

Chapter 6: Applications of Definite Integrals.

6.1. Volumes using Cross - Sections

DEFINITION The **volume** of a solid of integrable cross-sectional area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b ,

$$V = \int_a^b A(x) dx.$$

Solids of revolution: the disk Method

Volume by Disks for Rotation About the x -axis

$$V = \int_a^b A(x) dx = \int_a^b \pi [R(x)]^2 dx.$$

Volume by Disks for Rotation About the y -axis

$$V = \int_c^d A(y) dy = \int_c^d \pi [R(y)]^2 dy.$$

Solids of revolution: The washer method

Volume by Washers for Rotation About the x -axis

$$V = \int_a^b A(x) dx = \int_a^b \pi ([R(x)]^2 - [r(x)]^2) dx.$$

Volume by washer for rotation about the axis

$$V = \int_c^d \pi ([R(y)]^2 - [r(y)]^2) dy$$

6.2. Volumes using cylindrical shells

The shell method

Shell Formula for Revolution About a Vertical Line

The volume of the solid generated by revolving the region between the x -axis and the graph of a continuous function $y = f(x) \geq 0, L \leq a \leq x \leq b$, about a vertical line $x = L$ is

$$V = \int_a^b 2\pi \left(\frac{\text{shell}}{\text{radius}} \right) \left(\frac{\text{shell}}{\text{height}} \right) dx.$$

shell Formula for revolution about a vertical line

$$V = \int_a^b 2\pi \left(\frac{\text{shell}}{\text{radius}} \right) \left(\frac{\text{shell}}{\text{height}} \right) dy$$

the difference between shell and washer:

- **Shell method:** the line dividing the curve is parallel to the axis of revolution.
- **Washer method:** the line dividing the curve is perpendicular to the axis of revolution.

6.3. Arc length

DEFINITION If f' is continuous on $[a, b]$, then the **length (arc length)** of the curve $y = f(x)$ from the point $A = (a, f(a))$ to the point $B = (b, f(b))$ is the value of the integral

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx. \quad (3)$$

Formula for the Length of $x = g(y)$, $c \leq y \leq d$

If g' is continuous on $[c, d]$, the length of the curve $x = g(y)$ from $A = (g(c), c)$ to $B = (g(d), d)$ is

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy = \int_c^d \sqrt{1 + [g'(y)]^2} dy. \quad (4)$$

6.4. Areas of surfaces of revolution

DEFINITION If the function $f(x) \geq 0$ is continuously differentiable on $[a, b]$, the **area of the surface** generated by revolving the graph of $y = f(x)$ about the x -axis is

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} dx. \quad (3)$$

Surface Area for Revolution About the y -Axis

If $x = g(y) \geq 0$ is continuously differentiable on $[c, d]$, the area of the surface generated by revolving the graph of $x = g(y)$ about the y -axis is

$$S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy = \int_c^d 2\pi g(y) \sqrt{1 + (g'(y))^2} dy. \quad (4)$$