ASSIGNMENT-2

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Download all python codes from

https://github.com/Sowmyabandi99/Assignment_2/blob/main/assignment2.py

and latex-tikz codes from

https://github.com/Sowmyabandi99/Assignment_2/blob/main/main.tex

1 Question No. 2.40

Construct PLAN where $PL = 4, LA = 6.5, \angle P = 90°, \angle A = 110°$ and $\angle N = 85°$.

2 Solution

1) Let us assume vertices of given quadrilateral $PLAN$ as $\mathbf{P}$,$\mathbf{L}$,$\mathbf{A}$ and $\mathbf{N}$.
2) Let us generalize the given data:

$$\angle P = 90° = \theta \tag{2.0.1}$$
$$\angle A = 110° = \alpha \tag{2.0.2}$$
$$\angle N = 85° = \gamma \tag{2.0.3}$$
$$\angle E = 180° - \angle L = \delta, \tag{2.0.4}$$
$$\|\mathbf{L} - \mathbf{P}\| = 4 = a, \tag{2.0.5}$$
$$\|\mathbf{A} - \mathbf{L}\| = 6.5 = b, \tag{2.0.6}$$
$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{L} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{2.0.7}$$

3) Also, let us assume the other two sides as

$$\|\mathbf{N} - \mathbf{A}\| = c \tag{2.0.8}$$
$$\|\mathbf{P} - \mathbf{N}\| = \|\mathbf{N}\| = d \quad (\because \mathbf{P} = 0) \tag{2.0.9}$$

4) We know,sum of angles of a quadrilateral = 360°

$$\angle P + \angle L + \angle A + \angle N = 360° \tag{2.0.10}$$
$$90° + \angle L + 110° + 85° = 360° \tag{2.0.11}$$
$$\implies \angle L = 75° \tag{2.0.12}$$

- Now sum of all the angles given and (2.0.9) is 360°.So construction of given quadrilateral is **possible**.

Lemma 2.1. *The coordinates of A and N can be written as follows:*

$$\mathbf{A} = \mathbf{L} + b\begin{pmatrix} \cos E \\ \sin E \end{pmatrix} \tag{2.0.13}$$
$$\mathbf{N} = \mathbf{P} + d\begin{pmatrix} \cos P \\ \sin P \end{pmatrix} \tag{2.0.14}$$

Proof. • For finding coordinates of A:
The vector equation of line is given by:

$$\mathbf{A} = \mathbf{L} + b\begin{pmatrix} \cos E \\ \sin E \end{pmatrix} \tag{2.0.15}$$
$$\implies \|\mathbf{A} - \mathbf{L}\| = b \times \|\begin{pmatrix} \cos 105° \\ \sin 105° \end{pmatrix}\| \tag{2.0.16}$$
$$\implies \|\mathbf{A} - \mathbf{L}\| = b \tag{2.0.17}$$

- For finding coordinates of N:

The vector equation of line is given by:

$$\mathbf{N} = \mathbf{P} + d\begin{pmatrix} \cos P \\ \sin P \end{pmatrix} = d\begin{pmatrix} \cos P \\ \sin P \end{pmatrix} \quad (\because \mathbf{P} = 0) \tag{2.0.18}$$
$$\implies \|\mathbf{N}\| = d \times \|\begin{pmatrix} \cos 90° \\ \sin 90° \end{pmatrix}\| \tag{2.0.19}$$
$$\implies \|\mathbf{N}\| = d \tag{2.0.20}$$

Using inner products of vectors we get,

$$\frac{(A-N)^\top (A-L)}{\|A-N\| \times \|A-L\|} = \cos A \tag{2.0.21}$$
$$\frac{(A-N)^\top (P-N)}{\|A-N\| \times \|P-N\|} = \cos N \tag{2.0.22}$$

Now, dividing (2.0.21) and (2.0.22) we get:

$$\frac{(A-N)^\top (A-L)}{(A-N)^\top (P-N)} \times \frac{\|P-N\|}{\|A-L\|} = \frac{\cos A}{\cos N} \tag{2.0.23}$$
$$\frac{(A-N)^\top (A-L)}{(A-N)^\top (P-N)} \times \frac{d}{b} = \frac{\cos A}{\cos N} \tag{2.0.24}$$
$$\frac{A^\top A - A^\top L - N^\top A + N^\top L}{-A^\top N + N^\top N} = \frac{b}{d} \times \frac{\cos A}{\cos N} (\because \mathbf{P} = 0) \tag{2.0.25}$$

$$Let \quad S = b \times \frac{\cos A}{\cos N} \tag{2.0.26}$$

On computing,

$$A^{\top}A = a^2 + b^2 + 2ab\cos E \tag{2.0.27}$$
$$A^{\top}L = a^2 + ab\cos E \tag{2.0.28}$$
$$N^{\top}A = bd\sin P\sin E \tag{2.0.29}$$
$$N^{\top}L = 0 \tag{2.0.30}$$
$$A^{\top}N = bd\sin P\sin E \tag{2.0.31}$$
$$N^{\top}N = d^2 \tag{2.0.32}$$

From (2.0.25),

$$\implies d = \frac{S(d^2 - bd\sin P\sin E)}{b^2 + ab\cos E - bd\sin P\sin E} \tag{2.0.33}$$

□

By solving,we get

$$\implies d = \frac{-25.50(d^2 - 6.27d)}{42.25 - 6.72 - 6.27d} \tag{2.0.34}$$
$$\implies d = 6.47 \tag{2.0.35}$$

5) Putting values of b and d in (2.0.13) and (2.0.14)

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b\begin{pmatrix} \cos E \\ \sin E \end{pmatrix} \tag{2.0.36}$$
$$\implies \mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + 6.5\begin{pmatrix} \cos 105° \\ \sin 105° \end{pmatrix} \tag{2.0.37}$$
$$\implies \mathbf{A} = \begin{pmatrix} 2.31 \\ 6.27 \end{pmatrix} \tag{2.0.38}$$

$$and \quad \mathbf{N} = d\begin{pmatrix} \cos P \\ \sin P \end{pmatrix} \tag{2.0.39}$$
$$\implies \mathbf{N} = 6.47\begin{pmatrix} \cos 90° \\ \sin 90° \end{pmatrix} \tag{2.0.40}$$
$$\implies \mathbf{N} = \begin{pmatrix} 0 \\ 6.47 \end{pmatrix} \tag{2.0.41}$$

6) Now,the vertices of given Quadrilateral PLAN can be written as,

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{L} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 2.31 \\ 6.27 \end{pmatrix}, \mathbf{N} = \begin{pmatrix} 0 \\ 6.47 \end{pmatrix} \tag{2.0.42}$$

7) On constructing the quadrilateral *PLAN* we get:

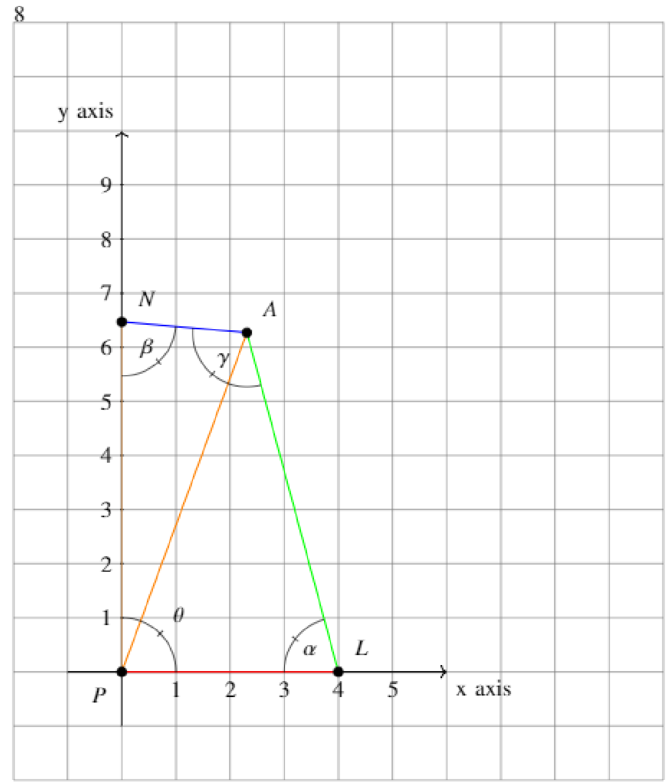

Fig. 2.1: Quadrilateral PLAN