ASSIGNMENT-2

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1 QUESTION No. 2.40

Construct PLAN where $PL = 4, LA = 6.5, \angle P = 90°, \angle A = 110°$ and $\angle N = 85°$.

2 SOLUTION

1) Let us assume vertices of given quadrilateral *PLAN* as **P,L,A** and **N**.

2) Let us generalize the given data:

$$\angle P = 90° = \theta \tag{2.0.1}$$
$$\angle A = 110° = \alpha \tag{2.0.2}$$
$$\angle N = 85° = \gamma \tag{2.0.3}$$
$$\angle E = 180° - \angle L = \delta, \tag{2.0.4}$$
$$\|L - P\| = 4 = a, \tag{2.0.5}$$
$$\|A - L\| = 6.5 = b, \tag{2.0.6}$$
$$P = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, L = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{2.0.7}$$

3) Also, let us assume the other two sides as

$$\|N - A\| = c \tag{2.0.8}$$
$$\|P - N\| = \|N\| = d \quad (\because P = 0) \tag{2.0.9}$$

4) We know,sum of angles of a quadrilateral = $360°$

$$\angle P + \angle L + \angle A + \angle N = 360° \tag{2.0.10}$$
$$90° + \angle L + 110° + 85° = 360° \tag{2.0.11}$$
$$\implies \angle L = 75° \tag{2.0.12}$$

- Now sum of all the angles given and (2.0.9) is $360°$.So construction of given quadrilateral is **possible**.

Lemma 2.1. *The coordinates of A and N can be written as follows:*

$$A = L + b \begin{pmatrix} \cos E \\ \sin E \end{pmatrix} \tag{2.0.13}$$

$$N = P + d \begin{pmatrix} \cos P \\ \sin P \end{pmatrix} \tag{2.0.14}$$

Proof. • For finding coordinates of A:
The vector equation of line is given by:

$$A = L + \lambda \begin{pmatrix} \cos E \\ \sin E \end{pmatrix} \tag{2.0.15}$$

$$\implies \|A - L\| = |\lambda| \times \left\| \begin{pmatrix} \cos 105° \\ \sin 105° \end{pmatrix} \right\| \tag{2.0.16}$$

$$\implies \|A - L\| = |\lambda| \times 1 \tag{2.0.17}$$

Using (2.0.6) we get:

$$\implies |\lambda| = b \tag{2.0.18}$$

• For finding coordinates of N:
The vector equation of line is given by:

$$N = P + \mu \begin{pmatrix} \cos P \\ \sin P \end{pmatrix} = \mu \begin{pmatrix} \cos P \\ \sin P \end{pmatrix} \quad (\because P = 0) \tag{2.0.19}$$

$$\implies \|N\| = |\mu| \times \left\| \begin{pmatrix} \cos 90° \\ \sin 90° \end{pmatrix} \right\| \tag{2.0.20}$$

$$\implies \|N\| = |\mu| \times 1 \tag{2.0.21}$$

Using (2.0.8) we get:

$$\implies |\mu| = d \tag{2.0.22}$$

Using inner products of vectors we get,

$$\frac{(A - N)^\top (A - L)}{\|A - N\| \times \|A - L\|} = \cos A \tag{2.0.23}$$

$$\frac{(A - N)^\top (P - N)}{\|A - N\| \times \|P - N\|} = \cos N \tag{2.0.24}$$

Now, dividing (2.0.23) and (2.0.24) we get:

$$\frac{(A-N)^\top(A-L)}{(A-N)^\top(P-N)} \times \frac{\|P-N\|}{\|A-L\|} = \frac{\cos A}{\cos N} \quad (2.0.25)$$

$$\frac{(A-N)^\top(A-L)}{(A-N)^\top(P-N)} \times \frac{d}{b} = \frac{\cos}{\cos} \quad (2.0.26)$$

$$\frac{A^\top A - A^\top L - N^\top A + N^\top L}{-A^\top N + N^\top N} = \frac{b}{d} \times \frac{\cos A}{\cos N} \quad (\because \mathbf{P} = \quad (2.0.27)$$

$$Let \quad S = b \times \frac{\cos A}{\cos N} \quad (2.0.28)$$

$$\implies d = \frac{S(-A^\top N + N^\top N)}{A^\top A - A^\top L - N^\top A + N^\top L} \quad (2.0.29)$$

$\square$

5) Now using (2.0.7), (2.0.13) and (2.0.28) and solving we get

$$\implies d = 6.47 \quad (2.0.30)$$

6) Putting values of b and d in (2.0.13) and (2.0.14)

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos E \\ \sin E \end{pmatrix} \quad (2.0.31)$$

$$\implies \mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 105° \\ \sin 105° \end{pmatrix} \quad (2.0.32)$$

$$\implies \mathbf{A} = \begin{pmatrix} 2.31 \\ 6.27 \end{pmatrix} \quad (2.0.33)$$

$$and \quad \mathbf{N} = d \begin{pmatrix} \cos P \\ \sin P \end{pmatrix} \quad (2.0.34)$$

$$\implies \mathbf{N} = 6.47 \begin{pmatrix} \cos 90° \\ \sin 90° \end{pmatrix} \quad (2.0.35)$$

$$\implies \mathbf{N} = \begin{pmatrix} 0 \\ 6.47 \end{pmatrix} \quad (2.0.36)$$

7) Now,the vertices of given Quadrilateral PLAN can be written as,

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{L} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 2.31 \\ 6.27 \end{pmatrix}, \mathbf{N} = \begin{pmatrix} 0 \\ 6.47 \end{pmatrix} \quad (2.0.37)$$
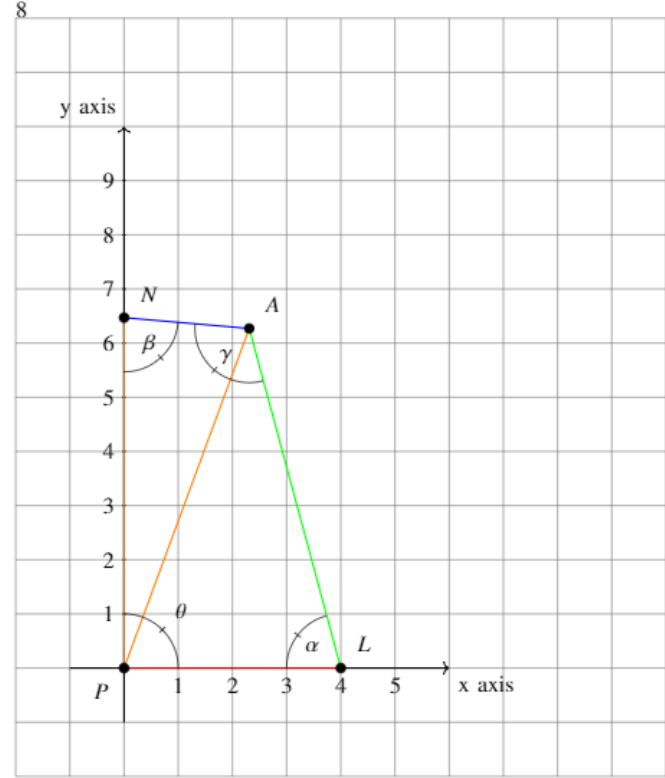
8) On constructing the quadrilateral *PLAN* we get:

Fig. 2.1: Quadrilateral PLAN