Lab 06: Millikan Oil Drop Experiment

Import Libraries & Setup

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
# rounding to 4th decimal place for easier reading
pd.options.display.float_format = '{:.4f}'.format
```

Reflection

I had no lab partner for this lab, so I was getting accustomed to doing a lab on my own. As well it would have been beneficial to consider how best to gather data before starting the experiment, as I changed methods in order to collect a reasonable ammount of data.

Data Analysis

Start by importing our data

```
day1 = pd.read_csv('data/day1.csv')
day1_long = pd.read_csv('data/day1_long.csv')
# day2 data is already in fall/rise format so convert it
day2_fall = pd.read_csv('data/day2_fall.csv').to_numpy()[:,1:]
day2_rise = pd.read_csv('data/day2_rise.csv').to_numpy()[:,1:]
```

`day1` and `day1_long` contain timestamps for when the oil drop hits a reticle. We need to calculate the fall and rise times from these timestamps.

```
# convert dataframes to numpy arrays
day1 = day1.to_numpy()
day1_long = day1_long.to_numpy()

# replace the Drop # column with zeroes to indicate t0
day1[:,0] = 0
day1_long[:,0] = 0

# calculate fall & rise times
day1_times = day1[:, 1:] - day1[:,:-1]
day1_fall = day1_times[:, ::2]
day1_rise = day1_times[:, 1::2]

day1_long_times = day1_long[:, 1:] - day1_long[:,:-1]
day1_long_fall = day1_long_times[:, ::2]
day1_long_rise = day1_long_times[:, 1::2]
```

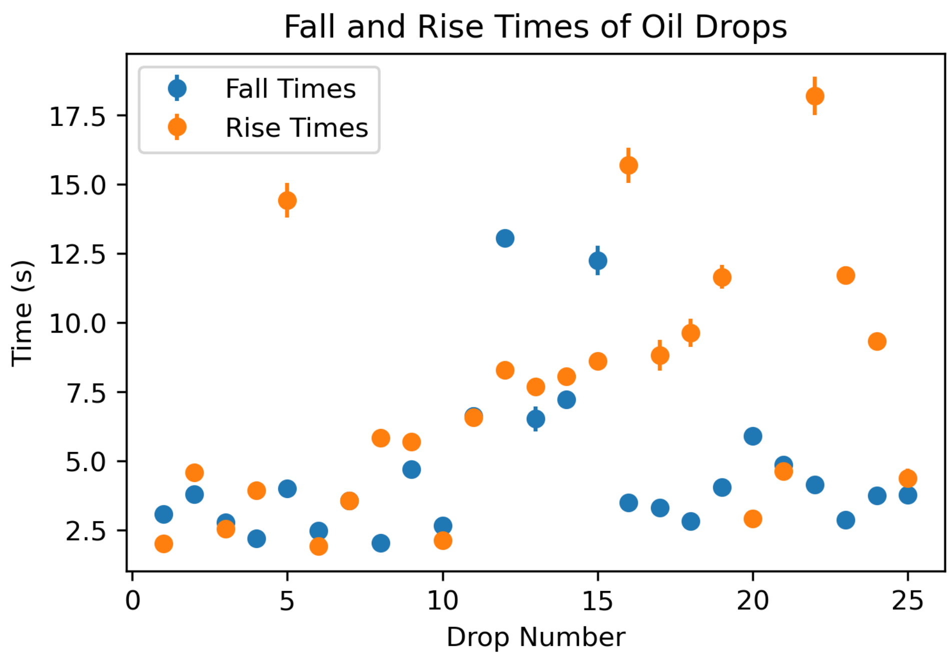
Now we get the mean fall and rise times for each drop

```
def combine_times(fall, rise, day):
    fall_means = np.mean(fall, axis=1)
    fall_stds = np.std(fall, axis=1)
    rise_means = np.mean(rise, axis=1)
    rise_stds = np.std(rise, axis=1)
    day_array = np.full(fall.shape[0], day)
    return np.stack((fall_means, fall_stds, rise_means, rise_stds, day_array), axis=1)
day1_combined = combine_times(day1_fall, day1_rise, 1)
day1_long_combined = combine_times(day1_long_fall, day1_long_rise, 1)
day2_combined = combine_times(day2_fall, day2_rise, 2)
drops = np.concatenate((day1_combined, day1_long_combined, day2_combined), axis=0)

# output to csv for use in lab report table
drops_df = pd.DataFrame(np.round(drops, 2), columns=['Fall Time', 'Fall Std', 'Rise Time', '
drops_df.insert(0, 'Drop #', np.arange(1, len(drops_df)+1))
drops_df.to_csv('data/drop_stats.csv', index=False)
```

Plot the fall and rise times of drops with error bars

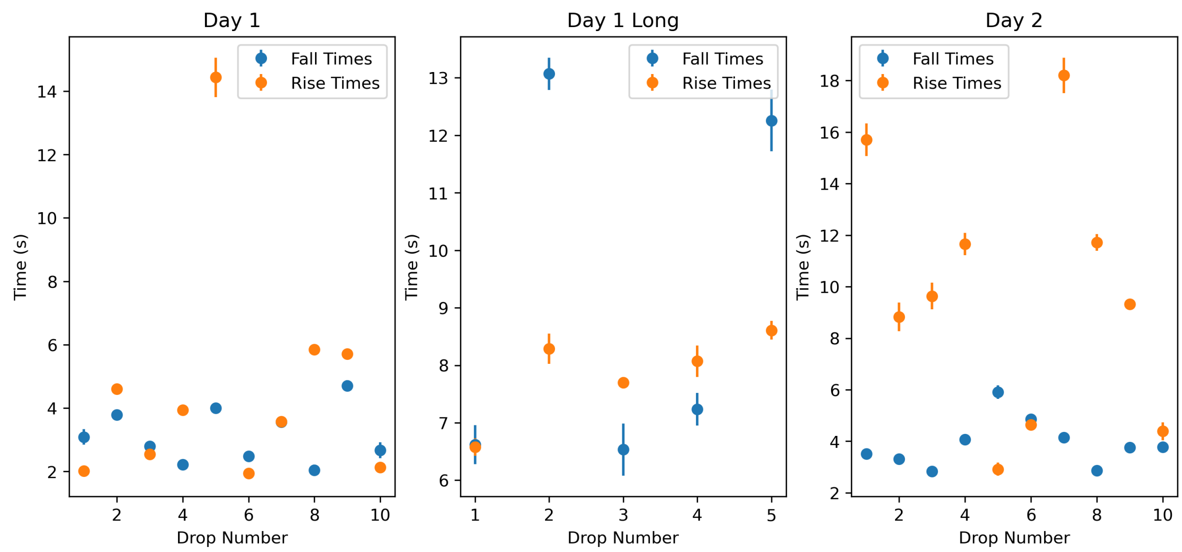
```
X = np.arange(0, len(drops), 1) + 1
plt.figure
plt.errorbar(X, drops[:,0], yerr=drops[:,1], fmt='o', label='Fall Times')
```

```
plt.errorbar(X, drops[:,2], yerr=drops[:,3], fmt='o', label='Rise Times')
plt.xlabel('Drop Number')
plt.ylabel('Time (s)')
plt.title('Fall and Rise Times of Oil Drops')
plt.legend()
plt.show()
```



As well here are two graphs of all three datasets separately to see if there are any differences between them.

```
titles = ['Day 1', 'Day 1 Long', 'Day 2']
plt.figure(figsize=(12, 5))
for i, curr_drops in enumerate([day1_combined, day1_long_combined, day2_combined]):
  plt.subplot(1, 3, i+1)
  X = np.arange(0, len(curr_drops), 1) + 1
  plt.errorbar(X, curr_drops[:,0], yerr=curr_drops[:,1], fmt='o', label='Fall Times')
  plt.errorbar(X, curr_drops[:,2], yerr=curr_drops[:,3], fmt='o', label='Rise Times')
  plt.xlabel('Drop Number')
  plt.ylabel('Time (s)')
  plt.title(titles[i])
  plt.legend()
plt.show()
```



Calculating Charge

Now we can calculate the charge on each drop using the formula provided in the lab manual. We will need to define some constants first.

```
# Constants
g = 9.81  # m/s^2
b = 8.23e-3  # Pa.m (constant for air viscosity correction)
eta_1 = 18.52e-6  # N.s/m^2 (viscosity of air at day 1 temp)
eta_2 = 18.48e-6  # N.s/m^2 (viscosity of air at day 2 temp)
P_1 = 93.83  # kPa (pressure day 1)
P_2 = 93.26  # kPa (pressure day 2)
P_err = 0.01 # kPa (error in pressure measurements)
dist_1 = (18.14-6.20-6.12)*10e-3  # m (distance between plates measured day 1)
dist_2 = (18.16-6.05-6.12)*10e-3 # m (distance between plates measured day 2)
raw_dist_err = np.sqrt((0.01e-3)**2 * 3) # m (error in the distance measurements)
dist = (dist_1 + dist_2) / 2 # Weighted average of distances
dist_err = raw_dist_err / np.sqrt(2) # Weighted error of distances
d = 0.5e-3  # m (distance between reticles)
V = 400  # V (voltage across plates)
rho_oil = 890  # kg/m^3 (density of oil)
```

Now we can calculate the charge on each drop

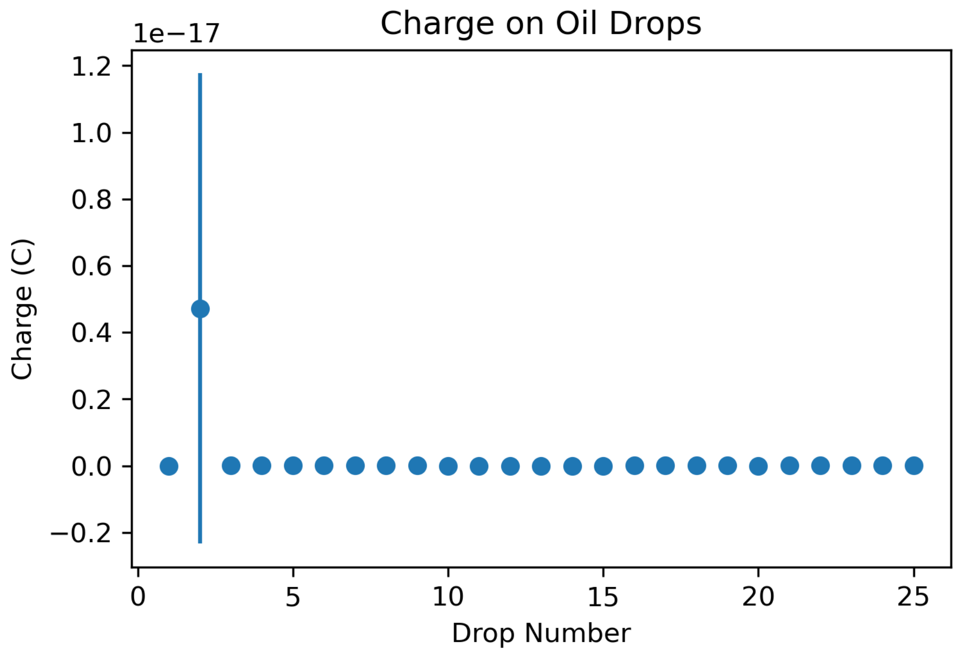
```
fall_time = drops[:,1]
fall_err = drops[:,1]
rise_time = drops[:,2]
rise_err = drops[:,3]
day = drops[:,4]
eta = np.where(day == 1, eta_1, eta_2)
P = np.where(day == 1, P_1, P_2)
dist_used = np.where(day == 1, dist_1, dist_2)
v_fall = d/fall_time
v_fall_err = d/(fall_time**2) * fall_err
v_rise = d/rise_time
v_rise_err = d/(rise_time**2) * fall_err
r = -b/(2*P) + np.sqrt((b**2)/(4*P**2) + (9 * eta * v_fall)/(2 * rho_oil * g))
r_err = np.sqrt(
  (P_err*(9*eta)/(8*rho_oil*g*(r+b/(2*P))))**2 +
  (v_fall_err * (b/(2*P**2) - (b**2)/(4*(P**3)*(r+b/(2*P)))))**2
)
E = V / dist
q = ((v_fall + v_rise)*4*np.pi*rho_oil*g*r**3)/(3*E)
q_err = q * np.sqrt(
  (v_rise_err/(v_fall+v_rise))**2 +
  (1/(2*v_fall) + 1/(v_fall + v_rise))**2 * v_fall_err**2
)

# output to csv for use in lab report table
charges_df = pd.DataFrame({ "Drop #": drops_df["Drop #"], "Charge": q, "Error": q_err})
charges_df.to_csv('data/drop_charges.csv', index=False, float_format='%.2e')
charges_df
```

	Drop #	Charge	Error
0	1	0.0000	0.0000
1	2	0.0000	0.0000
2	3	0.0000	0.0000
3	4	0.0000	0.0000
4	5	0.0000	0.0000
5	6	0.0000	0.0000
6	7	0.0000	0.0000
7	8	0.0000	0.0000
8	9	0.0000	0.0000
9	10	0.0000	0.0000
10	11	0.0000	0.0000

	Drop #	Charge	Error
11	12	0.0000	0.0000
12	13	0.0000	0.0000
13	14	0.0000	0.0000
14	15	0.0000	0.0000
15	16	0.0000	0.0000
16	17	0.0000	0.0000
17	18	0.0000	0.0000
18	19	0.0000	0.0000
19	20	0.0000	0.0000
20	21	0.0000	0.0000
21	22	0.0000	0.0000
22	23	0.0000	0.0000
23	24	0.0000	0.0000
24	25	0.0000	0.0000

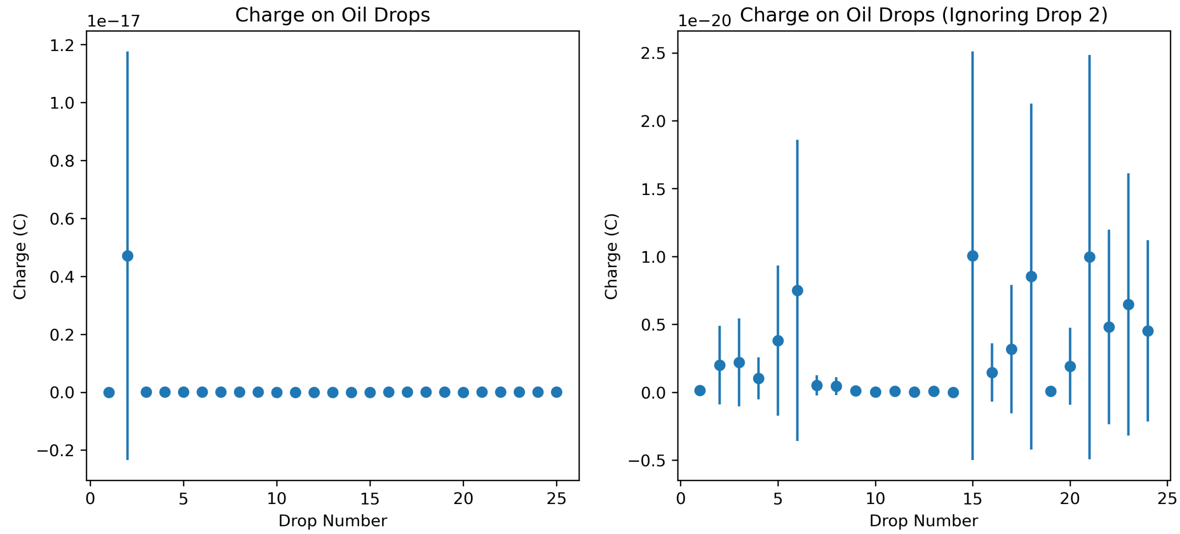
Now we can plot q

```
plt.figure()
X = np.arange(0, len(q), 1) + 1
plt.errorbar(X, q, yerr=q_err, fmt='o')
plt.xlabel('Drop Number')
plt.ylabel('Charge (C)')
plt.title('Charge on Oil Drops')
plt.show()
```



For some reason, we had a really large uncertainty on the 2nd drop, which means that it is hard to see the quantization of charge in this plot. However, if we ignore that point can see the quantization more clearly. Note we only ignore this point for visualization, not for any calculations.

```
# Plot both for easier comparison and to make the lab report look nicer
plt.figure(figsize=(12,5))
plt.subplot(1,2,1)
X = np.arange(0, len(q), 1) + 1
plt.errorbar(X, q, yerr=q_err, fmt='o')
plt.xlabel('Drop Number')
plt.ylabel('Charge (C)')
plt.title('Charge on Oil Drops')
plt.subplot(1,2,2)
Q = q[~(X==2)]
Q_err = q_err[~(X==2)]
X = np.arange(0, len(Q), 1) + 1
plt.errorbar(X, Q, yerr=Q_err, fmt='o')
plt.xlabel('Drop Number')
plt.ylabel('Charge (C)')
plt.title('Charge on Oil Drops (Ignoring Drop 2)')
plt.show()
```



from this it is easier to see a noticable pattern in the charges, which suggests that charge is quantized, despite the large uncertainties in some of the measurements. It seems that the more charged our oil drops are the larger the uncertainty in our measurements. This suggests that our error is systematic, likely due to some unaccounted for factor in our experiment, such as collisions with air molecules.

Let us divide by the smallest charge to see if we can find integer multiples

```
min_charge = np.min(q)
min_charge_err = q_err[np.argmin(q)]
ratios = q / min_charge
ratios_err = ratios * np.sqrt((q_err/q)**2 + (min_charge_err/min_charge)**2)
print(f"min_charge: {min_charge:.4e} ± {min_charge_err:.4e}")
pd.DataFrame({ "ratios": ratios, "err": ratios_err})
```

```
min_charge: 5.5501e-24 ± 7.9995e-24
```

	ratios	err
0	25.9869	52.1015
1	849360.0701	1764626.4440
2	358.5164	734.0145
3	396.6770	816.8321
4	187.3045	388.3179
5	687.0768	1404.1769
6	1352.8956	2791.1610

	ratios	err
7	92.5890	190.7172
8	80.6382	166.0034
9	20.6987	41.4934
10	6.0848	12.4436
11	12.7678	26.2591
12	1.9848	4.0503
13	12.3781	25.4478
14	1.0000	2.0383
15	1813.4288	3765.7410
16	260.5353	539.2404
17	573.0393	1187.5206
18	1537.1146	3189.6886
19	16.4364	33.2133
20	345.5885	712.5265
21	1793.8440	3725.9547
22	866.1322	1796.6662
23	1167.4052	2420.4848
24	814.0053	1680.4823

We do not have very good data, so it is hard to see integer multiples in these ratios, no matter if we divide by an integer or not. Our minimum charge is seen above and definetly does not agree with our expected value of the elementary charge of approximately 1.6×10^{-19} C. This is likely due to more systematic error in our measurements, most likely imprecise timing this time, as well as not having enough data to average out random errors.

Now we can try to estimate the elementary charge by dividing our charges by our ratios, rounded to the nearest integer, then we can look at the weigheted average of these estimates to get a better estimate of the elementary charge.

```
int_ratios = np.round(ratios)
e_estimates = q / int_ratios
e_estimates_err = e_estimates * np.sqrt((q_err/q)**2 + (ratios_err/ratios)**2)
weights = 1 / (e_estimates_err**2)
e_weighted_avg = np.sum(e_estimates * weights) / np.sum(weights)
e_weighted_err = np.sqrt(1 / np.sum(weights))
print(f"Estimated elementary charge: {e_weighted_avg:.4e} ± {e_weighted_err:.4e} C")
```

```
Estimated elementary charge: 5.5537e-24 ± 2.8038e-24 C
```

Sadly this doesn't yield much hope for us to get a good estimate of the elementary charge, as our value is still far from the expected value. Hopefully in the future we can improve our experimental methods to reduce systematic error and get better results.