$$

2 SOLUTION

Let the balanced version of (1.0.1) be

$$x_1NaOH + x_2H_2SO_4 \rightarrow x_3Na_2SO_4 + x_4H_2O \tag{2.0.1}$$

which results in the following equations:

$$(x_1 - 2x_3)Na = 0 \tag{2.0.2}$$
$$(x_1 + 4x_2 - 4x_3 - x_4)O = 0 \tag{2.0.3}$$
$$(x_1 + 2x_2 - 2x_4)H = 0 \tag{2.0.4}$$
$$(x_2 - x_3)S = 0 \tag{2.0.5}$$

which can be expressed as

$$x_1 + 0.x_2 - 2x_3 + 0.x_4 = 0 \tag{2.0.6}$$
$$x_1 + 4x_2 - 4x_3 - x_4 = 0 \tag{2.0.7}$$
$$x_1 + 2x_2 + 0.x_3 - 2x_4 = 0 \tag{2.0.8}$$
$$0.x_1 + x_2 - x_3 + 0.x_4 = 0 \tag{2.0.9}$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 1 & 4 & -4 & -1 \\ 1 & 2 & 0 & -2 \\ 0 & 1 & -1 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0} \tag{2.0.10}$$

where,

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \tag{2.0.11}$$

(2.0.10) can be reduced as follows:

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 1 & 4 & -4 & -1 \\ 1 & 2 & 0 & -2 \\ 0 & 1 & -1 & 0 \end{pmatrix} \xleftrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 4 & -2 & -1 \\ 0 & 2 & 2 & -2 \\ 0 & 1 & -1 & 0 \end{pmatrix} \tag{2.0.12}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{4}} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{4} \\ 0 & 2 & 2 & -2 \\ 0 & 1 & -1 & 0 \end{pmatrix} \tag{2.0.13}$$

$$\xleftrightarrow[R_4 \leftarrow R_4 - R_2]{R_3 \leftarrow R_3 - 2R_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{4} \\ 0 & 0 & 3 & -\frac{3}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{4} \end{pmatrix} \tag{2.0.14}$$

$$\xleftrightarrow{R_3 \leftarrow \frac{R_3}{3}} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{4} \end{pmatrix} \tag{2.0.15}$$

$$\xleftrightarrow[R_4 \leftarrow R_4 + \frac{R_3}{2}]{R_2 \leftarrow R_2 + \frac{R_3}{2}} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.16}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + 2R_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.17}$$

Thus,

$$x_1 = x_4, x_2 = \frac{1}{2}x_4, x_3 = \frac{1}{2}x_4 \tag{2.0.18}$$

$$\implies \quad \mathbf{x} = x_4 \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix} \tag{2.0.19}$$

by substituting $x_4 = 2$

$$\mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix} \tag{2.0.20}$$

Hence, (2.0.1) finally becomes

$$2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O \tag{2.0.21}$$