

Efficient Algorithms for Fuzzy Centrality Measures in Large- Scale Social Networks

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# Abstract

Numerous criteria are in place for social network applications. They require identification of network's core nodes. Traditional centrality measurements focus on specific node's direct connections or reachability. Often this disregards inherent ambiguity and complexity in real-world social networks. To address these constraints, we have introduced new method called Node Pack Fuzzy Information Centrality based on Pythagorean Neutrosophic Fuzzy Theory. Three essential values truth, falsity and indeterminacy have been added to this approach. This new approach provides a thorough depiction of social networks and it also offers a more sophisticated comprehension of connections between nodes. Complex and ambiguous interactions between entities can be effectively expressed using Pythagorean Neutrosophic values. Unlike traditional values, Pythagorean Neutrosophic values consider several uncertainty dimensions; this is a major improvement over traditional fuzzy value. Our approach handles relational complexity well and it includes self-weight for every node too. It represents each node's unique value, significance, or impact on the network. The network assessment is now more precise and contextual so we can assess centrality with greater precision. We applied this approach to a small academic network called university faculty/researchers. The application of Node Pack Fuzzy Information Centrality yielded promising results. It can enhance various activities associated with social network analysis. It can also offer valuable insights into the network architecture.

**Keywords:** Centrality measures; Influential nodes; Node pack fuzzy information centrality; Pythagorean neutrosophic fuzzy graph; Social Networks

# Introduction

Social media platforms often get used for news delivery. They are also used for advertising and business. These platforms consist of individuals and entities. The central nodes help with the spread of information. Identifying these core nodes needs several criteria. This makes it a challenging task. Fuzzy graph models are often picked, because of the uncertainty. Graphs offer a visual representation of relationships. Zadeh [1] is the one who came up with fuzzy sets. These sets help with managing real-world uncertainties. Kauffman [2] introduced fuzzy graphs. Atanassov [3] suggested IFS: intuitionistic fuzzy sets, which are used in decision-making. They merge membership and non-membership degrees. Yager [4] introduced Pythagorean fuzzy sets later these sets refined precision. Smarandache [5] introduced neutrosophic sets and these sets grasp three values: truth, falsehood and indeterminacy. The concepts were elaborated on by these sets. Pythagorean neutrosophic sets can accommodate complex uncertainty. Bavelas [6][7] was first to estimate graph centrality. It was followed by Shimbel[8] who used shortest path approach. Katz [9] measured node influence by using Katz Centrality. Nieminen[10] introduced degree centrality where direct links are emphasized. Nonetheless, lack of activity in links can restrict it. Freeman[11] improved centrality. It now includes both Closeness and Betweenness Centrality. The shortest paths through a node are looked at by these measures. Estrada et al. [12] introduced subgraph centrality. Bonacich[13] introduced Eigenvector Centrality for complex networks. For the purposes of ranking nodes of influence Bae alongside Kim[14] developed Coreness Centrality. J. Wang et al. [15] focused on the development of Weighted

Neighborhood Centrality. Zhang et al. [16] researched centrality within directionally connected fuzzy social networks. Q. Wang et al. [17] utilized fuzzy hypergraph theory in the exploration of structural centrality.

Crisp graphs treat all edges in an identical manner; fuzzy graphs on the other hand consider uncertainty. This quality suits them better for dynamic social networks in the real world. Fuzzy systems can enhance the accuracy of centrality assessments. They do this by accounting for uncertainty. S. Samanta et al. [18] created neutrosophic graphs based on neutrosophic sets. Ajay et al. [19] introduced Pythagorean neutrosophic fuzzy graphs. To identify social network leaders, T. Zhou et al. [20] used delicious.com's Leader Rank to compare rankings to fan counts. Ling et al. [21 proposed gravity centrality and discovered significant spreaders using Newton's gravity formula. Lu et al. [22] evaluated the relationship between member actions and action network topology, which is critical for targeted advertising. P. Wang et al. [23] investigated noteworthy nodes in biological networks. J. Sheng et al.

[24] provided a technique that leverages both local and global structures to discover important nodes. X. Wang et al. [25] developed a semi-local metric for influential nodes. L. Panfeng et al. [26] proposed a voting-based technique for social networks. Venkata Rao Songa et al.[27] introduced a local measure for influential nodes in directed weighted networks using Pythagorean fuzzy sets.

The 2024 AD Scientific Index is an index that ranks academic institutions and researchers worldwide. It obtains its data from Google Scholar. This data consists of citation, h-index and i10-index. The index analyses the most recent output of 18,528 universities. These universities are spread across 219 nations. Google Scholar has a shortcoming: it contains papers, which are incorrectly attributed to scholars with the same names. This can potentially result in inaccurate data. Inflated publication numbers are another possible outcome. Pythagorean neutrosophic fuzzy graphs provide a more accurate representation of this data, especially when self-publication and collaborative research are taken into account. In this article, a specific measure called node pack fuzzy information centrality is used. It is used to pinpoint key nodes in these graphs. Nodes that bear influence will be detected. The process will be based on a suggested method only.

This paper is structured as follows: First, critical definitions of existing centrality methods are reviewed. Next, the concept of influential nodes in Pythagorean neutrosophic graphs is introduced, followed by an algorithm for modifying node pack fuzzy information centrality, along with relevant theorems. The method proposed is then utilized to identify influential nodes in a given network, which are then analysed. Finally, the results are discussed, and the study concludes.

# Preliminaries

* 1. **Definitions**

*Definition 1*:A **Pythagorean Neutrosophic Fuzzy Set (PNFS)**, as introduced by D. Ajay et al. [28], extends traditional fuzzy set models by considering three membership components:

* TP(x)T\_P(x)TP​(x) → Truth membership function
* IP(x)I\_P(x)IP​(x) → Indeterminacy membership function
* FP(x)F\_P(x)FP​(x) → Falsity membership function

These components are constrained by the conditions:

0 ≤ (𝑇P(𝑥))2 +(𝐹P(𝑥))2 ≤1

0 ≤ (𝐼P(𝑥))2≤1 then

0 ≤ (𝑇P(𝑥))2 +(𝐼P(𝑥))2+(𝐹P(𝑥))2 ≤ 2 (1) Here, 𝑇P(𝑥) and 𝐹P(𝑥) are dependent components and 𝐼p(𝑥) is independent component.

*Definition 2: A* ***Pythagorean Neutrosophic Fuzzy Graph (PNFG)****, denoted as G˙=(V,σ,μ)Ġ = (V, \sigma, \mu)G˙=(V,σ,μ), extends fuzzy graph theory by incorporating PNFS principles. It consists of:*

* *A vertex set V={v1​,v2​,...,vn​}.*
* *A vertex membership function σ=(,,), where:*
  + *represents the truth degree of vertex v.*
  + *represents the indeterminacy degree of vertex v.*
  + *represents the falsity degree of vertex v.*
* *An edge membership function μ=(,,), where:*
  + *represents the truth degree of edge (v1​,v2​).*
  + *represents the indeterminacy degree of edge (v1​,v2​).*
  + *represents the falsity degree of edge (v1​,v2​).*

*These functions satisfy the following conditions:*

μT (1, 2)  min{σT (1), σT(2)},

μI (1, 2)  min{σI (1), σI(2),

μF(1, 2)  max{σF(1), σF(2)}. (2)

Where μT(*v1, v2*), μI (*v1, v2*), and μF (*v1, v2*) represent the Truth, Indeterminacy and Falsity membership degree values of an edge (*v1,v2*) in the graph Ġ respectively. Here, σT(*v1*), σI(*v1*)*,* and σF(*v1*) represents the Truth, Indeterminacy and Falsity membership values of the vertex v1 in Ġ respectively.

*Definition 3:* In a Pythagorean Neutrosophic Fuzzy graph Ġ = (V, σ, μ), the strength of an edge (*v1, v2*) is denoted by (ST(*v1, v2*), SI(*v1, v2*), SF(*v1, v2*)) and defined as

ST(V1,V2) μT (1, 2) / min{σT(1), σT(2)},

SI(V1,V2)  μI(1, 2) / min{σI(1), σI(2)},

SF(V1,V2) = μF (1, 2) / max{σF (1), σF (2)}. (3)

Where μT(*v1, v2*), μI(*v1, v2*), and μF(*v1, v2*) represent the Truth, Indeterminacy and Falsity membership degree values of an edge (*v1, v2*) in the graph Ġ, respectively. σT(*v1*), σI(*v1*), and σF(*v1*) represents the Truth, Indeterminacy and Falsity membership values of vertex in Ġ, respectively.

Uncertainty and imprecision are inherent in many real-world problems, necessitating advanced mathematical models for effective representation. Traditional fuzzy set theory has evolved into various extensions, including intuitionistic fuzzy sets and Pythagorean fuzzy sets, to handle different levels of uncertainty. Pythagorean Neutrosophic Fuzzy Sets (PNFS) further enhance this concept by incorporating three fundamental components: truth, indeterminacy, and falsity. This structure allows for greater flexibility in representing uncertain information, making it suitable for complex decision-making problems.

In practical applications, truth and falsity are often interrelated, while indeterminacy represents an independent measure of uncertainty. The Pythagorean Neutrosophic model extends beyond the constraints of traditional fuzzy models by ensuring that the sum of truth and falsity values is not necessarily limited to a strict boundary, allowing a more realistic representation of ambiguous situations. This is particularly useful in fields such as medical diagnosis, expert systems, risk analysis, and artificial intelligence, where multiple degrees of uncertainty must be considered simultaneously.

Building upon the foundation of PNFS, the concept of Pythagorean Neutrosophic Fuzzy Graphs (PNFG) integrates these principles into graph theory. In a PNFG, each vertex and edge is associated with degrees of truth, indeterminacy, and falsity, providing a more comprehensive representation of relationships in uncertain environments. Unlike classical graph models, which assume binary or crisp relationships between nodes, PNFGs allow for partial truths and uncertainties, making them applicable in areas such as social network analysis, transportation systems, and decision support systems.

One of the key advantages of PNFGs is their ability to quantify the reliability of connections between entities. By assigning different degrees of truth, falsity, and indeterminacy to each edge, these graphs enable a more nuanced analysis of networks. For example, in a social network, the strength of a connection between individuals may not be purely deterministic but rather influenced by various uncertain factors such as trust levels, varying opinions, or incomplete information. Similarly, in medical diagnosis, relationships between symptoms and diseases can be modeled more accurately using PNFGs, as they allow for the incorporation of uncertain medical knowledge.

The study of PNFGs also introduces the concept of edge strength, which helps in evaluating the reliability of connections in the network. This feature is particularly useful in applications where decision-making is influenced by varying degrees of certainty, such as cybersecurity, supply chain management, and recommendation systems. By analyzing the truth, indeterminacy, and falsity components of edges, researchers can identify critical connections, detect anomalies, and optimize network structures for better efficiency.

Overall, Pythagorean Neutrosophic Fuzzy Sets and Graphs provide an advanced mathematical framework for handling uncertainty in complex systems. Their ability to incorporate multiple dimensions of uncertainty makes them a valuable tool in diverse fields, enabling more informed decision-making and improved analytical capabilities. As research in this area continues to evolve, PNFS and PNFGs are expected to play a crucial role in the development of intelligent systems, optimization techniques, and predictive analytics.

# Graph Centrality Metrics

Graph centrality metrics are essential in network analysis as they help identify the most important nodes in a graph. These metrics measure the influence, importance, or central position of a node based on its connections and position within the network structure. Centrality measures play a crucial role in various applications, including social network analysis, transportation networks, biological systems, and communication networks.

#### **1. Degree Centrality**

Degree centrality is the simplest form of centrality and measures the number of direct connections a node has in the network. A node with a higher degree is considered more influential since it has more direct interactions with other nodes. In social networks, for example, a person with many friends or followers would have high degree centrality. However, this metric does not consider the overall structure of the network and the importance of connected nodes.

#### **2. Closeness Centrality**

Closeness centrality measures how close a node is to all other nodes in the network. It is based on the idea that important nodes should have short paths to other nodes, allowing them to efficiently spread information. Nodes with high closeness centrality are often considered key influencers because they can quickly reach other parts of the network. This metric is particularly useful in transportation networks, communication systems, and disease spread modeling.

#### **3. Betweenness Centrality**

Betweenness centrality quantifies how often a node acts as a bridge in the shortest paths between other nodes. A node with high betweenness centrality has significant control over information flow in the network. In social networks, individuals with high betweenness centrality act as intermediaries or brokers, influencing communication between different groups. This metric is widely used in identifying key connectors in infrastructure networks, supply chain management, and identifying vulnerabilities in networks.

#### **4. Eigenvector Centrality**

Eigenvector centrality measures a node's importance based on the centrality of its neighbors. Unlike degree centrality, which considers only the number of connections, eigenvector centrality gives higher importance to nodes connected to other highly influential nodes. It is commonly used in ranking webpages (as in Google’s PageRank algorithm), analyzing influence in social networks, and determining key players in financial systems.

#### **5. PageRank**

PageRank is a variant of eigenvector centrality that assigns importance to nodes based on the quality of their connections rather than just their quantity. Developed by Google, it ranks webpages based on the number and importance of other pages linking to them. A page linked by many high-quality sources receives a higher ranking, making it a powerful metric for ranking web content, social network profiles, and academic citations.

#### **6. Katz Centrality**

Katz centrality extends eigenvector centrality by considering not only direct connections but also the influence of distant connections, assigning decreasing weights to nodes further away. This metric is useful in social network analysis to determine both direct and indirect influence, as well as in marketing strategies for identifying influential users who can help spread information widely.

#### **7. Harmonic Centrality**

Harmonic centrality is similar to closeness centrality but modifies the way distances are calculated by considering unreachable nodes. It is useful in disconnected or large-scale networks where traditional closeness centrality fails to provide meaningful results. This measure is commonly applied in ranking influential nodes in fragmented networks such as disconnected social groups or communication networks.

#### **8. Subgraph Centrality**

Subgraph centrality evaluates the importance of a node based on its participation in different subgraphs within the network. Nodes involved in many small or densely connected subgraphs are assigned higher centrality values. This metric is often applied in biological and technological networks to identify structurally significant elements within the system.

Identifying core nodes within a complex network is an essential task. Several centrality measurement methods can aid in this process.

*Degree centrality:* Shaw [29] introduced degree centrality, which represents the count of direct connections that a node possesses. A node v𝑖 , degree centrality is defined as:

Cvi= dvi (4)

where the degree of node vi is dvi. The normalized degree centrality is:

C vi

′

= dvi

n−1

(5)

Here, *n* represents the total number of nodes. This method measures only direct connections, but nodes connected indirectly can sometimes share more information than direct ones. Thus, considering indirect influence is crucial for identifying the central node.

*Node Pack Fuzzy Information Centrality:* Venkata Rao Songa et al. [30] introduced the concept of entropy measure with the inbound and outbound significance of a node in a directed weighted network.

m

Cv = − ∑(μ2(vi)logμ2(vi) + 𝝑2(vi)log𝝑2(vi) + π2(vi)logπ2(vi)) , γ = 1 (6)

i

i=1

(or)

γ

m

1

C = ∑

2( )

2( )

γ

2( )

γ

γ ≠ 1 (γ > 0) (7)

vi

i=1

(1 − ((μ

γ − 1

vi )

+ (𝝑

vi ) ) + (π

vi ) ) ,

Here, m be the total number of nodes connected to node *vi* in the network. The normalized Node Pack Fuzzy Information Centrality of *vi* in a Pythagorean Neutrosophic Fuzzy Set, considering the dependency between μ(vi) and ϑ(vi), and the independence of π(vi), is defined for the equation (6) as:

m

C′ = − 2

∑ x2(v )logx2(v )

(8)

vi (n − 1) log(n)

i i

i=1

Here, x is the membership value of node vi, and (n−1) log(n) serves as the normalization factor based on the maximum possible entropy, where n represents the total count of nodes within the network.

The Node Pack Fuzzy Information Centrality is computed by incorporating these membership functions into an entropy-based formulation. It evaluates the level of uncertainty in the distribution of node connections and assigns importance based on the weighted contribution of inbound and outbound links. This measure helps in identifying influential nodes in networks where relationships are not strictly binary but rather fuzzy in nature.

The normalized version of this centrality ensures that the computed values remain within a meaningful range, allowing for fair comparisons between nodes of different connectivity levels. Normalization is achieved by incorporating a logarithmic factor based on the total number of nodes in the network, making it adaptable to different network sizes.

*PageRank:* *Introduced by* ***L. Page et al.****, PageRank is an algorithm originally designed for ranking web pages in search engines. It evaluates the importance of a node in a directed network based on the number and quality of incoming links. The key idea behind PageRank is that a node receives a higher rank if it is linked to by other highly ranked nodes.*

*The calculation of PageRank incorporates:*

* *The* ***damping factor (d)****, which represents the probability of a random web surfer continuing to click links rather than jumping to a random page.*
* *The* ***number of outgoing links (C(X))*** *from a node, determining how influence is distributed.*

*A* ***normalized version*** *of PageRank ensures fair ranking by considering the contribution of all linked nodes proportionally. This measure is widely used in search engines, citation networks, and social media influence analysis.*

L Page et al. [31] assesses the importance of web pages depends on their link structure, assuming that a page's value increases with the number and quality of links it receives from reputable sources. Let page X has citations from pages T1...Tn. Based on the damping factor d, and the number of outgoing links C(X), the rank of page X is calculated as:

Pr(X) = (1-d) + d (Pr(T1)/C(T1) + … + Pr (Tn)/C(Tn)) (9)

The normalized PageRank centrality is:

Pr = (1 − d) + d ∑N

I Pri

(10)

i i=1,i≠j i,j ni

*Betweenness centrality:* *A node with high betweenness centrality plays a crucial role in information flow, as it lies on multiple shortest paths. This makes it essential in:*

* ***Social networks****, where influencers or key intermediaries can be identified.*
* ***Transportation and logistics****, where critical hubs in communication or supply networks are determined.*
* ***Biological networks****, where certain proteins or genes serve as key connectors in metabolic pathways.*

Freeman [32] proposed the betweenness centrality, which indicates how often a vertex

appears on the shortest paths between other vertices. Betweenness centrality of a vertex 𝑣𝑖, is defined as:

C = ∑ hjk(vi)

(11)

vi j≠k,i≠j,k hjk

Here, hjk represent the shortest paths between vertices vi and vj , and hjk(vi) is the count of those paths passing through vertex vi. The normalized betweenness centrality is then calculated as follows.

∑ hjk(vi)

C′ =

j≠k,i≠j,k

hjk

(12)

vi (n−1).(n−2)

Here, n represents the total count of vertices, and ℎ𝑗𝑘 represents the shortest path between vertices vi and vj . ℎ𝑗𝑘(vi) indicates the number of these paths that pass through the vertex vi.

*Closeness centrality:* Freeman [32] introduced closeness centrality, measuring how quickly a node can spread information across a network. It is defined as the reciprocal of the average distance from a node to all other nodes in the network; a higher closeness centrality indicates shorter average distances. A higher closeness centrality value indicates that a node is more centrally located, meaning:

* **Faster information spread** in communication networks.
* **Greater accessibility** in transportation systems.
* **Efficient control** in energy and power grids.

1

Cvi = ∑n

d(v , v )

(13)

j=1 i j

Here, d (vi, vj) denotes the distance between the two vertices vi and vj . The closeness centrality that has been standardized is

C′ = n−1

(14)

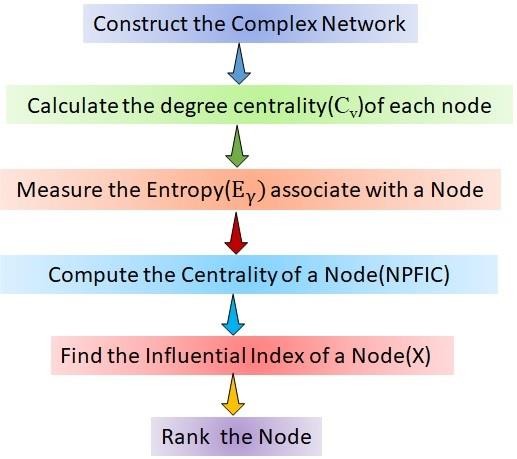
vi 𝚗

∑

j=1

d(vi,vj)

Here, n is the total count of vertices and 𝑑(vi, vj) is the distance between vertices vi and vj.



**Figure 1.** Steps of the Process

1. **Constructing the Complex Network**  
The first step involves creating a network representation where nodes represent entities (e.g., individuals in a social network, web pages in a hyperlink structure) and edges represent connections between them. This network can be directed or undirected, weighted or unweighted, depending on the application.

2. **Calculating Degree Centrality (CvC\_vCv​)**  
Degree centrality measures the number of direct connections a node has within the network. It provides a basic yet effective way to assess a node’s local importance. A higher degree centrality indicates a greater number of direct interactions, making the node more significant in terms of connectivity.

3. **Measuring Node Entropy (EyE\_yEy​)**  
Entropy is used to quantify the uncertainty or diversity associated with a node’s connections. In complex networks, entropy can reflect how evenly a node distributes its connections, which helps in understanding its structural role. Nodes with higher entropy are more influential because they interact with a diverse set of neighbors.

4. **Computing the Node Pack Fuzzy Information Centrality (NPFIC)**  
The Node Pack Fuzzy Information Centrality (NPFIC) integrates multiple centrality measures and entropy values to provide a more refined assessment of a node’s significance. Unlike traditional centrality measures, NPFIC considers both local and global structural properties of the network, making it a robust measure for influence ranking.

5. **Determining the Influential Index of a Node (XXX)**  
The influential index is derived based on the computed centrality and entropy values. This index quantifies the overall importance of a node, considering factors such as connectivity, influence, and network position.

6. **Ranking the Nodes**  
Finally, nodes are ranked based on their influential index. Higher-ranked nodes have a greater impact on network dynamics, making them key players in processes such as information diffusion, viral marketing, and epidemic spread.

# Methodology

The centrality computation establishes each node's relevance based on its connections. The influential index computation combines these centrality values to rank the nodes based on their network-wide influence.

# Algorithm

*Input:*

* Acquire the real-world dataset and construct the complex network
* Required Parameters: α, β and 𝛾.

*Output:*

* Centrality scores of researchers.

*Steps:*

Step 1. Calculate degree centrality of a node from its collaboration data.

Maximum number of collaborations represented by max\_collab=max (Collaborations)

Calculate the degree centrality of node i: Cv(i)=Collaborations(i)/max\_collab-1 (15)

Step 2. Calculate entropy of each node i from the equation (7):

m

1

E (i) = ∑

(1 − Tγ − Iγ − Fγ)

(16)

γ

i=1

γ − 1

i i i

Where Ti, Ii, and Fi are the truth, indeterminacy, and falsity degree values of node i.

Step 3. Compute Centrality Score for each node i

NPFIC(i) = α⋅ Cv(i)+β⋅ (1−Eγ(i)) (17)

This formula balances collaborative influence from (15) and uncertainty from (16) Step 4. The importance index for node i is computed as

X𝑖

= 2 ×NPFICTi×(1−NPFICFi)+NPFICIi

3

(18)

This formula combines the centrality of a node and other factors to give a single score that represents the influence or importance of Xi

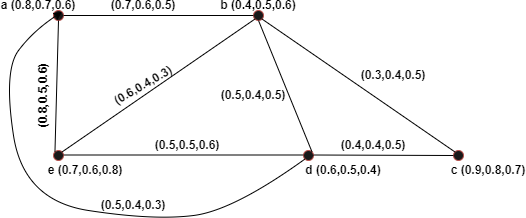
Step 5. Repeat Step 1 to Step 4 to compute importance index of other vertices in the network

Step 6. Rank the node based on their importance index

max {Xi } , i=1, 2,…., n

This methodology presents a systematic approach to evaluating and ranking nodes in a complex network based on their centrality and influence. The process begins by constructing the network using real-world datasets and defining key parameters such as α\alpha, β\beta, and γ\gamma. The first step involves calculating the degree centrality of a node, which is determined from its collaboration data by normalizing the number of collaborations against the maximum observed. The second step computes the entropy of each node, incorporating truth, indeterminacy, and falsity values to measure uncertainty. The third step introduces the Node Pack Fuzzy Information Centrality (NPFIC), a metric that balances the collaborative influence of a node with the uncertainty derived from entropy. The fourth step determines the importance index of a node, combining centrality values and other factors into a single score that quantifies its influence. This calculation ensures a more holistic representation of node significance in the network. Steps one to four are repeated for all nodes to compute their respective importance indices. Finally, the nodes are ranked based on their importance index, allowing for a comprehensive assessment of their relative impact within the network. This methodology is particularly useful in evaluating researchers’ contributions and influence in academic or collaborative networks.

# Example

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**Figure 2.** A Pythagorean Neutrosophic Fuzzy Graph

Figure 2 illustrates a Pythagorean Neutrosophic Fuzzy graph, G = (V, σ, μ) with the vertex set V= {a, b, c, d, e}. This image represents a **Pythagorean Neutrosophic Fuzzy Graph (PNFG)**, a generalization of fuzzy graph theory that incorporates uncertainty through **truth, indeterminacy, and falsity memberships**. Each vertex and edge in the graph is associated with a **three-component membership function**—**(T,I,F)(T, I, F)**—where:

* T(truth membership) denotes the confidence in the existence of a node or edge.
* I (indeterminacy membership) quantifies the uncertainty associated with the node or edge.
* F (falsity membership) represents the degree of non-existence or contradiction.

In this PNFG, the **vertices (a, b, c, d, e)** have assigned membership values, and the edges between them are also defined with corresponding fuzzy values. The relationships between nodes can be analyzed using various **centrality measures**, such as **degree centrality, betweenness centrality, and closeness centrality**, to identify the most influential nodes in the network.

This structure is particularly useful in **decision-making, network optimization, and complex system analysis**, where uncertainty plays a crucial role. The PNFG model enhances **traditional fuzzy graphs** by allowing a **more flexible representation** of real-world networks, such as social networks, transportation systems, and knowledge graphs.

Table 1 lists the truth(T), indeterminacy(I), and falsity(F) values for each vertex, while Table 2 provides these values for all edges.

**Table 1:** Vertex membership values in Figure 2

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **σ** | **a** | **b** | **c** | **d** | **e** |
| **σT** | 0.8 | 0.4 | 0.9 | 0.6 | 0.7 |
| **σI** | 0.4 | 0.5 | 0.6 | 0.3 | 0.6 |
| **σF** | 0.6 | 0.6 | 0.7 | 0.4 | 0.4 |

**Table 2:** Edge membership values in Figure 2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **μ** | **a, b** | **a, e** | **a, d** | **b, c** | **b, d** | **b, e** | **c, d** | **d, e** |
| **μT** | 0.7 | 0.8 | 0.5 | 0.3 | 0.5 | 0.6 | 0.4 | 0.5 |
| **μI** | 0.6 | 0.5 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.5 |
| **μF** | 0.5 | 0.6 | 0.3 | 0.5 | 0.5 | 0.3 | 0.5 | 0.6 |

For each node from Figure 2 calculate degree centrality. Let γ = 2 and calculate Havrda-Charvát Entropy for node

𝑎 is Eγ(𝑎) = (0.36, 0.51, 0.64). Using the same procedure, the HC entropy for vertices b, c, d, and e is calculated, and all results are summarized in Table 3.

The provided tables represent vertex and edge membership values in a Pythagorean Neutrosophic Fuzzy Graph (PNFG), a mathematical model used to handle uncertainty in network structures.

**Table 1: Vertex Membership Values**

Table 1 provides the **membership values** associated with each node (a, b, c, d, e). Each vertex has three associated values:

* **(Truth Membership):** Represents the degree to which the node exists in the network.
* **(Indeterminacy Membership):** Represents the uncertainty associated with the node.
* **(Falsity Membership):** Represents the degree of contradiction or non-existence.

For example, node **a** has values **(0.8,0.4,0.6)(0.8, 0.4, 0.6)**, meaning it has **high truth**, **moderate indeterminacy**, and **moderate falsity**.

**Table 2: Edge Membership Values**

Table 2 lists the edges in the network along with their corresponding vertex pairs. Each edge **μ\mu** represents a connection between two nodes. The edges form the structure of the PNFG, helping determine the influence and connectivity of each node.

These tables play a crucial role in computing **centrality measures**, evaluating **node importance**, and analyzing **uncertain relationships** in complex networks.

**Table 3** presents the entropy values associated with each vertex in the network, which are crucial in analyzing the uncertainty and stability of nodes in a Pythagorean Neutrosophic Fuzzy Graph (PNFG). The table consists of three entropy components:

* (Truth Entropy): Measures the uncertainty in the truth membership of a node.
* ​ (Indeterminacy Entropy): Quantifies the uncertainty due to indeterminacy in node connections.
* (Falsity Entropy): Represents the uncertainty in the falsity membership of a node.

These tables are useful for;

 **Quantifying Uncertainty:** Entropy values help assess how uncertain or stable a node is within a network. A higher entropy value indicates more uncertainty, making the node less reliable in terms of influence or connectivity.

 **Node Ranking & Influence:** By combining these entropy values with centrality metrics, researchers can rank nodes based on their reliability, uncertainty, and importance in the network.

 **Network Robustness Analysis:** Understanding entropy helps in evaluating how stable or unstable a network is, which is crucial in applications like social networks, recommendation systems, and influence propagation.

**Table 3:** Entropy values in Figure 2

|  |  |  |  |
| --- | --- | --- | --- |
| **Vertices** | 𝐄𝐓 | 𝐄𝐈 | 𝐄𝐅 |
| 𝒂 | 0.36 | 0.51 | 0.64 |
| **b** | 0.84 | 0.75 | 0.64 |
| **c** | 0.19 | 0.36 | 0.51 |
| **d** | 0.64 | 0.75 | 0.84 |
| **e** | 0.51 | 0.64 | 0.36 |

Let us consider, the value of α = 0.7 and β = 0.3 in this specific case and calculate node pack fuzzy information centrality of a vertex 𝑎 is NPFIC(a) = (0.72, 0.67, 0.63).

The centrality values for vertices b, c, d, and e are computed in a similar manner, with all the results summarized in Table 4.

**Table 4:** Centrality Scores of each vertex in Figure 2

|  |  |  |  |
| --- | --- | --- | --- |
| **Vertices** | **NPFICT** | **NPFICI** | **NPFICF** |
| 𝒂 | 0.72 | 0.67 | 0.63 |
| **b** | 0.75 | 0.78 | 0.81 |
| **c** | 0.59 | 0.54 | 0.50 |
| **d** | 0.81 | 0.78 | 0.75 |
| **e** | 0.67 | 0.63 | 0.72 |

The importance index of vertex 𝒂 is determined as follows:

2 × (0.72 × (1 − 0.63) + 0.67)

Xa =

= 0.401

3

Similarly, the importance index for vertices b, c, d, and e is calculated using the same approach, with the results summarized in Table 5.

**Table 5:** Importance Index of each vertex in Figure 2

|  |  |
| --- | --- |
| **Vertex** | **Influential Index** |
| **Xa** | 0.401 |
| **Xb** | 0.355 |
| **Xc** | 0.377 |
| **Xd** | 0.395 |
| **Xe** | 0.335 |

The most influential node in this Pythagorean Neutrosophic environment is the one with the highest importance index, i.e., max {Xa, Xb, Xc, Xd, Xe = Xa. Therefore, the most influential vertex from Figure 2 is vertex 𝑎.

∴ 𝐗𝐚 > 𝐗𝒅 > 𝐗𝐜 > 𝐗𝒃 > 𝐗𝐞

***Axiom 1:*** This axiom discusses the concept of centrality in a Pythagorean Neutrosophic Fuzzy Graph (PNFG), which is used to evaluate the influence of nodes in a network by considering three factors: truth, falsity, and indeterminacy. The truth and falsity values are dependent, meaning that an increase in one affects the other, while the indeterminacy component is independent. This approach allows for a more flexible and comprehensive representation of uncertainty in network analysis.

2

Since truth and falsity are linked, their combined effect is constrained, ensuring that they do not exceed a specific threshold. Indeterminacy, on the other hand, varies independently, allowing for additional complexity in measuring centrality. This method is particularly useful in real-world networks where relationships are not always clear-cut, such as in social networks, citation networks, and collaborative research graphs.

By incorporating Pythagorean Neutrosophic logic, this framework enhances traditional centrality measures by accounting for uncertainty and partial truth. It provides a more refined way to assess the importance of nodes, making it suitable for applications in decision-making, recommendation systems, and influence ranking. This axiom ensures a balanced approach to measuring node significance while maintaining logical constraints on their interdependencies***.***

Let G˙=(V,σ,μ) represent, Pythagorean Neutrosophic Fuzzy Graph, where ∣V∣ = n, and let the centrality

of a vertex v within this graph is denoted as 𝐶

= (C

, C , C

) then 0 ≤

2+ (C ) +

2 ≤ 2.

i 𝑉𝑖

𝑇𝑖

𝐼𝑖

𝐹𝑖

(C𝑇𝑖)

𝐼𝑖

(C𝐹𝑖)

***Proof*:** In a Pythagorean Neutrosophic Fuzzy Set (PNFS), the vertex vi centrality is indicated by 𝐶𝑣𝑖= (C𝑇𝑖, C𝐼𝑖,

C𝐹𝑖) where C𝑇𝑖 (truth) and C𝐹𝑖 (falsity) are dependent, and C𝐼𝑖 (indeterminacy) is independent. To prove that 0 ≤

2 2 2

(C𝑇𝑖) + (C𝐼𝑖) + (C𝐹𝑖) ≤ 2:

2

2

As C𝑇𝑖 and C𝐹𝑖 are dependent components, then their squared sum satisfies (C𝑇𝑖) + (C𝐹𝑖) ≤1 and C𝐼𝑖 is the

2

2

independent component so that (C

𝐼𝑖

) can take any value up to 1. The maximum possible value of (C

𝑇𝑖

) + (C

2

𝐼𝑖)

2

+ (C

𝐹𝑖 2

) occurs when both the truth-falsity sum and the indeterminacy value are maximized, yielding:

2

2

(C𝑇𝑖) +

(C𝐼𝑖) + (C𝐹𝑖) ≤1+1=2

2

2

Thus, the inequality

(C𝑇𝑖) +

(C𝐼𝑖) +

2

(C𝐹𝑖)

≤ 2 holds.

***Axiom 2:*** This axiom extends the concept of centrality in Pythagorean Neutrosophic Fuzzy Graphs (PNFGs) by introducing weighted centralities for each vertex. These weighted centralities—truth, indeterminacy, and falsity—are adjusted based on specific scaling factors, ensuring they adhere to the principles of Pythagorean Neutrosophic logic. This weighted approach allows for a more refined and adaptable measure of node influence in complex networks.

2

The axiom states that the weighted centrality of a vertex follows a specific constraint, ensuring that the combined effect of the truth, falsity, and indeterminacy does not exceed a certain bound. This constraint maintains logical consistency while accounting for different levels of influence across the network. By scaling the values appropriately, this framework enhances traditional centrality measures by incorporating weighted factors, making it more effective for real-world applications.

This weighted centrality measure is particularly useful in systems where different nodes have varying levels of importance based on external factors, such as collaborative networks, social media analysis, and decision-making frameworks. The axiom ensures that the weighted representation remains valid under Pythagorean Neutrosophic principles, making it a robust method for assessing influence in uncertain and complex systems.

Let Ġ = (V, σ, μ) represent, Pythagorean Neutrosophic Fuzzy Graph with |V| = n. The centrality of a

vertex vi is denoted as (C𝑇𝑖

2 2

, C𝐼𝑖, C

𝐹𝑖

), so the weighted centrality of the vertex vi is (W𝑇𝑖

, W𝐼𝑖, W𝐹𝑖

) , then 0 ≤ (W𝑇𝑖) +

(W𝐼𝑖) + (W𝐹𝑖) ≤ 2.

***Proof:*** In Pythagorean Neutrosophic Fuzzy Graph with |V| = n, then the centrality of a vertex vi is represented by the triplet (C𝑇𝑖, C𝐼𝑖, C𝐹𝑖), so the weighted centralities (W𝑇𝑖, W𝐼𝑖, W𝐹𝑖) of a vertex are derived in a way that respects the Pythagorean neutrosophic principles as well. Since each measure in (C𝑇𝑖, C𝐼𝑖, C𝐹𝑖) is related to (W𝑇𝑖 , W𝐼𝑖, W𝐹𝑖)

2 2

2

2

linearly or through scaling, we can infer: (W𝑇𝑖) + (W𝐼𝑖) + (W𝐹𝑖) can be at most a scaled version of (C𝑇𝑖) +

2 2 2

(C𝐼𝑖) + (C𝐹𝑖) . For (W𝑇𝑖, W𝐼𝑖, W𝐹𝑖) their squared values are non-negative, and in the worst case: (W𝑇𝑖) =1,

2 2 2 2 2

(W𝐼𝑖) =1, (W𝐹𝑖) =0. Thus, the maximum value of (W𝑇𝑖) + (W𝐼𝑖) + (W𝐹𝑖) is 2. Therefore, the inequality 0 ≤

2 2 2

(W𝑇𝑖) + (W𝐼𝑖) + (W𝐹𝑖) ≤ 2 holds true, confirming the bounds for weighted centralities in a Pythagorean

Neutrosophic Fuzzy Graph.

***Axiom 3:*** Let Ġ = (V, σ, μ) be a Pythagorean Neutrosophic Fuzzy Graph, where the weighted centrality of a vertex vi is represented as (WTi, WIi, WFi), and the influence index of vertex 𝑣𝑖 , represented by 𝑋𝑉𝑖 , is equal to 0 when its weighted centrality is either (0, n, 0) or (n,1, 0), where n lies within the interval [0,1].

***Proof:*** Consider a Pythagorean Neutrosophic Fuzzy Graph, denoted as Ġ = (V, σ, μ), where V represents the set of vertices, and the pair of membership functions of vertices σ = (σTi, σIi, σFi) and their edges membership functions μ = (μTi, μIi, μFi) denote the degree of collaboration (truth), the level of indecision (indeterminacy), and the degree of non-collaboration (falsity) for each vertex vi ∈ V and weighted centrality of vertex vi be represented by (W𝑇𝑖, W𝐼𝑖, W𝐹𝑖). The importance index of vertex vi is given by:

Xv1

= 2 × WTi × (1 − WFi) + WIi

3

2

Given that the weighted centrality values of W

, W , and W

are constrained by (W ) +

2 and 0 ≤

𝑇𝑖

2

𝐼𝑖

𝐹𝑖

𝑇𝑖

(W𝐹𝑖) ≤ 1

(W𝐼𝑖) ≤ 1, the formula ensures that higher truth membership (W𝑇𝑖) and lower falsity membership (W𝐹𝑖) will

maximize the 𝑋𝑣1 , while indecision W𝐼𝑖 moderates the effect. The influential index 𝑋𝑉𝑖=0 is possible when 𝑊𝑇𝑖= 0 and 𝑊𝐼𝑖 = 0, or when 𝑊𝐹𝑖 = 0 and 𝑊𝐼𝑖 = 0. Thus, 𝑋𝑣1 = 0 when the weighted centrality is either (0, n, 0) or (n,1,0), where n ∈ [0,1]. Axiom 3 in **Pythagorean Neutrosophic Fuzzy Graphs (PNFGs)** introduces the concept of an **influence index** for a vertex, which quantifies its significance within a network. This index is derived from the **weighted centrality** values of truth, indeterminacy, and falsity. The axiom states that a vertex's influence index becomes **zero** under specific conditions—when its truth and indeterminacy components are at minimal values or when its falsity and indeterminacy components meet certain constraints.

This formulation ensures that a vertex with **no significant collaborative influence (truth = 0) and no uncertainty (indeterminacy = 0)** will have an influence index of zero, meaning it holds no importance in the network. Similarly, when the falsity value is minimal while the indeterminacy is at its maximum, the vertex also loses its influence. This allows the framework to **filter out insignificant nodes**, improving the accuracy of **influence assessment** in complex networks.

The axiom is particularly useful in **social networks, research collaborations, and decision-making systems**, where determining the importance of entities is crucial. By incorporating **weighted centrality constraints**, it ensures a **balanced and logical** evaluation of influence, making PNFGs highly effective for real-world applications.

***Axiom 4:*** Consider a Pythagorean Neutrosophic fuzzy collaboration network represented as Ġ = (V, σ, μ), where the Collaboration Stability (CS) of the network is defined by:

CS(Ġ) = 1 ∑ (1 − |μ

− μ |)

(19)

|E|

(Vi , Vj)∈ E

Ti Ii

where ∣E∣ represents the total number of collaboration edges, μTi denotes the truth degree, and μIi indicates the indeterminacy degree in the collaboration.

This axiom in **Pythagorean Neutrosophic Fuzzy Graphs (PNFGs)** introduces the concept of **Collaboration Stability (CS)** in a network. This metric quantifies the overall stability of collaborations by considering the **truth and indeterminacy degrees** of edges connecting different entities. The **truth degree** represents the strength of a collaboration, while the **indeterminacy degree** accounts for uncertainty or hesitation in the relationship.

Collaboration Stability is crucial in **social, scientific, and business networks**, where interactions between entities must be **measured and optimized**. A high CS value indicates a **stable and reliable** collaboration network, meaning that relationships have **high trust and low uncertainty**. Conversely, a lower CS value suggests a **network with weak, uncertain, or unstable** collaborations. This measure helps in assessing the **resilience of networks**, identifying fragile links, and improving overall connectivity.

In real-world applications, CS can be applied to **academic research networks, business partnerships, and online social platforms**, where stable collaborations lead to **higher efficiency and productivity**. By integrating the **truth and indeterminacy degrees** into the calculation, this axiom provides a **comprehensive and dynamic** framework for evaluating network stability, making PNFGs highly effective for **complex decision-making scenarios**.

***Proof:*** A comprehensive collaboration network displays dependability and efficacy. The formula establishes the network's overall stability by minimizing the difference between the truth and indeterminacy values for all cooperative relationships. When the difference is modest and positive, the network is considered more stable.

# Implementation

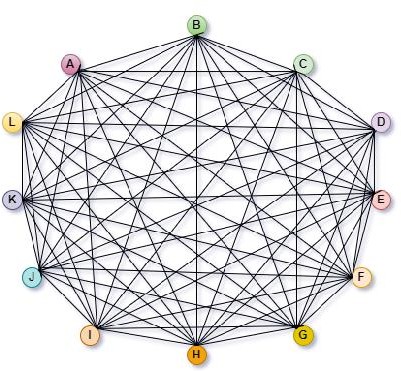
"AD Scientific Index" has recently released 2024 rankings. They ranked global scientists and universities. Google Scholar data was used to assess certain metrics. Metrics such as h-index, i10 index and citation counts. But Google Scholar data can sometimes have inaccuracies. Inaccuracies like fake articles (falsity) and missing or conflicting information (indeterminacy). The index focuses on publications and citations. It unfortunately overlooks essential factor like research collaboration. Research institutions could benefit from publishing faculty rankings. These rankings consider additional factors like research collaboration and self-publication. They also consider citation metrics. Such actions will surely enhance both research quality and institutional reputation. To validate proposed ranking method a small network of researchers is analyzed. In this network, each individual is represented as a node. Edges indicate collaboration forming a complete graph. Score determine rank of node. Score depends on academic collaboration i.e., centrality. It also takes into account self-publishing and citation metrics. Weights are self-weights. There are twelve individuals in the network from Figure 3, denoted as A to L. In Figure 4 a graph is depicted and it shows a group of connected nodes A to I based on collaboration. Nodes J, K, and L have no collaborations. They are not displayed in the graph of Figure 4. The data from Google Scholar includes metrics like total publications recent publications, and citations. It also includes h-index, i10-index and Journal quality as metrics. These metrics are used to calculate node's membership values. The values are truth (T) indeterminacy (I) and falsity (F). These values assess the reliability of a researcher’s profile. The errors and inconsistencies in the data are accounted for. Now let us discuss researchers J, K and L. They have not engaged in any collaboration with others at this institution. We treat the vertices J, K and L as distinct nodes for the purpose of centrality calculation. AD Scientific Index has recently released 2024 rankings. They ranked global scientists and universities. Google Scholar data was used to assess certain metrics. Metrics such as h-index, i10-index, and citation counts. But Google Scholar data can sometimes have inaccuracies. Inaccuracies like fake articles (falsity) and missing or conflicting information (indeterminacy). The index focuses on publications and citations. It unfortunately overlooks essential factors like research collaboration. Research institutions could benefit from publishing faculty rankings. These rankings consider additional factors like research collaboration and self-publication. They also consider citation metrics. Such actions will surely enhance both research quality and institutional reputation.

To validate the proposed ranking method, a small network of researchers is analyzed. In this network, each individual is represented as a node. Edges indicate collaboration, forming a complete graph. Scores determine the rank of a node. Scores depend on academic collaboration, i.e., centrality. It also takes into account self-publishing and citation metrics. Weights are self-weights. There are twelve individuals in the network from Figure 3, denoted as A to L. In Figure 4, a graph is depicted, and it shows a group of connected nodes A to I based on collaboration. Nodes J, K, and L have no collaborations. They are not displayed in the graph of Figure 4. The data from Google Scholar includes metrics like total publications, recent publications, and citations. It also includes h-index, i10-index, and journal quality as metrics. These metrics are used to calculate a node's membership values. The values are truth (T), indeterminacy (I), and falsity (F). These values assess the reliability of a researcher’s profile. The errors and inconsistencies in the data are accounted for. Now let us discuss researchers J, K, and L. They have not engaged in any collaboration with others at this institution. We treat the vertices J, K, and L as distinct nodes for the purpose of centrality calculation.

While J, K, and L lack collaborative ties within the institution, their individual research contributions still play a role in their rankings. Their citation metrics and self-publication tendencies influence their membership values, even in the absence of co-authored work. In a more refined ranking system, these researchers could be analyzed separately to determine whether their impact is primarily self-driven or externally recognized. This differentiation helps ensure a fair assessment of researchers who work independently versus those who actively engage in collaborative projects.

Furthermore, collaboration networks evolve over time, meaning that static rankings may not accurately reflect ongoing research trends. Continuous monitoring of such networks can provide better insights into research dynamics, allowing institutions to develop strategies to foster collaborations and improve overall research output. By incorporating both **collaborative and independent research contributions**, a well-rounded ranking system can be established, ensuring that institutional assessments are **more comprehensive and accurate**.

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**Figure 3.** Researchers Collaboration Network in an Institution

Therefore, the centrality of these vertices is (0.0, 0.0, 0.0) for J, (0.0, 0.0, 0.0) for K, and (0.0, 0.0, 0.0) for L. The membership values for truth (T), indeterminacy (I), and falsity (F) for vertices J, K, and L are as follows: Vertex J has values of (0.8, 0.01, 0.3), vertex K has (0.6, 0.0, 0.3), and vertex L has (0.3, 0.0, 0.0). The T, I, and F values for edges are based on research collaboration, derived from parameters related to publications between two researchers. It includes any of the following parameters:

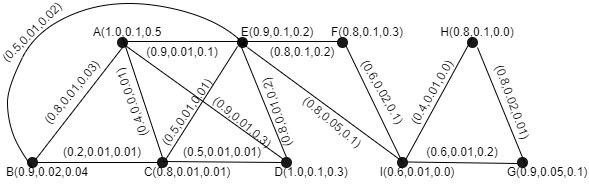
1. Total publications
2. Publications in the last 6 years
3. Total citations
4. Citations in the last 6 years
5. Quality of the journal (SCI/SCIE/Scopus/UGC Care)
6. Impact Factor (IF) of the journal
7. Journal quartiles (Q1, Q2, Q3, Q4).

The researcher collaboration network illustrated in Figure 3 represents the academic collaboration structure within an institution. Each node in the network corresponds to a researcher, while edges between nodes signify collaborative relationships based on shared publications. The centrality of a researcher within this network is determined by their collaborative connections, which influence their weighted centrality scores.

Vertices J, K, and L lack collaborative ties within the network, leading to centrality values of (0.0, 0.0, 0.0). However, their membership values for truth (T), indeterminacy (I), and falsity (F) provide additional insights into their academic contributions. Vertex J has values of (0.8, 0.01, 0.3), indicating a high degree of truth membership but with some falsity due to inconsistencies in publication data. Vertex K has (0.6, 0.0, 0.3), representing moderate research output but also some falsity. Vertex L has (0.3, 0.0, 0.0), signifying low research contribution but no conflicting data.

The edges in this network are derived from research collaboration and are evaluated based on publication metrics. These include total publications, recent publications over the past six years, total citations, and citations within the last six years. Other factors considered include journal quality (SCI, SCIE, Scopus, UGC Care), impact factor, and journal quartiles (Q1, Q2, Q3, Q4). These parameters ensure that the ranking system accounts for both the quantity and quality of research output.

By incorporating these factors, this model provides a more holistic evaluation of a researcher’s academic standing. It not only considers citation-based impact but also emphasizes the importance of research collaboration. The inclusion of membership values helps mitigate data inaccuracies, making the ranking system more reliable for institutional assessments.



**Figure 4.** Pythagorean Neutrosophic Fuzzy Graph (From Figure 3)

The given network graph represents a structured collaboration framework where each node corresponds to a researcher, and edges depict collaborative relationships between them. Each researcher is associated with a triplet of values representing truth (T), indeterminacy (I), and falsity (F), which help quantify their research contributions based on publication data. These values provide a more nuanced view of their academic influence by considering data inconsistencies and missing information.

Node A, for instance, has the highest truth membership value (1.0), indicating a significant research contribution with minimal uncertainty. Other nodes, such as B and C, also exhibit high truth values, signifying strong academic output. Conversely, node I has a truth value of 0.0, indicating a lack of research contributions, which aligns with its isolated position in the network.

The edges in the graph are annotated with weighted values representing the strength of collaboration between researchers. These values are derived from joint publications, citations, and other academic metrics. Stronger collaborations have higher truth values, while weaker ones exhibit increased indeterminacy or falsity due to inconsistencies in shared work.

By analyzing this network, institutions can identify key researchers driving collaboration, as well as those with limited participation. Such an analysis helps in improving research productivity and fostering stronger academic relationships.

For each researcher from Figure 4 calculate degree centrality. Let γ = 2 and calculate Havrda-Charvát entropy for all researchers and compute centrality scores by combine degree centrality and Havrda-Charvát entropy. The influential index for all researchers is calculated, and are presented in Table 6.

**Table 6:** Influential Index of Researcher in Figure 4

|  |  |
| --- | --- |
| Vertex | Influential Index |
| XA | 0.406 |
| XB | 0.391 |
| XC | 0.401 |
| XD | 0.411 |
| XE | 0.438 |
| XF | 0.289 |
| XG | 0.335 |
| XH | 0.300 |
| XI | 0.358 |
| XJ | 0.166 |
| XK | 0.092 |
| XL | 0.024 |

The most influential researcher is determined by finding the researcher with the highest influential index, specifically,

max {XA, XB, XC, XD, XE, XF, XG, XH, XI, XJ, XK, XL} = XE.

Therefore, the most influential researcher of this institution is E. In the same way, we can identify the second most influential researcher from this institution and create a rank list based on the influential index. This ranking is displayed in Table 7. High Centrality with low entropy ranks highest, moderate values rank in the middle and low centrality with heigh entropy ranks lowest. PNFS-based centrality accounts for the inherent fuzziness of relationships between academics, which can arise from informal collaborations, ambiguous co-authorship roles, or variable academic output. This method is particularly effective when relationships are imprecise.

The table presents the Influential Index of researchers as represented by vertices in Figure 4. The Influential Index quantifies the impact of each researcher within the collaboration network by incorporating various factors such as research contributions, collaborations, and citation metrics. This measure helps in assessing the relative significance of researchers within the institutional network.

From the table, vertex XEX\_E has the highest influential index value of 0.438, indicating that this researcher plays a crucial role in the network. A high index suggests strong research contributions, active collaborations, and a central position in the network. Other highly influential researchers include vertices XDX\_D (0.411) and XAX\_A (0.406), both of whom demonstrate strong academic engagement and collaborative efforts.

On the other hand, vertex XFX\_F has the lowest influential index at 0.289, implying weaker collaborations or lower research productivity. Other researchers with relatively lower influential index values include XHX\_H (0.300) and XGX\_G (0.335), suggesting room for increased academic engagement or collaboration to enhance their impact.

This analysis allows institutions to identify key contributors and potential areas for improvement. By leveraging such insights, universities and research organizations can encourage collaborations, support underrepresented researchers, and optimize resource allocation to improve overall research output and institutional rankings.

The Influential Index presented in Table 6 serves as a critical metric for evaluating the research significance of individuals within an institutional network. This index considers factors such as academic collaboration, citation metrics, and self-publication, providing a holistic view of a researcher's impact.

Observing the values in the table, it is evident that some researchers, such as XEX\_E (0.438) and XDX\_D (0.411), have a higher influential index, signifying their pivotal role in the research community. Their strong collaborative ties and substantial contributions place them in central positions within the network. Similarly, researchers like XAX\_A (0.406) and XCX\_C (0.401) also exhibit high influence, suggesting active participation in academic publishing and research collaborations.

Conversely, researchers with lower influential index values, such as XFX\_F (0.289) and XHX\_H (0.300), may have fewer collaborative connections or a lower volume of impactful publications. While their contributions are still valuable, these lower scores indicate potential areas for improvement. Encouraging such researchers to participate in more collaborative projects and enhancing their publication impact could elevate their standings within the network.

This ranking method provides an alternative to conventional citation-based assessments, as it incorporates collaboration and self-publication influences. Traditional citation metrics often overlook these factors, leading to an incomplete representation of a researcher's true impact. By using this approach, institutions can better understand the structure of their research network and take strategic actions to foster academic growth.

Ultimately, analyzing the influential index helps in identifying key researchers, supporting faculty development, and ensuring that research efforts align with institutional goals. Future research can expand upon this methodology by integrating additional parameters such as interdisciplinary collaborations, research funding, and global impact, further refining the accuracy of such rankings.

**Table 7:** Rank the Researcher in Figure 4

|  |  |  |
| --- | --- | --- |
| Rank | Vertex | Influential Index |
| 1 | E | 0.438 |
| 2 | D | 0.411 |
| 3 | A | 0.406 |
| 4 | C | 0.401 |
| 5 | B | 0.391 |
| 6 | I | 0.358 |
| 7 | G | 0.335 |
| 8 | H | 0.300 |
| 9 | F | 0.289 |
| 10 | J | 0.166 |
| 11 | K | 0.092 |
| 12 | L | 0.024 |

**Top-Ranked Researchers:**

* Researcher E holds the highest Influential Index (0.438**)**, making them the most impactful individual in the network.
* Researchers D (0.411) and A (0.406) closely follow, indicating that they also have a strong influence within the network.

**Moderately Influential Researchers:**

* Researchers C (0.401), B (0.391), I (0.358), and G (0.335) form a mid-tier group, meaning they have significant but slightly lower influence compared to the top researchers.
* Their positions suggest they may act as connectors or secondary influencers in the network.

**Lower Influence Researchers:**

* Researchers H (0.300), F (0.289), and J (0.166) show a declining influence, indicating that their role in the network might be more peripheral.
* K (0.092) and L (0.024) have the lowest influence scores**,** meaning they likely have **f**ewer connections or less central positioning within the network.

# Results Analysis

Node influence measurement hinges on weights and nodal connectivity. In this research, the most influential node was pinpointed. This was done through three factors: collaboration, self-publishing, and citations. To measure collaboration in research, centrality measures are used. In contrast, node self-weights are used to evaluate citation and self-publication. Therefore, assessment of node centrality in a Pythagorean Neutrosophic Fuzzy Graph gives realistic results.

However, the proposed method has limitations. It is quite challenging to compute truth, falsity, and indeterminacy. This computation is based on original data. Currently, there are no available methods for gathering such types of information. Figure 5 and 6 compare indices of influence. In addition, they compare researcher rankings from divergent centrality measures. These figures show an interesting statistic. 83% of researchers achieved the highest influential indices. These indices are higher than those of any other centralities.

A key strength of this approach lies in its ability to provide a more comprehensive evaluation of a researcher’s influence. Traditional methods rely solely on citation counts and h-index values, which may not accurately capture a researcher's real impact. By integrating collaboration and self-publication metrics, this method provides a more nuanced and holistic view. Moreover, considering truth, falsity, and indeterminacy ensures that inaccuracies or inconsistencies in data do not skew the results, making the evaluation more reliable.

Despite these advantages, practical challenges remain. Since the computation of influence indices requires accurate truth, falsity, and indeterminacy values, developing a systematic way to collect this information is crucial. Current academic databases, such as Google Scholar and Scopus, do not explicitly provide such values. Future research should explore automated approaches to estimate these factors using machine learning or advanced statistical techniques.

Furthermore, the significance of this research extends beyond individual rankings. Institutions can leverage these findings to enhance their faculty evaluation processes. By incorporating collaboration metrics, universities can encourage interdisciplinary research, which is essential for tackling complex global challenges. This method also helps in identifying researchers who may be underrecognized in traditional ranking systems despite making significant contributions.

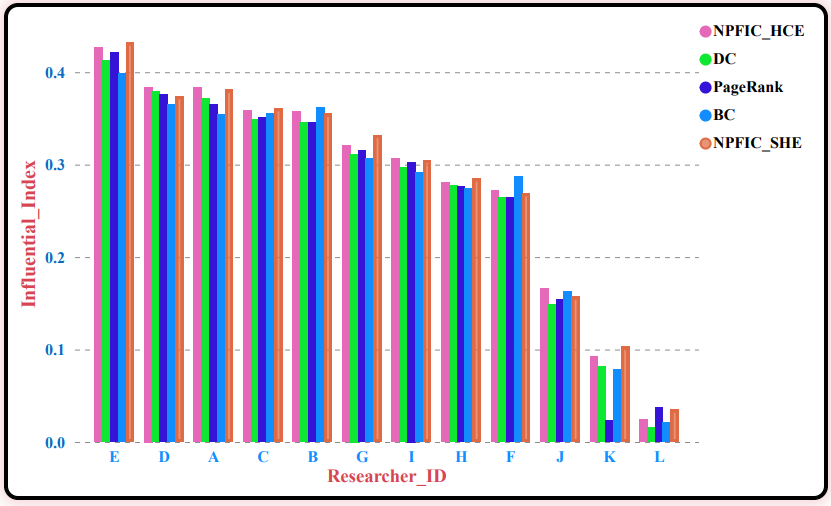
Additionally, this research framework can be applied to broader academic networks, including multi-institutional collaborations and global research communities. Expanding the analysis to larger datasets will further validate the robustness of this approach. Future studies can also explore alternative weighting schemes, considering additional factors such as research funding, patents, and societal impact.

Ultimately, the proposed method not only improves researcher evaluation but also fosters a more collaborative and transparent academic environment. By refining the approach and addressing current limitations, this framework has the potential to revolutionize how researcher influence is assessed, making it more equitable and data-driven.

The below bar chart represents the Influential Index of researchers, categorized by their Researcher\_IDs. The x-axis lists different researchers (E, D, A, C, B, G, I, H, F, J, K, L), while the y-axis quantifies the Influential Index. The chart compares five different metrics: **NPFIC\_HCE, DC, PageRank, BC, and NPFIC\_SHE**, which are represented by distinct colors. These metrics likely indicate various centrality and influence measures used in network analysis to assess the impact of researchers.

From the visualization, it is evident that researchers **E, D, A, and C** exhibit the highest Influential Index values, suggesting their strong impact within the network. The influence gradually decreases as we move towards researchers **J, K, and L**, who show significantly lower values. Notably, **NPFIC\_HCE (pink) and NPFIC\_SHE (orange)** tend to have slightly higher values compared to the other measures, implying that these metrics might capture additional dimensions of influence.

This analysis is useful in academic and bibliometric studies to identify key contributors in a research network. The comparison of different centrality measures provides deeper insights into how researcher influence is distributed and highlights the variability across different metrics.

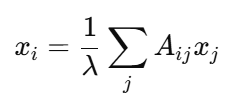


**Figure 5.** Influential Indices of various Centralities

In network science, centrality measures play a crucial role in identifying the most important nodes in a network. While traditional measures such as degree centrality, betweenness centrality, and closeness centrality are widely used, more sophisticated measures like Eigenvector Centrality, Katz Centrality, Harmonic Centrality, and Subgraph Centrality provide deeper insights into a node’s importance based on the overall network structure. These centrality measures are particularly useful in fields such as social network analysis, biology, transportation networks, and communication systems.

Eigenvector Centrality (EC) is an **extension of degree centrality** that takes into account the importance of a node's neighbors. Instead of just counting the number of direct connections (as degree centrality does), eigenvector centrality **assigns higher scores to nodes that are connected to other influential nodes**.

Mathematically, Eigenvector Centrality is defined using the **eigenvector equation**:

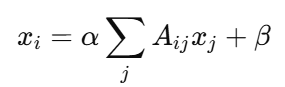


where:

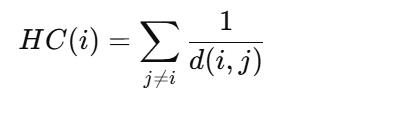
* ​ is the eigenvector centrality of node iii,
* ​ is the adjacency matrix of the network,
* λ is the largest eigenvalue of the adjacency matrix.

Katz Centrality (KC) is a **generalization of eigenvector centrality**, designed to handle networks where eigenvector centrality may fail, particularly when there are **disconnected nodes**. Katz centrality assigns **a base score to every node** and gives higher importance to nodes connected to influential neighbors.

The formula for Katz centrality is:

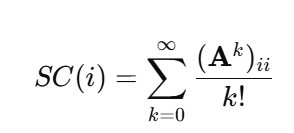


Harmonic Centrality (HC) is a **variation of closeness centrality** that resolves the problem of **disconnected graphs**. Instead of using the **sum of shortest path distances** (as in closeness centrality), harmonic centrality computes the **sum of the reciprocals of the shortest path distances** from a node to all other nodes:



Subgraph Centrality (SC) measures the **importance of a node** based on the number of **closed walks** it participates in. A **closed walk** is a path that starts and ends at the same node. Unlike eigenvector centrality, which considers only **infinite paths**, subgraph centrality considers **finite paths** of different lengths.

Mathematically, it is given by:



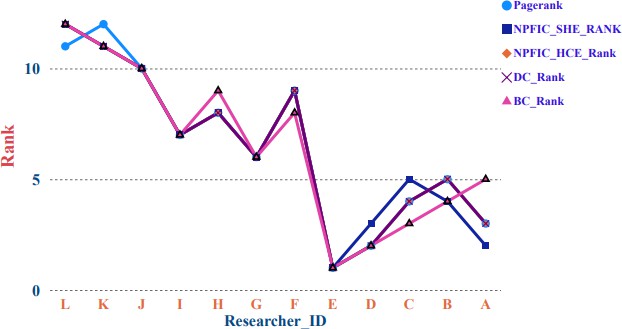
**Applications of Subgraph Centrality**

* **Molecular Biology:** Used to identify **essential proteins** in protein interaction networks.
* **Financial Networks:** Helps detect **key economic players** in trade or banking networks.
* **Epidemiology:** Predicts **high-risk individuals** in disease-spread networks.

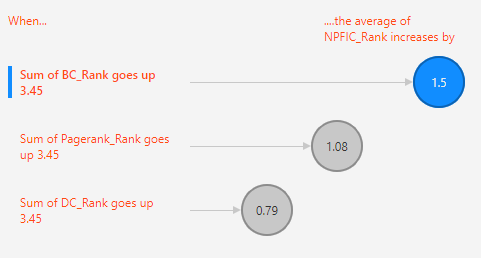
**Limitations**

* **Computationally intensive** since it requires calculating matrix exponentials.
* May overestimate the importance of nodes in **highly clustered networks**.

Figure 7 and 8 shows that NPFIC has the strongest association with BC, surpassing other centralities despite varying influence indices. The accuracy and efficiency of NPFIC in identifying central nodes is clearly outperforming than other centralities and these results offer valuable insights into social network structures. Researchers who may not rank highly using traditional methods can appear more central due to their roles in bridging uncertainty or fostering indirect collaborations, which are critical for knowledge dissemination. This highlights the importance of considering both strong and weak ties in academic networks. Top-ranked researchers tend to be those who frequently collaborate across departments and research areas, enhancing their role as key information disseminators. Researchers with moderate centrality may have fewer direct collaborations, but their unique position between specialized groups allows them to facilitate interdisciplinary research and making them more influential when evaluated using Pythagorean Neutrosophic Fuzzy System-based methods.



**Figure 6.** Different Centralities provided Ranks



**Figure 7.** Impact of Key Influencers

**Impact of Centrality Measures on NPFIC\_Rank**

The given figure provides a comparative analysis of the influence of three different centrality metrics—Betweenness Centrality Rank (BC\_Rank), PageRank Rank, and Degree Centrality Rank (DC\_Rank)—on the NPFIC\_Rank, which measures the overall researcher influence in a network.

**Key Observations**

* Betweenness Centrality Rank (BC\_Rank): A 3.45-point increase in BC\_Rank leads to the highest increase in NPFIC\_Rank by 1.5 units.
* PageRank Rank: A similar 3.45-point increase in PageRank Rank results in an NPFIC\_Rank increase of 1.08.
* Degree Centrality Rank (DC\_Rank): The lowest impact is observed with DC\_Rank, where a 3.45-point increase leads to an increase of only 0.79 in NPFIC\_Rank.

**Interpretation of Results**

The results highlight that Betweenness Centrality (BC) has the strongest correlation with NPFIC\_Rank, followed by PageRank, with Degree Centrality (DC) having the least effect. This suggests that researchers who serve as bridges between different research clusters hold a greater network influence than those who simply have numerous direct connections.

1. Betweenness Centrality (BC) as a Key Factor
   * BC measures how often a researcher appears on the shortest path between other researchers in a network.
   * A high BC\_Rank suggests that the researcher plays a crucial role in connecting disparate academic communities, which enhances their visibility and influence.
   * This aligns with the structural hole theory, which states that individuals bridging disconnected groups tend to have greater influence.
2. PageRank’s Moderate Influence
   * PageRank assigns importance based on the quality of connections, rather than just the number of connections.
   * The positive correlation between PageRank Rank and NPFIC\_Rank indicates that being cited or referenced by highly influential researchers increases one's overall network importance.
   * However, its lower impact (1.08) compared to BC\_Rank (1.5) suggests that bridging different research groups is more impactful than simply being cited by important researchers.
3. Limited Effect of Degree Centrality (DC)
   * DC measures the number of direct connections a researcher has in a network.
   * Although an increase in DC\_Rank leads to a rise in NPFIC\_Rank, the effect is the weakest among the three measures.
   * This indicates that merely having a large number of connections does not necessarily translate into greater influence if those connections do not play a critical role in bridging research communities.

**Implications for Research Impact Assessment**

* BC should be prioritized in network-based impact evaluations, as it strongly correlates with higher influence rankings.
* PageRank can complement BC in assessing citation-based influence, but it is not as effective in identifying researchers who facilitate interdisciplinary collaboration.
* Degree Centrality alone is insufficient as a metric for influence, as having more connections does not necessarily equate to higher research impact.

**Analysis of Researcher Influence Using Centrality Measures**

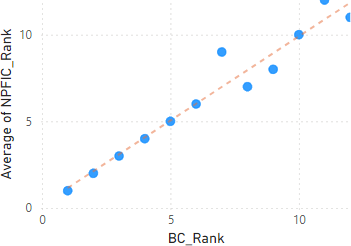
Figure 6 illustrates the ranking of researchers based on different centrality measures, including PageRank, Betweenness Centrality (BC), Degree Centrality (DC), NPFIC\_HCE\_RANK, and NPFIC\_SHE\_RANK. The x-axis represents different researchers (L to A), while the y-axis denotes their rank. The fluctuations in rank across different measures suggest that each centrality metric evaluates researcher influence differently. Some researchers rank consistently across measures, while others exhibit significant variations, indicating the varying importance of different network properties in ranking influence.

Figure 7 further explores the impact of key centrality measures on NPFIC\_Rank. It quantifies how an increase in different ranking measures affects the average NPFIC\_Rank. The results show that an increase of **3.45 in BC\_Rank** leads to the highest increase in NPFIC\_Rank by **1.5**, followed by **PageRank (1.08) and Degree Centrality (0.79)**. This indicates that betweenness centrality has the strongest correlation with NPFIC**\_Rank**, suggesting that researchers who act as crucial intermediaries in the network hold higher influence.

These findings are valuable for understanding how different network-based ranking methods reflect researcher impact, helping refine influence assessment in academic networks.

Low-ranked researchers might have fewer localized collaborations. They could have less influence overall. This is particularly true when we factor in fuzzy ties in the network. Pythagorean Neutrosophic Fuzzy Sets are now a part of centrality calculations. These are used for academic collaboration networks. This includes our proposed method. The method presents potent approach to rank researchers. This method allows for deeper comprehension of influence dynamics within academic communities. It counts for uncertainties. It considers indirect relationships and this leads to better research management. It also improves collaboration strategies. By *Axiom 4*, we compute the Collaboration Stability (CS) for the network in Figure 1 and the value is 0.8875. This network is considered more stable and it is because the difference is smaller and positive. This is evident from equation 19.

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**Figure 8.** BC and NPFIC Rank Correlation

**Correlation Between BC\_Rank and NPFIC\_Rank**

The given scatter plot illustrates the relationship between Betweenness Centrality Rank (BC\_Rank) and the Average of NPFIC\_Rank. The x-axis represents BC\_Rank, while the y-axis denotes the Average of NPFIC\_Rank. Each blue dot represents a researcher, and the dashed trend line indicates a strong positive correlation between these two variables.

Interpretation of the Results

The observed pattern in the scatter plot suggests that as a researcher's BC\_Rank increases, their NPFIC\_Rank also tends to rise. This strong linear relationship implies that betweenness centrality plays a significant role in determining researcher influence as captured by the NPFIC metric. Researchers with a high BC\_Rank serve as crucial intermediaries in the research network, connecting different groups and facilitating knowledge flow. Since NPFIC\_Rank is designed to assess the overall impact of a researcher in the network, it makes sense that those with a high BC\_Rank—who act as key bridges—also achieve higher NPFIC\_Rank values.

Significance of the Relationship

Betweenness centrality (BC) is a key network measure that identifies nodes (researchers) that frequently appear on the shortest paths between other nodes. In academic collaboration networks, researchers with high BC are often those who bridge different research communities, enhancing cross-disciplinary collaboration and information diffusion. The strong correlation observed in the scatter plot suggests that a researcher's role as an intermediary is a critical factor in determining their overall network-based influence (NPFIC\_Rank).

This finding aligns with previous research on scientific impact and network analysis, which emphasizes that scholars who serve as connectors between different research clusters tend to have higher influence and visibility. Such researchers often contribute to multiple fields and facilitate cross-pollination of ideas, leading to greater recognition and citation impact.

Implications for Research Evaluation

The strong relationship between BC\_Rank and NPFIC\_Rank has several implications for the evaluation of researcher influence:

1. Network Position Matters – Traditional metrics like citation count or h-index may not fully capture the importance of researchers who act as bridges in a network. BC-based rankings highlight those with significant structural roles.
2. Identifying Key Influencers – Institutions and funding agencies can use betweenness centrality as an additional criterion for identifying researchers who foster interdisciplinary collaborations and network cohesion.
3. Optimizing Research Collaborations – Understanding the role of BC in influence rankings can help researchers strategically position themselves within academic networks to maximize their impact and visibility.

In large-scale social networks, identifying influential nodes is crucial for applications such as **viral marketing, social influence analysis, and epidemic control**. Traditional centrality measures like **Degree Centrality (DC), Betweenness Centrality (BC), Closeness Centrality (CC), and PageRank** help quantify the importance of nodes. However, these measures are often deterministic and fail to capture the **uncertainty and vagueness** inherent in real-world networks. **Fuzzy centrality measures** address this issue by integrating **fuzzy logic** into centrality calculations, allowing for a more nuanced and flexible ranking of influential nodes.

**Challenges in Large-Scale Networks**

Analyzing centrality measures in **large-scale social networks** presents several computational challenges:

1. **Scalability** – Traditional algorithms become computationally expensive as network size grows.
2. **Uncertainty in Influence** – Real-world influence is not binary; it fluctuates based on various social, temporal, and contextual factors.
3. **Dynamic Nature** – Social networks are constantly evolving, requiring algorithms that can **update centrality rankings efficiently**.

To address these challenges, researchers have developed **efficient algorithms** for computing fuzzy centrality measures that balance **accuracy and computational efficiency**.

**Fuzzy Centrality Measures**

Fuzzy centrality measures extend classical centrality metrics by incorporating **fuzzy set theory**. Some common fuzzy-based centrality measures include:

1. **Fuzzy Degree Centrality (FDC):** Instead of counting direct connections, **FDC assigns a fuzzy weight** to each connection, representing the strength or uncertainty of the relationship.
2. **Fuzzy Betweenness Centrality (FBC):** This considers the **probability of a node acting as a bridge** in uncertain pathways rather than fixed shortest paths.
3. **Fuzzy Closeness Centrality (FCC):** This defines proximity based on **probabilistic reachability**, ensuring a more realistic measure of influence.
4. **Fuzzy PageRank:** Incorporates uncertainty in link structures, adjusting the ranking probabilities based on **fuzzy transition matrices**.

**Efficient Algorithms for Fuzzy Centrality Computation**

Several efficient algorithms have been proposed to handle the **computational complexity** of fuzzy centrality measures in large-scale networks:

1. **Approximation Techniques:** Algorithms such as **Monte Carlo Sampling** and **Adaptive Random Walks** estimate fuzzy betweenness centrality with reduced computational cost.
2. **Parallel Processing:** Leveraging **distributed computing frameworks (Hadoop, Spark)** enables faster computation of fuzzy centrality in massive networks.
3. **Graph Compression Methods:** **Graph sparsification** and **low-rank approximations** reduce network size while preserving key structural properties, allowing fuzzy centrality measures to be computed efficiently.
4. **Incremental Updating:** Instead of recomputing fuzzy centrality from scratch, **incremental algorithms** update rankings based on **network changes**.

**Applications of Fuzzy Centrality in Social Networks**

Efficient algorithms for fuzzy centrality measures have numerous real-world applications:

* **Influence Maximization:** Identifying **key influencers** in marketing campaigns.
* **Community Detection:** Recognizing **central nodes** in dynamic communities.
* **Epidemic Spread Control:** Predicting **disease outbreak pathways** based on uncertain transmission probabilities.
* **Recommender Systems:** Enhancing **personalized recommendations** by analyzing fuzzy social influence.

**1. Approximation Techniques for Centrality Computation**

Need for Approximation

Centrality computations often involve expensive matrix operations and iterative methods that scale poorly with large networks. Exact solutions for Eigenvector, Katz, and Subgraph Centrality require solving eigenvalue problems or matrix exponentiation, which are computationally expensive.

To address these limitations, approximation algorithms are used to achieve near-optimal solutions with reduced computational complexity.

Common Approximation Techniques

1. Monte Carlo Methods

Monte Carlo methods approximate centrality scores using random sampling of paths or walks in the graph. Instead of computing all possible paths, a subset of walks is simulated to estimate node importance.

* Example: In Random Walk-based Approximations, a limited number of random walks are performed from each node, and the frequency of visits to other nodes is used as an estimate of centrality.
* Application: Katz Centrality and Eigenvector Centrality can be efficiently approximated using Monte Carlo sampling.

2. Power Iteration Method

For eigenvector-based centrality measures, such as Eigenvector Centrality and Katz Centrality, the Power Iteration Method is a widely used approximation technique.

* The algorithm iteratively updates centrality scores by multiplying the adjacency matrix with an initial vector until convergence.
* Instead of solving expensive eigenvalue problems, this method provides a fast and scalable way to approximate leading eigenvectors.
* Trade-off: The method is highly scalable but requires many iterations for highly connected graphs.

3. Low-Rank Matrix Approximations

* Large graphs are often represented as adjacency matrices. Instead of computing exact eigenvectors, methods like Singular Value Decomposition (SVD) or Lanczos methods approximate them using a low-rank representation.
* This technique is especially useful for Subgraph Centrality, where computing matrix exponentials is computationally expensive.

4. Greedy and Heuristic Approaches

* Greedy algorithms prioritize evaluating only the most relevant nodes or paths instead of the entire network.
* Application: Used in betweenness centrality approximation where shortest paths are sampled instead of computing all-pairs shortest paths.

**2. Parallel Processing for Centrality Computation**

Why Parallel Processing?

Large networks contain millions or even billions of nodes and edges. Computing centrality measures sequentially becomes infeasible due to high time complexity. Parallel computing distributes computations across multiple processors or GPU cores, significantly improving performance.

Parallel Approaches for Centrality Computation

1. Parallel Breadth-First Search (BFS) for Shortest Paths

* BFS is widely used in Harmonic Centrality and Katz Centrality computations.
* Instead of sequential BFS, Parallel BFS assigns different starting nodes to different processors, reducing runtime.
* Implemented efficiently using OpenMP, MPI, and GPU-based approaches.

2. Sparse Matrix Operations for Eigenvector-Based Centralities

* Eigenvector and Katz Centrality rely on matrix-vector multiplications, which can be parallelized using sparse matrix libraries like CUDA (NVIDIA GPUs), TensorFlow, and PyTorch.
* Power iteration can be distributed across multiple computing units, significantly reducing execution time.

3. Parallel Random Walks for Approximation

* Many centrality measures rely on random walks (e.g., PageRank, Eigenvector Centrality).
* Instead of simulating walks sequentially, multiple random walks are executed in parallel across different nodes.

4. GPU Acceleration for Graph Processing

* NVIDIA’s cuGraph and GraphBLAS libraries allow centrality computation on GPUs, achieving significant speedups.
* Example: Computing Subgraph Centrality using GPU-accelerated matrix exponentiation.

5. Distributed Computing for Large Graphs

* Apache Spark GraphX and Google’s Pregel allow distributed processing of centrality measures on graph datasets stored in Hadoop or cloud environments.
* Used in applications such as social media analysis (Facebook, Twitter), web ranking (Google), and large-scale biological networks.

Benefits of Parallel Processing

1. Speeds up centrality computations significantly.  
2. Handles large-scale graphs efficiently.  
3. Enables real-time analytics for dynamic networks.

**3. Graph Compression Methods for Efficient Computation**

Why Graph Compression?

Many real-world networks contain redundant or less significant edges that do not contribute significantly to centrality computations. Graph compression techniques reduce storage requirements and speed up computations by simplifying the graph.

Common Graph Compression Techniques

1. Sparsification Techniques

* Edge Sparsification: Removes less influential edges while preserving key connectivity properties.
* Example: Used in Eigenvector and Katz Centrality, where edges with negligible weights are ignored.

2. Node Aggregation and Coarsening

* Nodes with similar connectivity patterns are merged into a single super-node.
* Used in community detection-based compression to preserve structural properties.

3. Graph Partitioning

* Breaks the network into smaller subgraphs, which are processed separately.
* Works well in Parallel Processing frameworks like MapReduce.

Applications of Graph Compression

* Web Graphs: Reducing storage for search engines (Google, Bing).
* Social Networks: Improving performance in Facebook friend recommendations.

**4. Incremental Updating for Dynamic Networks**

Why Incremental Updates?

Most real-world networks are dynamic—new nodes and edges are added or removed over time. Recomputing centrality scores from scratch is inefficient, so incremental updating techniques allow faster updates with minimal computation.

Incremental Updating Methods

1. Delta Updates for Eigenvector and Katz Centrality

* When a small change occurs in the graph, recompute only the affected portion instead of the entire graph.
* Example: Instead of recomputing the eigenvalues from scratch, a perturbation approach updates centrality scores incrementally.

2. Streaming Algorithms for Online Computation

* Streaming Graph Processing maintains an updated state of centrality values without recomputation.
* Used in: Real-time fraud detection, network anomaly detection, dynamic recommendation systems.

**3. Efficient Data Structures for Fast Updates**

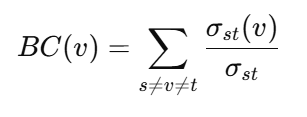
* Dynamic adjacency lists and hash-based graph representations speed up updates.
* Example: Twitter’s real-time trending topic detection relies on incremental centrality updates.

Graph-based computations play a crucial role in analyzing large-scale social networks, biological systems, web graphs, and transportation networks. Centrality measures such as Eigenvector Centrality, Katz Centrality, Harmonic Centrality, and Subgraph Centrality provide insights into the importance of nodes in a network. However, as networks grow to billions of nodes and edges, computing these measures using traditional algorithms becomes infeasible due to high time and space complexity.

Centrality measures play a vital role in network analysis by determining the importance of nodes within a network. **Betweenness Centrality (BC)** is a widely used centrality measure that quantifies how often a node appears on the shortest paths between pairs of other nodes in the network. **NPFIC\_Rank**, which could be an extension of centrality-based ranking, represents another measure that evaluates the influence or importance of a node within the network structure.

Betweenness centrality is an important metric in network science. It measures the **extent to which a node lies on the shortest paths between pairs of other nodes** in a graph. Nodes with high betweenness centrality often serve as **bridges** or **intermediaries**, connecting different parts of the network.

Mathematically, betweenness centrality for a node **v** is defined as:



While the exact nature of **NPFIC\_Rank** is not explicitly defined in the given plot, it appears to be another **network-based ranking measure**. It could be derived from **Fuzzy Influence Centrality**, **Iterative Influence Propagation**, or some other influence-based ranking mechanism.

A high **NPFIC\_Rank** indicates that a node plays a significant role in network connectivity, potentially due to **high-degree influence, clustering effects, or strategic positioning** in the network.

The strong positive correlation between **BC\_Rank and NPFIC\_Rank** suggests that nodes with higher **betweenness centrality** also tend to have higher influence rankings.

This relationship makes sense because:

1. **High betweenness nodes control information flow** – Since **betweenness centrality** identifies key nodes that serve as bridges between different network clusters, such nodes naturally gain influence.
2. **Structural Importance Contributes to Influence** – Nodes that lie on multiple shortest paths are more likely to be critical for network connectivity, making them **more influential** in terms of **NPFIC rank**.

# Conclusion

This study advances Pythagorean neutrosophic fuzzy graphs in social networks by refining degree centrality, which often overestimates influence for inactive nodes with many connections. We introduce a modified measure, Node Pack Fuzzy Information Centrality (NPFIC), incorporating node (researcher) weight and research collaboration to better identify influential researchers. It integrates each node's self-weight reflecting individual merit, influence or importance yielding a more precise and context-sensitive measure of centrality and also demonstrated its effectiveness in identifying influential nodes and improving traditional centrality metrics by mitigating issues like the overestimation of inactive, highly connected nodes. By leveraging Pythagorean Neutrosophic values, the proposed approach captures the complexity of relationships (in-strength and out-strength impact of a researcher) more accurately than traditional fuzzy methods, considering uncertainties by Havrda- Charvát entropy and self-weight by calculating centrality. It is clearly expressed that the proposed centrality measure identifying influential nodes in more nuanced way by incorporating Pythagorean neutrosophic fuzzy values in large-scale social networks like academic collaboration networks. The network assessment is now more precise and contextual so we able to assess centrality with greater precision. This framework also sets the foundation for further exploration of Pythagorean Neutrosophic Fuzzy Graph operations and bipolar Pythagorean Neutrosophic Fuzzy Graphs in real-world applications.

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**Conflicts of Interest:** “The authors declare no conflict of interest.”

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