Advanced Dynamics, Spring Semester 2022
ETH Zürich

Longitudinal Dynamics of a FS Car

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We would like to study the longitudinal dynamics of a Formula Student car, see Figure 1. This analysis will enable us to estimate the car performance and comfort level, and give guidelines to the engineers to improve the design.

Figure 1: Formula Student Car by ETH.

THE MODEL

The car is modelled as an assembly of rigid bodies: the frame, powertrain and engine are lumped in one rigid body. There are then two rigid links representing the suspension arms, two wheels and two rigid rims representing the tires, for a total of 7 rigid bodies.
A general view is given in Figure 2, while the rigid bodies constituting the model, together with the corresponding degrees of freedom, are shown in Figure 3.

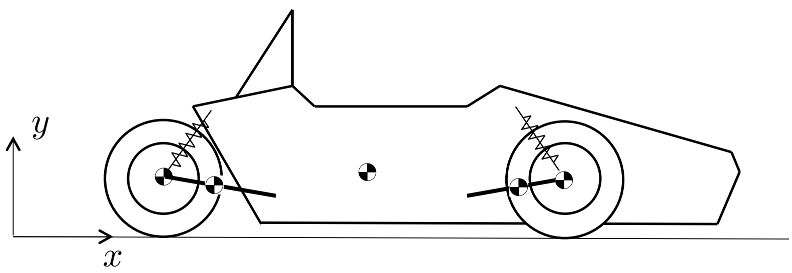

Figure 2: Car model

WHEEL MODEL

The wheels are schematized by two rigid rings (one representing the tire, one representing the wheel rim). The tire and the rim are connected by a continuous flexible layer (distributed radial springs) representing the rubber of the tire (see Figure 4). The natural length of the distributed springs is null in order to approximate the pretension effect of inflated tires.
The contact mechanics of the tire is in fact very complex, but here we simplify the behavior by assuming that the vertical contact stiffness is linear:

$$f_{cont} = k_{cont}\ y_{tF}$$

The same equation is satisfied for the rear tire.

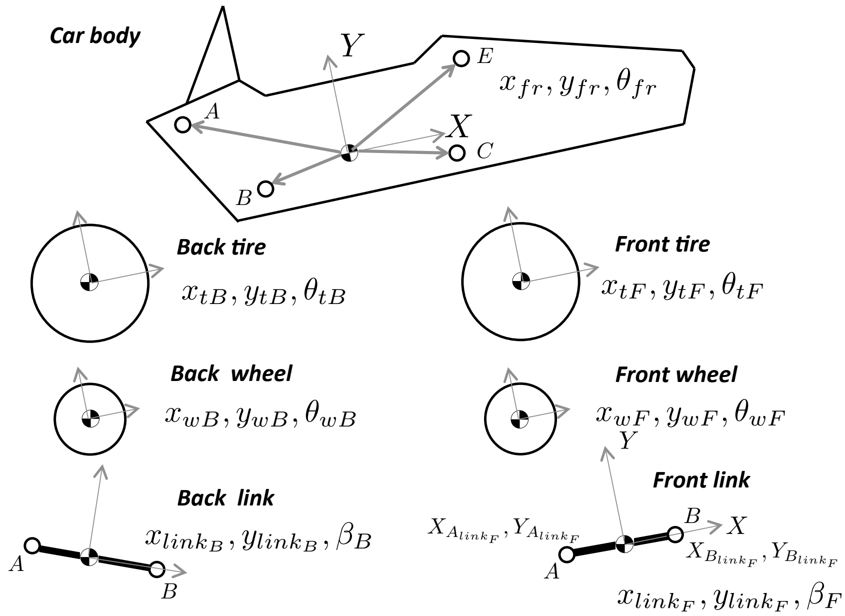

Figure 3: Rigid bodies composing the car model.

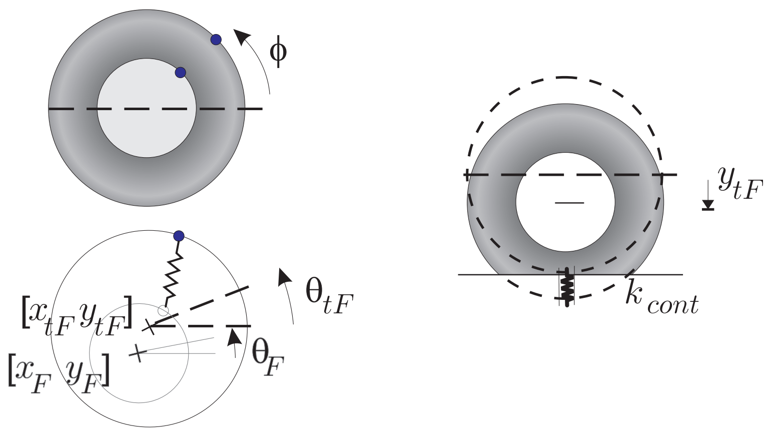

Figure 4: Simplified tire model

DATA

The necessary geometrical and mechanical data are defined in the file `car_data_2014.m`. The main file is `FS_2016.m`.

PART 1 - MODELING AND ANALYSIS STEPS

The analysis of the car is performed in the following steps:

1. **Kinematics**

   Consider the different constraints on the system and find the number of degrees of freedom remaining. Define the kinematic relations between the different variables and choose the generalized coordinates. (DONE)

2. **Add a driver**

   The car is still missing a driver. Model the driver as a point mass of 32.5 kg (we need to model half system because of symmetry!) and rigidly attach it to the car frame at some reasonable location. (DONE)

3. **Energies**

   Build the expressions for the kinetic $\mathcal{T}$ and potential energies $\mathcal{V}$. (DONE) In particular, explain in detail how the expression of the tires potential is derived.

4. **Equations of motion** In the absence of damping, and non-conservative forces, find the dynamic equations of motion by applying Lagrange equations. (DONE)

5. **Equilibrium**

   Find an equilibrium position of the car under gravity forces. For that, you need to suppress the rigid body motion present into the system. State how many rigid body modes are present and how you suppress them. Then verify that the solution found is also an equilibrium position when the suppressed rigid body motion are arbitrary prescribed. Explain why rigid body modes need to be suppressed in order to find and equilibrium. Find then the linearized mass and stiffness matrices around that equilibrium position. (DONE). Try to find two other equilibrium positions by choosing a different initial guess configuration. Plot the obtained configurations with the `car_plot.m` file. (TO BE DONE)

6. **Stability**

   Discuss the stability of all the equilibrium positions you found. (TO BE DONE)

7. **Linearized equations**

   Find the linearized mass and stiffness matrix around the stable equilibrium position found. (DONE)

8. **Eigenmodes and eigenfrequencies**

   Find the eigenmodes and the eigenfrequencies of the car around the stable equilibrium position. (DONE). Plot the mode shapes (i.e. plot the deformed configuration of the car) and describe them (TO BE DONE)

9. **Add damping and drag forces, and engine torque.**

   Add the suspension damper forces, the contact damping force, and the aerodynamic drag. A torque is added to the back wheel to simulate the action of the engine (DONE). Explain clearly how the various generalized forces are incorporated in the Lagrange equations.

10. **Nonlinear Time integration**

    We want now to estimate the acceleration and maximum speed performance of the car. In order to do this, we perform a time integration of the nonlinear equation of motion (DONE). This is done by writing the system in the state-space form and using the `ODE45` time integration function of Matlab. Examine the horizontal acceleration of the frame. Which is the maximum speed of the car? How much time it takes to reach 80 km/h? In particular, plot the following degrees of freedom (TO BE DONE):

    a) the horizontal displacement of the frame
    b) the vertical displacement of the frame
    c) the wheel-ground elastic forces
    d) the orientation of the rear linkage

PART 2 - ADD WINGS (TO BE DONE)

In order to increase the downforce for cornering, a front and rear wing are added. The two wings are rigidly attached to the car frame by very stiff supports and located as shown in Figure 5.

The drag and lift forces (indicated F_D and F_L respectively) provided by each wing are given by:

$$F_D = -\frac{1}{2}\rho V^2 S\left(c_{D0} + c_{D\alpha}\left(\alpha_0 - \theta_{fr}\right)^2\right)\underline{e}_x$$

$$F_L = -\frac{1}{2}\rho V^2 S c_{L\alpha}\left(\alpha_0 - \theta_{fr}\right)\underline{e}_y$$

where $V = \dot{x}_{fr}$ is the velocity of the car, ρ is the air density, and S is the wing surface. This formula accounts for the pitch of the car frame in effectively changing the angle of attack of the wing profile. Include the contribution of the aerodynamic forces in the Lagrange equations and explain clearly how this have to be implemented. The necessary data are included in the file `wing_data.m`. Redo the steps of Part 1. How much time it takes now to reach 80 Km/h? How is the maximum speed affected? Compare

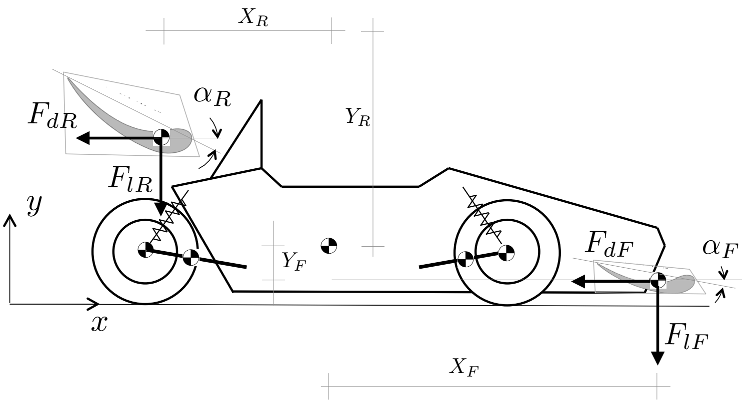

Figure 5: Front and rear wings layout.

the time history of the following degrees of freedom with the ones obtained for the car without wings:

a) the horizontal displacement of the frame
b) the vertical displacement of the frame
c) the wheel-ground elastic forces
d) the orientation of the rear linkage

and briefly comment the results.

PART 3 - HARMONIC RESPONSE (TO BE DONE)

Consider now the car without wings. Suppress the rigid motion of the car by constraining the rotation of the back wheel, and apply a harmonic, vertical force of 30 N at the center of mass of the frame at a frequency spanning the [0 - 50] Hz interval. Consider the linearized behavior by constructing the Frequency Response Function using modal superposition and a diagonal modal damping matrix. Plot the following degrees of freedom:

a) the vertical displacement of the frame
b) the rotation of the fame
c) the wheel-ground elastic forces

as a function of the forcing frequency, and briefly comment the results.

NOTE FOR THE EXAM

Be prepared to discuss each part of the assignment during the exam. Numerical results and equations can be shown by means of a presentation or a report. The actual form of the document is not very important - the important thing is your comprehension of the problem!