
FACULTAD DE CIENCIAS

Tarea 3

ANÁLISIS NÚMÉRICO

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Índice

1. Problemas de Computadora	2
1.1. 23	3
1.2. 24	4
1.3. 25	6
1.4. 26	7
1.5. 27	9
 2. Anexo	 11
2.1. Givens	11
2.2. GramSchmidt	12
2.3. HouseHolder	13
2.4. LeastSquares	13
2.5. BackwardSubstitution	14
2.6. CholeskyBanachiewicz	15
2.7. CholeskyGaussian	16
2.8. CompleteLUdecomposition	17
2.9. ForwardSubstitution	18
2.10. LUdecomposition	19

1. Problemas de Computadora

Una nota importante es que al inicio de CADA script se incluyen los algoritmos, porfavor cambia la primera linea de cada script para que el path sea el correcto, porfavor.

Esta linea:

```
1 getd( '/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework2/Code/Algorithms' )
```

Para ejecutar cada uno basta con hacer algo como:

```
1 exec( "/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework2/Code/22a.sce" , -1)
```

1.1. 23

Ejecuta los scripts que esta dentro de Code llamado: 23.sce

En este código muestra justo lo que se nos pide, por el método de ecuaciones normales veremos la solución aproximada para cada una de las 2 b que nos dan y vemos a aunque son muy parecidas los resultados son completamente diferentes, y si, aunque puede parecer que o el algoritmo esta mal o algo raro acaba de pasar basta con dar un vistazo a la matriz A y verla como lo que la estamos interpretando, un conjunto de vectores.

Recuerda que lo que estamos haciendo es encontrar una combinación lineal de estos vectores para aproximar a nuestros vectores b .

Pues si nuestros vectores no son ortogonales lo que pasa es que un pequeño cambio en el resultado nos da una combinación lineal muy diferente. Es justo lo que esta pasando estamos trabajando con vectores muy parecidos.

La mejor forma de explicar este fenomeno y mostrar porque la ortogonalidad es tan importante es ver este video que justo habla de porque $1, x, x^2, \dots$ es una base horrible para los polinomios. El link es <https://youtu.be/pYOGYQOXqTk>

Esto es un pdf y no me puedo quejar del espacio que utilizo, así que dejo el código:

```

1 // @Author: Rosas Hernandez Oscar Andres
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5
6 getd(' /Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework3/Code/Algorithms ')
7 clc;
8
9 A23 = [
10     0.16  0.10;
11     0.17  0.11;
12     2.02  1.29;
13 ]
14
15 b23 = [
16     0.26;
17     0.28;
18     3.31;
19 ]
20
21 b2_23 = [
22     0.27;
23     0.25;
24     3.33;
25 ]
26
27 disp("Ax = b")
28
29 disp("A:")
30 disp(A23)
31
32 disp("b:")
33 disp(b23)
34
35 x23 = LeastSquares(A23, b23)
36 disp("x:")
37 disp(x23)
38
39 disp("Ax:")
40 disp(A23 * x23)
41
42 disp("b_2:")
43 disp(b2_23)
44
45 x2_23 = LeastSquares(A23, b2_23)
46 disp("x_2:")
47 disp(x2_23)
48
49 disp("Ax_2:")
50 disp(A23 * x2_23)

```

1.2. 24

```

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5
6 getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework3/Code/Algorithms')
7 clc;
8
9 x24 = [
10     1.02;
11     0.95;
12     0.87;
13     0.77;
14     0.67;
15     0.56;
16     0.44;
17     0.30;
18     0.16;
19     0.01;
20 ]
21
22 y24 = [
23     0.39;
24     0.32;
25     0.27;
26     0.22;
27     0.18;
28     0.15;
29     0.13;
30     0.12;
31     0.13;
32     0.15;
33 ]
34
35 A24 = eye(10, 5)
36 b24 = x24
37
38 for i = (1 : 10)
39     A24(i, 1) = y24(i) * y24(i)
40     A24(i, 2) = x24(i) * y24(i)
41     A24(i, 3) = x24(i)
42     A24(i, 4) = y24(i)
43     A24(i, 5) = 1
44
45     b24(i) = x24(i) * x24(i)
46 end
47
48 disp("Ax = b")
49
50 disp("xs:")
51 disp(x24)
52
53 disp("ys:")
54 disp(y24)
55
56 disp("A:")
57 disp(A24)
58
59 disp("b:")
60 disp(b24)
61
62 estmedx24 = LeastSquares(A24, b24)
63 disp("x:")
64 disp(estmedx24)
65
66 disp("Ax:")
67 disp(A24 * estmedx24)
68
69 clf()
70
71
72
73 function [x] = solveEquation(a, b, c)
74     x = (-1 * b + sqrt(b*b - 4*a*c)) / (2 * a)
75 endfunction
76
77 function [y] = solve(coefficients, x)
78     a = coefficients(1)
79     b = coefficients(2)
80     c = coefficients(3)
81     d = coefficients(4)
82     e = coefficients(5)
83
84     reala = (a)
85     realb = (b * x + d)

```

```
86     realc = (c * x + e - x * x)
87
88     y = solveEquation(reala, realb, realc)
89 endfunction
90
91 someX = linspace(0, 1.10, 50)
92 someY = someX
93
94 for i = (1 : 50)
95     someY(i) = solve(estimatedx24, someX(i))
96 end
97
98 plot(someX, someY, 'r-')
99
100 scatter(x24, y24)
```

1.3. 25

```

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5
6 getd( '/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework2/Code/Algorithms' )
7 clc;
8
9
10 for k = (1 : 10)
11     epsilon = 10 ** (-2 * k)
12
13     A25 = [
14         epsilon 1;
15         1 1;
16     ];
17
18     b25 = [
19         1 + epsilon;
20         2;
21     ];
22
23     realX25 = [
24         1;
25         1;
26     ]
27
28     disp("A")
29     disp(A25)
30
31     disp("b")
32     disp(b25)
33
34     [x] = GaussianElimination(A25, b25);
35
36     disp("Estimated Solution: A \tilde x")
37     disp(A25 * x)
38
39     disp("Real Solution: A x")
40     disp(A25 * realX25)
41
42     disp("Difference of Error: \tilde x - x")
43     disp(x - realX25)
44
45     disp("Estimated Condition")
46     disp( Condition(A25, 10) )
47
48     disp("Real Condition")
49     disp( cond(A25) )
50
51 end

```

1.4. 26

```

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5
6 getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework2/Code/Algorithms')
7 clc;
8
9
10 digitsAltered = eye(12, 1)
11 digitsClassic = eye(12, 1)
12 digitsTwo = eye(12, 1)
13 data = eye(12, 1)
14
15 for i = (2 : 12)
16     H = eye(i, i)
17     data(i) = i
18
19     for i = (1 : i)
20         for j = (1 : i)
21             H(i, j) = 1 / (i + j - 1)
22         end
23     end
24
25
26 [Q26Classic, R26] = GramSchmidt(H, 0)
27 [Q26Altered, R26] = GramSchmidt(H, 1)
28 [Q26Two, R26] = GramSchmidt(Q26Classic, 0)
29
30 digitsClassic(i) = -log10(norm(eye(i, i) - Q26Classic' * Q26Classic))
31 digitsAltered(i) = -log10(norm(eye(i, i) - Q26Altered' * Q26Altered))
32 digitsTwo(i) = -log10(norm(eye(i, i) - Q26Two' * Q26Two))
33
34 disp("Q of GramSchmidt Classical:")
35 disp(Q26Classic)
36
37 disp("Digits:")
38 disp(digitsClassic)
39
40 disp("Q of GramSchmidt Altered:")
41 disp(Q26Altered)
42
43 disp("Digits:")
44 disp(digitsAltered)
45 end
46
47 plot(data, digitsClassic, '.b-')
48 plot(data, digitsAltered, '.r-')
49 plot(data, digitsTwo, '.g-')
50
51 hl=legend(['Classical GramSchmidt'; 'Altered GramSchmidt'; 'Two times GramSchmidt']);

```

```

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3 // @Author: Laurrabaquio Rodríguez Miguel Salvador
4 // @Author: Pahuá Castro Jesús Miguel Ángel
5
6 getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework2/Code/Algorithms')
7 clc;
8
9
10 digitsAltered = eye(12, 1)
11 digitsClassic = eye(12, 1)
12 digitsTwo = eye(12, 1)
13 digitsHouse = eye(12, 1)
14 data = eye(12, 1)
15
16 for i = (2 : 12)
17     H = eye(i, i)
18     data(i) = i
19
20     for i = (1 : i)
21         for j = (1 : i)
22             H(i, j) = 1 / (i + j - 1)
23         end
24     end
25
26
27 [Q26Classic, R26] = GramSchmidt(H, 0)
28 [Q26Altered, R26] = GramSchmidt(H, 1)
29 [Q26Two, R26] = GramSchmidt(Q26Classic, 0)
30 [Q26H, R26] = HouseHolder(H)
31

```



```

32 digitsClassic(i) = -log10(norm(eye(i, i) - Q26Classic' * Q26Classic))
33 digitsAltered(i) = -log10(norm(eye(i, i) - Q26Altered' * Q26Altered))
34 digitsTwo(i) = -log10(norm(eye(i, i) - Q26Two' * Q26Two))
35 digitsHouse(i) = -log10(norm(eye(i, i) - Q26H' * Q26H))
36
37 disp("Q of GramSchmidt Classical:")
38 disp(Q26Classic)
39
40 disp("Digits:")
41 disp(digitsClassic)
42
43 disp("Q of GramSchmidt Altered:")
44 disp(Q26Altered)
45
46 disp("Digits:")
47 disp(digitsAltered)
48 end
49
50 plot(data, digitsClassic, '.b-')
51 plot(data, digitsAltered, '.r-')
52 plot(data, digitsTwo, '.g-')
53 plot(data, digitsHouse, '.black-')
54
55 title("Digits of precision")
56
57 hl=legend(['Classical GramSchmidt'; 'Altered GramSchmidt'; 'Two times GramSchmidt'; 'HouseHolder']);

```

```

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4 // @Author: Pahuá Castro Jesús Miguel Angel
5
6 getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework2/Code/Algorithms')
7 clc;
8
9
10 digitsAltered = eye(12, 1)
11 digitsClassic = eye(12, 1)
12 digitsTwo = eye(12, 1)
13 digitsNormal = eye(12, 1)
14 data = eye(12, 1)
15
16 for i = (2 : 12)
17     H26 = eye(i, i)
18     data(i) = i
19
20     for j = (1 : i)
21         for k = (1 : j)
22             H26(i, j) = 1 / (i + j - 1)
23         end
24     end
25
26
27 [Q26Classic, R26] = GramSchmidt(H26, 0)
28 [Q26Altered, R26] = GramSchmidt(H26, 1)
29 [Q26Two, R26] = GramSchmidt(Q26Classic, 0)
30 [L26] = CholeskyGaussian(H26' * H26)
31 Q26N = H26 * L26'
32
33 digitsClassic(i) = -log10(norm(eye(i, i) - Q26Classic' * Q26Classic))
34 digitsAltered(i) = -log10(norm(eye(i, i) - Q26Altered' * Q26Altered))
35 digitsTwo(i) = -log10(norm(eye(i, i) - Q26Two' * Q26Two))
36 digitsNormal(i) = -log10(norm(eye(i, i) - Q26N' * Q26N))
37
38 disp("Q of GramSchmidt Classical:")
39 disp(Q26Classic)
40
41 disp("Q of GramSchmidt Altered:")
42 disp(Q26Altered)
43
44 end
45
46 plot(data, digitsClassic, '.b-')
47 plot(data, digitsAltered, '.r-')
48 plot(data, digitsTwo, '.g-')
49 plot(data, digitsNormal, '.black-')
50
51 title("Digits of precision")
52
53 hl=legend(['Classical GramSchmidt'; 'Altered GramSchmidt'; 'Two times GramSchmidt'; 'Normal
    equation']);

```

1.5. 27

Este proceso lo haremos hasta $\epsilon = 2^{-10}$ porque ϵ más pequeñas simplemente ya hacer que se trate a la matriz como singular pues se ve como una matriz con 3 columnas iguales, por lo tanto causa errores al tomarse como una matriz singular.

Lo que podemos ver que mientras epsilon más y más pequeña nos acercamos al claro resultado donde $x_1 = x_2 = x_3 = 0.333$

Algo curioso y diferente de los métodos anteriores, del parcial pasado es que prácticamente todos los métodos dan el mismo resultado, curiosamente GramSchmidt es el único diferente en ciertos ϵ .

```

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5
6 getd('Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework3/Code/Algorithms')
7 clc;
8
9
10 values = [
11     2;
12     1.1;
13     1;
14     0.9;
15     2e-01;
16     2e-02;
17     2e-05;
18     2e-08;
19     2e-10;
20     2e-11;
21 ]
22
23 [numberOfValues, _] = size(values)
24
25 for i = (1 : numberOfValues)
26
27     e = values(i)
28     disp("=====")
29     disp("epsilon:")
30     disp(e)
31
32     A27 = [
33         1 1 1;
34         e 0 0;
35         0 e 0;
36         0 0 e;
37     ]
38
39     b27 = [
40         1;
41         0;
42         0;
43         0;
44     ]
45
46     disp("Ax = b")
47
48     disp("A:")
49     disp(A27)
50
51     try
52         x27LeastSquares = LeastSquares(A27, b27)
53
54         [Q27HH, R27HH] = HouseHolder(A27)
55         x27HouseHolder = QRDecomposition(Q27HH, R27HH, b27)
56
57         [Q27GS, R27GS] = GramSchmidt(A27, 0)
58         x27GramSchmidt = QRDecomposition(Q27GS, R27GS, b27)
59
60         [Q27G, R27G] = Givens(A27)
61         x27Givens = QRDecomposition(Q27G, R27G, b27)
62
63
64     disp("x of Least Squares:")
65     disp(x27LeastSquares)

```

```
66
67     disp("Ax of Least Squares:")
68     disp(A27 * x27LeastSquares)
69
70     disp("x of HouseHolder:")
71     disp(x27HouseHolder)
72
73     disp("Ax of HouseHolder:")
74     disp(A27 * x27HouseHolder)
75
76     disp("x of GramSchmidt:")
77     disp(x27GramSchmidt)
78
79     disp("Ax of GramSchmidt:")
80     disp(A27 * x27GramSchmidt)
81
82     disp("x of Givens:")
83     disp(x27Givens)
84
85     disp("Ax of Givens:")
86     disp(A27 * x27Givens)
87 catch
88     disp("Error: Singular matrix")
89 end
90
91
92
93
94 end
```

2. Anexo

2.1. Givens

```
1 // Get the estimated solution to Ax = b using the HouseHolder transformation
2 // @param: A a matriz in  $M_{\{m \times n\}}$  where  $m > n$ 
3 // @return: Q a matriz in  $M_{\{m \times m\}}$  that is ortogonal
4 // @return: R a matriz in  $M_{\{m \times n\}}$  that is triangular superior
5
6 // @Author: Rosas Hernandez Oscar Andres
7 // @Author: Alarcón Alvarez Aylin Yadira Guadalupe
8 // @Author: Laurrabaquio Rodríguez Miguel Salvador
9 // @Author: Pádua Castro Jesús Miguel Angel
10
11 function [Q, R] = Givens(A)
12     [m, n] = size(A);
13     Q = eye(m, m);
14     R = A;
15
16     for j = (1 : n)
17         for i = (m : -1 : j + 1)
18             GivenMatrix = eye(m, m);
19             [c, s] = GivensRotations(R(i-1, j), R(i, j));
20             GivenMatrix([i-1, i], [i-1, i]) = [c -s; s c];
21
22             R = GivenMatrix' * R;
23             Q = Q * GivenMatrix;
24         end
25     end
26 end
27
28 function [c, s] = GivensRotations(a, b)
29     if (b == 0)
30         c = 1;
31         s = 0;
32     else
33         if abs(b) > abs(a)
34             r = a / b;
35             s = 1 / sqrt(1 + r**2);
36             c = s * r;
37         else
38             r = b / a;
39             c = 1 / sqrt(1 + r**2);
40             s = c * r;
41         end
42     end
43 end
44
45 end
```

2.2. GramSchmidt

```

1 // Get the estimated solution to  $Ax = b$  using the Gramm-Schmidt transformation
2 // @param: A a matriz in  $M_{\{m \times n\}}$  where  $m > n$ 
3 // @param: option if 1 then is the alterate else is the classic algorithm
4 // @return: Q a matriz in  $M_{\{m \times m\}}$  that is ortogonal
5 // @return: R a matriz in  $M_{\{m \times n\}}$  that is triangular superior
6
7
8 // @Author: Rosas Hernandez Oscar Andres
9 // @Author: Alarcón Alvarez Aylin Yadira Guadalupe
10 // @Author: Laurrabaquio Rodríguez Miguel Salvador
11 // @Author: Pahua Castro Jesús Miguel Angel
12
13 function [Q, R] = GramSchmidt(A, option)
14     [m, n] = size(A);
15
16     Q = zeros(m, n);
17     R = A;
18
19     for j = (1:n)
20
21         Q(:,j) = A(:, j)
22
23         for i = (1 : j - 1)
24             if (option == 1)
25                 R(i, j) = Q(:, i)' * Q(:, j);
26             else
27                 R(i, j) = Q(:, i)' * A(:, j);
28             end
29
30             Q(:, j) = Q(:, j) - R(i, j) * Q(:, i);
31         end
32
33         R(j, j) = norm(Q(:, j));
34         Q(:, j) = Q(:, j) / R(j, j);
35     end
36
37 endfunction

```

2.3. HouseHolder

```

1 // Get the estimated solution to Ax = b using the HouseHolder transformation
2 // @param: A a matriz in M_{m x n} where m > n
3 // @return: Q a matriz in M_{m x m} that is ortogonal
4 // @return: R a matriz in M_{m x n} that is triangular superior
5
6 // @Author: Rosas Hernandez Oscar Andres
7 // @Author: Alarcón Alvarez Aylin Yadira Guadalupe
8 // @Author: Laurrabaquio Rodríguez Miguel Salvador
9 // @Author: Pahua Castro Jesús Miguel Ángel
10
11 function [Q, R] = HouseHolder(A, b)
12     [m, n] = size(A);
13     Q = eye(m, m);
14
15     for i = (1 : n)
16         aei = zeros(m - i + 1, 1);
17
18         alpha = -sign(A(i,i)) * norm(A(i : m, i));
19         aei(1) = alpha
20
21         v = A(i : m, i) - aei;
22         HouseHolder = eye(m - (i-1), m-(i-1)) -2 * ((v * v') / (v' * v));
23
24         RealHouseHolder = eye(m, m);
25         RealHouseHolder(i : m, i : m) = HouseHolder;
26
27         A = RealHouseHolder * A;
28         Q = Q * RealHouseHolder;
29
30     end
31     R = A;
32
33 endfunction

```

2.4. LeastSquares

```

1 // Get the estimated solution to Ax = b using least squares
2 // @param: A a matriz in M_{m x n} where m > n
3 // @param: b a vector of m rows
4 // @return: x a estimated solution
5
6 // @Author: Rosas Hernandez Oscar Andres
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8 // @Author: Laurrabaquio Rodríguez Miguel Salvador
9 // @Author: Pahua Castro Jesús Miguel Ángel
10
11 function [x] = LeastSquares(A, b)
12     [L] = CholeskyBanachiewicz(A' * A, 1);
13
14     y = FowardSubstitution(L, A' * b);
15     x = BackwardSubstitution(L', y);
16 endfunction

```

2.5. BackwardSubstitution

```
1 // Solve a system  $Ux = y$  where  $U$  is an upper triangular
2 // using the famous algorithm backward substitution
3 // @param: U triangular superior matrix
4 // @param: b the b in  $Ux = b$ 
5 // @return: x the solution vector
6
7 // @Author: Rosas Hernandez Oscar Andres
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9 // @Author: Laurrabaquio Rodríguez Miguel Salvador
10 // @Author: Pádua Castro Jesús Miguel Ángel
11
12 function [x] = BackwardSubstitution(U, b)
13     [m, n] = size(U);
14     x = zeros(n, 1);
15
16     for i = (n : -1 : 1)
17         if (U(i, i) == 0)
18             error('Error: Singular matrix');
19             return;
20         end
21
22         x(i) = b(i) / U(i, i);
23
24         for j = (1 : i - 1)
25             b(j) = b(j) - U(j, i) * x(i);
26         end
27     end
28 endfunction
```

2.6. CholeskyBanachiewicz

```

1 // Factor A as A = L * L^T
2 // using the famous algorithm called Cholesky using this really awesome property
3 // First A = L U then we make U a unit upper triangular matrix so we have L D L' and then
4 // we do L D2 D2 L' were D2(i, j) = sqrt(D(i, j)) finally we associate and we have A = L2 * L2'
5 // where L2 = L * D2
6 // @param: A a positive defined matrix (so A is symmetric)
7 // @param: option if 1 then A = L * L else A = L * D * L
8 // @return: L lower triangle matrix
9
10 // @Author: Rosas Hernandez Oscar Andres
11 // @Author: Alarcón Alvarez Aylin Yadira Guadalupe
12 // @Author: Laurrabaquio Rodríguez Miguel Salvador
13 // @Author: Pahuá Castro Jesús Miguel Ángel
14
15 function [L, D] = CholeskyBanachiewicz(A, option)
16     [m, n] = size(A);
17     D = eye(n, n);
18     L = eye(m, n);
19     U = A;
20
21     for step = (1 : n - 1)
22         if (A(step, step) == 0)
23             error('Error: Singular matrix');
24             return;
25         end
26
27         for row = (step + 1 : n)
28             L(row, step) = U(row, step) / U(step, step);
29             for column = (1 : n)
30                 U(row, column) = U(row, column) - L(row, step) * U(step, column);
31             end
32         end
33     end
34
35     if option == 1
36         for step = (1 : n)
37             for row = (step : n)
38                 L(row, step) = L(row, step) * sqrt(U(step, step));
39             end
40         end
41     else
42         for step = (1 : n)
43             D(step, step) = U(step, step);
44         end
45     end
46
47
48
49 endfunction

```


2.7. CholeskyGaussian

```

1 // Factor A as A = L * L^T
2 // using the famous algorithm called Cholesky using a modification of Gaussian Elimination
3 // @param: A a positive defined matrix (so A is symmetric)
4 // @return: L lower triangle matrix
5
6 // @Author: Rosas Hernandez Oscar Andres
7 // @Author: Alarcón Alvarez Aylin Yadira Guadalupe
8 // @Author: Laurabaquio Rodríguez Miguel Salvador
9 // @Author: Pádua Castro Jesús Miguel Angel
10
11 function [L] = CholeskyGaussian(A)
12
13     [m, n] = size(A);
14     L = zeros(m, n);
15
16     for step = (1 : n)
17         A(step, step) = sqrt( A(step, step) );
18
19         for column = (step + 1 : n)
20             A (column, step) = A(column, step) / A(step, step);
21         end
22
23         for i = (step + 1 : n)
24             for j = (step + 1 : n)
25                 A(i, j) = A(i, j) - A(i, step) * A(j, step);
26             end
27         end
28     end
29
30     for row = (1 : n)
31         for column = (1 : n)
32             if (row >= column)
33                 L(row, column) = A(row, column);
34             end
35         end
36     end
37
38 endfunction

```

2.8. CompleteLUdecomposition

```

1 // Factor A as PAQ = LU
2 // @param: A a not singular matrix
3 // @return: L (not sure) lower triangle matrix
4 // @return: U upper triangle matrix
5 // @return: P permutation matrix
6 // @return: Q permutation matrix
7
8 // JUST BECAUSE I WRITE IT, IT DOES NOT MEAN IT IS FAST, IT IS NOT FAST!
9
10 // @Author: Rosas Hernandez Oscar Andres
11 // @Author: Alarcón Alvarez Aylin Yadira Guadalupe
12 // @Author: Laurrabaquio Rodríguez Miguel Salvador
13 // @Author: Pádua Castro Jesús Miguel Ángel
14
15 function [L, U, P, Q] = CompleteLUdecomposition(A)
16     [m, n] = size(A);
17     if (m ~= n) == 0 then
18         error('Error: Not square matrix');
19     end
20     P = eye(n, n);
21     Q = eye(n, n);
22     L = eye(n, n);
23     U = A;
24
25     for step = (1 : n - 1)
26         Qi = eye(n, n);
27         Pi = eye(n, n);
28
29         [maxIndex, index] = max(abs(A(step : n, step : n)));
30         index(1) = index(1) + step - 1;
31         index(2) = index(2) + step - 1;
32
33         if (maxIndex == 0)
34             error('Error: Singular matrix');
35         end
36
37         temporal = Pi(step, :);
38         Pi(step, :) = Pi(index(1), :);
39         Pi(index(1), :) = temporal;
40
41         temporal = Qi(:, step);
42         Qi(:, step) = Qi(:, index(2));
43         Qi(:, index(2)) = temporal;
44
45         temporal = U(step, :);
46         U(step, :) = U(index(1), :);
47         U(index(1), :) = temporal;
48
49         temporal = U(:, step);
50         U(:, step) = U(:, index(2));
51         U(:, index(2)) = temporal;
52
53         for row = (step + 1 : n)
54             L(row, step) = U(row, step) / U(step, step);
55             for column = (1 : n)
56                 U(row, column) = U(row, column) - L(row, step) * U(step, column);
57             end
58         end
59
60         Q = Q * Qi;
61         P = P * Pi;
62     end
63 endfunction
64

```

2.9. FowardSubstitution

```
1 // Solve a system Ly = b where L is triangular inferior
2 // using the famous algorithm foward substitution
3 // @param: L triangular inferior matrix
4 // @param: b the b in Ly = b
5 // @return: x the solution vector
6
7 // @Author: Rosas Hernandez Oscar Andres
8 // @Author: Alarcón Alvarez Aylin Yadira Guadalupe
9 // @Author: Laurrabaquio Rodríguez Miguel Salvador
10 // @Author: Pahua Castro Jesús Miguel Ángel
11
12 function [y] = FowardSubstitution(L, b)
13     [m, n] = size(L);
14     y = zeros(n, 1);
15
16     for i = (1 : n)
17         if (L(i, i) == 0)
18             error('Error: Singular matrix');
19             return;
20         end
21
22         y(i) = b(i) / L(i, i);
23
24         for j = (i + 1 : n)
25             b(j) = b(j) - L(j, i) * y(i);
26         end
27     end
28 endfunction
```

2.10. LUDecomposition

```
1 // Factor A as A = L * U
2 // @param: A a not singular matrix
3 // @return: L lower triangule matrix
4 // @return: U upper triangule matrix
5
6 // @Author: Rosas Hernandez Oscar Andres
7 // @Author: Alarcón Alvarez Aylin Yadira Guadalupe
8 // @Author: Laurrabaquio Rodríguez Miguel Salvador
9 // @Author: Pádua Castro Jesús Miguel Angel
10
11 function [L, U] = LUDecomposition(A)
12     [m, n] = size(A);
13     L = eye(m, n);
14     U = A;
15
16     for step = (1 : n - 1)
17
18         if (A(step, step) == 0)
19             error('Error: Singular matrix');
20             return;
21         end
22
23         for row = (step + 1 : n)
24             L(row, step) = U(row, step) / U(step, step);
25
26             for column = (1 : n)
27                 U(row, column) = U(row, column) - L(row, step) * U(step, column);
28             end
29         end
30     end
31 endfunction
32
```