

## Divide-and-conquer

### Questions

**Problem 1.** Write an algorithm of cost  $\Theta(\lg n)$  that, given a sorted table  $T$  with  $n$  different elements, and two further elements  $x$  and  $y$  with  $x \leq y$ , return the number of elements in  $T$  that fall between  $x$  and  $y$  inclusively.

**Problem 2.** A sequence  $A = \{a_1, \dots, a_n\}$  of length  $n \geq 3$  is called *unimodal* if there exists an index  $p$  with  $1 \leq p \leq n$  such that  $a_1 < a_2 < \dots < a_p$  and  $a_p > a_{p+1} > \dots > a_n$ . In other words, the sequence increase up to  $a_p$  and then decreases. The elements  $a_p$  is called its *top*. Write a divide-and-conquer algorithm that, given a unimodal sequence, finds its top in  $O(\lg n)$ . Argue that the suggested algorithm has time complexity of  $O(\lg n)$ , and prove its correctness.

### References

Atserias, Albert et al. (2022). *Data Structure and Algorithms Problem Set*. Universitat Politècnica de Catalunya.