REPORTE:
Diagramas de Bruijn y ecuaciones

Sistemas Complejos

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Capítulo 1

Diagramas de Bruijn

Los diagramas de Bruijn proporcionan una manera conveniente de describir las configuraciones de autómatas celulares (CA).

Sea Σ el alfabeto y $s \geq 1$ un número, y el grafo de Bruijn $B(s, \Sigma)$ como sigue: $B(s, \Sigma)$ tiene un conjunto de vértices Σ^s y vertices (ax, xb) para todo $a, b \in \Sigma, x \in \Sigma^s$.

Para calcular estos tuve que mostrar las reglas de una manera que me permitiera facilmente crear los diagramas, para eso cree este pequeño programa en `C++` que me permite mostrarlas de una manera comoda ss:

```cpp
#include <bitset>
#include <cstdint>
#include <iostream>
using namespace std;

auto show_rules(const uint8_t rule_id) {
  const auto rule = bitset<8>{rule_id};

  cout << "Rule " << rule.to_ulong() << endl;
  for (auto i = 0; i < 8; ++i)
    cout << bitset<3>(i) << " -> " << rule[i] << endl;

  cout << endl;
}

auto main() -> int {
  const auto rules = {15, 22, 30, 54, 90, 110};
  for (const auto rule : rules)
    show_rules(rule);

  return 0;
}
```
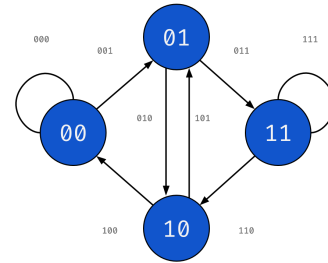
Dando este resultado:

```
Rule 15        Rule 22        Rule 30        Rule 54        Rule 90        Rule 110
000 -> 1       000 -> 0       000 -> 0       000 -> 0       000 -> 0       000 -> 0
001 -> 1       001 -> 1       001 -> 1       001 -> 1       001 -> 1       001 -> 1
010 -> 1       010 -> 1       010 -> 1       010 -> 1       010 -> 1       010 -> 1
011 -> 1       011 -> 0       011 -> 1       011 -> 0       011 -> 1       011 -> 1
100 -> 0       100 -> 1       100 -> 1       100 -> 1       100 -> 1       100 -> 0
101 -> 0       101 -> 0       101 -> 0       101 -> 1       101 -> 0       101 -> 1
110 -> 0       110 -> 0       110 -> 0       110 -> 0       110 -> 1       110 -> 1
111 -> 0       111 -> 0       111 -> 0       111 -> 0       111 -> 0       111 -> 0
```

Podemos editarlo un poco para saber el valor de las conexiones del diagrama:

```cpp
#include <bitset>
#include <cstdint>
#include <iostream>
using namespace std;

auto show_rules(const uint8_t rule_id) {
  const auto rule = bitset<8>{rule_id};

  cout << "Rule " << rule.to_ulong() << endl;
  for (auto i = 0; i < 8; ++i) {
    const auto total = bitset<3>(i).to_string();
    const auto start = total.substr(0, 2);
    const auto end = total.substr(1, 2);
    cout << start << " to " << end << " -> " << rule[i] << endl;
  }
  cout << endl;
}

auto main() -> int {
  const auto rules = {15, 22, 30, 54, 90, 110};
  for (const auto rule : rules)
    show_rules(rule);

  return 0;
}
```

```
Rule 15             Rule 30             Rule 90
00 to 00 -> 1       00 to 00 -> 0       00 to 00 -> 0
00 to 01 -> 1       00 to 01 -> 1       00 to 01 -> 1
01 to 10 -> 1       01 to 10 -> 1       01 to 10 -> 0
01 to 11 -> 1       01 to 11 -> 1       01 to 11 -> 1
10 to 00 -> 0       10 to 00 -> 1       10 to 00 -> 1
10 to 01 -> 0       10 to 01 -> 0       10 to 01 -> 0
11 to 10 -> 0       11 to 10 -> 0       11 to 10 -> 1
11 to 11 -> 0       11 to 11 -> 0       11 to 11 -> 0

Rule 22             Rule 54             Rule 110
00 to 00 -> 0       00 to 00 -> 0       00 to 00 -> 0
00 to 01 -> 1       00 to 01 -> 1       00 to 01 -> 1
01 to 10 -> 1       01 to 10 -> 1       01 to 10 -> 1
01 to 11 -> 0       01 to 11 -> 0       01 to 11 -> 1
10 to 00 -> 1       10 to 00 -> 1       10 to 00 -> 0
10 to 01 -> 0       10 to 01 -> 1       10 to 01 -> 1
11 to 10 -> 0       11 to 10 -> 0       11 to 10 -> 1
11 to 11 -> 0       11 to 11 -> 0       11 to 11 -> 0
```

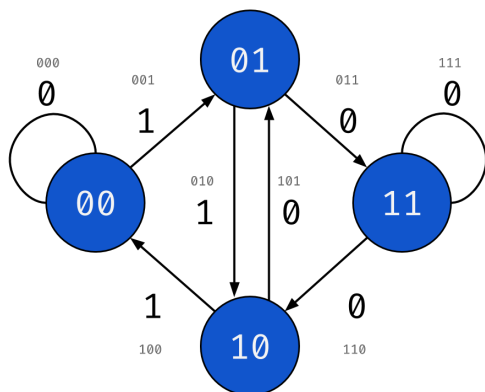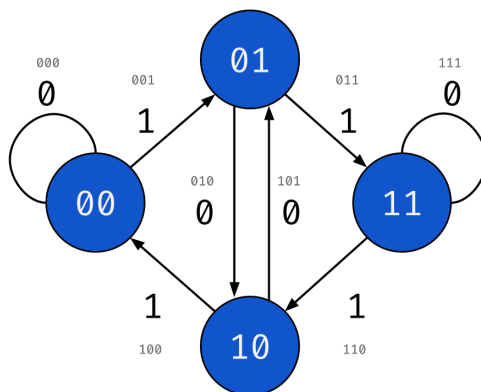Con esto listo podemos calcular facilmente los diagramas de Bruijn.

1.1.   Regla 15
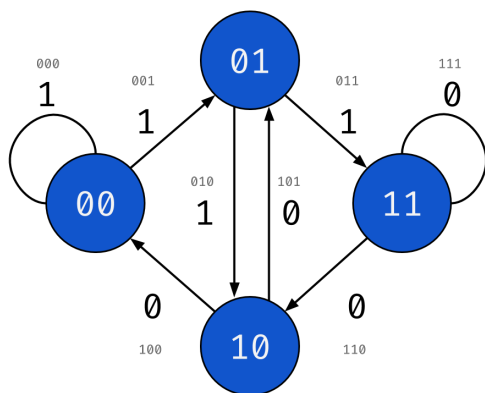
1.2.   Regla 22

1.3.   Regla 30

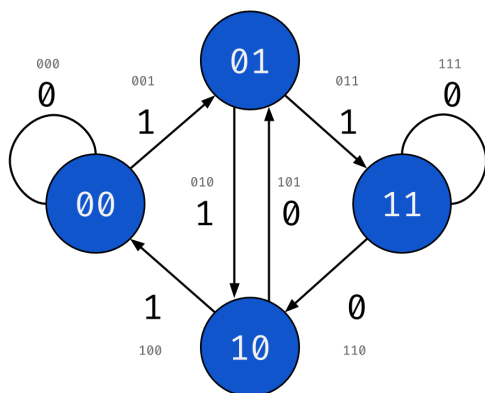## 1.4.   Regla 54
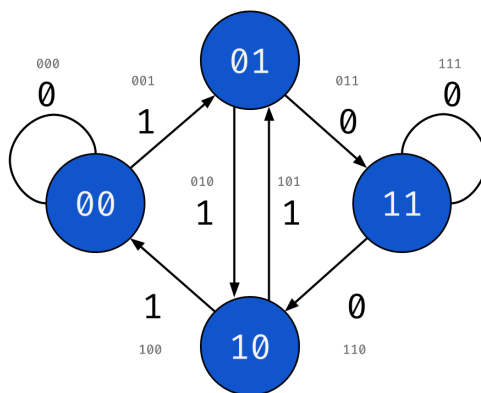
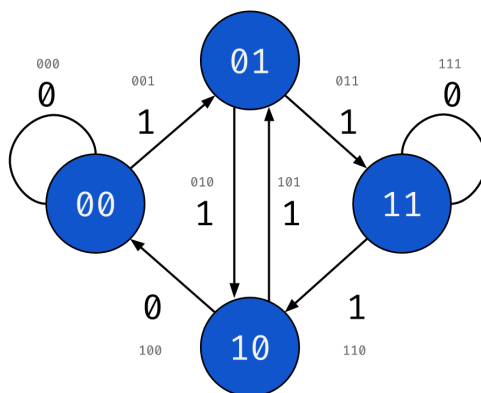## 1.5.   Regla 90

1.6.   Regla 110

Capítulo 2

Ecuaciones simbólicas (expresiones regulares)

Una vez que construyeron los diagramas podemos calcular todas las ecuaciones simbólicas (expresiones regulares). Hasta calcular la ecuación que representa todo el autómata.

La expresión mas importante es:

$$R_{i,j}^k = R_{i,j}^{k-1} + \ R_{i,k}^{k-1} \ (R_{k,k}^{k-1})^* \ R_{k,j}^{k-1} \tag{2.1}$$

Podemos crear un programa que se encarge de generar estas ecuaciones:

```
const nonEmpty = (x: string) => (x === "" ? "∅" : x);

const parentesis = (x: string) => {
  if (x.length < 2) return x;
  if (x[x.length - 1] === ")") return x;
  if (x.split("").every(e => e === "1" || e === "0")) return x;

  return `(${x})`;
};

const or = (x: Array<string>) => {
  const elements = x.filter(e => e).filter(e => e !== "∅");
  return [...new Set(elements)].join(" + ");
};

const and = (x: Array<string>) => {
  if (x.some(e => e === "∅")) return "∅";
  return x
    .filter(e => e)
    .filter(e => e !== "ε")
    .filter(e => e !== "ε^*")
    .map(e => parentesis(e))
    .join("");
};

const kleeneClosure = (x: string) => {
```

```typescript
  const data = x.split(" + ");
  const result = data.filter(e => e !== "ε").join(" + ");

  if (!result) return "";

  return `(${result}^*)`;
};

const memo = [
  ["", "", "", ""],
  ["", "", "", ""],
  ["", "", "", ""],
  ["", "", "", ""]
];

memo[0][0] = "0";
memo[0][1] = "1";
memo[1][2] = "1";
memo[1][3] = "0";
memo[2][0] = "1";
memo[2][1] = "0";
memo[3][2] = "0";
memo[3][3] = "0";

memo[1][1] = "ε";
memo[2][2] = "ε";

memo[0][2] = "∅";
memo[0][3] = "∅";
memo[1][0] = "∅";
memo[2][3] = "∅";
memo[3][0] = "∅";
memo[3][1] = "∅";

memo[0][0] = memo[0][0] + " + ε";
memo[3][3] = memo[3][3] + " + ε";

function R(i: number, j: number, k: number): string {
  if (k === 0) return memo[i][j];

  //console.log(`${a}+${b}(${c})^*${d}`);
  const a = R(i, j, k - 1);

  const b = parentesis(R(i, k, k - 1));
  const c = R(k, k, k - 1);
  let d = parentesis(R(k, j, k - 1));

  if (c === "ε" && d === "ε") d = "";

  const concat = and([b, kleeneClosure(c), d]);
  return nonEmpty(or([a, concat]));
}

for (let k = 0; k < 4; ++k) {
  for (let i = 0; i < 4; ++i) {
    for (let j = 0; j < 4; ++j) {
      console.log(`R(${i}, ${j}, ${k}) = ` + R(i, j, k));
    }
  }
  console.log();
}
```

Logrando estas ecuaciones para la regla 22:

```
R(0, 0, 0) = 0 + ε
R(0, 1, 0) = 1
R(0, 2, 0) = ∅
R(0, 3, 0) = ∅
R(1, 0, 0) = ∅
R(1, 1, 0) = ε
R(1, 2, 0) = 1
R(1, 3, 0) = 0
R(2, 0, 0) = 1
R(2, 1, 0) = 0
R(2, 2, 0) = ε
R(2, 3, 0) = ∅
R(3, 0, 0) = ∅
R(3, 1, 0) = ∅
R(3, 2, 0) = 0
R(3, 3, 0) = 0 + ε

R(0, 0, 1) = 0 + ε
R(0, 1, 1) = 1
R(0, 2, 1) = 11
R(0, 3, 1) = 10
R(1, 0, 1) = ∅
R(1, 1, 1) = ε
R(1, 2, 1) = 1
R(1, 3, 1) = 0
R(2, 0, 1) = 1
R(2, 1, 1) = 0
R(2, 2, 1) = ε + 01
R(2, 3, 1) = 00
R(3, 0, 1) = ∅
R(3, 1, 1) = ∅
R(3, 2, 1) = 0
R(3, 3, 1) = 0 + ε

R(0, 0, 2) = 0 + ε + 11(01^*)1
R(0, 1, 2) = 1 + 11(01^*)0
R(0, 2, 2) = 11 + 11(01^*)(ε + 01)
R(0, 3, 2) = 10 + 11(01^*)00
R(1, 0, 2) = 1(01^*)1
R(1, 1, 2) = ε + 1(01^*)0
R(1, 2, 2) = 1 + 1(01^*)(ε + 01)
R(1, 3, 2) = 0 + 1(01^*)00
R(2, 0, 2) = 1 + (ε + 01)(01^*)1
R(2, 1, 2) = 0 + (ε + 01)(01^*)0
R(2, 2, 2) = ε + 01 + (ε + 01)(01^*)(ε + 01)
R(2, 3, 2) = 00 + (ε + 01)(01^*)00
R(3, 0, 2) = 0(01^*)1
R(3, 1, 2) = 0(01^*)0
R(3, 2, 2) = 0 + 0(01^*)(ε + 01)
R(3, 3, 2) = 0 + ε + 0(01^*)00

R(0, 0, 3) = 0 + ε + 11(01^*)1 + (10 + 11(01^*)00)(0 + 0(01^*)00^*)(0(01^*)1)
R(0, 1, 3) = 1 + 11(01^*)0 + (10 + 11(01^*)00)(0 + 0(01^*)00^*)(0(01^*)0)
R(0, 2, 3) = 11 + 11(01^*)(ε + 01) + (10 + 11(01^*)00)(0 + 0(01^*)00^*)0 + 0(01^*)(ε
    + 01)
R(0, 3, 3) = 10 + 11(01^*)00 + (10 + 11(01^*)00)(0 + 0(01^*)00^*)(0 + ε + 0(01^*)00)
R(1, 0, 3) = 1(01^*)1 + (0 + 1(01^*)00)(0 + 0(01^*)00^*)(0(01^*)1)
R(1, 1, 3) = ε + 1(01^*)0 + (0 + 1(01^*)00)(0 + 0(01^*)00^*)(0(01^*)0)
R(1, 2, 3) = 1 + 1(01^*)(ε + 01) + (0 + 1(01^*)00)(0 + 0(01^*)00^*)0 + 0(01^*)(ε + 01)
R(1, 3, 3) = 0 + 1(01^*)00 + (0 + 1(01^*)00)(0 + 0(01^*)00^*)(0 + ε + 0(01^*)00)
R(2, 0, 3) = 1 + (ε + 01)(01^*)1 + (00 + (ε + 01)(01^*)00)(0 + 0(01^*)00^*)(0(01^*)1)
R(2, 1, 3) = 0 + (ε + 01)(01^*)0 + (00 + (ε + 01)(01^*)00)(0 + 0(01^*)00^*)(0(01^*)0)
R(2, 2, 3) = ε + 01 + (ε + 01)(01^*)(ε + 01) + (00 + (ε + 01)(01^*)00)(0 +
    0(01^*)00^*)0 + 0(01^*)(ε + 01)
R(2, 3, 3) = 00 + (ε + 01)(01^*)00 + (00 + (ε + 01)(01^*)00)(0 + 0(01^*)00^*)(0 + ε +
```

```
    0(01^*)00)
R(3, 0, 3) = 0(01^*)1 + (0 + ε + 0(01^*)00)(0 + 0(01^*)00^*)(0(01^*)1)
R(3, 1, 3) = 0(01^*)0 + (0 + ε + 0(01^*)00)(0 + 0(01^*)00^*)(0(01^*)0)
R(3, 2, 3) = 0 + 0(01^*)(ε + 01) + (0 + ε + 0(01^*)00)(0 + 0(01^*)00^*)0 + 0(01^*)(ε
    + 01)
R(3, 3, 3) = 0 + ε + 0(01^*)00 + (0 + ε + 0(01^*)00)(0 + 0(01^*)00^*)(0 + ε +
    0(01^*)00)
```

[3]

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