Math III-Calculus II: Mathematica Project

Instruction: The whole class is divided into 4 groups (Group i, Group ii, Group iii and Group iv). You are supposed to form a group by yourself, but should have more or less the same number of members. Each group has to solve one sub-problem of every problem (1, 2 and 3) which has the same number as the group number. For example, Group i has to solve problems 1.i, 2.i and 3.i. Group ii has to solve problems 1.ii, 2.ii and 3.ii. And so on. I will ask one member of each group to present the work next week. So all members of each group must be well prepared for the presentation.

1. **Ellipse**: The graph of the equation $(x/a)^2 + (y/b)^2 = 1,$ with $-a \leq x \leq a, -b \leq y \leq b$ is an ellipse.
   i. Find the enclosed area of this ellipse. **Hint**: First find the area of the first-quadrant portion, $y = b(1-(x/a)^2)^{1/2}, 0 \leq x \leq a,$ and multiply it by 4. Then, with $a = 3$ and $b = 5,$ plot the enclosed area of the ellipse using the built-in syntax of Mathematica.
   ii. Find the length of this ellipse. **Hint**: First find the length of the first-quadrant portion, $y = b(1-(x/a)^2)^{1/2}, 0 \leq x \leq a,$ and multiply it by 4. Then, with $a = 3$ and $b = 5,$ plot the ellipse using the built-in syntax of Mathematica.
   iii. Find the volume of the solid by rotating this ellipse about the x-axis. **Hint**: First find the volume of the solid of revolution of the first-quadrant portion, $y = b(1-(x/a)^2)^{1/2}, 0 \leq x \leq a,$ about the x-axis and multiply it by 2. Then, with $a = 3$ and $b = 5,$ plot the solid of revolution about the x-axis using the built-in syntax of Mathematica.
   iv. Find the area of the surface by rotating this ellipse about the x-axis. **Hint**: First find the area of the surface of revolution of the first-quadrant portion, $y = b(1-(x/a)^2)^{1/2}, 0 \leq x \leq a,$ about the x-axis and multiply it by 2. Then, with $a = 3$ and $b = 5,$ plot the surface of revolution about the x-axis using the built-in syntax of Mathematica.

Remark: use the same scale for x-axis and y-axis.

2. **Torus**:

   i. The disk $x^2 + y^2 \leq a^2$ is revolved about the line $x = b$ $(b > a)$ to generate a solid shaped like a doughnut called a *torus*. Find the formula of the volume of the *torus*. **Hint**: use the disk method by slicing along the y-axis.

   ii. The circle $x^2 + y^2 = a^2$ is revolved about the line $x = b$ $(b > a)$ to generate a surface shaped like a doughnut called a *torus*. Find the formula of the surface area of the *torus*.

   iii. Plot the torus with $a = 2$ and $b = 5$ using the built-in syntax of Mathematica.

   iv. Plot the torus with $a = 3$ and $b = 4$ using the built-in syntax of Mathematica.

3. **Astroid**: The graph of the equation $x^{2/3} + y^{2/3} = 1$, with $-1 \leq x \leq 1, -1 \leq y \leq 1$ is called an "asteroid" because of its starlike appearance. (see exercise 24 of section 6.3 in page 350 of the main textbook).

   i. Find the enclosed area of this asteroid. **Hint**: First find the area of the first-quadrant portion, $y = (1 - x^{2/3})^{3/2}, 0 \leq x \leq 1$, and multiply it by 4. Then, plot the enclosed region of the astroid using the built-in syntax of Mathematica.

   ii. Find the length of this asteroid. **Hint**: First find the length of the first-quadrant portion, $y = (1 - x^{2/3})^{3/2}, 0 \leq x \leq 1$, and multiply it by 4. Then, plot the astroid using the built-in syntax of Mathematica.

   iii. Find the volume of the solid by rotating this asteroid about the x-axis. **Hint**: First find the volume of the solid of revolution of the first-quadrant portion, $y = (1 - x^{2/3})^{3/2}, 0 \leq x \leq 1$, about the x-axis and multiply it by 2. Then, plot the solid of revolution about the x-axis using the built-in syntax of Mathematica.

   iv. Find the area of the surface by rotating this asteroid about the x-axis. **Hint**: First find the area of the surface of revolution of the first-quadrant portion, $y = (1 - x^{2/3})^{3/2}, 0 \leq x \leq 1$, about the x-axis and multiply it by 2. Then, plot the surface of revolution about the x-axis using the built-in syntax of Mathematica.