ASSIGNMENT-2

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Download all python codes from

https://github.com/Sowmyabandi99/Assignment_2/blob/main/assignment2.py

and latex-tikz codes from

https://github.com/Sowmyabandi99/Assignment_2/blob/main/main.tex

1 Question No. 2.40

Construct PLAN where $PL = 4, LA = 6.5, \angle P = 90^\circ, \angle A = 110^\circ$ and $\angle N = 85^\circ$.

2 Solution

1) Let us assume vertices of given quadrilateral $PLAN$ as $\mathbf{P}$,$\mathbf{L}$,$\mathbf{A}$ and $\mathbf{N}$.
2) Let us generalize the given data:

$$\angle P = 90^\circ = \theta \tag{2.0.1}$$
$$\angle A = 110^\circ = \alpha \tag{2.0.2}$$
$$\angle N = 85^\circ = \gamma \tag{2.0.3}$$
$$\angle E = 180^\circ - \angle L = \delta, \tag{2.0.4}$$
$$\|\mathbf{L} - \mathbf{P}\| = 4 = a, \tag{2.0.5}$$
$$\|\mathbf{A} - \mathbf{L}\| = 6.5 = b, \tag{2.0.6}$$
$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{L} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{2.0.7}$$

3) Also, let us assume the other two sides as

$$\|\mathbf{N} - \mathbf{A}\| = c \tag{2.0.8}$$
$$\|\mathbf{P} - \mathbf{N}\| = \|\mathbf{N}\| = d \quad (\because \mathbf{P} = 0) \tag{2.0.9}$$

4) We know,sum of angles of a quadrilateral = 360°

$$\angle P + \angle L + \angle A + \angle N = 360^\circ \tag{2.0.10}$$
$$90^\circ + \angle L + 110^\circ + 85^\circ = 360^\circ \tag{2.0.11}$$
$$\implies \angle L = 75^\circ \tag{2.0.12}$$

5) Now sum of all the angles given and (2.0.9) is 360°.So construction of given quadrilateral is **possible**.
6) Using cosine formula in $\triangle PLA$,we can find PA:

$$\implies \|\mathbf{A} - \mathbf{P}\|^2 = \|\mathbf{P} - \mathbf{L}\|^2 + \|\mathbf{L} - \mathbf{A}\|^2 - 2 \times \|\mathbf{P} - \mathbf{L}\| \times \|\mathbf{L} - \mathbf{A}\| \cos L \tag{2.0.13}$$

$$\implies PA = 6.69 \tag{2.0.14}$$
$$\implies \theta = 68.43^\circ \tag{2.0.15}$$

7) Now in $\triangle PNA$,we know

$$\angle P = 21.57^\circ, \angle N = 85^\circ. \tag{2.0.16}$$

We know that sum of the angles of a triangle is 180°

$$\implies \angle P + \angle N + \angle A = 180^\circ \tag{2.0.17}$$

$$\implies 21.57^\circ + 85^\circ + \angle A = 180^\circ \tag{2.0.18}$$

$$\implies \angle A = 73.43^\circ \tag{2.0.19}$$

8) Now applying sine rule, we get NP=6.47=d

Lemma 2.1. *The coordinates of A and N can be written as follows:*

$$\mathbf{A} = \mathbf{L} + b\mathbf{e} \tag{2.0.20}$$
$$\mathbf{N} = \mathbf{P} + d\mathbf{p} \tag{2.0.21}$$

Proof.
- For finding coordinates of A:

The vector equation of line is given by:

$$\mathbf{A} = \mathbf{L} + b \begin{pmatrix} \cos E \\ \sin E \end{pmatrix} \tag{2.0.22}$$
$$\implies \|\mathbf{A} - \mathbf{L}\| = b \times \| \begin{pmatrix} \cos 105^\circ \\ \sin 105^\circ \end{pmatrix} \| \tag{2.0.23}$$
$$\implies \|\mathbf{A} - \mathbf{L}\| = b \tag{2.0.24}$$

- For finding coordinates of N:

The vector equation of line is given by:

$$\mathbf{N} = \mathbf{P} + d\begin{pmatrix}\cos P\\ \sin P\end{pmatrix} = d\begin{pmatrix}\cos P\\ \sin P\end{pmatrix} \quad (\because \mathbf{P} = 0) \tag{2.0.25}$$

$$\implies \|\mathbf{N}\| = d \times \|\begin{pmatrix}\cos 90^\circ\\ \sin 90^\circ\end{pmatrix}\| \tag{2.0.26}$$

$$\implies \|\mathbf{N}\| = d \tag{2.0.27}$$

□

Putting values of b and d in (2.0.20) and (2.0.21)

$$\mathbf{A} = \begin{pmatrix}a\\ 0\end{pmatrix} + b\begin{pmatrix}\cos E\\ \sin E\end{pmatrix} \tag{2.0.28}$$

$$\implies \mathbf{A} = \begin{pmatrix}4\\ 0\end{pmatrix} + 6.5\begin{pmatrix}\cos 105^\circ\\ \sin 105^\circ\end{pmatrix} \tag{2.0.29}$$

$$\implies \mathbf{A} = \begin{pmatrix}2.31\\ 6.27\end{pmatrix} \tag{2.0.30}$$

$$and \quad \mathbf{N} = d\begin{pmatrix}\cos P\\ \sin P\end{pmatrix} \tag{2.0.31}$$

$$\implies \mathbf{N} = 6.47\begin{pmatrix}\cos 90^\circ\\ \sin 90^\circ\end{pmatrix} \tag{2.0.32}$$

$$\implies \mathbf{N} = \begin{pmatrix}0\\ 6.47\end{pmatrix} \tag{2.0.33}$$

Now,the vertices of given Quadrilateral PLAN can be written as,

$$\mathbf{P} = \begin{pmatrix}0\\ 0\end{pmatrix}, \mathbf{L} = \begin{pmatrix}4\\ 0\end{pmatrix}, \mathbf{A} = \begin{pmatrix}2.31\\ 6.27\end{pmatrix}, \mathbf{N} = \begin{pmatrix}0\\ 6.47\end{pmatrix} \tag{2.0.34}$$

On constructing the quadrilateral *PLAN* we get:

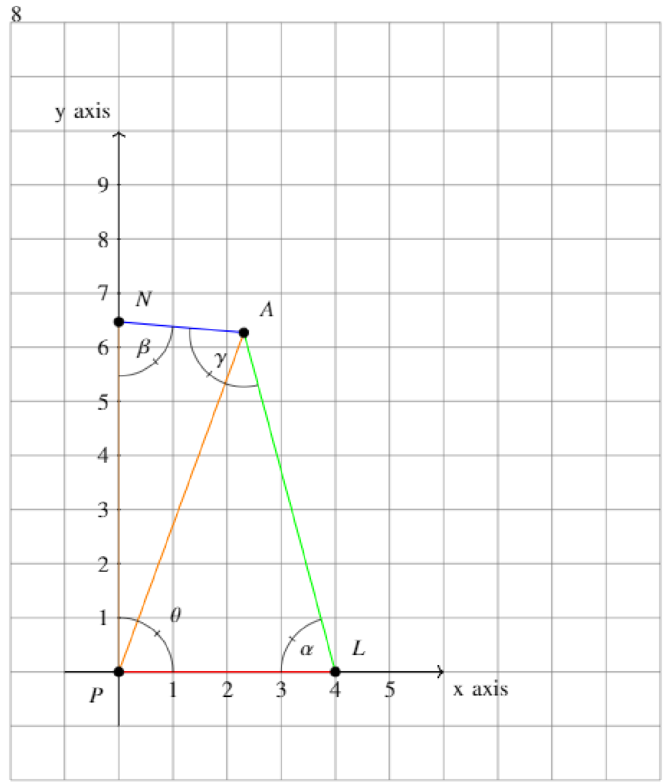

Fig. 2.1: Quadrilateral PLAN