Math III-Calculus II

Problem Set I: Chapter 6 Applications of Definite Integral

1. Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on the sides by $x = 0$ and $x = 1$.

A. 4/5 B. 5/6 C. 6/10 D. 12/17 E. 3/7

2. Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

A. 1/2 B. 1/3 C. 2/3 D. 3/5 E. 1/7

3. The region R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the x-axis. Find the volume of the resulting solid.

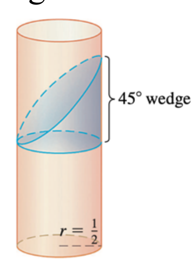
A. $\frac{2\pi}{5}$ B. $\frac{2\pi}{7}$ C. $\frac{3\pi}{7}$ D. $\frac{2\pi}{15}$ E. $\frac{2\pi}{17}$

4. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the y-axis.

A. $\frac{96\pi}{3}$ B. $\frac{94\pi}{5}$ C. $\frac{96\pi}{5}$ D. $\frac{106\pi}{3}$ E. $\frac{86\pi}{5}$

5. Consider a right-circular cylinder of diameter 1. Form a wedge by making one slice parallel to the base of the cylinder completely through the cylinder, and another slice at an angle of 45° to the first slice and intersecting the first slice at the opposite edge of the cylinder (see accompanying diagram). Find the volume of the wedge.

A. $\frac{\pi}{8}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{16}$ D. $\frac{\pi}{2}$ E. $\frac{3\pi}{8}$



6. Find the volume of the solid obtained by rotating the region bounded by $y = 2x^2 - x^3$ and $y = 0$ about the y-axis.

A. 3.5π B. 2.5π C. 5.2 π D. 5.4 π E. 6.5 π

7. Find the volume of the solid obtained by rotating the region between $y = x$ and $y = x^2$ about the y-axis.

A. π/2 B. π/3 C. π/4 D. π/5 E. π/6

8. Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.

A. π B. π/2 C. π/3 D. π/4 E. π/6

9. Find the length of the curve $y = \frac{x^3}{3} + x^2 + x + \frac{1}{4x+4}$, $0 \le x \le 2$.

A. 51/7 B. 47/3 C. 61/6 D. 59/6 E. 53/6

10. Find the length of the curve

$$y = \int_0^x \sqrt{\cos 2t}\, dt$$

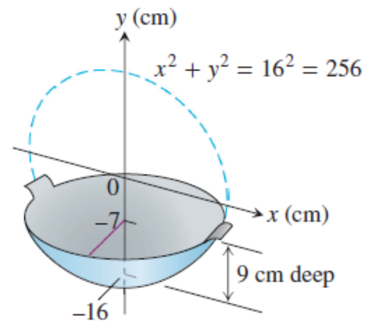
from $x = 0$ to $x = \frac{\pi}{4}$. **Hint**: first find $\frac{dy}{dx}$ using the Fundamental Theorem of Calculus.

A. 1/2 B. 2 C. 3 D. 1 E. 1/2

11. Find the area of the surface generated by revolving the curve $y = \sqrt{x+1}$, $(1 \le x \le 5)$, about the x-axis.

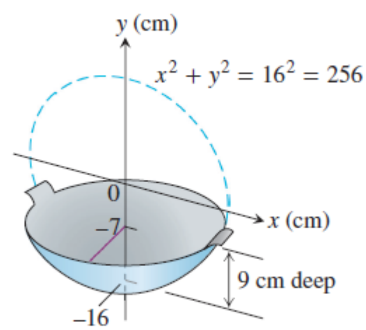
A. $\frac{49}{3}\pi$ B. $\frac{47}{3}\pi$ C. $\frac{59}{3}\pi$ D. $\frac{61}{3}\pi$ E. $\frac{67}{3}\pi$

12. You are designing a wok frying pan that will be shaped like a spherical bowl with handles. A bit of experimentation at home persuades you that you can get one that holds about 3 L if you make it 9 cm deep and give the sphere a radius of 16 cm. To be sure, you picture the wok as a solid of revolution, as shown here, and calculate its volume with an integral. To the nearest cubic centimeter, what volume do you really get? ($1\ L = 1000\ cm^3$.)



A. 2.98 L B. 3.13 L C. 3.31 L D. 3.52 L E. 3.21 L

13. Your company decided to put out a deluxe version of the successful wok you designed. The plan is to coat it inside with white enamel and outside with blue enamel. Each enamel will be sprayed on 0.5 mm thick before baking. How much enamel of each color is needed to coat 5000 such woks?



(Neglect waste and unused material and give your answer in liters. Remember that $1\ cm^3 = 1\ mL$, so $1\ L = 1000\ cm^3$.)

A. 262.2 L B. 226.2 L C. 274.2 L D. 260.2 L E. 224.2 L