Compilando Conocimiento

Tarea Random

Análisis Númerico

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1. Interpolacion por Hermite

1.1. Ideas previas

Recordemos que:

$$H_{2n+1}(x) = \sum_{j=0}^{n} f(x_j)H_j(x) + \sum_{j=0}^{n} f'(x_j)\hat{H}_j(x)$$

Donde:

- $H_j(x) = (1 - 2(x - x_j)L_j'(x_j))L_j^2(x)$
- $\hat{H}_j(x) = (x - x_j)L_j^2(x)$

1.2. Ejemplo

Sea $f(x) = e^{2x}$, por lo tanto $f'(x) = 2e^x$ Entonces:

- $x_0 = 0$
- $x_1 = 1$
- $x_2 = 2$

- $f(x_0) = 1$
- $f(x_1) = e^2$
- $f(x_2) = e^4$

- $f'(x_0) = 2$
- $f'(x_1) = 2e^2$
- $f'(x_2) = 2e^4$

Entonces veamos como se ve el interpolante de Hermite, para hacerlo tenemos que calcular $H_0, H_1, H_2, \hat{H}_0, \hat{H}_1, \hat{H}_2$.

Vamos paso por paso:

$$\begin{aligned} L_0(x) &= \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \\ &= \frac{(x-1)(x-2)}{(0-1)(0-2)} \\ &= \frac{(x-1)(x-2)}{2} \\ &= \frac{x^2-3x+2}{2} \end{aligned}$$

Ahora:

$$\begin{aligned} L_0'(x) &= (\frac{x^2-3x+2}{2})' \\ &= \frac{1}{2}(x^2-3x+2)' \\ &= \frac{1}{2}(2x-3) \end{aligned}$$

$$L_0'(0) = \frac{-3}{2}$$

$$\begin{aligned} L_1(x) &= \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \\ &= \frac{(x-0)(x-2)}{(1-0)(1-2)} \\ &= \frac{(x-0)(x-2)}{-1} \\ &= \frac{x^2-2x}{-1} \end{aligned}$$

Ahora:

$$\begin{aligned} L_1'(x) &= (\frac{x^2-2x}{-1})' \\ &= \frac{1}{-1}(x^2-2x)' \\ &= \frac{1}{-1}(2x-2) \end{aligned}$$

$$L_1'(1) = 0$$

$$\begin{aligned} L_2(x) &= \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \\ &= \frac{(x-0)(x-1)}{(2-0)(2-1)} \\ &= \frac{x^2-x}{2} \end{aligned}$$

Ahora:

$$\begin{aligned} L_2'(x) &= (\frac{x^2-x}{2})' \\ &= \frac{1}{2}(x^2-x)' \\ &= \frac{1}{2}(2x-1) \end{aligned}$$

$$L_2'(2) = \frac{3}{2}$$

Ahora si que podemos hacer las $H_i(x)$

- $H_0(x)$ y $\hat{H}_0(x)$

$$\begin{aligned} H_0(x) &= (1-2(x-x_0)L_0'(x_0))L_0^2(x) \\ &= (1-2(x-0)L_0'(0))L_0^2(x) \\ &= (1-2(x-0)\frac{-3}{2})L_0^2(x) \\ &= (1+3x)\left(\frac{x^2-3x+2}{2}\right)^2 \end{aligned}$$

$$\begin{aligned} \hat{H}_0(x) &= (x-0)L_0^2(x) \\ &= (x)\left(\frac{x^2-3x+2}{2}\right)^2 \end{aligned}$$

- $H_1(x)$ y $\hat{H}_1(x)$

$$\begin{aligned} H_1(x) &= (1-2(x-x_1)L_1'(x_1))L_1^2(x) \\ &= (1-2(x-1)L_1'(1))L_1^2(x) \\ &= (1-2(x-1)0)L_1^2(x) \\ &= L_1^2(x) \\ &= (x^2-2x)^2 \end{aligned}$$

$$\begin{aligned} \hat{H}_1(x) &= (x-1)L_1^2(x) \\ &= (x-1)(x^2-2x)^2 \end{aligned}$$

- $H_2(x)$ y $\hat{H}_2(x)$

$$\begin{aligned} H_2(x) &= (1-2(x-x_2)L_2'(x_2))L_2^2(x) \\ &= (1-2(x-2)L_2'(2))L_2^2(x) \\ &= (1-2(x-2)\frac{3}{2})L_2^2(x) \\ &= (1-2(x-2)\frac{3}{2})\left(\frac{x^2-x}{2}\right)^2 \end{aligned}$$

$$\begin{aligned} \hat{H}_2(x) &= (x-2)L_2^2(x) \\ &= (x-2)\left(\frac{x^2-x}{2}\right)^2 \end{aligned}$$

$$
\begin{aligned}
H_{2n+1}(x) &= \sum_{j=0}^{n} f(x_j)H_j(x) + \sum_{j=0}^{n} f'(x_j)\hat{H}_j(x) \\
&= f(x_0)H_0(x) + f(x_1)H_1(x) + f(x_2)H_2(x) + f'(x_0)\hat{H}_0(x) + f'(x_1)\hat{H}_1(x) + f'(x_2)\hat{H}_2(x) \\
&= 1\left(H_0(x)\right) + e^2\left(H_1(x)\right) + e^4\left(H_2(x)\right) + 2\left(\hat{H}_0(x)\right) + 2e^2\left(\hat{H}_1(x)\right) + 2e^4\left(\hat{H}_2(x)\right) \\
&= 1\left((1+3x)\left(\frac{x^2-3x+2}{2}\right)^2\right) + e^2\left((x^2-2x)^2\right) + e^4\left((1-2(x-2)\frac{3}{2})\left(\frac{x^2-x}{2}\right)^2\right) \\
&+ 2\left((x)\left(\frac{x^2-3x+2}{2}\right)^2\right) + 2e^2\left((x-1)(x^2-2x)^2\right) + 2e^4\left((x-2)\left(\frac{x^2-x}{2}\right)^2\right)
\end{aligned}
$$