

ASSIGNMENT-2

SOWMYA BANDI

Download all python codes from

https://github.com/Sowmyabandi99/Assignment_2/blob/main/assignment2.py

and latex-tikz codes from

https://github.com/Sowmyabandi99/Assignment_2/blob/main/main.tex

1 QUESTION No. 2.40

Construct $PLAN$ where $PL = 4$, $LA = 6.5$, $\angle P = 90^\circ$, $\angle A = 110^\circ$ and $\angle N = 85^\circ$.

2 SOLUTION

- 1) Let us assume vertices of given quadrilateral $PLAN$ as $\mathbf{P}, \mathbf{L}, \mathbf{A}$ and \mathbf{N} .
- 2) Let us generalize the given data:

$$\angle P = 90^\circ = \theta \quad (2.0.1)$$

$$\angle A = 110^\circ = \alpha \quad (2.0.2)$$

$$\angle N = 85^\circ = \gamma \quad (2.0.3)$$

$$\angle E = 180^\circ - \angle L = \delta, \quad (2.0.4)$$

$$\|\mathbf{L} - \mathbf{P}\| = 4 = a, \quad (2.0.5)$$

$$\|\mathbf{A} - \mathbf{L}\| = 6.5 = b, \quad (2.0.6)$$

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{L} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (2.0.7)$$

- 3) Also, let us assume the other two sides as

$$\|\mathbf{N} - \mathbf{A}\| = c \quad (2.0.8)$$

$$\|\mathbf{P} - \mathbf{N}\| = \|\mathbf{N}\| = d \quad (\because \mathbf{P} = 0) \quad (2.0.9)$$

- 4) We know, sum of angles of a quadrilateral = 360°

$$\angle P + \angle L + \angle A + \angle N = 360^\circ \quad (2.0.10)$$

$$90^\circ + \angle L + 110^\circ + 85^\circ = 360^\circ \quad (2.0.11)$$

$$\implies \angle L = 75^\circ \quad (2.0.12)$$

- Now sum of all the angles given and (2.0.9) is 360° . So construction of given quadrilateral is **possible**.

Lemma 2.1. The coordinates of A and N can be written as follows:

$$\mathbf{A} = \mathbf{L} + b \begin{pmatrix} \cos E \\ \sin E \end{pmatrix} \quad (2.0.13)$$

$$\mathbf{N} = \mathbf{P} + d \begin{pmatrix} \cos P \\ \sin P \end{pmatrix} \quad (2.0.14)$$

Proof. • For finding coordinates of A :
The vector equation of line is given by:

$$\mathbf{A} = \mathbf{L} + b \begin{pmatrix} \cos E \\ \sin E \end{pmatrix} \quad (2.0.15)$$

$$\implies \|\mathbf{A} - \mathbf{L}\| = b \times \left\| \begin{pmatrix} \cos 105^\circ \\ \sin 105^\circ \end{pmatrix} \right\| \quad (2.0.16)$$

$$\implies \|\mathbf{A} - \mathbf{L}\| = b \quad (2.0.17)$$

• For finding coordinates of N :
The vector equation of line is given by:

$$\mathbf{N} = \mathbf{P} + d \begin{pmatrix} \cos P \\ \sin P \end{pmatrix} = d \begin{pmatrix} \cos P \\ \sin P \end{pmatrix} \quad (\because \mathbf{P} = 0) \quad (2.0.18)$$

$$\implies \|\mathbf{N}\| = d \times \left\| \begin{pmatrix} \cos 90^\circ \\ \sin 90^\circ \end{pmatrix} \right\| \quad (2.0.19)$$

$$\implies \|\mathbf{N}\| = d \quad (2.0.20)$$

Using inner products of vectors we get,

$$\frac{(A - N)^\top (A - L)}{\|A - N\| \times \|A - L\|} = \cos A \quad (2.0.21)$$

$$\frac{(A - N)^\top (P - N)}{\|A - N\| \times \|P - N\|} = \cos N \quad (2.0.22)$$

Now, dividing (2.0.21) and (2.0.22) we get:

$$\frac{(A - N)^\top (A - L)}{(A - N)^\top (P - N)} \times \frac{\|P - N\|}{\|A - L\|} = \frac{\cos A}{\cos N} \quad (2.0.23)$$

$$\frac{(A - N)^\top (A - L)}{(A - N)^\top (P - N)} \times \frac{d}{b} = \frac{\cos A}{\cos N} \quad (2.0.24)$$

$$\frac{A^\top A - A^\top L - N^\top A + N^\top L}{-A^\top N + N^\top N} = \frac{b}{d} \times \frac{\cos A}{\cos N} \quad (\because \mathbf{P} = 0) \quad (2.0.25)$$

$$\text{Let } S = b \times \frac{\cos A}{\cos N} \quad (2.0.26)$$

On computing,

$$A^T A = a^2 + b^2 + 2ab \cos E \quad (2.0.27)$$

$$A^T L = a^2 + ab \cos E \quad (2.0.28)$$

$$N^T A = bd \sin P \sin E \quad (2.0.29)$$

$$N^T L = 0 \quad (2.0.30)$$

$$A^T N = bd \sin P \sin E \quad (2.0.31)$$

$$N^T N = d^2 \quad (2.0.32)$$

From (2.0.25),

$$\Rightarrow d = \frac{S(d^2 - bd \sin P \sin E)}{b^2 + ab \cos E - bd \sin P \sin E} \quad (2.0.33)$$

□

By solving, we get

$$\Rightarrow d = \frac{-25.50(d^2 - 6.27d)}{42.25 - 6.72 - 6.27d} \quad (2.0.34)$$

$$\Rightarrow d = 6.47 \quad (2.0.35)$$

5) Putting values of b and d in (2.0.13) and (2.0.14)

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos E \\ \sin E \end{pmatrix} \quad (2.0.36)$$

$$\Rightarrow \mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 105^\circ \\ \sin 105^\circ \end{pmatrix} \quad (2.0.37)$$

$$\Rightarrow \mathbf{A} = \begin{pmatrix} 2.31 \\ 6.27 \end{pmatrix} \quad (2.0.38)$$

$$\text{and } \mathbf{N} = d \begin{pmatrix} \cos P \\ \sin P \end{pmatrix} \quad (2.0.39)$$

$$\Rightarrow \mathbf{N} = 6.47 \begin{pmatrix} \cos 90^\circ \\ \sin 90^\circ \end{pmatrix} \quad (2.0.40)$$

$$\Rightarrow \mathbf{N} = \begin{pmatrix} 0 \\ 6.47 \end{pmatrix} \quad (2.0.41)$$

6) Now, the vertices of given Quadrilateral PLAN can be written as,

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{L} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 2.31 \\ 6.27 \end{pmatrix}, \mathbf{N} = \begin{pmatrix} 0 \\ 6.47 \end{pmatrix} \quad (2.0.42)$$

7) On constructing the quadrilateral PLAN we get:

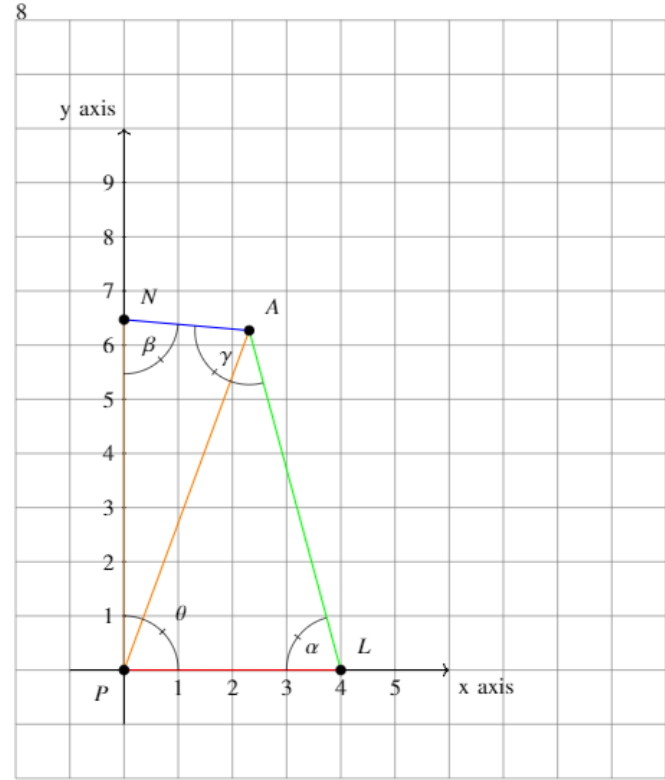


Fig. 2.1: Quadrilateral PLAN