Angular Correlation of Gamma Rays

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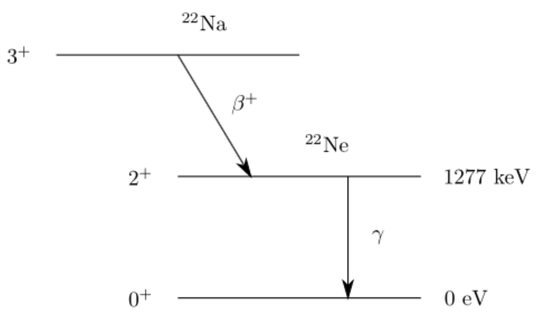
Introduction

Theoretical arguments predict that successive γ-rays emitted from the same spinning nucleus should be distributed anisotropically. Using a pair of scintillator detectors and a coincidence circuit, we will measure the angular correlation functions for γ-rays emitted from sodium and cobalt nuclei and verify theoretical predictions.

1. Theoretical Background

When two γ-rays are emitted in succession from an atomic nucleus, their directions are often correlated due to the physics of the emission process. In particular, when an excited nuclear state decays to the ground state through one or more intermediate states, the spin of the nucleus affects the angular distribution of the photons emitted during each transition. The relative probability that a photon will be emitted at an angle Θ with respect to a previously emitted photon is denoted $W(\Theta)$ and depends both on the angular momenta of the states involved in the transitions and on the multipole order of the emitted radiation. The angular correlation of successive γ-rays for all possible combinations of pure dipole and quandrupole radiations has been explicitly calculated by Hamilton and Goertzel [1, 2], and a



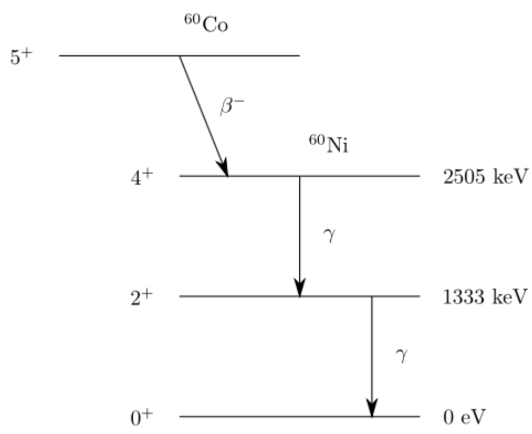


Figure 1: (*left*) Energy level diagram for the primary decay mechanism of ^{22}Na. The half-life is 2.6 yr, and the initial β^+ decay occurs with 90% probability. (*right*) Diagram for ^{60}Co decay. Half-life is 5.1 yr, and the intermediate state lives for $\sim$1 ps.

gneral discussion of successive radiations has been given by Yang [3]. In this practicum we will experimentally verify these results for ^{60}Co.

Nuclear processes, such as electron-positron pair annihilation, can also produce correlated γ-rays. An example of a simple angular distribution is particularly exhibited by ^{22}Na, illustrated by a diagram in figure 1, left. After undergoing β^{+} decay with a branching ratio of 90%, the resulting positron is captured by an electron, and the pair annihilates to produce a pair of 511 keV (rest mass of an electron) γ-rays. As the momentum of the positronium system must be concerved, the photons must be emitted in the opposite directions ('back-to-back'). Therefore, the angular distribution function for ^{22}Na is given by:

$$W(\Theta) = \delta(\Theta - \pi). \tag{1}$$

However, there is a finite width of angular correlation due to finite angular resolution of the detectors used in the experiment. Due to the sharp correlation of ^{22}Na, it is often used for calibration purposes.

As shown in figure 1, right, the ^{60}Co isotope decays through β^{-}-emission into an excited state of ^{60}Ni (2507 keV). This state then de-excites by a γ-ray cascade through an intermediate 1332 keV state, Since the lifetime of the intermediate ^{60}Ni state is on the order of only $\sim$1 ps, and the time resolution of the experimental setup is finite, the two γ-rays, with energies of 1332 keV and 1172 keV, will appear experimentally in coincidence.

The probability, per unit solid angle, that two successive γ-rays are emitted at an angle Θ is proportional to:

$$W(\Theta) = 1 + \sum_{1}^{l} a_i \, cos^{2i}\Theta, \tag{2}$$

where $2l$ is the order of the lowest multipole in the cascade. Thus, if both γ-rays are quadrupoles:

$$W(\Theta) = 1 + a_1 \, cos^2\Theta + a_2 \, cos^4\Theta, \tag{3}$$

A further restriction on the number of terms in $W(\Theta)$ is $a_i = 0$ for $i \; J_2$, where J_2 is the spin of the intermediate state in the cascade. Thus if J_2 is zero or $1/2$, the angular correlation will always be isotropic; if J_2 is 1 or $3/2$, the correlation will at most contain terms in $cos^2\Theta$.

The coefficients a_1 and a_2 have been given by Hamilton [1] for all possible combinations of angular momenta. The values of these coefficients for the values of J which are of interest in connection with our experiment are listed in figure 2. If a transition involves mixed multipoles, e.g. electric quadrupole and magnetic dipole components, the situation becomes very complicated and the coefficients depend not only on the relative intensities of the two components, but also on their relative phases [4].

2. Experimental Setup

In order to measure the angular correlation of γ-rays from decays of various radioactive sources, we will use a detector array composed of two sodium iodide (NaI) scintillation counters doped with thallium (Tl), shown in figure 3. They are placed on a goniometer table providing a measure of the angle Θ, where the radioactive source is placed in the middle, one

J_2	J_3	Multipoles	a_1	a_2
1	0	Dipole–Dipole	1	0
1	1	Dipole–Dipole	−1/3	0
1	2	Dipole–Dipole	−1/3	0
1	1	Quadrupole–Dipole	−1/3	0
1	2	Quadrupole–Dipole	3/7	0
1	3	Quadrupole–Dipole	−3/29	0
2	3	Dipole–Quadrupole	−3/29	0
2	2	Dipole–Quadrupole	3/7	0
2	1	Dipole–Quadrupole	−1/3	0
2	0	Quadrupole–Quadrupole	−3	4
2	1	Quadrupole–Quadrupole	5	−16/3
2	2	Quadrupole–Quadrupole	−15/13	16/13
2	3	Quadrupole–Quadrupole	0	−1/3
2	4	Quadrupole–Quadrupole	1/8	1/24

Figure 2: Coefficients for the angular correlation of γ-rays with the spin of the ground state $J_1 = 0$ [1].

of the detectors (larger one) has a fixed position, and another (smaller) can be moved around. When ionising radiation interacts with the NaI crystal, scintillation light with the wavelength of 410 nm is produced, and the number of generated photons is proportional to the deposited energy. This light falls on a photocathode of a photomultiplier tube that produces an electric pulse (see working principle in figure 3, right), again proportional to the energy loss by a particle.

The radioactive sources used in the experiment consist of a small fragment of radioactive material encapsulated in a cylindrical stainless steel container.

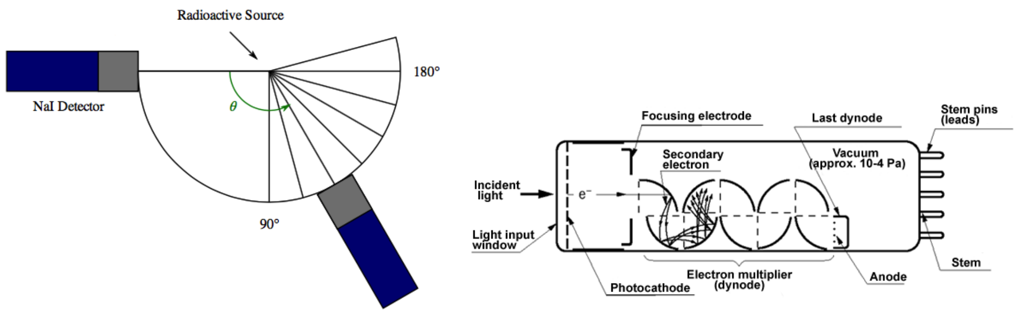


Figure 3: (*left*) Schematic of the experimental setup for the measurements of angular correlation of γ-rays with two NaI(Tl) scintillation detectors. (*right*) Working principle of a photomultiplier tube.

The scintillation light from particle interactions in the NaI(Tl) crystals is detected with photomultiplier tubes (PMTs). The operating voltage for the larger detector should not exceed 1.4 kV, and for the small one should be at maximum 1.0 kV. It is recommended that the high voltage (HV) power supply be turned on prior to the experiment and let warm up for about half an hour, in order to reduce the possibility of a shift in the PMT gain during data acquisition.

Our aim is to record the rate of γ-ray interactions coincident in both detectors for each angle, which then will be normalised (e.g. to the value corresponding to $\Theta = 90°$), and fitted with a function presented in equation 3 in order to extract the angular correlation coefficients and compare them with theoretical predictions from figure 2. The measurements are to be performed in a Θ range of 60° to 180° with a regular increment of 15°. In order to acquire sufficient statistics for analysis, the measurements at each angle has to run for about 20 min.

The schematic of the experimental setup is illustrated in figure 4. With the help of electronics module we will read-out, perform online analysis on the detector signals, set up coincidence, measure and record the data. The analog signals from both detectors will be split into two branches: 'energy branch', shown on the diagram with yellow color, which we will use to generate the trigger signal, and 'timing branch' (blue on the diagram), which will provide the signal to be measured and recorded with a computer.

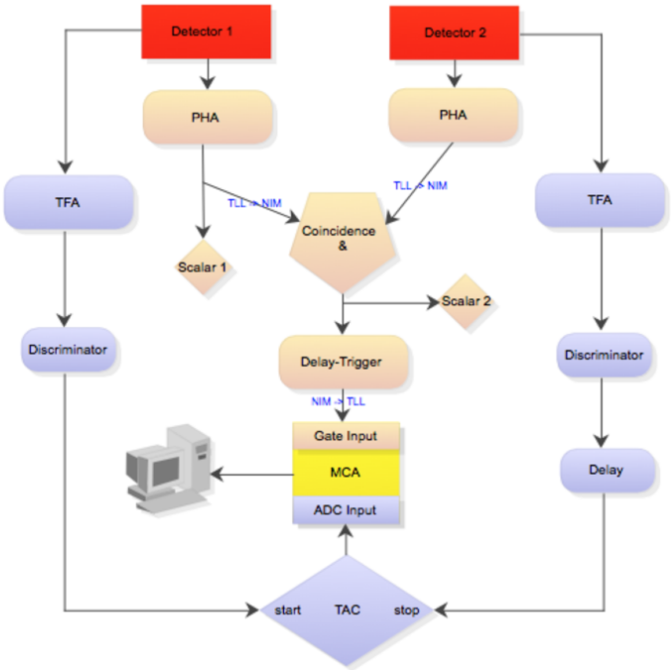


Figure 4: Schematic of the electronics chain for the angular correlation measurements. A series of NIM modules analyse the incoming pulses from each detector, set the requirements on particle energy, decide if pulses are in coincidence, and if so send a pulse corresponding to the time delay between the two detectors to an MCA for pulse height analysis.

The electronic modules that we will use to perform online analysis of the detectors' signals in our experiment are the following:

• Amplifier / Pulse height analyser (PHA).
The amplifier receives the linear (analog) pulse from NaI detector, shapes and amplifies it, and provides an output suitable for analysis by e.g. the muti-channel analyser. The pulse height analyser is an additional feature that works with the same principle as the discriminator module, where a standard logic pulse (TTL) is generated for every signal above some pre-defined lower threshold; the higher threshold can be set with a 'window' knob. We will use this type of amplifier for spectroscopy (energy spectra) measurements.

• Timing filter amplifier (TFA).
Timing filter amplifier is a wideband, pulse shaping amplifier designed to enhance timing analysis. By adjusting the time constant settings, the user can optimize pulse shape and slope-to-noise ratio when making timing measurements with moderate speed NaI radiation detectors. Independent integrate and differentiate controls allow flexible control of shaping with selection of different time constants.

• Discriminator.
The discriminator takes an analog input signal and outputs a standard logic pulse for every

signal above a preset threshold, with adjustable width. These standard (NIM) pulses are needed for the coincidence circuit, for the scaler modules, for the TAC, or other modules performing logic operations.

- Level adapter (LA).

The level adaptor takes an input logic pulse and converts it to the type of pulse needed by the electronic modules (TTL or NIM).

- Fan-out module.

The fan-out module allows to split the signal, thus providing several separate identical outputs, which can then be fed to several different circuits. A linear fan-out can be used with analog signals.

- Coincidence module.

The outputs from discriminators are fed into the coincidence circuit which generates a (NIM) output pulse whenever there is an overlap of input pulses from all enabled channels. A variable delay might be needed to adjust the timing of both detectors with respect to each other.

- Scaler.

The scaler records the rate of the incoming signals. We will use one to record the individual detector counting rate, and the other one to read the coincidence counting rate. This is primarily used as a cross-check of the experiment.

- Delay module.

A variable delay is a 'passive' module, where the input pulse can be routed via cables of different length, thus adjusting the signal delay.

- Delay trigger (DT).

Delay trigger is a circuit that generates a trigger signal with adjustable delay and length from an incoming rectangular signal, thus generating a 'gate'. The rising edge of the incoming signal is used to trigger the new trigger signal. This module makes it possible to delay the onset of a trigger pulse, and also change its length.

- Time-to-amplitude converter (TAC).

Time-to-amplitude converter is a module that generates a pulse with the amplitude proportional to the time delay between the 'stop' and 'start' NIM signals (in our case the pulses from both detectors, passing the TFA and discriminator modules).

- Multichannel analyser (MCA).

A multichannel analyzer is a laboratory instrument used to analyze an input signal that primarily consists of pulses. In pulse-height analysis (PHA) mode which we will be using, the pulses are counted based on amplitude. The number of different amplitudes that are counted depends on the number of channels of the MCA, but is normally in the range of a few thousand. In this way a histogram of frequency against pulse amplitude (or 'height') can be produced and sent to a computer.

All these modules belong to the Nuclear Instrumentation Module (NIM) standard, which defines mechanical and electrical specifications for electronics modules used in experimental particle and nuclear physics. This is the first, and perhaps the simplest, standard defined by the U.S. Atomic Energy Commission's report TID-20893 in 1968-1969, and most recently revised in 1990 (DOE/ER-0457T). It provides a common footprint for electronic modules (amplifiers, ADCs, DACs, discriminators, etc.), which plug into a larger chassis (NIM crate, or NIM bin). The crate must supply ±12 V and ±24 V DC power to the modules via a backplane; the standard also specifies ±6 V DC and 220 V or 110 V AC pins, but not all NIM bins provide them. Mechanically, NIM modules must have a minimum standard width of 1.35 inch (34 mm), a maximum faceplate height of 8.7 inch (221 mm) and depth of 9.7 inch (246 mm) [5]. They can, however, also be built in multiples of this standard width.

3. Measurements strategy

1. Measure the diameter of the NaI detectors and the distance from the source to the first detector. Calculate the distance to the second detector in order to have the same solid angle coverage, and hence the angular acceptance, as for the first detector.

2. Connect the SHV from the power supply to the NaI detectors.

3. Connect the BNC signal cables (PMT output) to the oscilloscope. Find baseline, set threshold to have a good sensitivity (sensitivity to single photoelecton emitted from the photocathode).

4. Set the voltage for the detectors, taking into account the limit settings written above. Do it in steps and monitor the signals from the PMTs - want to avoid any strange features and discharges which could point to a PMT failure.

5. Unplug the two signal cables from the oscilloscope, install PHA units in the NIM crate, and connect the detector signals to their input. Split the signal with a passive splitter and cables, terminate the not used output with a 50 Ohm resistor.

6. Connect the output of one PHA to the MCA input. Connect the corresponding logic (TTL) signal to the gate.

7. Install the ^{60}Co radioactive source into the holder in the middle of the table.

8. Load the Maestro software on the PC. Check the settings, the 'gate' should be off.

9. The acquisition can be started now, and the obtained spectra erased if any setting on the PHA is adjusted. Set the amplification gain such way that the resolution is best.

10. Acquire the energy spectrum of ^{60}Co with the first detector. Save into an ASCII file (create a sub-folder for you group). In the offline analysis, use the literature values for ^{60}Co γ-rays in order to calibrate the energy scale.

11. While the Adjust the 'lower level discriminator' and 'window/width' on PHA in order to have only full photoabsorption peaks. The 'gate' in Maestro settings should be enabled for this operation.

12. Perform the last 3 steps for the second detector. Now we have logic (TTL) signals from the two PHA units, corresponding to both detectors, which are 'true' when a γ-ray deposits energy in the NaI crystal in the energy range of full photoabsorption. Based on these signals, we will set the coincidence in the next step.

13. In order to check if the signals are in coincidence, we want to look at them with an oscilloscope. In addition, the coincidence module accepts only NIM standard. Hence, we first convert both signals from TTL to NIM with a level adapter, and then feed them into fan-out, such that we can have two copies.

14. Connect the NIM outputs of the level adapters to the coincidence module, the other copy into the oscilloscope. Into the 3rd channel in the oscilloscope plug the NIM output of the coincidence module. Adjust the width and delays such that the signals are in coincidence.

15. Unplug the cables from the oscilloscope. Take one copy of the signal from one of the detectors and feed it into a scaler. Now it measures the number of signals originating from particle interactions in this detector.

16. Install the second scaler module. Feed into it the NIM output from the coincidence module. For this you need a fan-out. In principle now we could already perform the experiment only using the scalers, but we will use it only as a cross-check of the results that we will obtain with an MCA.

17. Connect the so far unused split analog signal from the detectors to the TFA modules. Connect the output of the TFAs to the oscilloscope. Adjust the timing settings and gain to have clean signals with good timing characteristics.

18. Feed the outputs of the TFA modules into the discriminators. Adjust the threshold such way that the noise is not triggered.

19. Now we are ready to plug the NIM signals from the discriminators into the TAC module and measure the delay between them. Since the delay is essentially zero, and the TAC cannot measure negative time difference, we artificially shift the zero by adding a delay for one of the detectors' signals.

20. Perform time calibration. For this, split the NIM signal from one of the NaI detectors/discriminators and feed one copy through the delay box. Connect first copy into the 'start' inout of the TAC module, and the delayed copy into 'stop'. Connect the TAC output into the MCA ('gate' off), start data acquisition. Vary the delay, observe peaks in different ADC channels. Use 5-10 different settings, save the file.

21. Now we want to use the coincidence signal from the 'energy branch' as a gate signal for the data acquisition with the MCA. It should be fed into the delay trigger module. Connect the output of the module to the oscilloscope, as well as the output of the TAC (signal that we want to measure - delay time between the particle interactions in both NaI detectors). Adjust the delay trigger settings (width, delay time) such way that the TAC signal is always within the 'gate' signal generated by the delay trigger.

22. Plug the TAC output into the 'ADC in', the delay trigger into the gate. Switch on the gate in Maestro settings. Acquire the time spectrum. Done.

4. Laboratory Report

A common laboratory report should be prepared by each group performing the experiment. The main points to be included in the report are the following:

- Description of ^{60}Co decay, experimental goal.
- Theoretical considerations relevant for the present experiment: angular correlation function, nuclear state transitions, Clebsch-Gordan coefficients, function for fitting the measured data points.
- Measurement principle: schematic diagram of the experimental setup, coincidence, two branches (energy and time), what is the measured and what serves as a trigger, analog and logical signals. Can also include the pictures of the electronics rack (with a short description).
- Description of the NaI detectors and working principle of the photomultiplier tube.
- Short (1-2 sentences) description of the of the electronic/logic modules involved in the experiment.
- Angular acceptance of the measurements setup; role of the solid angle and its calculation.
- Energy spectra of ^{60}Co for both detectors, indicating the energy windows set with the pulse height analyser, energy calibration of the ADC. Describe what is seen in the spectra (full photoabsorption peaks, Compton continuum, etc.)
- Time measurement: why do we need the extra branch, origin and effect of random coincidences, time calibration of the ADC (include the calibration curve).
- Measurement protocols (original values written).
- Gaussian fit of the delay time measured with the ADC: describe ranges where you calculate integral rate of signal and background events, and how background subtraction is performed; elaborate on the origin of background events.
- Why do we use two scalers? What results can be obtained with them? Is there a difference between measurements with the TAC and the scalers; if yes, why?
- What is the time resolution of the measurement?
- Angular correlation function and description of the results.
- Determine the coefficients α and the nuclear state cascade
- Dead time: how large it is and what is the effect of it.
- Comment on the stability of the measurement.
- Discussion, conclusions, comments.

In addition, please consider the following:

- Include the settings used for various modules: high voltage bias for the NaI detectors, gains, shaping times for the amplifiers, energy and time windows, etc.
- Scale the rates from number of counts to reasonable units (e.g. Hz = events/sec), makes it easier to make comparison of various plots.

- Plot with the proper units for energy and time measurements.
- Error calculations are very important.

References

[1] D.R. Hamilton, Physical Review 58, 122 (1940).
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