Needell_Coen_HW3

February 9, 2020

[1]:
```
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
```

0.1 # Conceptual Exercises

0.2 ## Training/test error for subset selection

0.2.1 1

[2]:
```
np.random.seed(666)
# sd is 5 because I think it'll make the numbers more interesting.
df1 = {f'x{n}':np.random.normal(0, 5, 1000) for n in range(1, 21)}
df1 = pd.DataFrame(df1)
epsilon = np.random.normal(0, 5, 1000)

bzeros = np.random.randint(1, 20, 5) # decides random betas to be zero
betas = {f'x{n}': (np.random.normal(0, 5)
                   if n not in bzeros else 0) for n in range(1, 21)}

y = np.zeros(1000)
for i in range(1, 21):
    y += df1[f'x{i}'] * betas[f'x{i}']
y += epsilon
```

0.2.2 2

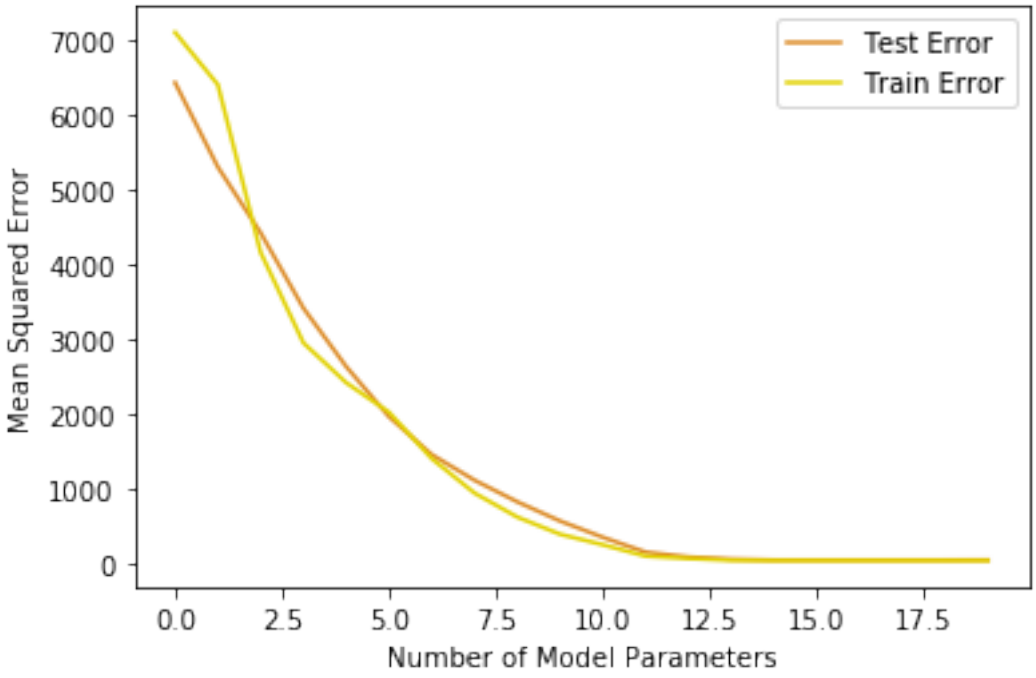
[3]:
```
from sklearn.model_selection import train_test_split

x_train, x_test, y_train, y_test = train_test_split(df1, y, test_size=900)
```

0.2.3 3

[4]:
```
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error as mse
from itertools import combinations
from functools import partial
import wes
```

[5]:
```
def process_subset(feats, xtr, xte, ytr, yte, returnmodel=False):
    train_feats = xtr[feats]
    test_feats = xte[feats]
    model = LinearRegression()
    model.fit(train_feats, ytr)
    if returnmodel:
        return model
    errte = mse(yte, model.predict(test_feats))
    errtr = mse(ytr, model.predict(train_feats))
    return errtr, errte

# I'm using a forward_stepwise algorithm, assuming that the features
# have some best order which is independent of other features.
# That assumption is baked into linear models though, so it's safe here.
# Also we know the data generating function, and we know that they're
# independant, so we know that both algorithms go to the same result.

# Also 2 ** 20 = 1,000,000 which is a little out-of-feasibility for python.
def forward_stepwise(xtr, xte, ytr, yte):
    n = len(xtr.keys())
    model = set()  # This is a set to which we will add features.
    params = []
    testerrs = []
    trainerrs = []
    models = []
    # This is a list where we'll record
    #the best process_subset outputs

    for k in range(0, n):
        augs = [str(st) for st in xtr.keys() if st not in model]
        results = []
        for x in augs:
            results.append(process_subset(list(model) + [x],
                           xtr, xte, ytr, yte))
        results = np.array(results)
        best = np.argmin(results[:, 1])
        model.add(augs[best])
        betr, bete = results[best]
        params.append(augs[best])
        testerrs.append(bete)
        trainerrs.append(betr)
    record = pd.DataFrame({"param":params,
                           "test_err":testerrs,
                           "train_err":trainerrs})
    return record

best_selec = forward_stepwise(x_train, x_test, y_train, y_test)
```

[7]:
```
wes.set_palette('FantasticFox1')
plt.plot(best_selec.test_err, label='Test Error')
plt.plot(best_selec.train_err, label='Train Error')
plt.xlabel('Number of Model Parameters')
plt.ylabel('Mean Squared Error')
plt.legend()
plt.show()
best_selec
```



[7]:

	param	test_err	train_err
0	x8	6416.784511	7089.241271
1	x16	5289.490369	6390.208384
2	x11	4409.229291	4147.456895
3	x12	3402.061119	2937.561717
4	x19	2620.542325	2405.103896
5	x9	1950.391660	2010.545592
6	x3	1443.183115	1389.383800
7	x10	1100.965204	928.892155
8	x15	811.961551	608.203719
9	x5	555.418121	378.544639
10	x20	337.576521	238.096790
11	x2	138.647058	85.752510
12	x17	74.697425	54.693290
13	x7	45.602417	30.619249
14	x1	34.774484	27.710232

```
15    x6    34.737711    27.688195
16   x13    34.740764    27.671565
17   x18    34.782783    27.651672
18   x14    35.063060    27.478137
19    x4    36.026689    27.269170
```

Unsurprisingly, it starts to even out around 13. If we look at the list of betas it's easy to see why, we know that 5 of the variables are useless, and another two have $|\beta| < 1$. If you look at the parameter that's added in for each step, and compare that to the list of betas, you can see the logic of it. The betas which are set to zero are associated with parameters that are included last.

[8]:
```
pd.DataFrame(betas, index=['beta']).transpose()
```

[8]:
```
          beta
x1  -0.717342
x2  -2.844386
x3  -4.660543
x4   0.000000
x5  -2.933093
x6  -0.002561
x7   1.038511
x8  -9.030795
x9  -5.204751
x10  3.710081
x11  6.849511
x12  6.229572
x13  0.000000
x14  0.000000
x15  3.321539
x16  5.709028
x17  1.531583
x18  0.000000
x19 -5.046486
x20 -2.791455
```

The high efficiency models (15 and 14) remove x1 and x6 plus the zeroes. Then the zeroes are added back in in order.

0.2.4 4

See plot for # 3. ### 5

[9]:
```
best_test = np.argmin(best_selec.test_err)
m = ','.join(best_selec.param[:best_test].sort_values())
m
```

[9]:
```
'x1,x10,x11,x12,x15,x16,x17,x19,x2,x20,x3,x5,x7,x8,x9'
```

So the best model is the model with 16 parameters. Why exactly it's at this number of parameters and not smaller or larger is somewhat interesting. We're measuring performance on the test set, for one, and chances are good that noise effects detected in the additional variables after the 16th are causing some overfitting when those variables are in the linear model. ### 6 We see that the best model contains all of the features except for those which have coefficents closest to zero. Give or take a few. Because of noise, we see one of the coefficient-zero parameters in the mix, as well as a close to zero parameter treated as if it's coefficient is zero.

[10]:
```
feats = best_selec.param[:best_test]
m = process_subset(feats, x_train, x_test, y_train, y_test, returnmodel=True)
coef_info = pd.DataFrame({'feats': feats, 'coefs': m.coef_, 'betas':␣
 ↪[betas[feat] for feat in feats]})
coef_info
```

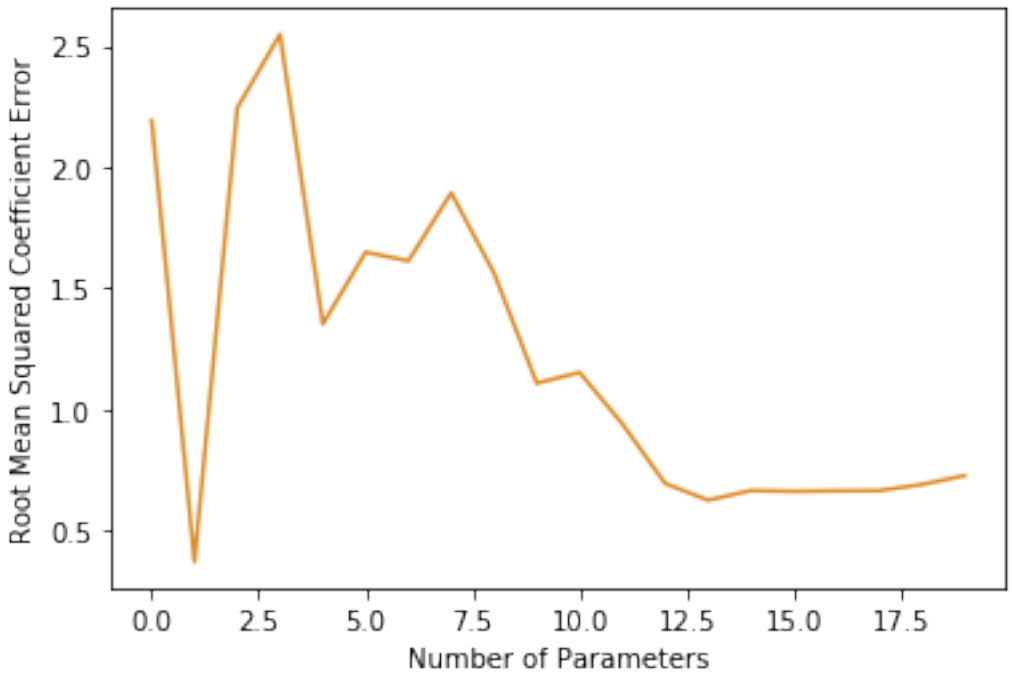
[10]:

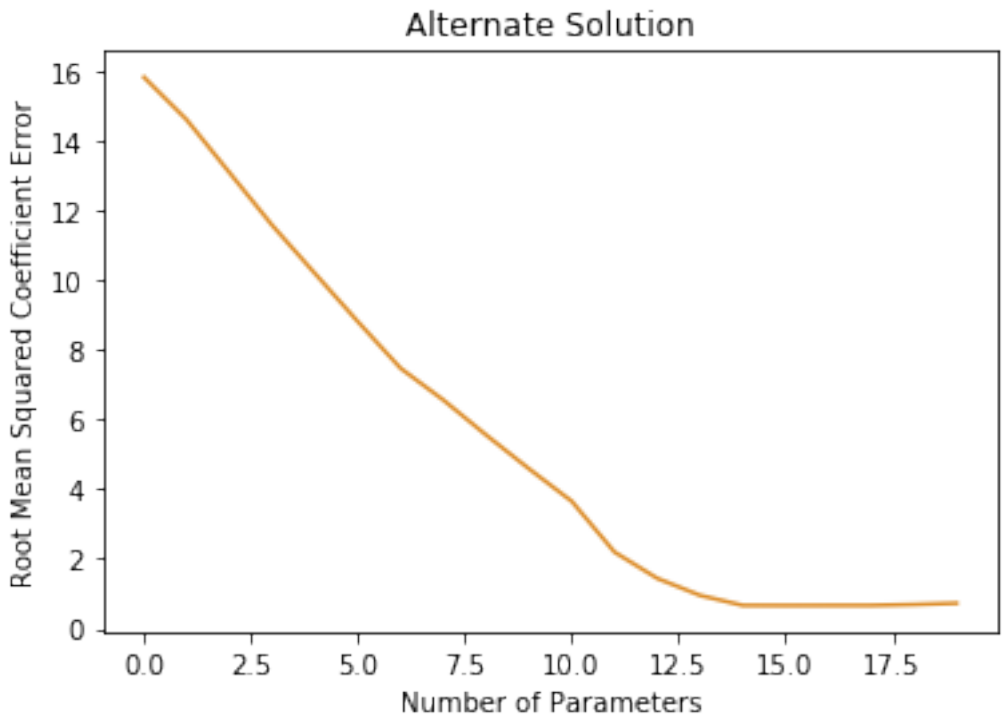
	feats	coefs	betas
0	x8	-9.423881	-9.030795
1	x16	5.525469	5.709028
2	x11	6.976860	6.849511
3	x12	6.230386	6.229572
4	x19	-4.830579	-5.046486
5	x9	-5.160344	-5.204751
6	x3	-4.709259	-4.660543
7	x10	3.847959	3.710081
8	x15	3.447054	3.321539
9	x5	-3.039797	-2.933093
10	x20	-2.759123	-2.791455
11	x2	-2.932852	-2.844386
12	x17	1.387939	1.531583
13	x7	0.886101	1.038511
14	x1	-0.424171	-0.717342

0.2.5 7

[13]:
```
rmcoefaccs = []
for r in range(1, 21):
    feats = best_selec.param[:r]
    m = process_subset(feats, x_train, x_test, y_train, y_test,␣
 ↪returnmodel=True)
    bhat = m.coef_
    b = np.array([betas[f] for f in feats])
    inner = (b - bhat) ** 2
    outer = np.sqrt(np.sum(inner))
    rmcoefaccs.append(outer)

plt.plot(rmcoefaccs)
plt.ylabel('Root Mean Squared Coefficient Error')
plt.xlabel('Number of Parameters')
```

```
plt.show()


rmcoefaccs2 = []
for r in range(1, 21):
    feats = best_selec.param[:r]
    m = process_subset(feats, x_train, x_test, y_train, y_test,␣
 ↪returnmodel=True)
    coefs = {f: co for f, co in zip(feats, m.coef_)}
    bhat = np.array([coefs[f] if f in coefs else 0 for f in betas])
    b = np.array(list(betas.values()))
    inner = (b - bhat) ** 2
    outer = np.sqrt(np.sum(inner))
    rmcoefaccs2.append(outer)

plt.plot(rmcoefaccs2)
plt.ylabel('Root Mean Squared Coefficient Error')
plt.xlabel('Number of Parameters')
plt.title('Alternate Solution')
plt.show()
```





The first thing we notice is this huge hump between 5 parameters and 10. This is due to the fact that the linear regression algorithm is using the available parameters to spuriously explain other parameters. We can compare this to the MSE plot where the error pretty much declines steadily as we add in more parameters, in this case we see a sort of distribution of error, before we get into the more accurate models about $p = 15$.

Since there was some widespread confusion about the meaning of this problem, I've included an alternate answer for the case where the Root Mean Squared Coefficient Error should include features that are not in the model. For these cases I have set $\hat{\beta}$ to zero for betas which are not included in the model. In this case it's easier to see that the model is off. But this information seems trivial. If you're judging the model based on things that aren't even included, then you just end up with a value that's remarkably similar to MSE. This measure is artificially inflated for smaller models, whereas the first graph shows how close to 'correct' the model gets for the features that are included.

0.3 # Application Exercises

Please note that `sklearn` calls the λ parameter α. And the α parameter l_1. I will use this convention.

[15]:
```
from sklearn.linear_model import RidgeCV, LassoCV, ElasticNetCV
gss_tr = pd.read_csv('data/gss_train.csv')
gss_te = pd.read_csv('data/gss_test.csv')
x_tr = gss_tr.drop('egalit_scale', axis=1)
y_tr = gss_tr.egalit_scale
```

```
x_te = gss_te.drop('egalit_scale', axis=1)
y_te = gss_te.egalit_scale
```

0.3.1 1

[16]:
```
def report_mse(model):
    model.fit(x_tr, y_tr)
    err = mse(model.predict(x_te), y_te)
    return err, model
```

[17]:
```
print(report_mse(LinearRegression())[0])
```

```
63.213629623014995
```

0.3.2 2

[18]:
```
print(report_mse(RidgeCV(alphas=np.logspace(-5, 5)))[0])
```

```
62.202521583380204
```

A little better...

[19]:
```
mes, mod = report_mse(LassoCV(alphas=np.logspace(-5, 5)))
print(f'mse is {mes}')
print(f'There are {sum(np.isclose(mod.coef_, 0))} non-zero coefficent␣
 ↪estimates')
```

```
mse is 62.90446289883912
There are 54 non-zero coefficent estimates
```

woof

[21]:
```
mes, mod = report_mse(ElasticNetCV(l1_ratio=np.linspace(.1,1,11)))
print(f'mse is {mes}')
print(f'There are {sum(np.isclose(mod.coef_, 0))} non-zero coefficient␣
 ↪estimates')
```

```
mse is 62.81645879274796
There are 54 non-zero coefficient estimates
```

Gotta say, we're not exactly killing it on the accuracy front. The feature ranges from 1 to 35, and the best of these methods, `RidgeCV`, was off by 7.8 points on average. Which I guess isn't so bad, but a more complicated method would, I'm sure, do a lot better. One of the interesting things from this exercise is that all of these MSEs are within 1 point of each other, which tells us that the regression method isn't having too much of an effect on the results. This implies that what we really need is a non-linear model, as we've probably hit the upper bound on what is possible with linear regressions.

I do also find it interesting that the Elastic Net considers more coefficients than the Lasso.