Linear Recurrence:

A linear recurrence is a function or a sequence such that each term is a linear combination of previous terms.

Steps to solve linear recurrence:

1. Determine k , number of terms on which f(i) depends
   (more precisely k is the minimum integers such that f(i) does not depend on f(i-M) for all M>k
2. Determine the F_1 vector the initial values
   F_1 is a column vector with k x 1 values

$$F_1 = \begin{bmatrix} f(1) \\ f(2) \\ \vdots \\ f(K) \end{bmatrix}$$

   and similarly define F_i whose first row is f(i) , next is f(i+1) and so on upto f(i+k-1)

3. Determine transformation matrix T such that

$$T\,F_i = F_{i+1}$$

Suppose that

$$f(i) = \sum_{j=1}^{K} c_j f(i-j)$$

and hence

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_K & c_{K-1} & c_{K-2} & c_{K-3} & \cdots & c_1 \end{bmatrix}.$$

4. Determine F_n

$$F_n = T^{(n-1)} F_1$$

and use fast exponentiation for determining the above

Note: In the solved example sequence a_i is defined as :

$$a_i = b_i \text{ (for } i <= k)$$
$$a_i = c_1 a_{i-1} + c_2 a_{i-2} + \ldots + c_k a_{i-k} \text{ (for } i > k)$$

and we need to determine a(n) % 10^9.