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Stochastic Optimization of Asset Management Project Portfolios: A Risk-Informed Approach

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Abstract

Asset management within the nuclear industry has become an increasingly relevant topic as safety requirements have tightened and energy security has become more important. Asset management ensures performance and reliability in a nuclear facility by balancing costs, opportunities, and risks to get the most out of assets. Asset management processes can often be modeled as capital budgeting problems, where investments are evaluated based on costs and benefits, which establishes a link to mathematical optimization. This study addresses asset management at the Swedish nuclear power plant, Forsmark, and aims to implement an optimization model to improve the project selection related to maintenance and replacement of assets at the plant. First, the most relevant areas of nuclear asset management are identified through a comprehensive literature review. The most relevant method, identified as a mix between risk-informed asset management and capital budgeting, is then implemented to fit the prerequisites at Forsmark. Several models of different complexity are developed and the resulting stochastic model incorporates uncertainty of input variables by assuming underlying distributions. Finally, a methodology to incorporate a quantitative risk measure in the optimization is suggested through the use of conditional value at risk. Results are generated based on simulated data and illustrate the potential of implementing the method at Forsmark.

Keywords Nuclear asset management, Risk-informed asset management, Portfolio optimization, Project selection, Knapsack problem, Monte Carlo simulation, Conditional Value at Risk

Sammanfattning

Svensk titel: *Stokastisk optimering av projektportföljer för tillgångsförvaltning: en riskinformerad metod*

Tillgångsförvaltning inom kärnkraftsindustrin har blivit alltmer aktuellt i takt med att säkerhetskraven har skärpts och tillförlitlighet i energiproduktionen blivit viktigare. Effektiv tillgångshantering säkerställer prestanda och reliabilitet i ett kärnkraftverk genom att hitta en balans mellan kostnader, möjligheter och risker för att maximera nyttan av tillgångar. Projekturval i tillgångsförvaltningen kan ofta modelleras som ett kapitalbudgeteringsproblem, där investeringar utvärderas utifrån kostnader och uppsida, vilket påvisar en koppling till matematisk optimering. Denna studie behandlar tillgångshantering vid det svenska kärnkraftverket Forsmark och syftar till att implementera en optimeringsmodell för att förbättra projekturvalet relaterat till underhåll av tillgångar vid anläggningen. Det första steget i studien bearbetar den befintliga litteraturen inom området för att få en uppfattning av relevanta metoder. Den mest relevanta metoden identifierades som en mix mellan riskinformerad tillgångsförvaltning och kapitalbudgetering. En metod baserad på de generella principerna för dessa områden utvecklades och anpassades för de specifika förutsättningarna på Forsmark. Flera modeller av olika komplexitet utvecklades och den slutgiltiga stokastiska modellen inkorporerar osäkerhet i de mest relevanta ingångsvariablerna genom att anta sannolikhetsfördelningar. Slutligen föreslås en metod för att implementera ett kvantitativt riskmått i optimeringen genom att använda CVaR. Resultaten genereras utifrån simulerade data och illustrerar potentialen i att implementera metoden på Forsmark.

Nyckelord Tillgångsförvaltning, Riskinformerad tillgångsförvaltning, Portföljoptimering, Projekturval, Kappsäcksproblem, Monte Carlo simulering, Conditional Value at Risk

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Abbreviations

FKA = Forsmark Kraftgrupp AB

NPP = Nuclear Power Plant

NAM = Nuclear Asset Management

RIAM = Risk-Informed Asset Management

RCM = Reliability Centered Management

RCAM = Reliability Centered Asset Management

NPV = Net Present Value

VaR = Value at Risk

CVaR = Conditional Value at Risk

MCMC = Markov Chain Monte Carlo

DM = Decision Maker

BIR = Benefit Investment Ratio

CDF = Cumulative Distribution Function

PDF = Probability Density Function

IP = Integer programming

LP = Linear programming

MIP = Mixed integer programming

EWD = Exponentiated Weibull distribution

KS = Kolmogorov-Smirnoff

AR = Avoided risk

TR = Taken risk

RR = Rest risk

SEK = Swedish Kronor

Contents

1 Introduction

Nuclear power has been a method of generating energy and part of the power mix since the 1950s. In its early days, commercial nuclear power generation was seen as an attractive option because of its reliability and low carbon emissions. However, accidents such as the partial meltdown at Three Mile Island in 1979 and the nuclear disaster at Chernobyl 1986, surged public opposition to nuclear plants and led to heavy regulations [61]. As a consequence, global installed capacity of nuclear energy slowed down from 1990 until today. In Sweden, there are today six nuclear reactors in operation compared to the historical maximum of twelve commercial reactors [63].

Nuclear power is a subject which have been frequently debated. In recent years however, the discourse have to some extent shifted following mitigated public opposition. In today's world where climate change, energy security, and sustainable development are major concerns, nuclear power has regained its consideration as a valuable source of clean energy. The high energy security of nuclear energy makes it a viable complement to intermittent renewable energy sources to meet the increasing global demand for electricity while limiting greenhouse gas emissions to mitigate climate change. Geopolitical tensions with the war in Ukraine has also brought attention to the vulnerability of the current energy systems and regional energy flow dependence.

Consequently, countries are revising their national energy policies, to investigate opportunities of extending the operational lifetime of existing reactors as well as construction of new advanced reactors. Another driver is the technological advancement enabling time and cost efficient deployment of small modular reactors to balance regional energy systems [60]. This trend is also highly relevant in Sweden, where the government in January 2023 announced it will investigate opportunities of amending current laws to enable construction of nuclear reactors at new locations, while also overseeing the maximum limit of ten active reactors [65].

As such, nuclear energy generation is expected to increase in the coming years [60]. Hence, the asset management of existing and new power plants will play a significant role in the future. The asset management of a nuclear power plant (NPP) is important to ensure safe, reliable and efficient operations as well as to maintain the trust and confidence of the public. Through well-functioning asset management, the risk of accidents can be minimized while also optimizing plant performance and asset lifetimes to create financial incentives. The establishment of efficient processes for asset management is also important due to the industry experiencing a generational change in human resources following consistent downsizing of the industry during the 21st century, where much of the knowledge could be lost as industry professionals retire.

Commercial nuclear power facilities are subject to a set of constraints which constitute the prerequisites for maintaining an operating licence. These include regulatory frameworks determined by governing forces and economic conditions relating to a large and complex group of shareholders including government, corporations and employees. Nuclear asset management (NAM) is a fundamental process in the management of facilities, enabling effective capacity generation and economic performance [19]. The International Atomic Energy Agency (IAEA) describes nuclear asset management similar to non-nuclear and refers to the Institute of Asset Management's definition of the process as a balancing act of costs, opportunities and risk, to achieve the required performance of assets that meet company objectives. Furthermore, NAM is identified as a crucial area for achieving efficiency and sustainability in nuclear power production [55].

Most of the existing processes in place for NAM today are to some extent based on the industry guidance document, AP-940, where the Nuclear Energy Institute compiled input from industry experts to outline a process map for the critical functions of NAM [24]. The functions include important aspects such as strategic planing for generation and long-term purposes, project evaluation and ranking, budgeting and plant/fleet valuation, as well as additional sub-functions for each. These are considered important to value and evaluate assets in NPPs to support business decisions. The AP-940 NAM model is provided on an overview level and can be combined with models focused on implementation, such as the risk-informed asset management (RIAM). In contrast, it describes a more structured approach to analyze investment alternatives for project portfolios, and a RIAM evaluation comprises of a safety model such as a probabilistic risk assessment, a reliability model such as a generation risk assessment and a cost model [19].

1.1 Forsmark Nuclear Power Plant

Forsmark's nuclear power plant is the largest producer of electricity in Sweden. The plant consists of three reactors with a combined output of 3 283 MW and an annual capacity of approximately 25 TWh. The plant is owned and operated by Forsmark Kraftgrupp AB (FKA) which in turn is owned by Vattenfall, Mellansvensk Kraftgrupp AB and E:ON Kärnkraft. Forsmark has an annual turnover of SEK 5.7 billion and employ a total of 1 200 people. As a result of the recent political landscape, SEK 13 billion is being invested to meet new requirements of safety and reliability, to prolong the operating life of the plant and to increase the output of electricity by 410 MW [64].

1.2 Forsmark Asset Management Initiative

This work is part of a larger initiative at FKA to improve the process of asset management throughout the plant. The process can be decomposed into risk and opportunity management, solutions planning, optimization and project management. The first part defines the appropriate measures to track in order to reflect potential risks. These are to be analyzed and the most influential ones selected to constitute input data for the formation of intervention projects. The solutions planning phase define interactions to mitigate risks which need to be prioritized based on criteria related to cost, risk and performance. This project is part of the optimization process that enables effective selection of relevant asset management projects at Forsmark based on a trade off between the aforementioned criteria.

The aim of the project is to initiate the work with mathematically based optimization in the asset management process at Forsmark and suggest a novel approach to be implemented. A central part of the project is thus focused on mapping out the asset management domain, that is the different areas and methods that could be relevant for the situation at Forsmark.

1.3 Purpose

The purpose of this study can be divided into two parts, first to explore and analyze the most relevant methods described in previous research, and second to develop a novel optimization approach compliant with the prerequisites at Forsmark that incorporates cost, risk and performance related to the asset management process. The proposed model is intended to answer questions regarding the feasibility of mathematical theory for asset management purposes at Forsmark. The purpose can be formulated as the process of answering the following research questions

- What are the most prominent methods available today within nuclear asset management?
- To what extent can mathematical optimization be implemented to improve the asset management process at Forsmark and what are the possible effects of a new method?

As suggested by the research questions and the larger perspective of the study described in Section 1.2, the focus of this study is to develop a methodology that can be implemented at Forsmark rather than obtaining realistic results in terms of data.

2 Theory

In this section, mathematical theory relating to key aspects covered in this study is discussed. Initially, optimization theory and different types of programming are discussed as optimization constitutes the basis of this study. Further, measures of risk are described as well as some relevant applications of reliability theory. The chapter concludes with the introduction of Bayesian inference and some of its common statistical methods, followed by the concept of three-point estimation.

2.1 Optimization Theory

Optimization theory involves defining an objective function which is to be maximized or minimized under a set of constraints that limit the possible solutions. This study will consider an optimization problem seeking to maximize the value of the objective function. Consequently, the following optimization problems will maximize the objective function. Several different problem types exist depending on the nature of the objective and constraint functions, as well as allowed variable values. The following sections describe some variants of these optimization problems. Furthermore, some algorithms to solve these problems are mentioned.

2.1.1 Linear Programming

Linear programming (LP), or linear optimization is a technique for optimizing a linear objective function with linear constraints. Hence, the objective is to attain the optimal solution given a model with linear relationships. An LP problem can be expressed in canonical form as

$$\begin{aligned} \max_{x_j \in R^n} \quad & \sum_{j=1}^{n} c_j x_j \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad i = 1, 2, \ldots, m \end{aligned} \tag{1}$$

where n represents the number of decision variables considered in the problem and m is the number of constraints. The decision variables are represented by x_j, which are to be determined in order to optimize the problem. Further c_j are the coefficients of the objective function while a_{ij} represent the coefficients of the constraint equations which are limited by the right-hand side values b_i. The constraints limit the values that the decision variables can be assigned to ensure that the attained solution satisfies the requirements formulated in the problem, meaning that the solution is feasible.

There are several different algorithms to solve LP problems. Some examples are the simplex method, the primal-dual method, and interior-point methods [37]. Generally these are algorithms that move from one feasible solution to the next until an optimum is found, and vary in efficiency depending on the characteristics of the LP problem.

2.1.2 Binary Integer Programming

In the previous section describing the LP problem, variables are assumed to be continuous. However, in many real-world problems in settings such as scheduling, inventory and manufacturing, the use of continuous variables is often unrealistic as the variables are discrete. Integer programming (IP) is a type of discrete optimization problem. An IP problem is a optimization program where the decision variables are constrained to only take integer values. A common version of the IP problem is the integer linear programming problem, which is an LP problem with variables restricted to integers. The objective function and constraint equations must however still be linear. In essence, the integer linear programming problem is the same as described in (1), with the exception of constraining the decision variables as $x_j \in Z$, where Z is the set of integers.

A further variant of IP is binary IP. As the name suggests, a binary IP problem is a problem where the integer variables are further restricted to only take values 0 or 1, described as

$$x_j \in \{0,1\} \quad j = 1,2,\dots,n \tag{2}$$

Consequently, rewriting (1) as a binary integer linear programming problem with variable constraint (2) yields the following optimization problem

$$\begin{aligned} \max_{x_j \in R^n} \quad & \sum_{j=1}^{n} c_j x_j \\ \text{s. t.} \quad & \sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad i = 1,2,\dots,m \\ & x_j \in \{0,1\} \quad j = 1,2,\dots,n \end{aligned} \tag{3}$$

Binary IPs are very useful when the integer variables are to reflect decisions, e.g. whether to make an investment or not where in general 0 indicates no and 1 yes. As this is the case in this study, a binary integer linear programming approach will be applied to solve the problem. Generally problems of this form can be fairly difficult to solve, and auxiliary search techniques apart from the simplex method are required [25]. Some of these techniques are discussed in the following section.

2.1.3 Optimization Algorithms

Solving an optimization problem involves finding the optimal solution among a set of feasible solutions, which translates to finding the values of the decision variables that maximize the stated objective function while staying within the confounds of the model constraints. There is a variety of algorithms to consider for solving a given problem and the choice depends on factors such as the complexity, size and characteristics of the problem. In this section a overview is provided of the most common algorithms for solving a mixed integer programming (MIP) problem using the IBM ILOG CPLEX Optimizer software. A MIP problem is type of programming problem where only some variables are constrained to integer values and others limited to non-integers, e.g. continuous variables. CPLEX is an optimizer software for MIP problems provided by IBM.

Branch and Bound

Branch and bound is a frequently used enumerative relaxation algorithm for solving optimization problems including discrete variables such as binary or integer variables. The algorithm is used in CPLEX as the default to solve MIP problems. The general idea of branch and bound involves dividing a large set of feasible solutions, S into smaller subsets or branches $\{S^i, i = 1, ..., k\}$, as it might be to difficult or computationally expensive to optimize over S. This creates smaller sub-problems which are handled and solved by the algorithm with the help of relaxation. Feasible solutions remain feasible in the relaxation as well, and an upper bound of the original problem can be represented by the optimal solution of the relaxed problem. The results are then combined to delimit the problem and bound the search space. The algorithm runs this procedure until the optimal solution is found or is terminated based on some criteria, usually in terms of a gap between the incumbent solution and new optimal solution following relaxation, or allowed number of processed nodes.

The division into branches can be illustrated as a tree structure of nodes, where each is created recursively. Not every node is analyzed, and certain criteria help determine which nodes to prune and which to examine in detail. Pruning criteria can be divided into feasibility, optimality and value dominance and they determine whether specific parts of the search space are eliminated from consideration. The feasibility condition says branches that can not lead to a feasible solution will be pruned, i.e. if the solution violates the model constraints. Pruning due to an optimality criteria is done if a branch can not lead to an optimal solution, which is determined by comparing the lower bound of the current solution with the upper bound of the solved relaxation of the problem. Value dominance refers to pruning of a branch with the same feasible region as another branch but with a worse objective value [2].

If a node is in no need of further branching due to optimality or feasibility conditions being met, it can be considered fathomed. The algorithm can be terminated when all nodes are fathomed or when an optimal solution is found. The computational speed of the algorithm can be enhanced by using heuristic methods such as the best bound rule where the nodes are ranked and solved in order of the lower bound of the objective value. A general branch and bound algorithm can be outlined in the following steps

- Initialization - Initiate the algorithm by setting the current best solution as the initial solution and determine upper and lower bounds.
- Branching - The feasible region is divided into sub-problems and the corresponding relaxations are defined.
- Bounding - The upper and lower bounds are calculated for each node. These help determine the extent and order of further exploration of the nodes.
- Pruning - Nodes are pruned according to the conditions described above. The algorithm then back-tracks to previous nodes to explore other solutions.
- Termination - The algorithm terminates when an optimal solution that satisfies the optimization criteria have been found or when all nodes have been explored.

Cutting plane algorithm
When IP and MIP problems are under consideration, another common method for solving such problems is the cutting plane algorithm. The general concept of cutting plane is to solve the non-integer linear relaxations of an IP problem. The solution search space can then be iteratively refined by introducing linear inequality cuts. This is based on the fact that for linear models, corner points or extreme points in the feasible region are possible optimal solutions as well as possible integers. For such optimal points that are not integer, there exists a linear inequality or a cut that separates the true feasible region from the non-integer optimum. This process is repeated until the optimal point found is an integer solution. The algorithm can be summarized in the following steps [46].

- Formulate a relaxation of the original IP problem and solve to obtain an initial solution.
- If the obtained optimal solution consists only of integer valued variables the algorithm is stopped as it is also the solution to the integer problem.
- If not, find a constraint in the form of a linear inequality that all feasible solutions satisfy, i.e. a cut.
- Add the constraint to the existing model and repeat the process from the second step.

2.1.4 Stochastic Programming

To go beyond deterministic optimization where parameters are assumed to be known, uncertainty needs to be incorporated to represent variations found in real world problems. A framework for optimization modelling is provided in the stochastic optimization literature as stochastic programming. Instead of known parameters stochastic programming relies on that the underlying probability distributions of parameters are known or can be estimated. The aim is to find feasibility in the solution for the majority of the possible parameter outcomes by optimizing some function of decision variables and a random variables. In general mathematical terms, stochastic programming is constituted by a set of functions, $f_i(x, w)$, that formulate the objective function and the constraints. Here, x is the decision variable and w is the random outcome of the variable which models variation and uncertainty for the random parameters. This can be formulated as a basic stochastic program according to

$$\max F_0(x) = \mathbb{E}[f_0(x, w)] \tag{4a}$$
$$\text{s.t. } F_i(x) = \mathbb{E}[f_i(x, w)] \leq 0, i = 1, ..., m \tag{4b}$$

The stochastic programming problem is convex if $f_i(x, w), i = 0, 1, \ldots, m$ for each w are convex in x, since this entails that $F_i(x)$ is also convex. Only in certain cases can analytical solutions be found to such problems. In other cases the solution needs to be approximated using Monte Carlo sampling [58].

2.1.5 Knapsack Problem

The knapsack problem is a combinatorial optimization problem based on the simple principle of filling a knapsack of fixed size with the best combination of items. The problem is well known in resource allocation applications where a set of projects are to be chosen with consideration of a fixed budget. There are several different formulations of the knapsack problem where the prerequisites and objectives differ but the most simple formulation, denoted as the 0-1 knapsack, has the following formulation

$$\max \sum_{j=1}^{n} p_j x_j \tag{5a}$$
$$\text{s.t. } \sum_{j=1}^{n} w_j x_j \leq c \tag{5b}$$
$$x_j \in \{0, 1\}, j = 1, ..., n \tag{5c}$$

where x_j is the decision variable taking values 0 and 1, p_j is the profit of item j, w_j is the weight of item j and c is the capacity [59]. Examples of further extensions involve having unbounded item availability and introducing multiple choice.

2.2 Monte Carlo

Monte Carlo simulation is a sampling method for which an underlying distribution of outcomes, for a decision or variable, is used to repeatedly generate values and recalculate the value of interest as a function of sampled variables. Each sample consists of different randomly generated values for the input variables used to determine an outcome. Input variables can be seen as a representation of the available options that affect an outcome. Monte Carlo simulation is generally applied in four steps, namely developing the model, identifying uncertainty, simulating the model and making decisions. The first step models the situation at hand by formulating the relation between inputs and output. Next, the uncertain variables that are crucial to the model need to be identified. The third step refers to the actual sampling of values that simulate the model for a given number of random combinations, where the simulations often exceed 1 000. This results in statistics in terms of outcome probabilities that help make informed decisions [5].

2.3 Measures of Risk

There are several potential measures of risk associated with optimization of asset management. Risk is defined and measured extensively at NPPs, but there are also quantitative measures related to the optimization process. These often originate from the financial sector but have useful applications in other fields as well.

2.3.1 Value at Risk

Value at Risk (VaR) is a statistic to measure the extent of potential financial losses within a given time frame and a common measure of risk within the financial industry. As a risk measure VaR provides a quantification of random loss, L, by calculating the percentiles of the loss distribution. Given a confidence level, $\alpha \in (0, 1)$

the VaR of L can be seen as the α-quantile of L. Thus, α-VaR can be considered as an upper bound for the largest losses and it is defined such that the probability of a loss larger than VaR is at most $(1-\alpha)$ over the specified time horizon. The formal definition of VaR can be expressed as

$$VaR_\alpha(L) = \min_x \mathbb{P}(L \leq x) \geq \alpha \tag{6}$$

Although easy to understand and interpret, VaR has been criticised in its capacity as a risk measure. In [8], the authors define four axioms as criteria for a risk measure to be considered coherent, and show that VaR in some cases fail to satisfy the axiom related to subadditivity. That is, the principle stated as "a merger does not create more risk", expressed mathematically as $\mathbb{P}(X_1 + X_2) \leq \mathbb{P}(X_1) + \mathbb{P}(X_2)$. Meanwhile, in [35] it is showed that VaR complies with an updated axiom of subadditivity although the hard-to-categorize robustness to the loss distribution's tail behaviour makes it hard to use in practice.

2.3.2 Conditional Value at Risk

Another common measure of downside risk can be found in Conditional Value at Risk (CVaR), also referred to as expected shortfall. CVaR measures the average loss in the tail beyond the VaR threshold, at a given probability or confidence level. The measure can be interpreted as the expected value of the worst cases, thus capturing the variability of possible outcomes. The most trivial formulation of CVaR is as follows

$$CVaR_\alpha(L) = \mathbb{E}[L | L \geq VaR_\alpha(L)] \tag{7}$$

While in [10], the CVaR definition is extended to following formal definition

$$CVaR_\alpha(L) = \frac{1}{(1-\alpha)} \int_{f(x,y)\geq VaR_\alpha} f(x,y)p(y)dy \tag{8}$$

where $f(x, y)$ is the loss expressed as a random variable given decision vector x and uncertainties described by the random vector $y \in \mathbb{R}^m$. Furthermore, $p(y)$ denotes the probability density of the underlying probability distribution of Y in $\mathbb{R}^m$. The authors formulate CVaR in the optimization context in relation to the auxiliary function $F_\alpha(x, y)$

$$F_\alpha(x,\gamma) = \gamma + \frac{1}{(1-\alpha)} \int_{f(x,y)\geq\gamma} [f(x,y)-\gamma]^+ p(y)dy \tag{9}$$

where γ is equivalent to VaR_α. In the study it is proved that $F_\alpha(x, y)$ is convex with respect to γ and that it equals CVaR at its minimum. Thus, CVaR can be obtained as

$$CVaR_\alpha = \min_\gamma F_\alpha(x,\gamma) = \min_\gamma \gamma + \frac{1}{(1-\alpha)} \int_{f(x,y)\geq\gamma} [f(x,y)-\gamma]^+ p(y)dy \tag{10}$$

CVaR is considered to have better properties than VaR, and it is seen as a coherent risk measure as defined in [8]. Properties of CVaR include that it is positively homogeneous, convex, translation-equivariant and monotonic with respect to stochastic dominance of order 1 and 2 [9]. Further, the CVaR risk measure has an advantage in its implementation as it not only produces a convex feasible region, which allows for global optimal solutions, but it can also be represented using linear relations. As discussed in Section 2.1.1, LP is well-understood in theory and efficient solving techniques are available. The LP representation is applicable when there are discrete underlying distributions, or when sampling techniques are employed. Consequently, CVaR is a suitable and highly useful choice of risk measure for stochastic optimization [32].

2.4 Reliability Theory

In broad terms, reliability theory studies an objects ability to function without failure. A general explanation of reliability is that it refers to the probability that a system or component perform its intended function under specified conditions during a given period of time. When a component or device stops fulfilling its function, it is considered to have failed [44]. Of course, this can entail a wide variety of more in-depth descriptions of what defines a failure. In this study, failure will be assumed to be an incident in which an asset in the NPP causes the plant not to be able to produce as normal.

The reliability of any system is consequently of critical importance to maintain satisfactory performance. An important aspect of the field of study is understanding the pattern for when failure occurs under different conditions and how mechanisms can affect this. To address the reliability of an asset, this uncertainty regarding failure patterns requires a probabilistic approach. This approach starts with considering the lifetime of an asset as the time interval of the asset operating without failure. In the coming sections, the introduction of asset lifetime as a random variable will be presented as well as mathematical theory commonly applied in reliability theory, such as the use of probability distributions to model the risk of failure, and a goodness of fit test.

2.4.1 Asset Lifetime

Using the concepts of reliability theory as described in [30], let $L \geq 0$ be a continuous random variable describing the lifetime of an asset, with cumulative distribution function (CDF) $F_L(t)$ and probability density function (PDF) $f_L(t)$. The CDF describes the probability that the lifetime is less than or equal to t, meaning that a failure occurs in the interval $[0, t]$. Consequently, the survival function, also called the reliability function, can be defined as

$$R_L(t) = P(L > t) = 1 - F_L(t) \tag{11}$$

As such, the survival function gives the probability that the asset survives past a specific time t. Further, let us consider the probability that failure occurs between t and $t + h$, given that failure has not occurred before t, derived as

$$P(t < L \leq t + h | L > t) = \frac{P(L < t \leq t + h)}{P(L > t)} = \frac{F_L(t + h) - F_L(t)}{R_L(t)} \approx \frac{f_L(t)h}{R_L(t)} \tag{12}$$

Therefore, one can define the failure rate $\lambda_L(t)$ as the proportionality factor described above

$$\lambda_L(t) = \frac{f_L(t)}{R_L(t)} \tag{13}$$

The rate can be interpreted as probability of failure per time unit at t given failure has not occurred before t. So, assuming the approximate relationship in (12), the probability of failure between t and $t + h$ can be estimated as

$$P(t < L \leq t + h | L > t) \approx \lambda_L(t)h \tag{14}$$

Finally, if a discrete setting is considered with time intervals of $h = 1$, the probability of failure between t and $t + 1$ can be estimated with failure rate function according to

$$P(t < L \leq t + 1 | L > t) \approx \lambda_L(t) \tag{15}$$

2.4.2 Exponentiated Weibull Distribution

The exponentiated Weibull distribution (EDW) is a family of probability distributions which extend the original Weibull family by adding another shape parameter. It was initially introduced in [3], and has proven to be very useful in modelling of failure rates as it accommodates a variety of different curves depending on its parameters. Assuming a scale parameter λ with first shape parameter k and second shape parameter α, all positive, the PDF of the distribution is given by

$$f(t) = \alpha\frac{k}{\lambda}[1 - \exp\left(-(t/\lambda)^k\right)]^{\alpha-1} \exp\left(-(t/\lambda)^k\right) \left(\frac{t}{\lambda}\right)^{k-1}, \quad t > 0 \tag{16}$$

Consequently, the CDF of the EWD is given by

$$F(t) = [1 - \exp\left(-(t/\lambda)^k\right)]^{\alpha}, \quad t > 0 \tag{17}$$

As one may identify, if $\alpha = 1$, the original Weibull distribution is obtained, while $k = 1$ corresponds to a exponentiated exponential distribution. By applying formula (11), the survival function of the distribution is given by

$$R(t) = 1 - [1 - \exp\left(-(t/\lambda)^k\right)]^{\alpha}, \quad t > 0 \tag{18}$$

Similarly, the failure rate function for the EWD is derived from (13) as

$$\lambda(t) = \alpha\frac{k}{\lambda}[1 - \exp\left(-(t/\lambda)^k\right)]^{\alpha-1} \exp\left(-(t/\lambda)^k\right) \left(\frac{t}{\lambda}\right)^{k-1} \left[1 - [1 - \exp\left(-(t/\lambda)^k\right)]^{\alpha}\right]^{-1}, \quad t > 0 \tag{19}$$

As previously mentioned, a useful property of the exponentiated Weibull family is that it offers a wide flexibility in modelling different types of failure functions, which can be summarized as [4]:

1. If $k \geq 1$ and $k\alpha \geq 1$, $\lambda(t)$ is monotone increasing.
2. If $k \leq 1$ and $k\alpha \leq 1$, $\lambda(t)$ is monotone decreasing.
3. If $k > 1$ and $k\alpha < 1$, $\lambda(t)$ is bathtub shaped.
4. If $k < 1$ and $k\alpha > 1$, $\lambda(t)$ is unimodal.

The monotonic functions are strict in all cases except for $\alpha = k = 1$, which corresponds to the negative exponential distribution.

As such, the exponentiated Weibull family is very useful as it allows the modelling of a variety of failure rate functions by using one single assumed distribution family. This flexibility is also beneficial when the nature of the hazard rate is unknown, since the fitted parameter values could indicate the nature of the hazard rate according to above criteria.

2.4.3 Kolmogorov-Smirnov Test

The Kolmogornov-Smirnov (KS) test is a statistical nonparametric test to evaluate the goodness of fit of a probability distribution. The one-sample KS-test is used to compare a sample of data with a reference distribution. The null hypothesis states that the random samples conform to a specific continuous distribution. In essence, the test is based on quantifying the maximum limit-superior distance between the empirical distribution function of the sample and CDF of the given distribution, which corresponds to the test statistic D_n.

Assuming a sample of random independent and identically distributed observations $X_1, X_2, \ldots, X_n$, the empirical distribution is defined as

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I\{X_i \leq x\} \tag{20}$$

where I denotes the indicator function which is equal to one if the statement is true and equal to zero otherwise. The KS-statistic for a theoretical CDF $F(x)$ is calculated as

$$D_n(x) = \sup_x |F_n(x) - F(x)| \tag{21}$$

where *sup* represents the supremum of the distances within the set. Consequently, a larger D_n corresponds to a greater discrepancy between the theoretical distribution and samples. As such, the objective is to identify the distribution which minimizes the test statistic, thus warranting a goodness goodness of fit of the proposed distribution and its parameters [53].

2.5 Bayesian Inference

Bayesian inference is a statistical method for updating probabilities based on new information becoming available. The method allows for incorporating prior beliefs and uncertainties regarding the parameters of a model, making it a useful tool for prediction and decision-making. As such, the Bayesian approach is very different from the frequentist inference which is based on the idea that probability represents the long-run frequency of certain events, and is purely driven by the data. A key difference between the approaches is the view on uncertainty, with Bayesian inference considering uncertainty in terms of posterior distribution while incorporating prior information.

Consequently, given a data set X, we are looking to infer the parameters θ of some function that is likely to have generated the observed data. Initially, a prior belief is suggested regarding the prior distribution of the parameters, denoted $p(\theta)$, representing the belief of what parameters likely generated the data prior to actually observing any data. As new data is observed, the believed distribution is updated to more appropriately match likely parameter values. To this end, the likelihood distribution is defined as $p(X|\theta)$, modelling the likelihood of data X belonging to some model. Next, with observed data points and a formulated prior distribution and likelihood distribution, Bayes' theorem is applied to find the joint posterior distribution as

$$p(\theta|X) = \frac{p(X|\theta)p(\theta)}{p(X)} \tag{22}$$

When evaluating the posterior distribution, the normalizer, $p(X)$, is essential. Also known as the model evidence, it is the scalar denominator in equation (22) integrated as

$$p(X) = \int p(X|\theta)p(\theta)d\theta \tag{23}$$

A common reference to the above integration is marginalizing of the likelihood over parameters θ, which explains the alternative name marginal likelihood. Removing the normalizer from equation (22) yields

$$p(\theta|X) \propto p(X,\theta)p(\theta) \tag{24}$$

Meaning that the non-normalized posterior distribution is proportional to the likelihood multiplied by the prior distribution. For simple models where the likelihood and prior are conjugate distributions, marginalization can be solved analytically with basic calculus. However, as the models become more complex, the integration becomes infeasible from analytical terms and instead an approximation approach is necessary [34].

2.5.1 Markov Chain Monte Carlo

Markov Chain Monte Carlo (MCMC) methods are a group of approximation algorithms that are highly useful for estimating probability distributions that are difficult to sample from directly. Consequently, MCMC methods are useful techniques for modelling the posterior distribution in Bayesian inference described in equation (22), when the integration is very complicated as previously mentioned.

In essence, they are types of sampling algorithms that sample from a ergodic Markov chain, where the stationary distribution is the target distributions that are of interest. This means that with a sufficient number of simulations, the sampling will eventually start converging to the actual distribution. In general terms, the algorithms work by starting at some point in the distribution. Then a new point is sampled, where if its posterior probability is greater than the starting point, a jump is performed to the new point. If the probability is not higher, sampling is repeated until a jump is performed. This process is iterated to move closer to the true distribution, and where the density of the posterior distribution is high. Important concepts for the sampling algorithms are the sample size, deciding how many iterations are performed, and burn-in, which means the number of iterations that should be ignored when calculating the ergodic average. A certain number of the first iterations are discarded as these will be very spread out and can significantly influence the ergodic average [6]. As previously mentioned, MCMC methods encompass several different algorithms, with primary differences in how new locations in the distribution are generated and the criteria for deciding whether to jump or not.

2.5.2 Metropolis-Hastings Algorithm

The Metropolis-Hastings algorithm is one of the most general MCMC algorithms and also one of the most commonly used. The algorithm works as follows given initial parameter values θ_t, target density f and proposal density q:

1. Generate candidate θ^*_{t+1} from proposal density $q(\cdot|\theta_t)$.
2. Let
$$\theta_{t+1} = \begin{cases} \theta^*_{t+1} & \text{with probability } \rho(\theta_t, \theta^*_{t+1}) \\ \theta_t & \text{with probability } 1 - \rho(\theta_t, \theta^*_{t+1}) \end{cases}$$
   where
$$\rho(\theta, \theta^*) = min\Big\{\frac{f(\theta^*)q(\theta|\theta^*)}{f(\theta)q(\theta^*|\theta)}, 1\Big\}$$

The algorithm is iterated until convergence toward the target distribution is achieved with adequate accuracy. The probability $\rho(\theta, \theta^*)$ is referred to as the Metropolis-Hastings acceptance probability [27]. The term $\frac{f(\theta^*)q(\theta|\theta^*)}{f(\theta)q(\theta^*|\theta)}$ in the acceptance probability measures the relative likelihood of transitioning from θ to θ^* versus transitioning from θ^* to θ. In essence, the acceptance probability measures the level of agreement between the target distribution f and the proposal distribution q. If candidate θ^*_{t+1} yields a higher density under the target distribution than the current value θ_t, and the proposal distribution can generate values close to θ^*_{t+1} with high probability, it means that the acceptance probability will be close to 1, suggesting that the candidate should be accepted. Similarly, if the candidate has a lower density under f compared to the current value or q is improbable of generating values in the neighborhood of θ^*_{t+1}, then $\rho(\theta, \theta^*)$ will be less than 1 and the probability of rejecting the candidate increases.

Overall, the Metropolis-Hastings algorithm balances the exploration of the proposal distribution and approaching the target distribution. The acceptance probability $\rho(\theta, \theta^*)$ plays a key role in determining which proposed values are accepted and which are rejected.

2.6 Three-Point Estimation

Three-point estimation is an estimation technique commonly used in project management. The method is used by professionals to construct approximate probability distributions to represent the outcome of uncertain events based on limited information in terms of prior experience or best guesses. As such, it is a method to incorporate uncertainty in future events within management applications. The technique entails formulating three numerical estimates as the name suggests according to a *Best case estimate*, a *Worst-case estimate*, and a *Most likely estimate*. These three points are then used to estimate the expected value and standard deviation of some assumed underlying distribution. Commonly used distributions in the three-point estimation are the normal distribution, triangular distribution, and PERT distribution. Thus, the expected value and standard deviation depend on the choice of distribution. For a PERT distribution, the values are calculated as [66]

$$E = \frac{Minimum + 4 \times Likely + Maximum}{6}, \quad SD = \frac{Maximum - Minimum}{6} \tag{25}$$

3 Literature Study

In the following section, findings from previous academic works will be presented and discussed. The literature study is divided into two main parts, the first one addressing asset management practices within nuclear energy generation and closely related fields within utilities and heavy industry. The second part considers areas within financial mathematical theory that have relevance in the physical asset management space.

3.1 Asset Management

Asset management within the nuclear industry has been studied to an increasing extent over the last decades as new regulations have emphasised the importance of safety and efficiency in production. However, as nuclear applications to some extent have been limited over the last decades, the asset management literature on other asset heavy industries will be studied as well to capture the full range of previous work. The field referred to as engineering asset management, encompasses previous research in the asset management domain with applications in industries where reliability and risk play a vital role, as within the nuclear industry. Such areas include other energy sources, infrastructure and heavy industry, which all rely on complex systems.

3.1.1 Reliability Centered Asset Management

One central aspect of asset management in the context of complex systems is focused on reliability. For asset heavy industries such as for NPPs, the role of reliability is centered around maintaining a low frequency of interruption in production. A well established methodology within this setting relates reliability to the optimization of maintenance policies to ensure reliability at minimum cost, namely Reliability Centered Maintenance (RCM). The method aims to preserve system functionality by examining different failure scenarios and determining the appropriate measure to manage failures through maintenance such as preventive maintenance interventions. Although praised for its simplicity, the RCM method lacks the analytical rigor to fully explain the potential benefits in terms of relating reliability efforts to the related costs [12].

The impact on reliability and costs of a system is linked to RCM principles in [18], with the proposal of a quantitative extension of the method as Reliability Centered Asset Management (RCAM). In the study RCAM is implemented in power distribution systems and the impact of different preventive maintenance strategies are evaluated on a cost basis relating to the total maintenance costs of ensuring component and system reliability. The applicability of RCAM in the nuclear industry is established in [29] where an opportunistic maintenance optimization approach is applied to replacement schedules of shaft seals in the nuclear feed-water pump systems at Forsmark. The total cost for all components of this particular system is incorporated in the model, including detailed costs of preventive maintenance, corrective maintenance and production loss, and it is minimized based on a discount factor. The authors further extend their study in [31], with the addition of more time steps and an extended model. The different policies suggested are then evaluated using a stochastic framework where the life limits have been sampled from a Weibull distribution. A reliability focused application of greater complexity is presented in [42] where the authors introduce a stochastic optimization model that take into account three hierarchical decision layers based on short-term, mid-term and real-time stages.

Although effective in linking reliability of assets to its monetary consequence, the RCAM approach requires extensive data on component level to make the relation to total system reliability. As described in [29] and [31], detailed data on component level, including failure probabilities, frequencies, detailed costs of different actions and knowledge of the relation between different assets in a system, is required to model the optimization problem accurately. While this might be available when studying smaller systems within a NPP, the information becomes more difficult to access when the analysis is extended to entire NPPs over long periods of time.

3.1.2 Risk-informed Asset Management

Another approach to manage assets in a manner that maximizes reliability and availability is RIAM. According to the Electric Power Research Institute, RIAM is a risk-based decision support model for evaluation of projects and assets at plant-level [13]. It is based on the general concept of using a rigorous risk-informed approach to analyze, assess, monitor and predict performance. Both economic and operational performance is considered while ensuring safety requirements are maintained. RIAM models incorporate probabilistic quantification of decision indicators to aid decision making related to plant improving investment options and to prioritize resources. Decision indicators include measures such as Net Present Value (NPV), projected earnings and costs, power production and efficiency, which are projected in the RIAM model using a risk-informed approach.

In [20] the institute proposes an implementation of RIAM in the NPP setting through presenting the requirements from a business, functional and non-functional perspective for an in-house developed software. The software is a risk-based evaluator of assets and projects primarily focused on nuclear power plants, with the objective of providing project prioritization and life-cycle management. A rigorous framework is presented with modular interactions between revenue and cost calculations to generate a profit module which allows management to prioritize between actions to maximize profitability under a set of constraints such as budget, labor capacity and scheduled outage time. Another RIAM approach is suggested in [54] where structure, system and component replacements are optimized under constraints of budgeting, reliability and data uncertainty by using a multidimensional multiple choice knapsack problem. The study is extended to involve a scenario-based approach to form a risk-hedge capital allocation portfolio. The resulting model is two-stage stochastic, focusing on a first stage project prioritization and a second stage scenario analysis of each portfolio. For a more detailed description of the model and its modules, please see the report [48] where it is explained in its entirety.

A two-stage stochastic model was also proposed in [32] to optimize the investment portfolio of real assets at a Brazilian gas utility company with constraints related to demand and supply as well as network capacity. The model considers demand as an uncertainty and the volatility is hedged by using a coherent risk measure in the form of Conditional Value at Risk, with a comparison of price decomposition and Benders' decomposition to handle coupling effects in the two-stage model to mitigate computing requirements. Similarly [57] propose a model to aid the process of defining and allocating an efficient budget for asset management in a heterogeneous multi-asset system. A two-stage stochastic optimization model with mean–variance constraints is implemented to minimize the number of scenarios with an insufficient budget, where uncertainty lies in the stochastic degradation of heterogeneous assets. In [26], the authors design a model for minimizing expected maintenance cost in a setting considering both preventive and corrective maintenance options in a system of items with a limited budget constraint for preventive maintenance. The nonlinear nature of the model is handled through reformulating the problem as an efficient binary integer program which is applied to optimize a maintenance policy in connection to a feed water system at a NPP.

A common theme in the above studies is that uncertainty lies in the risk of component or asset failure and costs associated with a potential failure. Thus, a crucial part of the optimization problem is deriving the probability of failure for items and the development of these rates over time. This can be done as in [43], where the authors consider two different types of failure rate functions in connection to a preventive optimization problem. By using real observed failure data from a US nuclear power plant, Bayesian modelling is used for the different failure functions. Markov Chain Monte Carlo algorithms are employed to parametrically model an increasing function with an extended gamma process, while an exponentiated Weibull distribution is adopted for a bathtub shape. For continuously increasing failure or degradation processes, gamma processes appear to be a recurring approach used by researchers as in [41] and [50] thanks to its non negative and independent properties. Assuming a Weibull distribution for asset lifetime is however not unusual either, as in for example [31].

3.1.3 Multi-Asset Systems

Asset management optimization in asset heavy industries often depend on complex systems consisting of several assets that work together to obtain functionality. The large amount of assets and the interaction between them have made optimization modeling a difficult issue for many applications. The theory and previous research of multi-asset systems in the asset management context have gained importance to extend the scope of study from a few assets to complete systems. In [49] asset dependencies are characterized as structural, stochastic and economic. Structural dependencies are asset interventions that require the intervention of other assets as well, while stochastic regard interactions in terms of failure between assets. Economic dependencies concern if there are positive or negative effects of joint maintenance efforts. A distinction between a set of homogeneous assets, labeled as a fleet, and heterogeneous assets constituting a portfolio, is also made for asset management purposes.

Previous work within the area include the aforementioned application of two-stage stochastic optimization in [57] to take on budget priori definition while considering heterogeneous stochastic asset degradation combined with a combinatorial optimization problem. A heuristic approach to optimization of a homogeneous vessel fleet within a chemical plant is presented in [45]. The authors incorporate risk in a total cost model as a business disruption cost to enable joint minimization of intervention cost and system-wide risk. The degree of disruption is measured by considering a k-out-of-N system where the number of non-operational assets affect the degree of system output. The k-out-of-N system approach, where system functionality is defined by the threshold of having k operational assets, is extended in [51] using an optimization model for long-term maintenance planning. Asset degradation is modeled in several states with different contributions to total system performance, to incorporate a dynamic risk of production loss.

3.1.4 Capital budgeting

Capital budgeting is the process of evaluating projects and investments. As such, it can be considered a central part of engineering asset management where most interventions are considered large projects with appurtenant investment costs. Capital budgeting often entails assessing the cash flows incurred by a project over its lifetime to analyze returns compared to other available options [62]. Previous studies often incorporate capital budgeting when other methodologies are considered as in [54] and [48], where the problem was structured as a capital budgeting one in a RIAM setting. A typical capital budgeting approach is modeled in [28], where the authors create a priority list by scoring projects using economic measures such as NPV. The common approach of utilizing the multi-knapsack approach, which is a combinatorial optimization formulation, is then applied when modeling the problem. Uncertainty is incorporated in the model by analyzing different scenarios with varying budgets and NPV cases to ensure the generated portfolio performs under varying conditions.

Other applications in literature focusing on the capital budgeting context also resort to some version of a knapsack formulation to solve the problem at hand. In [1] a capital budgeting scenario, where only one project out of a group can be chosen within the bounds of budgetary constraints, is considered and modeled as a 0-1 knapsack problem. The optimization objective maximizes the portfolio return of the chosen projects under the imposed constraints, which is the typical formulation in the context. A 0-1 knapsack problem is formulated in [11] as well, where information system project selection in health service institutions is investigated. By implementing the knapsack problem in the decision model several factors can be incorporated including decision maker (DM) preferences and priorities, project risk, costs, benefits and availability of resources.

3.2 Mathematical Relevance

The following section addresses relevant mathematical applications which originates from the field of financial mathematics. Apart from the portfolio optimization related to asset management, there are several other areas of financial methods that have been extended and applied in other non-financial fields.

3.2.1 Risk

Risk is a central aspect of engineering asset management and a common aim is to minimize risk related to system failure and production loss. To account for this type of risk, it needs to be considered in the project selection by being incorporated into the optimization model. A common approach is to calculate a monetary measure of risk and include it in the objective function as a cost. This is normally done by estimating an asset failure rate which is combined with a cost of failure or production loss as a result of asset failure. Such methodologies are utilised in several studies such as in the previously mentioned [54], [31], and [20]. Another approach takes from financial mathematics and implements a coherent risk measure in the optimization model. This is proposed by [57] and [32] which both use CVaR in combination with a Monte Carlo based stochastic model to capture the uncertainty of outcome in the worst cases, described by CVaR. The approach provides quantitative risk regulation and enables implementation of a risk appetite. CVaR can be included in the constraints or integrated into the objective function as a penalizing term.

3.2.2 Modern Portfolio Theory

Methods traditionally applied within the field of financial mathematics have also been adjusted to fit a broader range of applicability. Modern portfolio theory is implemented in several studies to aid the decision making process when considering project selection applications. Given the financial nature of the method, where the objective is to construct a portfolio of diversified assets such that the expected return is maximized for a given level of risk, alterations are necessary to fit the extended application. The approach has gained some traction within the energy industry where it can be used to identify an optimal power mix as well as in project decision making. Suggested adjustments to the model to fit the revised purpose include moving from a risk-return objective to a risk-cost frontier [21]. The feasibility of the methodology is tested within the oil and gas sector by [39] to construct an efficient portfolio of petroleum assets. Each asset is linked to an upstream oil and gas project, of which returns are measured as the expected monetary value of each project. Risk is maintained as the standard deviation of returns although alternative risk measures are suggested, including target level uncertainty and semi-standard deviation. In another study [36] follows a similar objective, to construct an optimal portfolio of nuclear power plants, taking into consideration the operations and management costs, construction costs and related uncertainties. Return distributions for each NPP is determined through simulations and depend on uncertainties relating to revenues and costs of the plants. The approach includes another significant method with mathematical relevance, namely real option analysis, which is utilized for plant level optimization.

3.2.3 Cost-Benefit Analysis

The process of formulating an optimization model is often based on the concept of analyzing costs and benefits. In contrast to objectives that are based on minimizing total costs, the cost-benefit analysis also includes upside such as revenue and profits. Commonly used measures include NPV and benefit investment ratio (BIR). A traditional profit - cost NPV is utilised in [28] where for each project, its costs are subtracted from its discounted revenues. In [52], an NPV measure weighing risk reduction benefits against the initial capital commitment and the running cost of operation, is used as the basis for project prioritization. Such an option is relevant to this study as the asset management at Forsmark works under the premise that there is no revenue derived from the actual projects since it can be difficult to connect maintenance measures to production output and since the produced electricity is sold to the market through the organisation's owner. Another NPV variant that does not rely on revenue is presented in [57] where the trade off between implementing a project today and waiting until a later date is transformed to a numeric value. This is done by monetizing the increasing risk of not implementing a project. BIR often occurs as a complement or alternative to NPV and is based on the same premise but formulated as a ratio instead.

3.2.4 Real Option Analysis

Another approach to applying theory from financial markets is the development of real option analysis in the project valuation space. In essence, real option analysis is a form of conceptual adaption of the financial options applied to investments in real assets. In both instances, the primary feature is the valuation of assets under uncertain conditions. However, financial options are explicit contracts with definitive conditions while real options instead need to be conceptualized depending on what is considered. A financial options is a contract where the owner has the right but not the obligation to buy or sell an underlying asset at a specified price in the future. Similarly, a real option is considered as the investment holder having the right but not obligation to undertake a business decision which takes advantage of an emerging opportunity, which can be seen as the source of cash flows generated by the decisions. As such, a real option could for example give the holder the choice to defer an investment into the future, or not to undertake the investment at all. Consequently, real option analysis incorporates the value of managerial flexibility in potential investment projects [7].

One approach to integrating uncertainty in real option valuation is the use of fuzzy numbers. In essence, a fuzzy number is the generalization of a real number by instead attributing it as a set of possible values, each weighted between 0 and 1 depending on how likely the outcome is. Consequently, the approach offers a practical way of dealing with uncertainties in accurate representation of numbers. A fuzzy approach to valuing real options have been proposed in [14]. The method uses trapezoidal fuzzy numbers to value real options in a similar manner to the Black-Scholes pricing formula for financial European options, which is the underlying inspiration of the approach. Practical adaptions of the fuzzy set approach are presented in [23] and [33], both valuating IT projects from a real option approach to prioritize selection. Another valuation technique is suggested by [22] as the Datar-Mathews method, where the uncertainty instead is reflected in min, max, likely scenarios of future profits. Through assuming a triangular distribution, a Monte Carlo simulation is performed to consider the variability of future cash flows. The method is primarily intended for R&D projects, but has also been applied to model the option value of very large real estate investments [15].

The application of real options analysis in flexible investment projects related to NPPs has been previously studied. In [17], two different investment projects are compared, the first being a flexible option of constructing multiple small module reactors and the other consisting of one large-scale plant. Through uncertainty in future electricity prices, the option value deriving from modular flexibility in the first project is evaluated. [16] analyzes the value of investing in different power generation projects from sources that do not emit carbon dioxide through a real options model. The study evaluates how future uncertainties in the price of carbon emission credits affect decision-making in the investment process of different power generation techniques.

Real option valuation in connection to optimal maintenance or replacement scheduling at NPPs is however limited, while studies have been conducted in other settings within the utility sector. Optimal scheduling for maintenance of wind turbines have been modelled by [47] with uncertainties in the production process as well as asset deterioration following postponed maintenance. The intermittent nature of wind production here poses as an evident source of uncertainty which is not present in the production of nuclear energy. Elsewhere, [56] presents a real option based framework for assessing the optimal time to implement seismic vulnerability reduction actions (maintenance) of infrastructure assets given uncertainties about the state of the asset, future hazards and total costs. Assets are described by five different states where a Markov process is used to model the condition of the assets. Stochastic simulation is a tool which is highly relevant within real option analysis. For example, [40] proposes a valuation method using dynamic programming and stochastic simulation in the form of Least-square Monte Carlo to value investments in power transmission projects with options of deferral.

4 Methodology

This section starts with presenting the setting of the considered optimization problem. Here the dimensions of interest are discussed and connected to the data available from FKA to perform an optimization. What follows is a description of the proposed approach to optimizing the investment portfolio of FKA. The approach starts by evaluating the projects to enable an initial selection of options for each project. Next, the different steps of the approach are explained in more detail and conclude with the introduction of an optimization model and build-outs of the initial model.

4.1 Problem Formulation

As for many other applications, one of the most important aspects of formulating a portfolio optimization problem in the asset management context is the underlying data. The measurements of metrics in NPPs that constitute the available data is highly dependent on the scope of current processes in place. There are also difficulties related to measurements of NPP components due to the high complexity of certain systems. As a result, NPP data tends to be qualitative rather than quantitative and restrictive rather than abundant. While figures such as costs of performing a project are rather straight forward, metrics related to component and system risk often require input from industry experts to make qualified estimations. To capture uncertainty in the outcome of included metrics each variable is provided with a minimum, maximum and most likely scenario. With these values, a three point estimation can be performed to derive an expected value and standard deviation for each variable as

$$E = \frac{Minimum + 3 \times Likely + Maximum}{5}, \quad SD = \frac{Maximum - Minimum}{5} \tag{26}$$

The above formulas can be seen as a modification of the distribution described in Section 2.6, and have been chosen in agreement with FKA as an appropriate three point estimation method. The expected value and standard deviation will further be used to assume underlying normal distributions for the parameters. These estimates of variable expected value and standard deviation are the basis of how uncertainty is handled throughout this study, with different approaches based on these values presented later in Section 4.4. The objective to incorporate cost, risk and performance in the optimization is reflected in the available data provided for each category. A vital aspect of the problem formulation relates to setting the structure of the optimization given the data prerequisites for each area, i.e. embedding measures of cost, risk and performance in the objective and constraints of the proposed model.

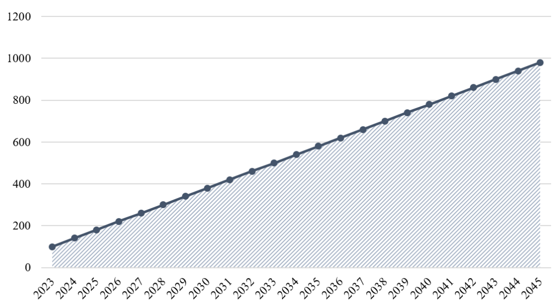
The information related to the situation at FKA is expected to include data on a total of 100+ different projects that are in the pipeline for the remaining life cycle of the plant. The projects are divided into different categories where one includes interventions required for the NPP to comply with industry regulations and ultimately maintain its operating licence. Another category refers to projects with passed approval that have been initiated or are waiting to be scheduled and performed. Finally, the regular projects with no previous decision, constitute the last category. When the choice of selecting the most crucial projects, the optimal implementation times for each project and the prioritization order between projects is considered, the problem can be seen as a capital budgeting problem. This structure to approach asset management optimization provides a good foundation for model development and it is compliant with previous studies observed in the literature [54], [48], [28], and [38].

4.1.1 Cost

The data related to the cost aspect of asset management interventions is rather straight forward. It is provided as values corresponding to the cost to perform the project and an associated project risk, pricing in uncertainties related to the time and cost of each project. The cost reflects the duration of the project by distributing the total project cost over the number of years it takes to perform. In an optimization context total costs are subject to minimization and they are often weighted against the available means in terms of capital allocated for asset management interventions, which is reflected in annual budget definitions. Data is provided in SEK and compounded to reflect the future value using an industry hurdle rate of 7%.

4.1.2 Risk

Risk is a central aspect of NPP operations due to the high demands on security and reliability. Different nuclear industry processes often express risk in terms of probabilities of failure related to different components and systems, which can be transformed into monetary terms by considering intervention costs. However, due to the lack of component specific data in this case, risk has an alternative representation. It is provided as two measures, total risk and residual risk, both represented in monetary terms. The total risk corresponds to the consequence of not doing a project and it is increasing over time. It is estimated as curve based on a provided value of risk at the end of the NPP life cycle and in some cases including a break point value for a certain year, where the gradient increases. The risk values for each time period have been estimated as linear functions between these points, and the two different options are illustrated in Figure 1. The total risk can be partitioned into risk that is carried while the project is considered and the remaining risk which is avoided when the project is implemented. The nominal values of these two parts are determined by the time of exercise, where early implementation generally entails higher avoided risk, which is evident when observing the development of costs in Figure 1 below. Although project interventions eliminate risk to some extent, it can not be considered fully removed. The risk remaining after a project has been implemented is referred to as the residual risk. When risk is considered in the project prioritization and selection process, it can be considered as a monetary cost and compared to varying levels of risk appetite.

(a)


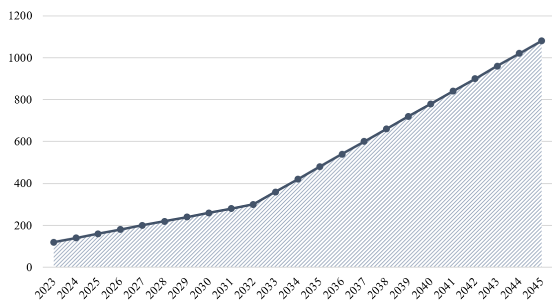
(b)


Figure 1: Illustration of possible total risk development over the project life cycle

4.1.3 Performance

Performance variables reflect the capacity targets of the NPP production. There is a total capacity to strive towards which is provided as a percentage of the total capacity, and can be translated to the number of days available for production stop as a result of maintenance and replacement interventions. At the project level, performance limitations are represented by the number of days of production stop each project requires. Combined with additional information on which interventions that are eligible for joint implementation, i.e. can be performed simultaneously, and the cost of a production stop, performance can also be expressed in monetary terms.

The preferred scenario in terms of performance is to achieve the highest possible capacity, although it is represented as a percentage threshold due to the recurring need to perform projects when the NPP is not in production. An additional aspect of performance is reflected in the availability of resources, previously mentioned in relation to yearly capital budgets. Another resource constraint to consider is the available manpower in the NPP. If the manpower is fully employed, the implementation of selected projects can be limited.

4.1.4 Sample Data

Although the methodology is developed to fulfil the end purpose of optimizing the actual data related to the asset management process at Forsmark, a sample data set is used to test the models and illustrate results. This is due to restrictions in terms of confidentiality of data and parallel work of developing the right processes to obtain the right data. The fictional data have been developed to approximately simulate correct nominal values of NAM projects where both small and large projects have been sampled to obtain some variation in the set. The sampled data set has been limited to fewer projects than what will be considered in a real scenario to enable a detailed illustration of possible outputs.

4.2 Outline of Approach

In the following sections, a novel methodology is proposed. The method has been derived by attempting to best match the objectives of FKA with findings from previous studies that have been presented in Section 3. As such, the novel methodology is essentially divided into two steps. The first step consists of valuing the individual projects, which is discussed in detail in Section 4.3. A method is proposed to identify the value of projects based on a trade off between risk and cost, which also considers the timing of performing a project. For each individual project, the valuation is performed for every possible time of initiation, meaning that a set of project values, denoted P, is obtained for the time interval. This means that for a project with a considered remaining life of T years for the NPP, a set of project values is obtained as

$$P = \{p_1, p_2, \ldots, p_T\} \tag{27}$$

where p_t represents the project value of performing the project at time t. To clarify, $t = 1$ represents performing the project today and $t = T$ means that the project is deferred until after the lifetime of the plant and thus never completed. Further let V be the set of all project values in P that are among the three largest elements in P and the value corresponding to the option of not doing a project at all p_T, defined as

$$V = \{\{p_t \in P\colon\ p_t \text{ is one of the three largest elements in } P\} \cup p_T\} \tag{28}$$

Obtaining the set V is done through ranking the project values in P and selecting the three timing options yielding the highest project values and then adding the option T, which is illustrated in Figure 2. As such, V is a set of four elements reflecting potential project values associated with three different timing options, and an option to never perform the project. This selection process of attaining the project set V is performed for every project in the portfolio. This is done to mitigate the computational requirements in the optimization process, which is the second step described in detail in Section 4.4. What follows is that the optimization process considers all projects with the top three options derived from the valuation process. These constitute the options in which the optimization aims to select the project mix which maximizes the total project value based on a set of constraints that FKA are faced with. This process is illustrated in Figure 3.

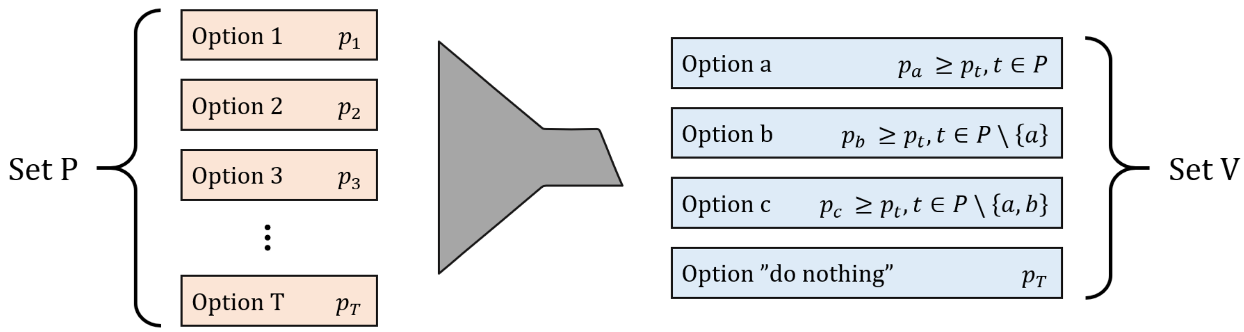


Figure 2: Illustration of option selection process to obtain set V for an individual project

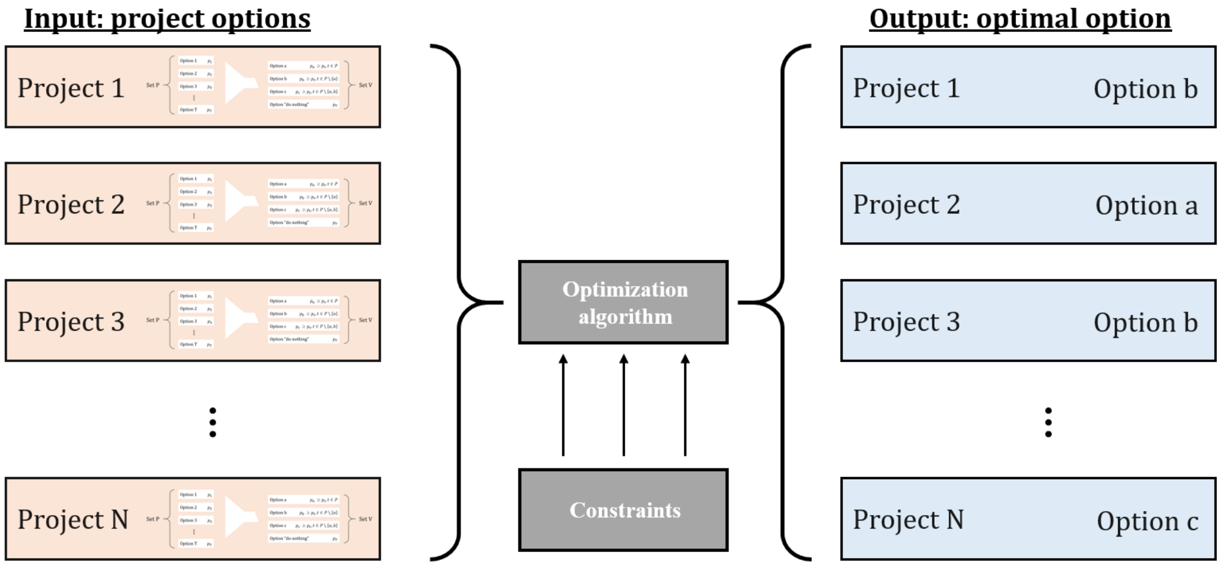


Figure 3: Illustration of optimization process to obtain optimal portfolio of project options

4.3 Project Valuation

In order to optimize the selection of projects, valuating the individual projects is a pivotal component of formulating an appropriate model. As will be presented in section 4.4, the objective function of the optimization model is based on the present value of the projects, meaning that the valuation process is the main driver of the outcome in the project selection process. In this section, different approaches of valuing the projects are discussed. A cost-benefit concept is applied to evaluate outcomes associated with different decisions. Benefits are in general considered as potential savings incurred by performing an action, as there is no revenue upside connected to the projects. There is only information regarding costs and risk-associated costs which we seek to minimize.

As this subject is highly unexplored by FKA, different methods are presented to both suggest potential future approaches based on what is at the forefront of the academic literature, and what could actually be implemented by FKA today based on current procedures and obtainable data.

4.3.1 Net Present Value

Net Present Value is a common approach in financial analysis to aid in the decision-making whether an investment is attractive or not. In essence, the current costs are compared to the current value of future cash flows arising from the investment using a discount rate. If the present value of the future cash flows is greater than the current investment costs, an investment is considered profitable given that the assumptions are accurate.

Using NPV as the basis for project valuation, two different methods are proposed. Both methods are based on the same logical reasoning regarding cash flows associated with performing an intervention, but require different input variables to calculate the value. Proposal 1 derives from what the authors believe is the most effective way of optimizing the project selection process. As such, the required inputs and intended use of this model will be presented, but it will not be tested as the necessary information currently is not available. Instead, an alternative method is presented in Proposal 2 which takes into account what information FKA has available today. Proposal 2 will be the underlying method used to value projects in the optimization model presented in Section 4.4.

Proposal 1: Based on Academic Literature
Considering the conditions presented in the problem formulation 4.1, a RIAM approach similar to that presented in [54] seems fitting to capture the effects of different implementation times for the projects. We picture the problem in the following way; at every time point, there is an option to either execute the project or defer it. The options generate different cash flow developments as a consequence of investment costs and risk reductions. If the project is performed today, the majority of the risks associated with the project are removed, and only the residual risk remains. In other words, the action removes most of the potential future costs as the problem has been proactively handled. Consequently, a project can only be performed once. Only a one-time investment cost, denoted C_p of preventively performing the action is incurred yielding the following cash flow.

Figure 4: Graphical illustration of performing preventive action today

On the other hand, if a project is deferred there is a risk that the asset associated with the project might fail, thus causing economic damage. So if an intervention is not performed today, there is a probability of failure until next time step that is represented by a failure rate function $\lambda_L(t)$, where L is a random variable describing the lifetime of the asset. Similarly, a survival function, $R(t)$ is introduced to represent the probability of the asset surviving past t. The failure rate and survival function are discussed in more detail in Section 4.3.3. Economic damage manifests as a consequence of failure in two different ways. Firstly, a corrective intervention cost, C_c, is incurred to repair the unit. A project's preventive and corrective costs can differ, in general with higher costs associated with corrective actions. In addition, failure risks leading to downtime in the production process, thus representing a cost as revenue lost from production outage. As such, each project has an associated number of downtime days, D, in case of failure and a daily cost of production being down, C_d. What follows is that at each time step we defer the intervention there is a risk of failure, which is only removed once the preventive action is performed, assuming failure has not already occurred in which case corrective action takes place at that moment. Hence, if we assume the intervention is performed at time T_0, the following cash flow is generated

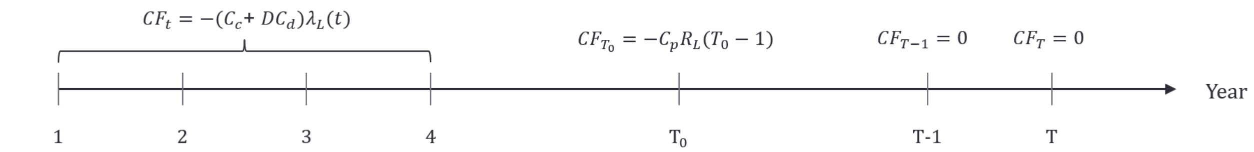


Figure 5: Graphical illustration of performing preventive action at T_0

As can be observed in Figure 5, if $T_0 = 1$, the cash flow development would equal that of Figure 4. Consequently, the NPV of the expected cash flows of performing an intervention at T_0 is

$$\widehat{NPV}_{T_0} = -\left(\sum_{t=1}^{T_0-1} \frac{(C_c + DC_d)\lambda_L(t)}{(1+r)^{t-1}} + \frac{C_p R_L(T_0 - 1)}{(1+r)^{T_0-1}}\right) \tag{29}$$

where the first term represents the expected costs of deferring the project until T_0 and the second term the costs of performing the preventive action at T_0, given a failure has not already occurred. The other option is to defer throughout the planning period, denoted $\widetilde{NPV}$. This means that the project is never performed and the risk of corrective and downtime costs are incurred over the entire period, with NPV calculated as

$$\widetilde{NPV} = -\sum_{t=1}^{T-1} \frac{(C_c + DC_d)\lambda_L(t)}{(1+r)^{t-1}} \tag{30}$$

Now the option of performing intervention at T_0 can be compared to the option of never performing an intervention as the "net" NPV between the aforementioned options. As such, the net NPV is calculated as

$$NPV_{T_0} = \widehat{NPV_{T_0}} - \widetilde{NPV} = \sum_{t=T_0}^{T-1} \frac{(C_c + DC_d)\lambda_L(t)}{(1+r)^{t-1}} - \frac{C_p R_L(T_0 - 1))}{(1+r)^{T_0 - 1}} \tag{31}$$

where

T	=	remaining number of years
T_0	=	year of intervention
C_p	=	cost of performing preventive action
C_p	=	cost of performing preventive action
C_d	=	daily downtime cost
D	=	number of downtime days following failure
r	=	discount rate
$R_L(t)$	=	survival function of random lifetime variable L
$\lambda_L(t)$	=	failure rate function of random lifetime variable L

In essence, the formula suggests that if $NPV_{T_0} > 0$, the project is worth committing to at T_0 as the potential cost associated with future asset failure outweighs the preventive cost. Similarly, a negative NPV suggests the project is worth postponing at T_0. Thus, when comparing multiple projects, the project with the greatest positive NPV is the project which creates most value if implemented at T_0.

Perhaps the most difficult variable to estimate in this proposal is the function associated with the lifetime variable L, as it requires deep understanding of the performance and condition of the item, asset or component associated with the project. In general, using a lifetime probability is a very effective way of modelling the risk associated with not performing an intervention, as it does a decent job of representing the "real" risk of system failure following degradation. This is however heavily dependent on being able to accurately model the failure probability, which in turn implies having sufficient historical data on asset failures. Previous academic studies have applied such failure functions in RCAM and RIAM optimization settings to incorporate risk, although in smaller contexts looking at individual component types. At this moment, this is not something that FKA can model in connection to the separate projects, or at least not something that is considered in the scope of this study. Especially considering a portfolio of 100+ projects, where most of these presumably would require individual failure rate functions. However, as this study is exploratory and intended to serve as a foundation for the development of future asset management practices at Forsmark, modelling a failure rate function based on empirical data will be addressed in Section 4.3.3. This to illustrate how this modelling can be performed if necessary data were available.

It is worth mentioning that even though this model is currently not feasible, it is still a fairly simple NPV model in relation to previous formulations in the academic sphere. A multitude of potential extensions could be introduced to more accurately reflect the real world case depending on what data is available and how the real situations are described. For example, different intervening actions could be introduced, such as differentiating between performing a replacement and maintenance action. Intuitively, different actions would then result in different risk reduction measures, meaning that the risk or probability of failure is not entirely removed, which is often the real-life case even when replacing a unit. Another potential extension could be taking into account interdependence between projects. Some projects may not be actionable until another project has been completed, which is something that currently is not considered. Downtime correlations between projects could also be included. Logically, some projects can be performed simultaneously during downtime which means that the counted days would not stack on each other as is currently the case.

Proposal 2: Based on FKA Data Availability

The rationale behind cash flow developments in this proposal is similar to those of Proposal 1, however this proposal is adjusted to illustrate the same problem but with data that is obtainable for FKA. As such this proposal adopts the data information described in Section 4.1 to formulate a valuation methodology which is implementable in the final optimization model. In this approach, there is no failure probability and no differentiation between preventive and corrective investment costs. Further, there is no cost of downtime associated with projects as this cost is already incurred in the investment cost of performing a project. Instead there is a nominal risk associated with not performing a project, which is estimated by industry experts. Similar to Proposal 1, a method considering the trade off between risk reduction and investment cost at each time step is proposed to yield the net value of investing in the project.

The risk is modelled as presented in Section 4.1.2. In this section some notation will be introduced to formalize the NPV calculation. As illustrated in Figure 1, there is a total risk associated with each project, which is the entire sum of the area under the graph. The total risk can at each time step be divided into two parts; taken risk (TR) and avoided risk (AR). The size of each component is dependent on when the project is performed, and illustrated as an example in Figure 6 where the project is initiated at $t = 10$. The left red side represents the risk that has been taken up until intervention TR_{10}, while the right grey area under the graph represents the risk that has been removed by intervening AR_{10}. The blue on the right side corresponds to the residual risk, meaning that even though a risk reduction measure is performed it does not necessarily remove the entire risk associated with the asset. The residual risk is calculated as a percentage of the final risk value, denoted Y, and in Figure 6 defined as the value of the graph at $t = 23$ which is then equally distributed over all time intervals, in this example 22. Finally, the considered residual risk for $t = 10$ corresponds to the sum of the blue areas under the graph. The percentage is denoted by α.

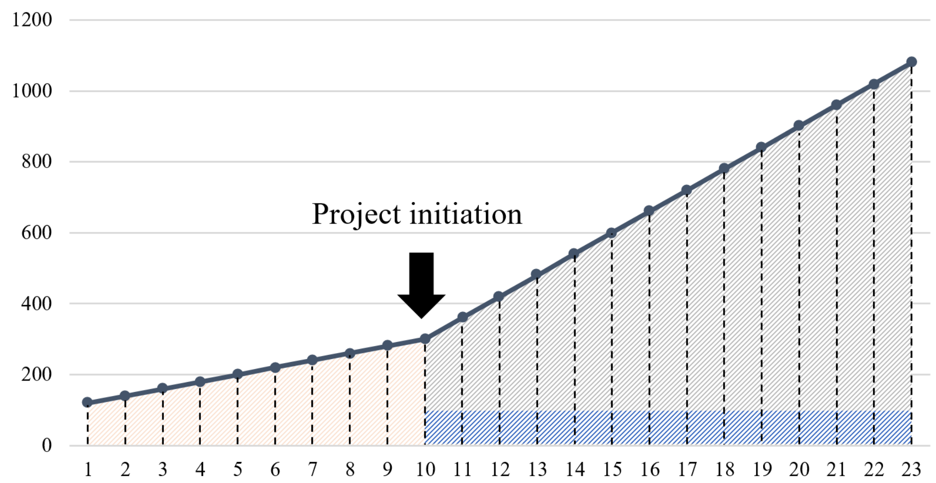


Figure 6: Example of risk distribution of taken (red), avoided (grey) and residual risk (blue) for a project intervention at $t = 10$.

Following this reasoning, if a project is performed today, the taken risk would be zero and avoided risk the same as the total risk. Of course there would still be a residual risk, but that is regarded as a separate unit from the avoided. Similarly, if the project is deferred until the end of the period, the taken risk would be equivalent to the total risk and the avoided risk zero. If a project is performed at t=T_0, the taken and avoided risk can, respectively, be calculated as

$$TR_{T_0} = \sum_{t=1}^{T_0} \Delta TR_{t-1} \quad , \quad AR_{T_0} = \sum_{t=T_0+1}^{T} \Delta AR_{t-1} \tag{32}$$

where ΔTR_{t-1} and ΔAR_{t-1} represent the incremental increase in risk between $t-1$ and t, in Figure 6 illustrated as the area under the graph between the interval of two time-points. Further, $TR_0 = 0$, and $AR_0 = 0$. Now attention is turned to the investment costs associated with the project. A project is assumed to have investment costs spread over N number of years, meaning that if a project is initiated at T_0, costs will be incurred during the interval $[T_0, T_0 + N]$. As such, the NPV of investment costs if the project is started at T_0 is

$$C_{T_0} = \sum_{n=1}^{N} \frac{C_n}{(1+r)^{T_0+n-2}} \tag{33}$$

where C_n is the nominal investment cost year n and r is the discount rate. Consequently, using the same reasoning as in Proposal 1, the net value of performing an intervention at T_0 is seen as the difference between the avoided risk and the investment cost, taken and residual risk. The value is calculated as

$$\begin{aligned} NPV_{T_0} &= AR_{T_0} - C_{T_0} - TR_{T_0} - \alpha Y \left(\frac{T-T_0}{T-1}\right) \\ &= \sum_{t=T_0+1}^{T} \Delta AR_{t-1} - \sum_{n=1}^{N} \frac{C_n}{(1+r)^{T_0+n-2}} - \sum_{t=1}^{T_0} \Delta TR_{t-1} - \alpha Y \left(\frac{T-T_0}{T-1}\right) \end{aligned} \tag{34}$$

where

T	=	remaining number of years
T_0	=	year of intervention
N	=	number of investment years
C_n	=	cost of performing action at $n \in [1, N]$
ΔTR_{t-1}	=	increase in taken risk between $t \in [t-1, t]$
ΔAR_{t-1}	=	increase in avoided risk between $t \in [t-1, t]$
Y	=	Final risk value at $t = T$
α	=	percentage residual risk
r	=	discount rate

As in the first case, $NPV_{T_0} > 0$ indicate that a project is worth performing at T_0 since the avoided risk then is larger than the associated costs with intervening. The taken and avoided risks are both assumed to be the estimates of the monetary real value of costs associated with the risks of a project. As such, these are not discounted as it is already incorporated in the calculations.

This method to value a project has been developed in collaboration with FKA to yield a feasible approach. Proposal 2 is the valuation method used in the presented use case of optimizing the portfolio selection process described later in this study. Next, an alternate approach to the NPV calculation is discussed, with data requirements aligned to what has been presented here in Proposal 2 to suggest another potential feasible valuation approach that could be implemented by FKA.

4.3.2 Benefit Investment Ratio

Another approach to evaluating individual projects is through a BIR. The metric is expressed as the ratio between the present value of the future benefits and the current costs of an investment. Using the available data and rationale behind Proposal 2 in the previous section, the BIR of performing an action at T_0 is proposed as

$$BIR_{T_0} = \frac{AR_{T_0}}{C_{T_0} + TR_{T_0} + \alpha Y \left(\frac{T-T_0}{T-1}\right)} = \frac{\sum_{t=T_0+1}^{T} \Delta AR_{t-1}}{\sum_{n=1}^{N} \frac{C_n}{(1+r)^{T_0+n-2}} + \sum_{t=1}^{T_0} \Delta TR_{t-1} + \alpha Y \left(\frac{T-T_0}{T-1}\right)} \tag{35}$$

A $BIR_{T_0} < 1$ indicates that a project is not profitable performing at T_0 as the costs are greater than the yielded benefits. Generally, the objective is to maximize the BIR since it means that the highest benefit per invested unit is realized. Consequently, the metric is highly useful when comparing different projects as the ratio imply which projects generate the highest return on the investment. A significant caveat of using the ratio is however that it risks causing large projects with high risk to be discarded, as the significant risk is countered by high investment costs. This means that the ratio could prioritize small projects that are cheap and do not remove much risk, while expensive projects that have large associated risks are not considered. Since the objective is to both reduce costs and the absorbed risk, this measure fails to completely capsulate both aspects due to its relative nature. The ratio however works well as a complementary metric in the evaluation and comparison of intervention times, as well as between different projects.

4.3.3 Failure Rate Estimation

As previously mentioned in Proposal 1 of Section 4.3.1, being able to model the asset lifetime is a useful component in optimizing NAM procedures. This can be done by using concepts of reliability theory as described in Section 2.4.1. Considering the setting described in the NPV calculation of Proposal 1, the failure rate function works as an adequate proxy of the probability of failure at certain years according to equation (15). What follows is the need to model the failure rates if Proposal 1 ever would be actualized. As previously mentioned, implementing the model would require failure rate modelling of each asset included in the project portfolio. This is not possible given the current available information. Instead, external data on component lifetimes is used to illustrate how failure rates could potentially be modelled in the future using reliability theory covered in Section 2.4 and Bayesian inference discussed in Section 2.5.

Two different sets of data have been gathered from other studies, which have observed real life data of component lifetime in NPPs. The first set of failure times are collected from [31] relating to shaft seals at Forsmark, and the second set is gathered from [43] with failure times of low pressure switches from the South Texas Project Nuclear Operating Company. These observations are presented in Tables 1 and 2. It can be recognized that the failure pattern tends be a little different between the two components. The shaft seals display a pattern of increasing probability of failure as time proceeds, while the pressure switches are subject to an early failure which then stabilizes in the middle and then increase in the long-run as a wear out effect. Consequently, the failure rate function of shaft seals can be believed to be monotonously increasing, while pressure switches exhibit a bathtub pattern.

Table 1: Failure times of shaft seals

Time to failure (days)
464.00
460.29
924.86
484.17
780.50
540.14
390.58

Table 2: Failure times of low pressure switches

Time to failure (days)
50.77
112.02
16.02
1164.33
24.51
1261.91
1309.64
1180.84
237.40

As discussed in Section 2.4.2, the exponentiated Weibull distribution can satisfy both a monotone increasing and bathtub shaped hazard rate depending on its parameters. Thus, the hazard rates of both components is modelled using the distribution. The parameters of the distribution are estimated through Bayesian inference using a Metropolis-Hastings algorithm as described in Section 2.5.2. The algorithm was run for 100 000 iterations with a burn-in of the first 1 000 samples. The goodness of fit of the derived probability distributions will also be tested using a KS-test with a significance level of $\alpha = 0.05$.

4.4 Optimization Model

The process of formulating an optimization model has been approached gradually to account for the varying prerequisites and different levels of complexity. The concept of starting with a simple model has set a foundation on which elements of increasing complexity can be added. The gradual development of the model can be seen as moving from a deterministic starting point to a stochastic model which incorporates uncertainty. All models have been constructed in IBM ILOG CPLEX Optimizer using the Python API.

4.4.1 Deterministic Model

The deterministic nature of the first model, corresponds to assuming that financial metrics related to the projects, i.e. project costs, risk costs and annual budgets, are known to the DM with some degree of certainty. Thus the deterministic model will ignore randomness related to the variation of actual outcomes. The included values are the mean of the assumed normal distributions, calculated using three point estimation as described in Section 4.1. As such, they can be considered to represent uncertainty of outcome in a deterministic way, i.e. assuming the mean values. The optimization model utilizes the NPV calculations and ranking of options detailed in Section 4.3.1. Thus the model input is constituted of a list of projects and the preferred options of which year to implement the project, i.e. option 1 can correspond to implementation year t while implementation according to option 2 might entail year $t+2$. The model aims to select an optimal project portfolio which maximizes total NPV of the included projects, and in a consecutive step the projects are prioritized.

Sets:

I: set of all projects

J: set of all options for each project

Z: set of all options for each project, excluding do nothing option

T: set of all time points

Parameters:

$c_{i,j,t}$: cost of selecting project i through option j in year t

$d_{i,j,t}$: downtime days of selecting project i through option j in year t

$p_{i,j}$: net present value of selecting project i through option j

B_t: annual budget for year t

D_t: maximum number of downtime days for year t

$z_i \in 0,1$: binary variable indicating whether project i is mandatory or not

Decision variable:

$x_{i,j} \in 0,1$: binary variable indicating whether project i is selected through option j

Formulation:

$$\max_{x_{i,j} \in 0,1} \sum_{i \in I} \sum_{j \in J} p_{i,j} x_{i,j} \tag{36a}$$

s. t.

$$\sum_{i \in I} \sum_{j \in J} c_{i,j,t} x_{i,j} \leq B_t, \forall t \in T \tag{36b}$$

$$\sum_{i \in I} \sum_{j \in J} d_{i,j,t} x_{i,j} \leq D_t, \forall t \in T \tag{36c}$$

$$\sum_{j \in Z} x_{i,j} z_i \geq z_i, \forall i \in I \tag{36d}$$

$$\sum_{j \in J} x_{i,j} = 1, \forall i \in I \tag{36e}$$

The objective function (36a) of the first model stated above maximizes the total NPV of all selected options for each project in the optimal portfolio. The portfolio is indicated by the decision variables $x_{i,j}$ and can be compiled as a vector of recommended options for each project. Constraint (36b) ensures that the total cost incurred by the selected projects each year remains within the boundaries of the yearly budget. Furthermore, constraint (36c) ensures that the number of downtime days of the selected projects through each option in year $t \in T$ cannot exceed the annual maximum number of days D_t. Inclusion of mandatory projects is guaranteed by constraint (36d), while the last constraint (36e) limit the selection of options to one for each project.

Heuristic approach to project prioritization

While the deterministic model provides an effective way of choosing the optimal subset of projects to perform, it lacks the process of prioritizing the order of the projects. Priority lists are often desired by DMs and have potential to aid decision making if conditions are changing, i.e. if new budget levels are defined. The prioritization task can be approached heuristically by considering different scenarios of altering budget levels. Assuming the yearly budgets are the same for every period, the process is initiated by running the model with an initial budget level considered as the lower end of the budget spectra. Next, the budget is increased slightly and the optimization run again. Repeating this process for a number of trials will enable more projects to be selected within the confound of the successively increasing budget. According to the heuristic approach, the projects chosen in the first trial are given the highest priority, and projects added to the portfolio as the budget increases are given priority rankings in the order they enter.

The process of obtaining a priority ranking can be adjusted according to other similar approaches to handle potential selection biases. The approach starting from the lowest budget level will in some cases not consider very large and costly projects as being of higher priority, i.e. if their cost exceeds the budget level. An alternative is to start from a high budget level and decrease it successively, with the constraint to only select projects included in the initial portfolio also for the lower budget levels. According to this process, a different priority list can be obtained.

4.4.2 "What if" Model

A fist step towards a stochastic model can be taken by introducing some stochastic elements using the approach outlined in literature as "what if" analysis. Using the developed deterministic model described in Section 4.4.1 in its current formulation, uncertainty can be incorporated to a greater extent by varying the input. By continuously feeding random input to the model, which can be seen as a black box, results for different scenarios are generated. As a result the output will also be random which enables analysis of statistics such as NPV distributions and project selection frequencies. The most influential parameters chosen for random sampling were the yearly project costs, the underlying risk values used to calculate risk, i.e. the break point value and ending value, and the annual budget levels.

The uncertainty of input variables related to costs, monetized risk and performance resources is reflected in the high, low and likely levels. While the deterministic model assumed the inputs where normally distributed, the deterministic approach only incorporated randomness in the usage of mean values. In the "what if" analysis, inputs are instead sampled from their assumed normal distributions. For the annual budgets there are no given values and the variation is introduced to analyze the effect of different scenarios. Thus, it has been represented by a uniform distribution from 40 million to 50 million. The sampling process can be done many times by implementing Monte Carlo simulation, which will generate results for a number of scenarios with different inputs. The implementation of the method increases understanding and transparency of the optimization model without implementing different scenarios in the objective function.

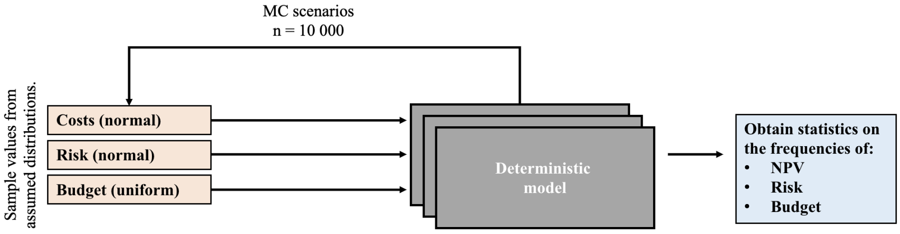


Figure 7: Illustration of the "What if" model

The extended analysis of the first model enables a new approach for project prioritization. The outcomes from the different scenarios, whit slight variations in input parameters, produce different portfolio compositions. A priority list can be obtained by scoring the projects, and the options through which they were selected, by their selection frequencies. That is, if a project is selected through one option in every scenario it will achieve the highest priority score. The portfolios consisting of different project and option combinations can also be scored in a similar manner to achieve a portfolio priority.

4.4.3 Full Stochastic Model

Moving towards a full stochastic model entails integration of randomness in the consideration of the objective function. While the "what if" model evaluated variations in input data, the actual model was still a fairly simple deterministic one. The stochastic model is based on probabilistic weights linked to each scenario where the most influential inputs have been sampled for each. Thus, we extend the formulation of the deterministic model in Section 4.4.1 by introducing the set W which denotes the set of all scenarios, and let q^w represent the probability of scenario w occurring. Thus the formulation is as follows.

Formulation:

$$\max_{x^w_{i,j} \in 0,1} \sum_{w \in W} q^w \sum_{i \in I} \sum_{j \in J} p^w_{i,j} x^w_{i,j} \tag{37a}$$

s. t.

$$\sum_{i \in I} \sum_{j \in J} c^w_{i,j,t} x^w_{i,j} \leq B^w_t, \forall t \in T \tag{37b}$$

$$\sum_{i \in I} \sum_{j \in J} d^w_{i,j,t} x^w_{i,j} \leq D^w_t, \forall t \in T \tag{37c}$$

$$\sum_{j \in Z} x^w_{i,j} z^w_i \geq z^w_i, \forall i \in I \tag{37d}$$

$$\sum_{j \in J} x^w_{i,j} = 1, \forall i \in I \tag{37e}$$

$$\sum_{w \in W} q^w = 1 \tag{37f}$$

The objective function (37a) is extended from the deterministic model by adding the NPV of each project from every possible scenario given that it is realized. Constraints (37b) - (37e) are similar to those in model (36), with added dimension of scenarios w. Finally (37f) ensures that the sum of probabilities of all possible scenarios are equal to one. For the sample data used in this study, the stochastic model has been implemented with 1 000 scenarios. The scenarios are all assumed to have equal probability, thus $q^w = 0.001, \forall w \in W$. Scenario costs and risk are still randomly sampled from a normal distribution, while the budget is assumed to be a random uniform variable between 40 million and 50 million.

4.4.4 CVaR Stochastic Model

As described in Section 2.3.2, CVaR can be integrated into the optimization model to introduce an active risk measure in the formulated problem. That is, the former maximization of NPV will be performed for a given level of risk. Thus, CVaR optimization can be seen as introducing a risk appetite through the aversion parameter λ and an additional term in the objective function. It will determine a certain degree of penalisation for the risk quantified by CVaR. Looking at the objective function only, it can be formulated broadly according to

$$\max_{x^w_{i,j}\in 0,1}(1-\lambda)\sum_{w\in W} q^w \sum_{i\in I}\sum_{j\in J} p^w_{i,j}x^w_{i,j} - \lambda CVaR_\alpha \tag{38}$$

where the effect of penalising risk on the objective value is evident. Using the auxiliary formulation of CVaR from equation (10), and adding to the constraints, the stochastic model with CVaR constraints can be expressed according to the following formulation. Here both γ and ν^w are auxiliary variables.

Formulation:

$$\max_{x^w_{i,j}\in 0,1}(1-\lambda)\sum_{w\in W} q^w \sum_{i\in I}\sum_{j\in J} p^w_{i,j}x^w_{i,j} - \lambda[\gamma + \frac{1}{(1-\alpha)}\sum_{w\in W} q^w\nu^w] \tag{39a}$$

s. t.

$$-\sum_{i\in I}\sum_{j\in J} p^w_{i,j,t}x^w_{i,j} - \gamma \leq \nu^w, \forall w \in W \tag{39b}$$

$$\gamma + \frac{1}{(1-\alpha)}\sum_{w\in W} q^w\nu^w = CVaR_\alpha \tag{39c}$$

$$\sum_{i\in I}\sum_{j\in J} c^w_{i,j,t}x^w_{i,j} \leq B^w_t, \forall t \in T \tag{39d}$$

$$\sum_{i\in I}\sum_{j\in J} d^w_{i,j,t}x^w_{i,j} \leq D^w_t, \forall t \in T \tag{39e}$$

$$\sum_{j\in Z} x^w_{i,j}z^w_i \geq z^w_i, \forall i \in I \tag{39f}$$

$$\sum_{j\in J} x^w_{i,j} = 1, \forall i \in I \tag{39g}$$

$$\sum_{w\in W} q^w = 1 \tag{39h}$$

$$\nu^w \geq 0, \forall w \in W \tag{39i}$$

Most constraints are the same as in the stochastic model. Although, constraints (39b) and (39c) are included to determine the value of the auxiliary variables and calculate the value of CVaR. Due to increased computational complexity, the model was implemented for only 20 scenarios to illustrate the difference between the stochastic model for a reasonable run time. The scenarios are still assumed to have equal probability, thus $q^w = 0.05, \forall w \in W$. Scenario costs and risk are sampled from the same distribution, and the budget is attained from the same uniform interval between 40 million and 50 million.

5 Results

The developed models have been implemented to comply with data resembling the actual data related to asset management projects at Forsmark as described in Section 4.1.4. That is, up to 100 projects with possible implementation over the remaining NPP life time of 23 years are considered. The size and dimensions of the data have not been an issue for the models with regards to computational efforts, but the results from larger sets are long and difficult to analyze on project level. Thus, to provide an intuitive display of the resulting portfolios and the model results in general, a focused case that zooms in on a shorter time period and for a smaller data set has been implemented.

This case considers 12 projects with possibilities for implementation over a 6 year term as a longer time period, such as the remaining NPP life time, requires a lot of projects for an optimization to be relevant. Each project is considered for implementation through several options with varying implementation times and thus some variation in costs and NPV due to a discount factor. Some projects are mandatory and need to be performed, while others have the option of not doing the project which also entails an NPV value. A summary of the projects and corresponding options used as input to the optimization can be found in Table 12. As a result of the current measures in place at FKA, more specifically the interplay between maintenance as a result of different projects, the aspect of performance has been excluded from the illustration of results. Performance is represented by the days of production stop incurred by a project, and while the models are formulated to handle such constraints the data is not yet adapted to simulate realistic outcomes. Here in the results, the measure rest risk (RR) is introduced, which refers to the total risk that is not avoided even though a project is implemented and is calculated as the risk taken plus the residual risk.

5.1 Deterministic Model

A deterministic model as described in 4.4.1, was constructed using the CPLEX API in python. The optimization of the model was tested on different values for certain parameters, which yielded a base case for the annual budget levels. The following Table 3 describes the results for the model when the annual budgets were set to 30 million. A full summary of costs related to each option of the included projects used as input for the optimization can be found in the Appendix 8, Table 12.

Table 3: Results of deterministic optimization given annual budgets of 30m, all values in millions

Proj.	**Opt.**	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**	**Tot. costs**	**NPV**	**RR**	**AR**
1	B		9,7	7,2				16,9	46,5	5,4	66,3
2	B	2,2	1,5	0,7				4,4	2,3	0,3	6,7
3	A				1,8	0,8	0,1	2,6	-0,5	0,3	2,3
4	C			14,9	12,2	4,0		31,1	36,1	7,7	72,2
5	Don't do							0,0	-19,1	19,1	0,0
6	A	4,3						4,3	15,8	0,7	20,0
7	C			5,1	10,7	1,2		17,0	22,2	6,0	44,0
8	A	21,3	10,4					31,6	60,2	2,1	91,8
9	A		3,7	1,7				5,4	1,6	0,9	7,1
10	B	1,0	0,9	0,2				2,0	0,0	0,2	2,1
11	Don't do							0,0	-74,0	74,0	0,0
12	Don't do							0,0	-61,5	61,5	0,0
Total		28,8	26,2	29,8	24,7	6	0,1	115,3	29,6	178,2	312,5

The Table shows costs, time of initiation, NPV, rest risk and avoided risk for each chosen project and option. For the given budget level all projects except 5, 11 and 12 are chosen. The projects that are excluded from the optimal portfolio still incur negative NPVs as the total risk of the projects need to be accepted by the DM. Next, the optimization model was solved for different budget levels to illustrate the effect on the resulting portfolios.

Here the NPV, rest risk and avoided risk refers to the entire portfolio based on the chosen projects. The effect of being forced to not do more projects as the budget levels decrease can be seen on the total NPV,

Table 4: Resulting portfolio composition of project options, NPV, RR and AR in millions for different budget scenarios

Budget	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**NPV**	**RR**	**AR**
10m	C	B	A			B			C	B			-329,5	394,8	95,7
15m	A		A	C		B			C				-206,1	321,2	169,4
20m	A	A	A			A	C		B	C		B	-140,3	282,9	207,6
25m	B		A			B	C	A	B	B		C	-13,5	207,5	283,1
30m	B	B	A	C		A	C	A	A	B			29,7	178,0	312,6
35m	A		A	C	A	B	C	A	B	A		C	129,1	115,4	375,1
40m	A	B	A	C		B	A	A	A		C	C	215,5	60,0	430,5
45m	A	C	B	C	A	B	C	A	A		C	A	248,0	37,8	452,8
50m	A	A	A	C	A	A	B	A	A	A	C	A	257,2	31,6	458,9
55m	A	C	A	A	C	A	A	A	B	A	C	A	261,3	28,8	461,8
60m	A	A	A	C	A	A	B	A	B	A	A	A	262,0	28,5	462,0

which turns negative when the annual budgets are decreased from 30m to 25m. The risks that constitute the base for NPV follow similar patterns as the budget changes. Another observation is the difference between portfolios for different budgets in terms of included project and options. Projects such as 2, 4, 5, 10 and 12 move in and out of the portfolio as budgets increase, while almost all projects alter between options to fit the budget constraints for some levels. This can be explained by the size of the projects and their NPVs. Looking at the metrics displayed in Table 12, it is clear why project 4 replaces project 10 when the budget allows a yearly spend of 15m. While project 7, 10 and 12 provide better NPVs in the next step where project 4 is excluded once again. To form a priority list of the portfolio following the approach described in 4.4.1, additional constraints were imposed on the model when running the different budget scenarios. The model was first solved for an annual budget of 10m and then increased with the condition that all projects from the first portfolio had to be included in the next solution through either option. For many scenarios this reduced the optimal NPV value as the model had less freedom to optimize the selection which can be seen in Table 5.

Table 5: Heuristic prioritization results of portfolio compositions, NPV, RR, and AR in millions for different budget scenarios

Budget	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**NPV**	**RR**	**AR**
10m	C	B	A			B			C	B			-329,5	394,8	95,7
15m	A	A	A			A			B	A		C	-225,2	333,7	156,8
20m	A	A	A			A	C		B	C		B	-140,3	282,9	207,6
25m	A	A	A		C	A	C		A	A		A	-115,4	263,9	226,6
30m	A	C	A		B	A	C	B	B	C		C	20,9	182,5	308,0
35m	A	A	A		C	B	C	A	B	A		B	34,8	174,8	315,8
40m	A	A	A	C	B	A	C	B	B	A		A	152,8	101,1	389,4
45m	A	A	A	C	A	A	A	A	B	C	C	C	242,2	39,8	450,7
50m	A	A	A	C	A	A	B	A	A	A	C	A	257,2	31,6	458,9
55m	A	C	A	A	C	A	A	A	B	A	C	A	261,3	28,8	461,8
60m	A	A	A	C	A	A	B	A	B	A	A	A	262,0	28,5	462,0

From Table 5, a heuristic priority list could be extracted by taking the first level picks as the highest priority, the additional projects in the next step as the second highest and continuing this process for all steps. The approach yielded the priority list displayed in Table 6.

The priority list provides an example of how it could be generated for an optimization. It can be done in many different ways, and by starting from the highest budget, or decreasing the step size between budgets, another priority list could have been produced.

Table 6: Priortity order from heuristic approach to budget constraint

	Heuristic prioritization order
Priority 1	{1, 2, 3, 6, 9, 10}
Priority 2	{12}
Priority 2	{7}
Priority 4	{5}
Priority 5	{8}
Priority 6	{4}
Priority 7	{11}

5.2 "What if" Analysis

The extended "what if" model utilises the optimization model used in the deterministic approach but varies the input parameters to obtain more descriptive results. Risk, costs and budgets where sampled as described in Section 4.4.2, and given the different inputs the uncertainty is captured in the outcome of outputs as well. The resulting distributions of NPV, rest risk and avoided risk are shown in Figure 8 for 10 000 input scenarios. Each value refers to the portfolio value in that scenario.

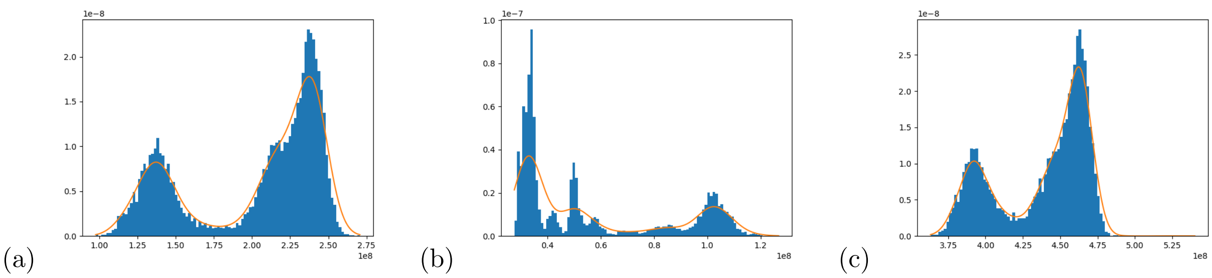


Figure 8: Distribution of (a) NPV (b) Rest risk (c) Avoided risk from 10 000 scenarios in "what if" analysis

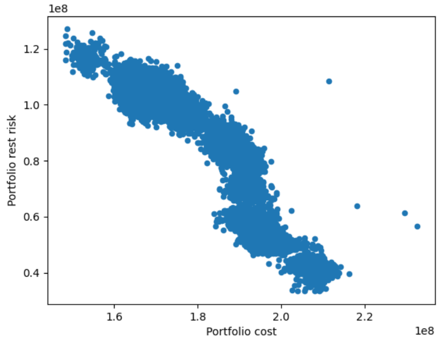
The inputs are sampled from a mix of normal distributions and a uniform distribution for the budget levels. Thus, the resulting distributions are difficult to attribute to a given family o distributions. All distributions shown in Figure 8, have clusters of outcomes around two different values where one is more prevalent. Statistical measures of the different samples displayed can be seen in Table 7 below.

Table 7: Statistical summary of NPV, rest risk and avoided risk in millions for 10 000 "what if" scenarios

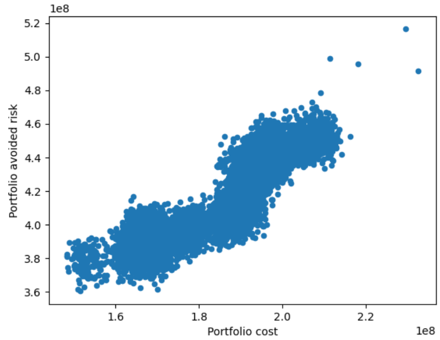
	Mean	Min	Max	SD	5th percentile	95th percentile
NPV	166,1	91,5	252,8	37,3	120,1	222,0
Rest Risk	80,2	33,4	127,0	24,4	47,3	109,9
Avoided Risk	412,8	360,4	516,4	25,1	380,4	451,9

Another dimension of interest is the relationship between investment costs and yielded values of rest and avoided risk. As both risk and costs are subject to uncertainties, the relation illustrated in Figure 9 can give insight in probable needed investment costs to attain a certain level of avoided and taken risk. Apart from a few outliers of extreme scenarios, there is a coherent pattern of how investment costs affect the risks under uncertain conditions.

The robustness obtained by analyzing 10 000 samples, each contributing to an optimal solution, enables more detailed prioritization of projects. Looking at the frequency of occurrence for different projects through their respective options, a ranking can be made based on probability of inclusion. The resulting priority list is displayed in Table 13 and except for some options that are never chosen, all options obtain individual rankings in contrast to the heuristic prioritization made for the deterministic model. The bottom ranked options include the option to not do one of the mandatory projects, which will always be mandated to perform



(a)



(b)

Figure 9: Rest and avoided risk vs total cost for 10 000 "what if" scenarios

according to the constraints. A similar prioritization can be done for the most frequent portfolio compositions. Table 8 shows the frequency and obtained statistics for the 5 most probable portfolio compositions. Similarly, Table 14 in the Appendix 8 illustrates the option allocation of these 5 portfolios according to the "what if" analysis.

Table 8: Statistics of 5 most common project portfolios under 10 000 "what if" scenarios, NPV RR, and AR in millions

		Project portfolios				
		1	2	3	4	5
	Frequency	155	133	123	117	89
	Probability	0,0155	0,0133	0,0123	0,0117	0,0089
NPV	Mean	139,9	135,3	139,1	146,0	137,5
	Min	122,5	119,4	121,7	127,4	124,8
	Max	157,7	153,3	161,4	166,9	149,8
	SD	6,6	7,2	6,6	7,2	6,0
Rest risk	Mean	96,1	101,3	96,4	88,1	101,3
	Min	89,3	95,5	89,7	82,4	94,8
	Max	102,4	106,1	101,4	93,3	105,7
	SD	2,6	2,0	2,6	2,3	1,9
Avoided Risk	Mean	396,8	391,5	396,2	402,8	392,5
	Min	380,8	374,4	380,1	389,5	382,8
	Max	415,3	406,8	416,4	420,3	405,4
	SD	6,1	6,4	6,4	6,5	5,2

5.3 Stochastic Model

For the stochastic model similar statistics were produced to identify distributions and frequencies in the output of the model. The results turn out differently from the "what if" model as each scenario is considered in the optimization at the same time. This also entailed a much higher computational complexity, making 10 000 sampled scenarios computationally costly. Thus the Monte Carlo simulation was limited to 1 000 scenarios for each uncertain parameter.

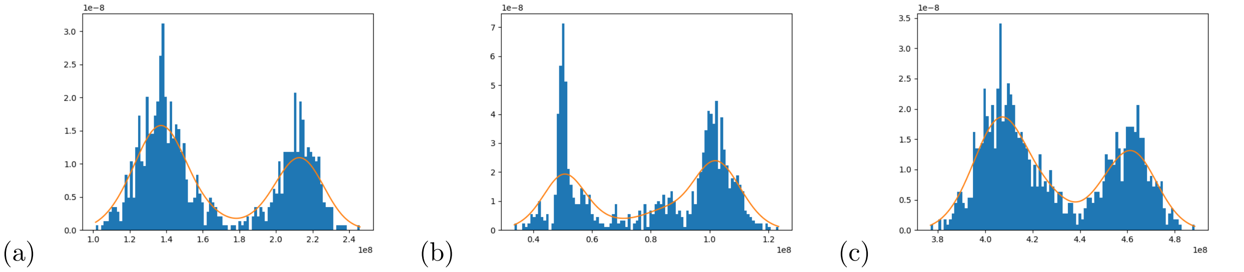


Figure 10: Distribution of (a) NPV (b) Rest risk (c) Avoided risk from 1 000 scenarios of stochastic program

As can be seen from Figure 10, the NPV distribution has a more even clustering around 130 million and 210 million, and the lower NPV cluster has higher frequencies. The risk distributions are instead more scattered than the corresponding plots in the "what if" model. The statistical moments of each metric is displayed in Table 9.

Table 9: Statistical summary of NPV, rest risk and avoided risk in millions for 1 000 scenarios of stochastic program

	Mean	Min	Max	SD	5th percentile	95th percentile
NPV	165.9	101.4	245.9	37.8	120.2	222.9
Rest Risk	80.3	33.6	123.4	24.5	47.8	110.3
Avoided Risk	428.8	376.9	488.5	27.0	394.6	470.7

Observing the scatter plots in Figure 11, a similar partition as for the "what if" analysis can be seen for the rest risk vs cost plot. For portfolio costs in the range 180-190 million there is larger spread in rest risk for similar cost values. The avoided risk and cost plot resembles a linear curve.

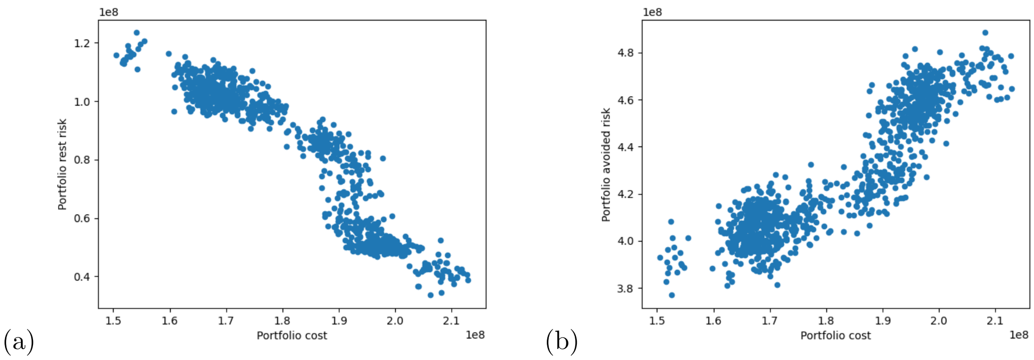


Figure 11: Rest and avoided risk vs total cost for 1 000 scenarios of stochastic program

Robust priority lists have also been compiled from the stochastic programming model. Table 15 depicts the priority of individual project options, which differ some to the "what if" analysis but in general show a consistent prioritization rationale. Below Table 10 show the key statistics of the 5 most occurring portfolio compositions in the stochastic setting. The option compositions of these 5 portfolios are presented in Table 16 in the Appendix 8.

Table 10: Statistics of 5 most common project portfolios under 1 000 scenarios of stochastic program, NPV RR and AR in millions

		Project portfolios				
		1	2	3	4	5
	Frequency	20	13	12	10	10
	Probability	0,0200	0,0130	0,0120	0,0100	0,0100
NPV	Mean	139,7	217,8	145,6	117,5	216,1
	Min	124,4	209,3	136,9	108,5	206,8
	Max	150,7	226,7	158,5	130,3	235,8
	SD	5,7	5,2	6,3	6,7	8,3
Rest risk	Mean	96,3	49,4	87,4	107,0	49,3
	Min	92,9	48,2	84,1	103,7	47,8
	Max	103,7	50,8	93,9	110,1	51,1
	SD	2,6	0,8	2,6	2,0	1,1
Avoided Risk	Mean	412,6	463,4	418,0	394,4	463,1
	Min	398,2	453,6	404,0	385,0	452,8
	Max	425,4	470,8	426,7	404,9	481,7
	SD	6,7	5,4	6,2	6,0	8,1

5.4 CVaR Stochastic Model

The inclusion of CVaR as a risk measure in the stochastic model depends on the probability distribution of the random variables included in the model, and it adds a layer of complexity to the optimization. As can be seen in Section 4.4.4, an additional term is added to the objective function, and more constraints are included, which should correspond to increased complexity. Another effect of incorporating CVaR could be identified in the run time of the model, which increased significantly. Thus, the computational power at hand is unable to handle larger numbers of simulations, and the results generated in this section are generated based on only 20 samples.

Table 11: Output based on the same data for the original stochastic model and the CVaR model, for 20 simulations with $\lambda = 0.5$ and $\alpha = 0.75$

		mean	min	max	std
NPV	Original	158,4	120,1	224,2	34,9
	CVaR	170,2	116,7	260,6	42,4
Rest risk	Original	84,7	50,9	107,4	20,8
	CVaR	79,5	48,9	110,5	22,9
Avoided risk	Original	423,5	398,4	471,2	25,0
	CVaR	438,0	392,1	602,6	46,2

The results of running the stochastic model and the CVaR model on the same data are illustrated in Table 11. To capture the tail of the distribution given less simulations, a lower α was set and the above illustration corresponds to the 0.75-CVaR. An arbitrary value of 0.5 was chosen for λ, but this value can be adjusted to reflect the risk appetite of the DM. It is evident that the CVaR model achieves better results, with higher mean values for NPV and avoided risk and lower rest risk. However, the variance of the CVaR model is higher for all metrics, as can be seen from the standard deviation.

5.5 Failure Rates

In this section, the results from the failure rate estimation with a Bayesian parametric model are presented. As discussed in Section 4.3.3, two models are presented based on real life data of failure times of components in NPPs. Both models are fitted to an exponentiated Weibull distribution using the Metropolis-Hastings algorithm. For the dataset containing lifetimes of shaft seals the estimated posterior distributions of the EWD parameters are given in Figure 12. As can be seen, the mean of the posterior distributions are $\lambda = 780.46$, first shape $k = 3.89$ and second shape $\alpha = 0.71$. The adequacy of the fitted distribution is tested with a Kolmogorov-Smirnov test. The resulting test statistic was found to be $D_n = 0.3824$, with a corresponding p-value $= 0.1977$. Based on this, we fail to reject the null hypothesis, and we can assume that the fitted distribution is suitable for our purposes.

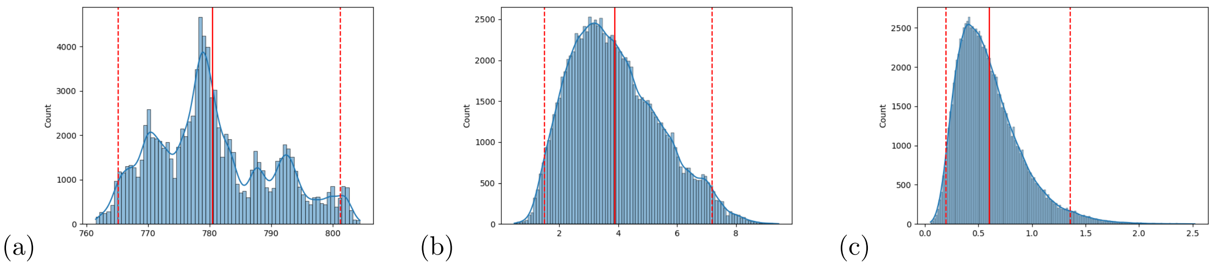


Figure 12: Posterior distribution, mean and 95% interval of parameters (a) λ (b) k (c) α

To evaluate whether the failure times of shaft seals follow the pattern of an increasing failure rate, the parameter conditions modelling the pattern described in Section 2.4.2 are applied. Hence, it can be inferred that the distribution of observations conform to a monotone increasing failure rate since $k = 3.89 > 1$, and $k\alpha = 2.77 > 1$. Figure 13 displays the fitted failure rate function, which depicts a failure rate strictly increasing as time progresses, indicative of the component having an aging process.

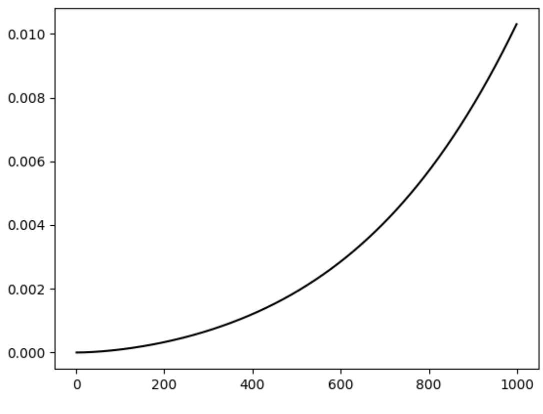


Figure 13: Fitted failure rate function $\lambda(t)$ for shaft seals

In terms of data related to low pressure switches, the posterior distributions of the parameters are presented in Figure 14. Means of the posterior distributions are scale $\lambda = 1704.35$, first shape $k = 5.02$ and second shape $\alpha = 0.12$. Further, a KS-test yielded test statistic $D_n = 0.3423$ and p-value $= 0.1916$, meaning that we can not reject the null-hypothesis. Consequently, the fitted distribution is assumed to be adequate.

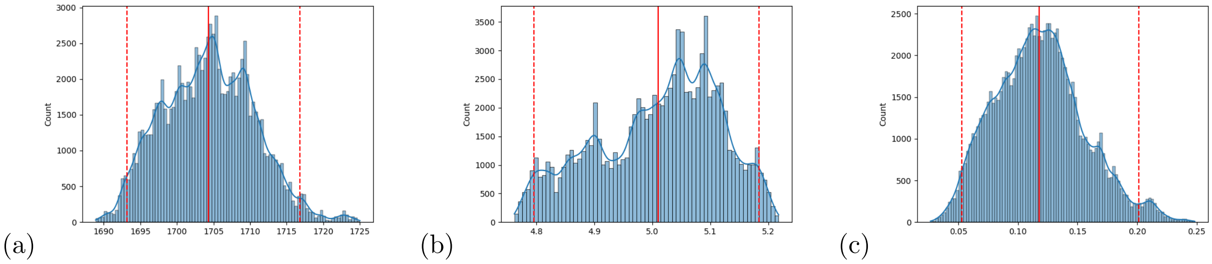


Figure 14: Posterior distribution, mean and 95% interval of parameters (a) λ (b) k (c) α

Now, again using the previously mentioned shape parameter conditions, it is implied that the pressure switch lifetime distribution indeed follows a failure rate equivalent to a bathtub shape since $k = 5.02 > 1$ and $k\alpha = 0.61 < 1$. The fitted failure rate function is shown in Figure 15, which clearly shows an initial decrease in failure rate early on which then increases long-term from wear-out effects.

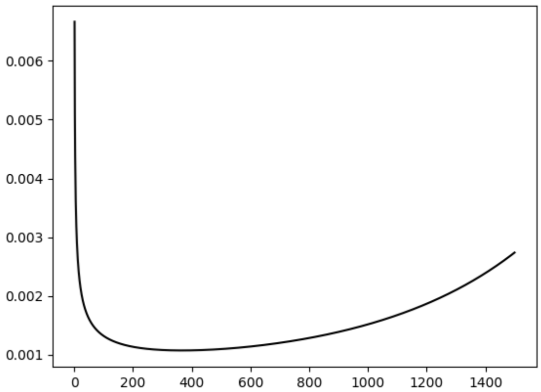


Figure 15: Fitted failure rate function $\lambda(t)$ for low pressure switches

6 Discussion

The discussion starts by reviewing the main takeaways from the literature review and what methods were found most relevant for the problem formulated in this study. The developed novel methodology is also discussed, considering both its strengths and weaknesses. The different approaches to incorporating uncertainty in the optimization are compared in their effectiveness while also accounting for their computational requirements.

6.1 Literature Review

The objective of the literature review was to identify the most relevant methods for NAM described and developed in previous academic work. The relevant literature included in the review constituted the foundation on which this study builds, and it set the initial direction of which area to base the methodology. As described in Section 3, several relevant areas were identified in the literature. It was established that the situation at hand resembled a capital budgeting problem and the method needed to comply with the setup of such a problem. Furthermore, the available data at FKA posed additional restrictions on the method selection as some previous implementations differed significantly in terms of the required input. Based on these realizations, RCAM could be ruled out as it required component data on a detailed level to model system reliability. A similar argument was made for multi-asset system methods as accurate modeling of asset dependencies, which has a central role for methods within the field, was identified as a complex issue given the available information. The remaining group of RIAM methods proved to be the best fit in terms of data. Additionally, some applications of RIAM had been implemented to model capital budgeting problems in previous studies, indicating the feasibility of effectively modeling the problem at hand.

The methods developed in this study, described in Section 4 are based on the general principles of capital budgeting and RIAM. Thus, the methods used in this study resemble RIAM methods in previous work, and other studies have proposed deterministic and stochastic models to solve knapsack problems in capital budgeting settings. However, aspects such as the NPV formulation, which has a central role in the optimization, are unique to this study and tailored to fit the specific needs at FKA. One example is the approach to circumvent the need for project-specific failure probabilities, which can be difficult to obtain in real-life applications. Furthermore, the application of methods from the field of financial mathematics in this study provides an addition to the current work. More specifically, the inclusion of CVaR as a way to adjust risk, is a new approach in capital budgeting methods according to the coverage of the literature examined in this study. Although, other asset management methods have implementations using CVaR in similar ways.

6.2 Methodology

As previously mentioned, the methodology developed in this study seeks to provide FKA with a tool to aid in the project selection process. The novel methodology has been based on relevant academic literature seeking to solve similar problems, in combination with formulating a method that is feasible given the available information concerning the NPP and its projects. As discussed in Section 6.1, a combined RIAM and capital budgeting approach was deemed to be a suitable starting point. The second part of the approach, namely the actual optimization of project selection was formulated with a knapsack problem as it was deemed to adequately represent the problem at hand.

The first part of the general problem is the valuation of individual projects. Here, the question of what aspects to consider in the valuation also arises. In this study, the primary dimensions have been risk, cost, and performance to cover the evaluation of projects. In the end, this has been done by transforming risk into monetary terms to yield a comparative capability between investment costs and consequent risk effects. The same could be done with performance if the information on lost revenue associated with downtime days is obtained. However, since downtime days have been disregarded in the example use case due to the current lack of information, this conversion has not been considered. As such, the valuation of projects can be aggregated into a single monetary measure. In this study, the NPV has been chosen as an adequate method of deriving this value. While ratios such as BIR can be helpful in finding the most value per SEK invested, the evident need of managing risk levels suggests the importance of primarily considering nominal terms instead. A caveat of an NPV approach is that it can fail to properly incorporate drastic changes from one year to another, as it considers the present value of all future cash flows. As such, a model looking forward at the incremental increase in risk between now and the year ahead could also be considered to help prioritize projects short-term. However, the approach implemented in this study looks cohesively on the long-term planning, and thus NPV is deemed adequate.

In terms of the NPV calculation, Proposal 1 has been formulated in accordance with common practice within the literary discourse on the subjects of RIAM. In general, the incorporation of failure probabilities appears to be a sound approach to incorporate risks associated with deferring projects, since it does a decent attempt at reflecting the real-life uncertainty connected to the faulting of physical objects. The complexity of this approach is also fairly scalable depending on available information, with a multitude of potential extensions discussed in Section 4.3.1. As previously mentioned, Proposal 2 has instead been derived as a feasible approach for FKA to value projects with current information. Even though Proposal 2 is recommended in the short-term, a long-term ambition of implementing a version of Proposal 1 could be desirable to more accurately reflect the risk provided that the necessary data was obtainable.

An important step between the two key parts of the outlined approach is obtaining the set of project options to consider in the optimization. Theoretically, this step could be disregarded if including all potential timing options were of interest. Of course, this would be desirable as it means the optimization problem would consider all potential combinations of timing options to identify the optimal portfolio composition. In reality this could however create capacity problems as it would significantly increase the combinatorial requirements. A practical case for Forsmark with a remaining lifetime of 23 years, would generate 23 options compared to 4 in the proposed method. Also taking into account that there is a portfolio of 100+ projects, increasing options for each project from 4 to 23 would clearly have a significant impact on the total number of combinations needed to consider. The decision of 4 options is however rather arbitrary and based on an intuitive trade off between having enough options and reducing computational requirements. The number of included options could easily be increased and the methodology would still work in the same way.

At the same time, assuming all projects are possible to initiate anywhere over the NPP lifetime is not always reasonable. For many projects, a handful of years usually constitute the reasonable time-window of when a project can be performed. This due to regulatory reasons and current project planning practices at FKA. As the problem is formulated to maximize the total NPV of the projects, the rationale of selecting the timing options generating the highest NPV is evident.

6.3 Models

The different models included in the study vary in complexity and thus the results and their relevance to FKA vary as well. By observing the output of the deterministic model, displayed in Table 3, it is evident that the model produces an optimal portfolio. However, due to the lack of incorporated uncertainty, discussed further in Section 4.4.1, the robustness of the solution is questionable. The applicability of a model with no variation is limited as there is valuable information in the analysis of different outcomes. As many measures in a NPP are hard to obtain and in some cases rely on rough estimates, uncertainty is essential to include for a model to be comprehensive. Additional shortcomings of the deterministic model include the heuristic priority list that was extracted by implementing some simple budget scenarios. Although it provides useful guidance on which projects to choose, it is inadequate in explaining the full ranking order of projects as it fails to consistently decide between individual projects.

In the transition towards a full stochastic model, the "what if" model provided more statistical rigor and demonstrated the usefulness of the deterministic model with the aid of Monte Carlo simulation. By continuously varying the uncertain input variables, the results of 10 000 scenarios of the deterministic model produced output with uncertainty incorporated. According to the NPV and risk distributions displayed in Figure 8, it is evident that max NPV and avoided risk converge towards two different portfolio compositions. As the input values, i.e. annual costs, risks, and annual budgets, are varied in rather small intervals the difference between the peaks of max NPV are interesting to observe. The more frequent NPV outcome in the higher peak is almost double that of the lower peak, indicating large potential benefits of slight input variations. It should be noted that the metric distributions depend on the assumptions of the underlying distributions corresponding to the sampling process of each parameter. All parameters are assumed to have normal distributions apart from the budget levels which are sampled from a uniform distribution. Thus the output, especially for NPV and avoided risk, resembles normal distributions where the uniform budget samples have split the distribution into two apparent clusters. Although the most frequent portfolio NPV values are similar according to the distribution, it can be seen in Table 14 in the Appendix 8 that there is some variation in the most frequent portfolio compositions. The variation is mainly in the projects corresponding to lower costs and NPV, as larger projects such as 1, 4, 6, and 8 are chosen consistently through similar options. This indicates that there is less room for flexibility in the asset management process at FKA, as some larger projects will be worth doing at all times. Finally, the priority list obtained from the "what if" model provides a more detailed overview, as can be seen in Table 13 in the Appendix 8. The ranking of each project option gives the DM more support in the project selection. The inclusion of the larger projects is also confirmed as they are chosen in every scenario.

In the full stochastic model, the occurrence of different scenarios is considered in the actual optimization. Thus, each outcome is produced with knowledge of other possible scenarios and their estimated likelihood. As a result, the DM can be considered to have more basis for implementing the portfolio suggested by the model and it should be more relevant in a realistic scenario. However for this to hold in practice, accurate probabilities need to be derived for each scenario, which can be difficult in some cases. Comparing the results of the stochastic model to those of the "what if", it seems to achieve lower values of NPV and avoided risk while the rest risk looks higher in Figure 10. However, observing the values in Table 9, the mean values are almost identical for the two models. The only indication of better performance for the stochastic model is the values corresponding to the most frequent portfolios. As can be seen in Table 16 the values are a lot better than for the corresponding portfolios obtained from the "what if" model. This indicates that the stochastic model obtains better performance and thus is better suited for FKAs asset management applications. However, a comparison between the two should be done with some caution as the stochastic model is sampled from only 1 000 scenarios compared to the 10 000 scenarios used in the "what if" model. Finally, observing the results of the CVaR model in Section 5.4, the higher mean values for NPV and avoided risk indicate that the model is effective in minimizing the worst-case scenarios from consideration and thus obtaining better statistics for each measure. According to the result, there are potential benefits of managing risk through the implementation of CVaR. Although, the statistical robustness of the CVaR model is significantly lower than the stochastic model as the computational complexity limited the number of simulations to 20.

6.4 Restrictions

As previously mentioned in Section 6.2, the selection of a set of timing options is part of the process to handle the combinatorial complexity that arises with a large pool of projects. The effects of this process become increasingly noticeable as the degree of incorporated uncertainty and complexity increased in the models. For the stochastic program, 1 000 scenarios were chosen because 10 000 required a run time of the algorithm which was deemed unreasonable. The "what if" analysis of 10 000 scenarios is computationally acceptable but still requires some run time to complete. For the CVaR model, computational complexity increased more rapidly as the number of simulations where increased and the run time for 20 simulations was several hours long.

It is important to state that the above is true for the use case of 12 projects and 4 options. A real implementation of the FKA portfolio of 100+ projects would thus dramatically increase the computational requirements of these models. Consequently, as the number of projects or options are increased, the general complexity of the model may need to be reduced by regressing to a "what if" analysis or reducing the number of scenarios. Of course, this also depends on the available software and what is deemed as an acceptable run time of the algorithm. Since the scope of this study considers the remaining lifetime of a NPP and projects that span over several years, there is no adamant need of running this algorithm on a high frequency since updates to the plan will rather happen on a yearly basis. As such, the computational requirements of a complete FKA portfolio could perhaps be acceptable for the DMs if the project prioritization process is performed a few times a year. Meanwhile, potential computational mitigation alternatives could be further investigated. For stochastic programs, alternatives such as decomposition methods could be applied to mitigate the problem. Examples are Benders' decomposition and price decomposition, which both can be used in a two-stage stochastic program to help separate variables when both the first stage and second stage need to be considered.

Returning to the NPV calculation of Proposal 1, the presented theoretical method of estimating failure rates with Bayesian inference is dependent on the availability of observed times until failure. Being able to accurately model a distribution for the lifetime variables is highly dependent on having enough data points. Considering that both examples gathered from other studies presented in Tables 1 and 2 contained very few samples, it suggests that obtaining a large dataset for the components may be difficult. This means that having accurate estimates of the failure probabilities may be unfeasible, which would discredit Proposal 1 as it is reliant on these. Further, the KS-test is also dependent on having a large enough sample size to accurately indicate whether the proposed distribution is a good fit for the observations. Thus, a larger set of observations would be desirable to better model the failure distributions and validate their adequacy.

6.5 Future Work at Forsmark

As mentioned in Section 1.2, the optimization phase within asset management is part of a larger initiative at FKA where the outcome of other processes can affect the impact of the optimization model. A prerequisite for the optimization is the data used in the model. The process of obtaining an appropriate data set is grounded in the decisions of what to measure and where to allocate resources to estimate data that is difficult to attain. Essential metrics include accurate project cost forecasts, risk values, and downtime days. The model is flexible to add new data by simply building on the constraints of the model. By including two approaches for calculating NPV, FKA has the option to go with the monetized risk costs or implement a process to estimate the failure rates of components in the NPP. The former is realistic considering the expertise at hand and previous similar estimations of risk. However, there are alternatives for the other option as well. One such option is presented by EPRI, which provide a software for failure rate estimation of systems and components, among other functions.

Looking forward, there is also a need of being able to evaluate the results of an optimization process as proposed in this study compared to current practices in place. This is to understand and identify the actual value creation and risk reduction potential of future implementations. To do this, a collection of historical decision-making regarding projects and associated estimations of risk, cost, and performance is necessary. With this, the novel methodology could be compared with how decisions have been made previously to yield quantitative results.

Finally, the restrictions of the model due to the computational complexity needs to be addressed when the scope of the optimization process have been established. The previous steps of the asset management initiative at FKA will set the boundaries for what will be considered in an optimization. When the nominal values of projects and options to consider have been determined, alternatives to mitigate computational complexity can be investigated as described in Section 6.4.

7 Conclusion

To conclude the study, it can be established that mathematical optimization can be implemented in the asset management process at FKA, and improved to achieve better results when considering metrics such as total NPV, avoided risk, and rest risk. The successive additions to the optimization models have resulted in several model options of different complexity. Although all can be utilized in the project selection at FKA, the final model has the highest potential according to the results. To relate to the research questions of the study, the first question addresses the current field of asset management methods. In the study, the most relevant methods were presented, reviewed, and evaluated based on their relevance for FKA. Available data and mathematical feasibility constituted the most influential criteria, and a mix of RIAM and capital budgeting was deemed suitable to model the problem at hand.

The second research question addressed the extent to which mathematical optimization could be implemented to aid asset management and the possible effects of implementing a new method. Before the optimization could be initialized, data pre-processing proved to be a vital part. The process focused mainly on project valuation and encompassed NPV calculations through two alternative approaches. One option also required additional preparation in the form of a failure rate estimation. However, when developing the optimization models it could be established that all the relevant constraints could be included to develop a model for project selection. Adding some additional elements to the model enabled the incorporation of uncertainty to provide a more robust optimization, first through the "what if" model and then through the stochastic model. Finally, risk could be incorporated quantitatively using CVaR as a risk measure and introducing a risk appetite in the optimization. While the optimization models in the study could be successively developed to provide a more comprehensive decision basis for FKA, it also entailed a significant increase in computational complexity. Thus, future work to be done at FKA, apart from the efforts in the larger asset management initiative, includes actions to reduce or handle the computational complexity to achieve an efficient implementation of the optimization model.

8 Appendix

Table 12: Summary of project option sets used in the optimization model; all values in millions

ProjectNr	Option	Mand.	c1	c2	c3	c4	c5	c6	NPV	BIR	RR	AR
1	A	No	10,4	7,8					50,3	3,4	3,2	71,6
1	B	No		9,7	7,2				46,5	3,1	5,4	68,8
1	C	No			9,0	6,7			31,2	2,1	13,4	60,2
1	Don't do	No							-71,6	0,0	71,6	0,0
2	A	No		2,0	1,3	0,7			2,5	1,6	0,3	6,9
2	B	No	2,2	1,5	0,7				2,3	1,5	0,3	7,0
2	C	No			1,9	1,2	0,6		2,3	1,5	0,5	6,6
2	Don't do	No							-7,0	0,0	7,0	0,0
3	A	Yes				1,8	0,8	0,1	-0,5	0,8	0,3	2,4
3	B	Yes			1,9	0,8	0,1		-0,5	0,8	0,2	2,5
3	C	Yes		2,1	0,9	0,1			-0,7	0,8	0,2	2,6
3	Don't do	Yes							-2,6	0,0	2,6	0,0
4	A	No		16,1	13,2	4,3			40,4	2,1	4,7	78,6
4	B	No	17,4	14,2	4,6				39,4	2,0	4,3	79,9
4	C	No			14,9	12,2	4,0		36,1	1,9	7,7	74,8
4	Don't do	No							-79,9	0,0	79,9	0,0
5	A	No		11,9	1,6				3,7	1,2	1,5	18,8
5	B	No	12,9	1,8					3,0	1,2	1,5	19,1
5	C	No			11,0	1,5			3,1	1,2	2,2	17,8
5	Don't do	No							-19,1	0,0	19,1	0,0
6	A	Yes	4,3						15,8	4,2	0,7	20,7
6	B	Yes		3,9					14,6	3,8	1,4	19,9
6	C	Yes			3,6				10,0	2,4	3,7	17,4
6	Don't do	Yes							-20,7	0,0	20,7	0,0
7	A	No	5,9	12,5	1,4				28,1	2,3	2,0	50,0
7	B	No		5,5	11,6	1,3			27,6	2,3	2,8	48,8
7	C	No			5,1	10,7	1,2		22,2	2,0	6,0	45,2
7	Don't do	No							-50,0	0,0	50,0	0,0
8	A	No	21,3	10,4					60,2	2,8	2,1	93,9
8	B	No		19,7	9,6				58,3	2,8	4,0	91,6
8	C	No			18,2	8,9			15,1	1,3	26,5	68,7
8	Don't do	No							-93,9	0,0	93,9	0,0
9	A	Yes		3,7	1,7				1,6	1,3	0,9	7,9
9	B	Yes			3,5	1,6			1,8	1,3	0,9	7,7
9	C	Yes	4,0	1,8					1,1	1,2	1,0	8,0
9	Don't do	Yes							-8,0	0,0	8,0	0,0
10	A	No		0,9	0,8	0,2			0,1	1,1	0,2	2,2
10	B	No	1,0	0,9	0,2				0,0	1,0	0,2	2,3
10	C	No			0,8	0,8	0,2		0,1	1,0	0,3	2,1
10	Don't do	No							-2,3	0,0	2,3	0,0
11	A	No		13,6	21,3	4,0			27,2	1,6	6,5	72,5
11	B	No	14,6	23,0	4,3				25,8	1,5	6,2	74,0
11	C	No			12,6	19,7	3,7		22,5	1,5	9,6	68,1
11	Don't do	No							-74,0	0,0	74,0	0,0
12	A	No	10,1	7,1	4,6				37,3	2,5	2,5	61,5
12	B	No		9,4	6,5	4,3			34,4	2,4	4,4	59,0
12	C	No			8,7	6,1	4,0		21,7	1,7	11,3	51,7
12	Don't do	No							-61,5	0,0	61,5	0,0

Table 13: Priority list of project options from "what if" analysis of 10 000 scenarios

ProjectNr	Mandatory	Option	Priority	Probability (%)
1	No	A	3	0.7883
		B	29	0.1940
		C	38	0.0177
		Don't do	42	0.0000
2	No	A	20	0.2508
		B	21	0.2466
		C	17	0.2619
		Don't do	23	0.2407
3	Yes	A	5	0.5726
		B	22	0.2423
		C	30	0.1851
		Don't do	42	0.0000
4	No	A	32	0.1375
		B	34	0.0628
		C	2	0.7991
		Don't do	41	0.0006
5	No	A	18	0.2617
		B	24	0.2335
		C	36	0.0236
		Don't do	9	0.4812
6	Yes	A	6	0.5532
		B	11	0.3875
		C	35	0.0593
		Don't do	42	0.0000
7	No	A	16	0.2750
		B	37	0.0180
		C	8	0.4852
		Don't do	27	0.2218
8	No	A	1	0.9105
		B	33	0.0895
		C	42	0.0000
		Don't do	42	0.0000
9	Yes	A	26	0.2259
		B	10	0.3967
		C	12	0.3774
		Don't do	42	0.0000
10	No	A	25	0.2325
		B	14	0.3154
		C	28	0.2010
		Don't do	19	0.2511
11	No	A	42	0.0000
		B	39	0.0091
		C	4	0.6266
		Don't do	13	0.3643
12	No	A	7	0.5523
		B	15	0.2882
		C	31	0.1571
		Don't do	40	0.0024

Table 14: Allocation of most common project portfolio composites from what if analysis of 10 000 scenarios

ProjectNr	Mandatory	Option	Project portfolios				
			1	2	3	4	5
1	No	A	x	x	x	x	x
		B					
		C					
		Don't do					
2	No	A	x		x		
		B					
		C		x			x
		Don't do				x	
3	Yes	A		x		x	x
		B					
		C	x		x		
		Don't do					
4	No	A					
		B					
		C	x	x	x	x	x
		Don't do					
5	No	A		x		x	
		B					x
		C					
		Don't do	x		x		
6	Yes	A		x			x
		B	x		x	x	
		C					
		Don't do					
7	No	A					
		B					
		C		x			x
		Don't do	x		x	x	
8	No	A	x	x	x	x	
		B					x
		C					
		Don't do					
9	Yes	A	x		x		
		B		x		x	
		C					x
		Don't do					
10	No	A			x		
		B	x				x
		C		x			
		Don't do				x	
11	No	A					
		B					
		C	x		x	x	
		Don't do		x			x
12	No	A	x		x	x	x
		B		x			
		C					
		Don't do					

Table 15: Priority list of project options from stochastic analysis of 1 000 scenarios

ProjectNr	Mandatory	Option	Priority	Probability (%)
1	No	A	2	0.8020
		B	30	0.1770
		C	37	0.0210
		Don't do	42	0.0000
2	No	A	15	0.2770
		B	20	0.2450
		C	19	0.2580
		Don't do	27	0.2200
3	Yes	A	6	0.5680
		B	25	0.2260
		C	28	0.2060
		Don't do	42	0.0000
4	No	A	32	0.1410
		B	34	0.0650
		C	3	0.7930
		Don't do	41	0.0010
5	No	A	18	0.2600
		B	21	0.2330
		C	37	0.0210
		Don't do	8	0.4860
6	Yes	A	7	0.5470
		B	11	0.3980
		C	35	0.0550
		Don't do	42	0.0000
7	No	A	15	0.2770
		B	36	0.0230
		C	9	0.4710
		Don't do	23	0.2290
8	No	A	1	0.9060
		B	33	0.0940
		C	42	0.0000
		Don't do	42	0.0000
9	Yes	A	24	0.2280
		B	10	0.4150
		C	13	0.3570
		Don't do	42	0.0000
10	No	A	22	0.2310
		B	13	0.3570
		C	29	0.1870
		Don't do	26	0.2250
11	No	A	42	0.0000
		B	39	0.0050
		C	4	0.6300
		Don't do	12	0.3650
12	No	A	5	0.5690
		B	17	0.2680
		C	31	0.1610
		Don't do	40	0.0020

Table 16: Allocation of most common project portfolio composites from stochastic analysis of 1 000 scenarios

ProjectNr	Mandatory	Option	Project portfolios				
			1	2	3	4	5
1	No	A	x	x	x	x	x
		B					
		C					
		Don't do					
2	No	A	x	x			
		B				x	x
		C					
		Don't do			x		
3	Yes	A			x	x	x
		B		x			
		C	x				
		Don't do					
4	No	A					
		B					
		C	x	x	x	x	
		Don't do					x
5	No	A			x	x	
		B					
		C					
		Don't do	x	x			x
6	Yes	A				x	x
		B	x	x	x		
		C					
		Don't do					
7	No	A		x			
		B					
		C				x	x
		Don't do	x		x		
8	No	A	x	x	x	x	x
		B					
		C					
		Don't do					
9	Yes	A	x			x	
		B		x	x		x
		C					
		Don't do					
10	No	A					x
		B	x	x		x	
		C					
		Don't do			x		
11	No	A					
		B					
		C	x	x	x		x
		Don't do				x	
12	No	A	x	x	x		x
		B					
		C				x	
		Don't do					

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