ADVANCED DYNAMICS, SPRING SEMESTER 2023
ETH ZÜRICH

# Assignment 1

Paolo Tiso

## 1 WHEEL ROLLING WITHOUT SLIPPING ON A 2D TRACK

A thin wheel of radius $R$ rolls without slipping on a track on the $x_1 - x_2$ plane, defined by $x_2 = f(x_1)$. The wheel plane stays vertical and tangent to such track at the contact point $P$. Denote with $\alpha$ the angle the disk plane forms with the $x_2$ axis, and with $\phi$ the rotation of the disk about its axis $\mathbf{e}_\phi$. The position of the center of the disk $C$ is indicated by $x_1^C$, $x_2^C$ and $x_3^C$. Assume a set of generalized coordinate $\mathbf{q} = [x_1^C \; x_2^C \; x_3^C \; \alpha \; \phi]$.

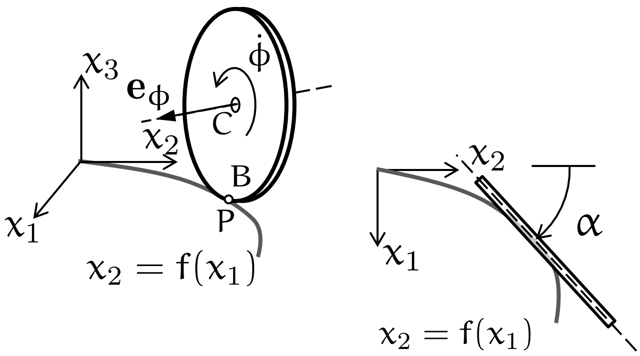


Figure 1.1: Wheel rolling without slipping on a track.

1. State all the constraints acting on the disk.

2. Determine whether the constraints are holonomic or non-holonomic.

3. Determine the degrees of freedom of the system.

## 2 Two bars linkage

Two bars $AB$ and $BC$ of equal length $L$ are hinged at point $B$, and move in the plane spanned by the unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$. The velocity $\mathbf{v}_C$ of point $C$ is required to be directed towards point $A$ at all times, as shown. Show that such constraint is non-holonomic. Use $x_1^B$, $x_2^B$, $\theta_1$ and $\theta_2$ as generalized coordinates.

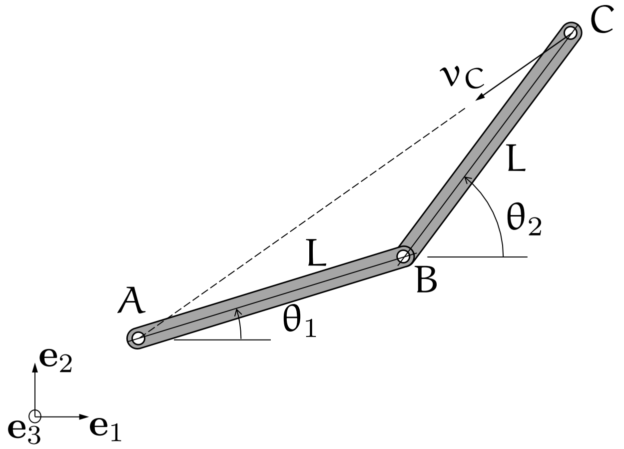


Figure 2.1: A two bar linkage in 2D. The velocity $v_C$ must be directed towards $A$ at all times.