Assignment 3

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1 ROTATING BENT ARM WITH SQUARE PLATE

In the system sketched in Figure 1, the bent arm ABC is welded to the vertical shaft, which rotates at the constant rate Ω. The square plate of sides L and total mass M rotates freely about axis BC at angular speed $\dot{\phi}$, with $\phi = 0$ corresponding to the plate being situated in the vertical plane, as shown in the sketch. Assume that the shafts AB and BC are thin rods of equal mass m. Gravity is acting downwards.

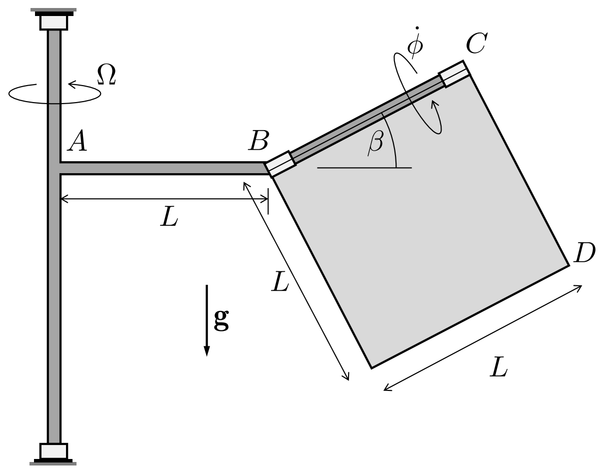

Figure 1.1: Sketch of problem 1.

1. Determine the kinetic and potential energy of the system.
2. Determine the differential equation governing ϕ.

3. Assume now that the vertical shaft is not driven anymore, but is free to rotate about the vertical axis. Denote with θ its rotation. Derive the equation of motion of such system.

4. Assume now that there exists a force distribution on the square plate, that is proportional to the velocity. That is to say, said $\mathbf{x}$ a generic point of the plate, the force per unit area $\mathbf{f}(\mathbf{x})$ on the plate is given by

$$\mathbf{f}(\mathbf{x}) = -c\dot{\mathbf{x}}, \tag{1.1}$$

   where c is a positive constant.

   Find the corresponding generalized forces when θ and ϕ are the degrees of freedom of the system and add it to the equations found in step 2.

5. Let $L = 0.25$ [m], $\beta = \pi/6$, $M = 0.5$ [kg], $m = 0.2$ [kg], $g = 9.81$[m/s^2] and $c = 0.1$[Ns/m^3].

6. Find the equilibrium configuration(s) for the system found in 1., 2. and 3..

7. Integrate the equations of motion in time for two different set of initial conditions of your choice. Can the system reach equilibrium after a long enough time span?