

UNAL ICPC Team Notebook (2024)

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1 Data Structures

1.1 DSU

```

const int N = 1e5+5;
int dsu[N];
int cc;

int find (int node){
    if(dsu[node] == -1) return node;
    return dsu[node] = find(dsu[node]);
}

bool connected(int A, int B){
    return find(A)==find(B);
}

void join (int A, int B){
    A = find(A);
    B = find(B);
    dsu[A] = B;
    cc--;
}

memset(dsu, -1, sizeof dsu);

```

1.2 DSU Pesos

```

int parent[MAX];
int rango[MAX];
int n;
void Init( int _n ){
    n = _n;
    for( int i = 0 ; i < n ; ++i ){
        parent[i] = i;
        rango[i] = 0;
    }
}

int Find( int x ){
    if( x == parent[ x ] )
        return x;
    else
        return parent[ x ] = Find( parent[ x ] );
}

void Union( int x , int y ){
    int xRoot = Find( x );
    int yRoot = Find( y );
    if( rango[ xRoot ] > rango[ yRoot ] )
        parent[ yRoot ] = xRoot;
    else{
        parent[ xRoot ] = yRoot;
        if( rango[ xRoot ] == rango[ yRoot ] )
            rango[ yRoot ]++;
    }
}

int countComponents(){
    int c = 0;
    for( int i=0; i<n; i++ )
        if( parent[i] == i )
            c++;
    return c++;
}

vector<int> getRoots(){
    vector<int> v;
    for( int i=0; i<n; i++ )
        if( i == parent[i] )
            v.push_back(i);
    return v;
}

int countNodesInComponent( int root ){
    int c = 0;
    for( int i=0; i<n; i++ )
        if( Find(i) == root )
            c++;
    return c++;
}

bool sameComponent( int x, int y ){
    return Find(x) == Find(y);
}

```

2 Graphs

2.1 Strongest Connected components

```
vector<bool> visited; // keeps track of which vertices are already visited

// runs depth first search starting at vertex v.
// each visited vertex is appended to the output vector when dfs leaves it.
void dfs(int v, vector<vector<int>> const& adj, vector<int> &output) {
    visited[v] = true;
    for (auto u : adj[v])
        if (!visited[u])
            dfs(u, adj, output);
    output.push_back(v);
}

// input: adj -- adjacency list of G
// output: components -- the strongly connected components in G
// output: adj_cond -- adjacency list of G's SCC (by root vertices)
void strongly_connected_components(vector<vector<int>> const& adj,
                                   vector<vector<int>> &components,
                                   vector<vector<int>> &adj_cond) {

    int n = adj.size();
    components.clear(), adj_cond.clear();

    vector<int> order; // will be a sorted list of G's vertices by exit time

    visited.assign(n, false);

    // first series of depth first searches
    for (int i = 0; i < n; i++)
        if (!visited[i])
            dfs(i, adj, order);

    // create adjacency list of G^T
    vector<vector<int>> adj_rev(n);
    for (int v = 0; v < n; v++)
        for (int u : adj[v])
            adj_rev[u].push_back(v);

    visited.assign(n, false);
    reverse(order.begin(), order.end());

    vector<int> roots(n, 0); // gives the root vertex of a vertex's SCC

    // second series of depth first searches
    for (auto v : order)
        if (!visited[v]) {
            std::vector<int> component;
            dfs(v, adj_rev, component);
            components.push_back(component);
            int root = *min_element(begin(component), end(component));
            for (auto u : component)
                roots[u] = root;
        }

    // add edges to condensation graph
    adj_cond.assign(n, {});
    for (int v = 0; v < n; v++)
        for (auto u : adj[v])
            if (roots[v] != roots[u])
                adj_cond[roots[v]].push_back(roots[u]);
}
```

2.2 SCC Tarjan

```
struct TarjanScc{
    vector<bool> marked;
    vector<int> id;
    vector<int> low;
    int pre;
    int count;
    stack<int> stck;
    vector<vector<int>> >G;

    TarjanScc( vector<vector<int>> >g, int V ){
        G=g;
        marked = vector<bool>(V, false);
        stck = stack<int>();
        id= low = vector<int>(V, 0);
        pre=count=0;
        for(int u=0; u<V; u++)
            if( !marked[u] ) dfs(u);
    }

    void dfs( int u ){
        marked[ u ] = true;
        low[ u ] = pre++;
        int min = low[ u ];

        stck.push( u );
```

```
        for( int w=0; w<G[u].size(); w++){
            if( !marked[G[u][w]] ) dfs( G[u][w] );
            if( low[ G[u][w] ] < min ) min = low[ G[u][w] ];
        }
        if( min<low[u] ){
            low[u] = min;
            return;
        }
        int w;
        do{
            w = stck.top();stck.pop();
            id[ w ] = count;
            low[ w ] = G.size();
        }while( w != u );
        count++;
    }

    int getCount() { return count; }

    // are v and w strongly connected?
    bool stronglyConnected(int v, int w) {
        return id[v] == id[w];
    }

    // in which strongly connected component is vertex v?
    int getId(int v) { return id[v]; }
};

//Ejemplo de Uso
int main( ){
    int u, v, N, M, cas, k=0;
    for (cin>>cas; k<cas; k++){
        scanf("%d %d", &N, &M);
        //cin>>N>>M;
        vector<vector<int>> >G(N);

        for(int i=0; i < M; i++){
            scanf("%d %d", &u, &v);
            //cin>>u>>v;
            u--;v--;
            G[u].PB(v);
        }

        TarjanScc tscc(G, N);
        //Encontrar cuantos nodos tienen grado de entrada 0
        vector<int> indegree(tscc.getCount(), 0);
        int idu, idv;
        for ( u = 0; u < N; u++){
            idu = tscc.getId( u );
            for ( v = 0; v < G[u].size(); v++){
                idv = tscc.getId( G[u][v] );

                if( idu!=idv ){
                    indegree[idv]++;
                }
            }
        }
        int res=0;
        for(int i=0; i<indegree.size(); i++){
            if(indegree[i]==0)res++;
        }
        printf("Case %d: %d\n",k+1,res);
    }
    return 0;
}
```

2.3 Topological sort

```
int n; // number of vertices
vector<vector<int>> adj; // adjacency list of graph
vector<bool> visited;
vector<int> ans;

void dfs(int v) {
    visited[v] = true;
    for (int u : adj[v]) {
        if (!visited[u])
            dfs(u);
    }
    ans.push_back(v);
}

void topological_sort() {
    visited.assign(n, false);
    ans.clear();
    for (int i = 0; i < n; ++i) {
```

```

        if (!visited[i]) {
            dfs(i);
        }
    }
    reverse(ans.begin(), ans.end());
}

```

2.4 Floyd-Warshall

```

//O(n^3)

//inicializar todo en INF previo a la lectura
for (int k = 0; k < n; ++k) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
        }
    }
}

//Si se tienen pesos negativos:
for (int k = 0; k < n; ++k) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (d[i][k] < INF && d[k][j] < INF)
                d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
        }
    }
}

//Pesos reales
if (d[i][k] + d[k][j] < d[i][j] - EPS)
    d[i][j] = d[i][k] + d[k][j];

/*Identificar ciclos negativos:
Si al final del algoritmo d[i][i] es negativo.*/

```

2.5 Dijkstra

```

//O(n^2+m)
for (int i = 1; i <= n; i++) distance[i] = INF;
distance[x] = 0;
q.push({0,x});
while (!q.empty()) {
    int a = q.top().second; q.pop();
    if (processed[a]) continue;
    processed[a] = true;
    for (auto u : adj[a]) {
        int b = u.first, w = u.second;
        if (distance[a] + w < distance[b]) {
            distance[b] = distance[a] + w;
            q.push({-distance[b],b});
        }
    }
}

```

2.6 Shortest Path Fast algorithm

```

//O(nm)
const int INF = 1000000000;
vector<vector<pair<int, int>>> adj;

bool spfa(int s, vector<int>& d) {
    int n = adj.size();
    d.assign(n, INF);
    vector<int> cnt(n, 0);
    vector<bool> inqueue(n, false);
    queue<int> q;

    d[s] = 0;
    q.push(s);
    inqueue[s] = true;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        inqueue[v] = false;

```

```

        for (auto edge : adj[v]) {
            int to = edge.first;
            int len = edge.second;

            if (d[v] + len < d[to]) {
                d[to] = d[v] + len;
                if (!inqueue[to]) {
                    q.push(to);
                    inqueue[to] = true;
                    cnt[to]++;
                    if (cnt[to] > n)
                        return false; // negative cycle
                }
            }
        }
    }
    return true;
}

```

3 Dynamic Programming

3.1 Coin Exchange Problem

```

#include <bits/stdc++.h>

using namespace std;

// Returns total distinct ways to make sum using n coins of
// different denominations
int count(vector<int>& coins, int n, int sum)
{
    // 2d dp array where n is the number of coin
    // denominations and sum is the target sum
    vector<vector<int>> dp(n + 1, vector<int>(sum + 1, 0));

    // Represents the base case where the target sum is 0,
    // and there is only one way to make change: by not
    // selecting any coin
    dp[0][0] = 1;
    for (int i = 1; i <= n; i++) {
        for (int j = 0; j <= sum; j++) {

            // Add the number of ways to make change without
            // using the current coin,
            dp[i][j] += dp[i - 1][j];

            if ((j - coins[i - 1]) >= 0) {

                // Add the number of ways to make change
                // using the current coin
                dp[i][j] += dp[i][j - coins[i - 1]];
            }
        }
    }
    return dp[n][sum];
}

// Driver Code
int main()
{
    vector<int> coins{ 1, 2, 3 };
    int n = 3;
    int sum = 5;
    cout << count(coins, n, sum);
    return 0;
}

```

4 Flows

4.1 Dinic

```

struct FlowEdge {
    int v, u;
    long long cap, flow = 0;
    FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(cap) {}
};

struct Dinic {

```

```
const long long flow_inf = 1e18;
vector<FlowEdge> edges;
vector<vector<int>> adj;
int n, m = 0;
int s, t;
vector<int> level, ptr;
queue<int> q;

Dinic(int n, int s, int t) : n(n), s(s), t(t) {
    adj.resize(n);
    level.resize(n);
    ptr.resize(n);
}

void add_edge(int v, int u, long long cap) {
    edges.emplace_back(v, u, cap);
    edges.emplace_back(u, v, 0);
    adj[v].push_back(m);
    adj[u].push_back(m + 1);
    m += 2;
}

bool bfs() {
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (int id : adj[v]) {
            if (edges[id].cap - edges[id].flow < 1)
                continue;
            if (level[edges[id].u] != -1)
                continue;
            level[edges[id].u] = level[v] + 1;
            q.push(edges[id].u);
        }
    }
    return level[t] != -1;
}

long long dfs(int v, long long pushed) {
    if (pushed == 0)
        return 0;
    if (v == t)
        return pushed;
    for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {
        int id = adj[v][cid];
        int u = edges[id].u;
        if (level[v] + 1 != level[u] || edges[id].cap - edges[id].flow < 1)
            continue;
        long long tr = dfs(u, min(pushed, edges[id].cap - edges[id].flow));
        if (tr == 0)
            continue;
        edges[id].flow += tr;
        edges[id + 1].flow -= tr;
        return tr;
    }
    return 0;
}

long long flow() {
    long long f = 0;
    while (true) {
        fill(level.begin(), level.end(), -1);
        level[s] = 0;
        q.push(s);
        if (!bfs())
            break;
        fill(ptr.begin(), ptr.end(), 0);
        while (long long pushed = dfs(s, flow_inf)) {
            f += pushed;
        }
    }
    return f;
}
};
```

4.2 MinCost Flow

```
struct Edge
{
    int from, to, capacity, cost;
};

vector<vector<int>> adj, cost, capacity;

const int INF = 1e9;

void shortest_paths(int n, int v0, vector<int>& d, vector<int>& p) {
    d.assign(n, INF);
```

```
d[v0] = 0;
vector<bool> inq(n, false);
queue<int> q;
q.push(v0);
p.assign(n, -1);

while (!q.empty()) {
    int u = q.front();
    q.pop();
    inq[u] = false;
    for (int v : adj[u]) {
        if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v]) {
            d[v] = d[u] + cost[u][v];
            p[v] = u;
            if (!inq[v]) {
                inq[v] = true;
                q.push(v);
            }
        }
    }
}

int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t) {
    adj.assign(N, vector<int>());
    cost.assign(N, vector<int>(N, 0));
    capacity.assign(N, vector<int>(N, 0));
    for (Edge e : edges) {
        adj[e.from].push_back(e.to);
        adj[e.to].push_back(e.from);
        cost[e.from][e.to] = e.cost;
        cost[e.to][e.from] = -e.cost;
        capacity[e.from][e.to] = e.capacity;
    }

    int flow = 0;
    int cost = 0;
    vector<int> d, p;
    while (flow < K) {
        shortest_paths(N, s, d, p);
        if (d[t] == INF)
            break;

        // find max flow on that path
        int f = K - flow;
        int cur = t;
        while (cur != s) {
            f = min(f, capacity[p[cur]][cur]);
            cur = p[cur];
        }

        // apply flow
        flow += f;
        cost += f * d[t];
        cur = t;
        while (cur != s) {
            capacity[p[cur]][cur] -= f;
            capacity[cur][p[cur]] += f;
            cur = p[cur];
        }
    }

    if (flow < K)
        return -1;
    else
        return cost;
}
```

5 Math

5.1 Primes

```
/*
2      3      5      7      11      13      17      19      23      29      31      37      41
97     101    103    107    109    113    127    131    137    139    149    151    157
227    229    233    239    241    251    257    263    269    271    277    281    283
367    373    379    383    389    397    401    409    419    421    431    433    439
509    521    523    541    547    557    563    569    571    577    587    593    599
661    673    677    683    691    697    701    709    719    727    733    739    751
      757    761    769    773    787    797    809    811    821    823    827
```

829	839	853	857	859	863	877	881	883	887	907	911	919
	929	937	941	947	953	967	971	977	983	991	997	
1009	1013	1019	1021	1031	1033	1039	1049	1051	1061	1063	1069	1087
	1091	1093	1097	1103	1109	1117	1123					
1129	1151	1153	1163	1171	1181	1187	1193	1201	1213	1217	1223	1229
	1231	1237	1249	1259	1277	1279	1283					
1289	1291	1297	1301	1303	1307	1319	1321	1327	1361	1367	1373	1381
	1399	1409	1423	1427	1429	1433	1439					
1447	1451	1453	1459	1471	1481	1483	1487	1489	1493	1499	1511	1523
	1531	1543	1549	1553	1559	1567	1571					
1579	1583	1597	1601	1607	1609	1613	1619	1621	1627	1637	1657	1663
	1667	1669	1693	1697	1699	1709	1721					
1723	1733	1741	1747	1753	1759	1777	1783	1787	1789	1801	1811	1823
	1831	1847	1861	1867	1871	1873	1877					
1879	1889	1901	1907	1913	1931	1933	1949	1951	1973	1979	1987	1993
	1997	1999										

*/

5.2 Log Utils

```
ln: log()
log base 10: log10()
e: exp()
primos aproximados hasta x: x/ln(x) o x/(ln(x) - 1 ,08366)
```

5.3 Line Representation

```
//si b=0: es como si fuera oo o -oo
//si a=0: la fraccion es 0

struct frac{
    ll a,b;
    frac(ll_a, ll_b): a(_a), b(_b){
        if(b<0) a*=-1, b*=-1;
        if(b==0) a = 1;
    }

    bool operator < (frac other){
        return a * other.b < b * other.a;
    }
};

map<frac,frac> mp;
```

5.4 Criba

```
int criba[N];

for(int i=2; i < N; i++){
    if(!criba[i]){
        for(int k=i+i; k<N; k+=i){
            criba[k] = i;
        }
    }
}
```

5.5 FastCriba

```
#define forr(i,a,b) for(int i=(a); i<(b); i++)
typedef long long ll;
typedef pair<int,int> ii;

#define MAXP 100000 //no necesariamente primo
int criba[MAXP+1];
void crearcriba(){
    int w[] = {4,2,4,2,4,6,2,6};
    for(int p=25;p<=MAXP;p+=10) criba[p]=5;
    for(int p=9;p<=MAXP;p+=6) criba[p]=3;
    for(int p=4;p<=MAXP;p+=2) criba[p]=2;
    for(int p=7,cur=0;p<=MAXP;p+=w[cur++%7]) if (!criba[p])
        for(int j=p*p;j<=MAXP;j+=(p<<1) if(!criba[j]) criba[j]=p;
```

```
}
vector<int> primos;
void buscarprimos(){
    crearcriba();
    forr (i,2,MAXP+1) if (!criba[i]) primos.push_back(i);
}

//~ Useful for bit trick: #define SET(i) ( criba[(i)>>5]|=1<<(((i)&31) ), #define INDEX(i) ( (criba[i]
>>5)>>((i)&31))&1 ), unsigned int criba[MAXP/32+1];

int main() {
    freopen("primos", "w", stdout);
    buscarprimos();
    cout << '{';
    bool first=true;
    forall(it, primos){
        if(first) first=false;
        else cout << ',';
        cout << *it;
    }
    cout << "};\n";
    return 0;
}
```

5.6 Fast Exp.

```
ll binpow(ll a, ll b) {
    //si se necesita la potencia modulo m: aplicar el modulo a todas
    las multiplicaciones y a 'a' al antes del loop*/
    ll res = 1;
    while (b > 0) {
        if (b & 1)
            res = res * a;
        a = a * a;
        b >>= 1;
    }
    return res;
}
```

5.7 Fast Matrix Exp.

```
#define forn(i,n) forr(i,0,n)
#define SIZE 350
int NN;
double tmp[SIZE][SIZE];
void mul(double a[SIZE][SIZE], double b[SIZE][SIZE]){ zero(tmp);
    forn(i, NN) forn(j, NN) forn(k, NN) res[i][j]+=a[i][k]*b[k][j];
    forn(i, NN) forn(j, NN) a[i][j]=res[i][j];
}

void powmat(double a[SIZE][SIZE], int n, double res[SIZE][SIZE]){
    forn(i, NN) forn(j, NN) res[i][j]=(i==j);
    while(n){
        if(n&1) mul(res, a), n--;
        else mul(a, a), n/=2;
    }
}
```

5.8 Euclidean algorithm

```
//Iterative
int gcd(int a, int b, int& x, int& y) {
    x = 1, y = 0;
    int x1 = 0, y1 = 1, a1 = a, b1 = b;
    while (b1) {
        int q = a1 / b1;
        tie(x, x1) = make_tuple(x1, x - q * x1);
        tie(y, y1) = make_tuple(y1, y - q * y1);
        tie(a1, b1) = make_tuple(b1, a1 - q * b1);
    }
    return a1;
}
```

5.9 GaussJordan

```

const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity or a big number

int gauss (vector < vector<double> > a, vector<double> & ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;

    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }

    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }

    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}

```

5.10 Chinese Remainder Theorem

```

struct Congruence {
    long long a, m;
};

long long chinese_remainder_theorem(vector<Congruence> const& congruences) {
    long long M = 1;
    for (auto const& congruence : congruences) {
        M *= congruence.m;
    }

    long long solution = 0;
    for (auto const& congruence : congruences) {
        long long a_i = congruence.a;
        long long M_i = M / congruence.m;
        long long N_i = mod_inv(M_i, congruence.m);
        solution = (solution + a_i * M_i % M * N_i) % M;
    }
    return solution;
}

```

6 Range Queries

6.1 BIT

```

const int N = 200005;
int BIT[N];

void update(int idx, int val){
    for(; idx < N; idx += idx & (-idx)){
        BIT[idx] += val;
    }
}

```

```

int query (int idx){
    ll ret = 0;
    for(; idx > 0; idx-=idx&(-idx)){
        ret += BIT[idx];
    }
    return ret;
}

int query (int left, int right){
    return query(right) - query(left-1);
}

int lower_find(int val){
    int id = 0;
    for(int i = 31-__builtin_clz(n); i >= 0; --i){
        int nid = id | (1 << i);
        if(nid <= n && BIT[nid] <= val){
            val -= BIT[nid];
            id = nid;
        }
    }
    return id;
}

iota(idx+1, idx+n+1, 1);
sort(idx+1, idx+n+1, [](int i_a, int i_b) { return arr[i_a] > arr[i_b];});

//Update range [l,r] to v
update(l,v);
update(r+1,-v);

//Update specific value at pos k to u
ll prev = query(k)-query(k-1);
update(k,u);
update(k, -prev);

//Inversions
for(int i=1; i <=n; i++){
    forward[i] = query(values[i]);
    update(1,1);
    update(values[i],-1);
}
memset(BIT, 0, sizeof BIT);

for(int i=n; i > 0; i--){
    backward[i] = query(values[i]);
    update(values[i]+1, 1);
}

//Dimension change
sort(difval, difval+ind);
map<int,int> idx;
int cnt = 0;
idx[difval[0]] = cnt;
cnt++;
for(int i=1; i < ind; i++){
    if(difval[i] != difval[i-1]){
        idx[difval[i]] = cnt;
        cnt++;
    }
}

```

6.2 Segment Tree Range Query

```

const int N = 1e5; // limit for array size
int n; // array size
int t[2 * N];

void build() { // build the tree
    for (int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
} //O(n)

void modify(int p, int value) { // set value at position p
    for (t[p] += value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];
} //O(log(n))

int query(int l, int r) { // sum on interval [l, r]
    int res = 0;
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) res += t[l++];
        if (r&1) res += t[--r];
    }
    return res;
} //O(log(n))

```

```
int main() {
    scanf("%d", &n);
    for (int i = 0; i < n; ++i) scanf("%d", t + n + i);
    build();
    modify(0, 1);
    printf("%d\n", query(3, 11));
    return 0;
}
```

6.3 Segment Tree Range Update

```
void modify(int l, int r, int value) {
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) t[l++] += value;
        if (r&1) t[--r] += value;
    }

    int query(int p) {
        int res = 0;
        for (p += n; p > 0; p >>= 1) res += t[p];
        return res;
    }

    /*Push to inspect modifications*/

    void push() {
        for (int i = 1; i < n; ++i) {
            t[i<<1] += t[i];
            t[i<<1|1] += t[i];
            t[i] = 0;
        }
    }
}
```

6.4 Segment Tree Lazy Propagation

```
const int N = 500'005;
const int MOD = 998244353;

int add ( int A, int B ) { return A+B<MOD? A+B: A+B-MOD; }
int mul ( int A, int B ) { return 1l(A)*B % 1l(MOD); }
int sub ( int A, int B ) { return add ( A, MOD-B ); }

int n, q, h;

int sum[2*N];
pii lazy[2*N];
int lengths[2*N];

pii combine ( pii A, pii B ) {
    return {mul(A.ff, B.ff), add(mul(A.ss, B.ff), B.ss)};
}

void apply(int p, pii value) {
    sum[p] = add(mul(sum[p], value.ff), mul(lengths[p], value.ss));
    if (p < n) lazy[p] = combine(lazy[p], value);
}

void build_t() { // build the tree
    for (int i = n - 1; i > 0; --i) {
        sum[i] = add(sum[i<<1], sum[i<<1|1]);
        lengths[i] = lengths[i<<1]+lengths[i<<1|1];
        lazy[i] = {1,0};
    }
}

void build(int p) {
    while (p > 1){
        p >>= 1;
        if(lazy[p] == pii(1,0)) sum[p] = add(sum[p<<1], sum[p<<1|1]);
    }
}

void push(int p) {
    for (int s = h-1; s > 0; --s) {
        int i = p >> s;
        if (lazy[i] != pii(1,0)) {
            apply(i<<1, lazy[i]);
            apply(i<<1|1, lazy[i]);
            lazy[i] = {1,0};
        }
    }
}
```

```

    }
}

void modify(int l, int r, pii value) {
    l += n, r += n;
    int l0 = l, r0 = r;
    push(l0);
    push(r0 - 1);
    for (; l < r; l >>= 1, r >>= 1) {
        if (l&1) apply(l++, value);
        if (r&1) apply(--r, value);
    }
    build(l0);
    build(r0 - 1);
}

int query(int l, int r) {
    l += n, r += n;
    push(l);
    push(r - 1);
    int res = 0;
    for (; l < r; l >>= 1, r >>= 1) {
        if (l&1) res = add(res, sum[l++]);
        if (r&1) res = add(sum[--r], res);
    }
    return res;
}

//Initialization:
scanf ( "%d%d", &n, &q );
h = sizeof(int) * 8 - __builtin_clz(n);
//cout << "h: " << h << endl;

for ( int i = n; i < 2*n; ++i ){
    scanf ( "%d", &sum[i] );
    lengths[i] = 1;
}

build_t();
}
```

7 Strings

7.1 Borders

```
const int N = 1e6+5;
int b[N];
int sz;

void borders(string p){
    b[0] = -1;
    p = '#' + p;
    for(int i=1; i <= sz; i++){
        int j=b[i-1];
        while(j>0 && p[i] != p[j+1]) j = b[j];
        b[i] = j+1;
    }

    //Encontrar periodos:
    sz = s.size();
    int aux = sz;
    borders(s);

    while(aux){
        cout << sz-b[aux] << " ";
        aux = b[aux];
    }
}
```

7.2 Hashing

```
const int p = 283;
const int M = 1e9+7;
const int N = 1e6+1;

int P[N], h[N];

1l binpow(1l a, 1l b) {
    1l res = 1;
```

```

a %= M;
while (b > 0) {
    if (b & 1)
        res = (res * a) % M;
    a = (a * a) % M;
    b >>= 1;
}
return res;
}

void prepareP(int n){
    P[0] = 1;
    for(int i = 1; i < n; ++i){
        P[i] = ((1ll)P[i-1]*p) % M;
    }
}

void computeRollingHash(string T){
    for(int i=0; i < (int)T.size(); ++i){
        if(i!=0) h[i] = h[i-1];
        h[i] = (h[i]+((1ll)(T[i]-'a'+1)*P[i])) % M;
    }
}

int hash_fast(int L, int R){
    if(L==0) return h[R];
    int ans = 0;
    ans = ((h[R]-h[L-1]) % M + M) % M;
    ans = ((1ll)ans*binpow(P[L],M-2)) % M;
    return ans;
}

```

7.3 Manacher

```

//Find palindromes
vector<int> manacher_odd(string s) {
    int n = s.size();
    s = "$" + s + "~";
    vector<int> p(n + 2);
    int l = 1, r = 1;
    for(int i = 1; i <= n; i++) {
        p[i] = max(0, min(r - i, p[l + (r - i)]));
        while(s[i - p[i]] == s[i + p[i]]) {
            p[i]++;
        }
        if(i + p[i] > r) {
            l = i - p[i], r = i + p[i];
        }
    }
    return vector<int>(begin(p) + 1, end(p) - 1);
}

```

7.4 Z-Algorithm

```

vector<int> z(string s) {
    int n = s.size();
    vector<int> z(n);
    int x = 0, y = 0;
    for (int i = 1; i < n; i++) {
        z[i] = max(0, min(z[i-x], y-i+1));
        while (i+z[i] < n && s[z[i]] == s[i+z[i]]) {
            x = i; y = i+z[i]; z[i]++;
        }
    }
    return z;
}

```

8 Trees

8.1 LCA

```

/*
Binary lifting:
O(nlogn) para preprocesamiento
O(logn) para cada query
*/

```

```

int n, l;
vector<vector<int>>> adj;

int timer;
vector<int> tin, tout;
vector<vector<int>>> up;

void dfs(int v, int p)
{
    tin[v] = ++timer;
    up[v][0] = p;
    for (int i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];

    for (int u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }

    tout[v] = ++timer;
}

bool is_ancestor(int u, int v)
{
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}

int lca(int u, int v)
{
    if (is_ancestor(u, v))
        return u;
    if (is_ancestor(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!is_ancestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}

void preprocess(int root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<int>(l + 1));
    dfs(root, root);
}

```


9 algorithm

#include <algorithm> #include <numeric>

Algo	Params	Funcion
sort, stable_sort	f, l	ordena el intervalo
nth_element	f, nth, l	<i>void</i> ordena el n-esimo, y particiona el resto
fill, fill_n	f, l / n, elem	<i>void</i> llena [f, l) o [f, f+n) con elem
lower_bound, upper_bound	f, l, elem	<i>it</i> al primer / ultimo donde se puede insertar elem para que quede ordenada
binary_search	f, l, elem	<i>bool</i> esta elem en [f, l)
copy	f, l, resul	hace $resul+i=f+i \forall i$
find, find_if, find_first_of	f, l, elem / pred / f2, l2	<i>it</i> encuentra $i \in [f, l)$ tq. $i=elem$, $pred(i)$, $i \in [f2, l2)$
count, count_if	f, l, elem/pred	cuenta elem, $pred(i)$
search	f, l, f2, l2	busca $[f2, l2) \in [f, l)$
replace, replace_if	f, l, old / pred, new	cambia old / $pred(i)$ por new
reverse	f, l	da vuelta
partition, stable_partition	f, l, pred	$pred(i)$ ad, $!pred(i)$ atras
min_element, max_element	f, l, [comp]	<i>it</i> min, max de [f, l]
lexicographical_compare	f1, l1, f2, l2	<i>bool</i> con $[f1, l1)_i [f2, l2]$
next/prev_permutation	f, l	deja en [f, l) la perm sig, ant
set_intersection, set_difference, set_union, set_symmetric_difference,	f1, l1, f2, l2, res	[res, ...) la op. de conj
push_heap, pop_heap, make_heap	f, l, e / e /	mete/saca e en heap [f, l), hace un heap de [f, l)
is_heap	f, l	<i>bool</i> es [f, l) un heap
accumulate	f, l, i, [op]	$T = \sum / oper$ de [f, l)
inner_product	f1, l1, f2, i	$T = i + [f1, l1) \cdot [f2, \dots)$
partial_sum	f, l, r, [op]	$r+i = \sum / oper$ de $[f, f+i] \forall i \in [f, l)$
__builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
__builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
__builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
__builtin_popcount	unsigned int	Cant. de 1's en x.
__builtin_parity	unsigned int	1 si x es par, 0 si es impar.
__builtin_XXXXXXll	unsigned ll	= pero para long long's.

10 Math

10.1 Identidades

$$\begin{aligned}
 \sum_{i=0}^n ni &= 2^n \\
 \sum_{i=0}^n ini &= n * 2^{n-1} \\
 \sum_{i=m}^n i &= \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2} \\
 \sum_{i=0}^n i &= \sum_{i=1}^n i = \frac{n(n+1)}{2} \\
 \sum_{i=0}^n i^2 &= \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \\
 \sum_{i=0}^n i(i-1) &= \frac{6}{6} \left(\frac{n}{2}\right) \left(\frac{n}{2} + 1\right) (n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par} \\
 \sum_{i=0}^n i^3 &= \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} = \left[\sum_{i=1}^n i\right]^2 \\
 \sum_{i=0}^n i^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30} \\
 \sum_{i=0}^n i^p &= \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^p \frac{B_k}{p-k+1} \binom{p}{k} (n+1)^{p-k+1} \\
 r &= e - v + k + 1 \\
 \text{Teorema de Pick: (Area, puntos interiores y puntos en el borde)} \\
 A &= I + \frac{B}{2} - 1
 \end{aligned}$$

10.2 Ec. Caracteristica

$$\begin{aligned}
 a_0 T(n) + a_1 T(n-1) + \dots + a_k T(n-k) &= 0 \\
 p(x) &= a_0 x^k + a_1 x^{k-1} + \dots + a_k \\
 \text{Sean } r_1, r_2, \dots, r_q &\text{ las ra\xedas distintas, de mult. } m_1, m_2, \dots, m_q \\
 T(n) &= \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n
 \end{aligned}$$

10.3 Tablas y cotas (Primos, Divisores, Factoriales, etc)

Factoriales

0! = 1	11! = 39.916.800
1! = 1	12! = 479.001.600 (∈ int)
2! = 2	13! = 6.227.020.800
3! = 6	14! = 87.178.291.200
4! = 24	15! = 1.307.674.368.000
5! = 120	16! = 20.922.789.888.000
6! = 720	17! = 355.687.428.096.000
7! = 5.040	18! = 6.402.373.705.728.000
8! = 40.320	19! = 121.645.100.408.832.000
9! = 362.880	20! = 2.432.902.008.176.640.000 (∈ tint)
10! = 3.628.800	21! = 51.090.942.171.709.400.000

$$\begin{aligned}
 \text{max signed tint} &= 9.223.372.036.854.775.807 \\
 \text{max unsigned tint} &= 18.446.744.073.709.551.615
 \end{aligned}$$

Primos cercanos a 10^n

9941 9949 9967 9973 10007 10009 10037 10039 10061 10067 10069 10079
 99961 99971 99989 99991 100003 100019 100043 100049 100057 100069
 999959 999961 999979 999983 1000003 1000033 1000037 1000039
 9999943 9999971 9999973 9999991 10000019 10000079 10000103 10000121
 99999941 99999959 99999971 99999989 100000007 100000037 100000039
 100000049
 999999893 999999929 999999937 1000000007 1000000009 1000000021 1000000033

Cantidad de primos menores que 10^n

$$\begin{aligned}
 \pi(10^1) &= 4 ; \pi(10^2) = 25 ; \pi(10^3) = 168 ; \pi(10^4) = 1229 ; \pi(10^5) = 9592 \\
 \pi(10^6) &= 78.498 ; \pi(10^7) = 664.579 ; \pi(10^8) = 5.761.455 ; \pi(10^9) = 50.847.534 \\
 \pi(10^{10}) &= 455.052.511 ; \pi(10^{11}) = 4.118.054.813 ; \pi(10^{12}) = 37.607.912.018
 \end{aligned}$$

Divisores

Cantidad de divisores (σ_0) para *algunos* $n/\neg\exists n' < n, \sigma_0(n')\sigma_0(n)$
 $\sigma_0(60) = 12 ; \sigma_0(120) = 16 ; \sigma_0(180) = 18 ; \sigma_0(240) = 20 ; \sigma_0(360) = 24$
 $\sigma_0(720) = 30 ; \sigma_0(840) = 32 ; \sigma_0(1260) = 36 ; \sigma_0(1680) = 40 ; \sigma_0(10080) = 72$
 $\sigma_0(15120) = 80 ; \sigma_0(50400) = 108 ; \sigma_0(83160) = 128 ; \sigma_0(110880) = 144$
 $\sigma_0(498960) = 200 ; \sigma_0(554400) = 216 ; \sigma_0(1081080) = 256 ; \sigma_0(1441440) = 288$
 $\sigma_0(4324320) = 384 ; \sigma_0(8648640) = 448$

Suma de divisores (σ_1) para *algunos* $n/\neg\exists n' < n, \sigma_1(n')\sigma_1(n)$
 $\sigma_1(96) = 252 ; \sigma_1(108) = 280 ; \sigma_1(120) = 360 ; \sigma_1(144) = 403 ; \sigma_1(168) = 480$
 $\sigma_1(960) = 3048 ; \sigma_1(1008) = 3224 ; \sigma_1(1080) = 3600 ; \sigma_1(1200) = 3844$
 $\sigma_1(4620) = 16128 ; \sigma_1(4680) = 16380 ; \sigma_1(5040) = 19344 ; \sigma_1(5760) = 19890$
 $\sigma_1(8820) = 31122 ; \sigma_1(9240) = 34560 ; \sigma_1(10080) = 39312 ; \sigma_1(10920) = 40320$
 $\sigma_1(32760) = 131040 ; \sigma_1(35280) = 137826 ; \sigma_1(36960) = 145152 ; \sigma_1(37800) = 148800$
 $\sigma_1(60480) = 243840 ; \sigma_1(64680) = 246240 ; \sigma_1(65520) = 270816 ; \sigma_1(70560) = 280098$
 $\sigma_1(95760) = 386880 ; \sigma_1(98280) = 403200 ; \sigma_1(100800) = 409448$
 $\sigma_1(491400) = 2083200 ; \sigma_1(498960) = 2160576 ; \sigma_1(514080) = 2177280$
 $\sigma_1(982800) = 4305280 ; \sigma_1(997920) = 4390848 ; \sigma_1(1048320) = 4464096$
 $\sigma_1(4979520) = 22189440 ; \sigma_1(4989600) = 22686048 ; \sigma_1(5045040) = 23154768$
 $\sigma_1(9896040) = 44323200 ; \sigma_1(9959040) = 44553600 ; \sigma_1(9979200) = 45732192$