Optimal Broadcast Auctions with a new Cost Model

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1. INTRODUCTION

2. COST MODEL AND SETTINGS

In this section, we define our cost model and explain its motivation. Our cost model is inspired from online second-hand item transactions, including examples like eBay, craigslist and universities' mailing-lists.

DEFINITION 1. *In our settings, one seller is selling one item to n buyers (bidders) whose valuations v_i $(1 \leq i \leq n)$ are independently and identically distributed (i.i.d.) over $[0, 1)$ with PDF $f(x)$ and CDF $F(x)$. The seller can broadcast a message to all bidders with cost b (the broadcast cost) for the seller. A bidder may reply to that broadcast with cost c (bidding cost) or remain silent with no cost. Such bidding cost c may contain two parts, β_1, β_2 where $\beta_1 + \beta_2 = c$. The first part β_1 is charged to the seller while the second is charged to the corresponding bidder. The bidder's reply is deterministic with respect to the seller's broadcast (for simplicity, we only consider pure strategy equilibrium).*

The major difference of this model are cost and broadcast capability. We'll explain bidding cost c, broadcast capability and broadcast cost b in the following text.

The bidding costs c may be caused by communication and other verification required to bid. For example, the bidder may have to input his credit card number and prepay an amount of money. Without such verification, a bidder may bid very high and refuse to pay in the end. A verification for the seller might also be needed. For example, a bidder may want to set up an appointment with the seller to check the item. Setting up such an appointment might be costly because they need to dicuss time and place via emails or phone calls. Attending that appointment may also cost travel fees and time. Note that our bidding cost is different from conventional participation cost studied before [8, 9]. In our model, it's free for bidders to participate without bidding, which is more close to most Internet platforms. In another word, you don't have to buy a ticket to walk in and observe.

We introduce broadcast capability because it's shared by many auctions. For example, a Vickrey auction or a first-price auction with reserve price can both be described as a broadcast auction with only one broadcast: telling every bidder the reserve price. The bisection auction [3] is an example with many rounds of broadcasts.

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In each round , it will broadcast a price and ask bidders to reply whether their valuation is beyond or below that price. In real world, sellers make such broadcasts via sending emails to a bunch of receivers (typically a mailing-list), listing items on a platform such as craigslist and eBay, or even showing ads on Internet/TV. Such broadcast activities costs either money (e.g. a list or ads fee) or time and effort (e.g. writing and sending emails).

Note that in our model, we only give sellers broadcast capability so they cannot find or communicate with each bidder one by one. The first reason to have this constraint is there are too many potential bidders on the Internet where our model is originated so it's hard to explicitly find them one by one. By contrast, in offline cases where the set of bidders are small and explicit (e.g. the government want to sell a land to one of three companies), it might be helpful to let the seller communicate with bidders one by one [5]. The second reason to have this constraint is that we want to focus on mechanisms that avoids time consuming bargaining between two individuals. It makes the mechanism simple, fair and convenient.

Finally, we define optimal mechanisms to be the ones that maximze seller's profit since when facing many different auction mechanisms, a rational seller will choose the one that gives him maximum profit.

DEFINITION 2. *We say a mechanism is optimal if it gives the seller maximum profit which equals to all the value payed to this seller (revenue) minus all the cost charged to this seller. A class of mechanisms are optimal if they contain one optimal mechanism.*

3. OPTIMAL MECHANISMS WITH EFFICIENCY AND ONLY SELLER'S COST

In this section, we consider a simplified optimizing problem with efficiency constraint and only seller's cost. Though these two constraints simplify our problem a lot, they are reasonable in real cases such as craigslist or moving sales in mailing-lists:

1. In many cases, sellers only have 0 valuation for the item. For example, some second-hand items will be tossed if they cannot be sold by a particular day, e.g. the day the seller moves out the house. We encounter many free items during second-hand sales as well, which is another demonstration of zero valuation. Additionally, sellers cannot commit to withhold the item or prevent re-sales between buyers. Under such circumstances, an efficient mechanism not only maximzes the social welfare but also maximizes the seller's revenue [1].

2. The bidding cost for each bidder sometimes seems [1] to be negligible compared to bidding cost charged to the seller. For

[1]But actually it's not that negligible as our example seems to claim. In later section, theorem 5 will show exactly how important it is.

example, if 100 bidders replied to the seller by a 1-minue call, each bidder only has a tiny 1 minue cost. But for the seller, it's a big 100 minutes cost which is very annoying. It's also necessary to remove bidder's bidding cost to achieve efficiency (without compensation). Otherwise, the item may not be able to allocate to the highest bidder when that highest valuation is below the bidder's bidding cost.

The rest of this section is organized as follows. First of all, we introduce a mechanism called Multi-round Vickrey Auction (MVA) based on what's been used in realworld online second-hand item transactions. Then we prove that MVAs are optimal (so there exists a MVA that's optimal). After that we'll try to find the specific MVA that achieves the optimality. Finally, we conduct some experiments to compare the optimal MVA with other mechanisms.

3.1 Multi-round Vickrey Auctions

A Multi-round Vickrey Auction (MVA) has multiple rounds of Vickrey auctions with progressively decreasing reserve prices. This kind of auction effectively occurs on eBay. The seller may set up a reserve price and let buyers bid for this item. The proxy bidding functionality makes such an auction equivalent to a Vickrey auction with a reserve price. If no buyers bid for a given reserve price, the seller may lower the reserve price, which makes the whole process equivalent to an MVA.

DEFINITION 3. *(Multi-round Vickrey Auction, MVA)*
In a Multi-round Vickrey Auction (MVA), there's a sequence of reserve prices $r_1, r_2, \ldots, r_k$ where $r_i > r_{i+1}$. The seller creates a Vickrey auction with a reserve price r_i at time i (or round i). In each Vickrey auction, if only one buyer bids, he/she gets the item and pays reserve price. Otherwise, the buyer with the highest bidding gets the item and pays the second highest bidding.

Since there's no cost charged to buyers, it's obvious to see that whenever a bidder decides to bid, he/she must bid truthfully. Thus the Bayesian Nash Equilibria (BNE) for MVAs can be described as k thresholds $a_1, a_2, \ldots, a_k$ where $a_i > a_{i+1}$. Whenever a bidder's valuation for the item is greater than a_i, he/she is going to bid in round i whose reserve price is r_i. Because of efficiency constraint we also have $r_k = a_k = 0$.

In later analysis, we firstly decide thresholds a_i since they are more meaningful for bidders to make decisions and for us to make analysis. Then we determine the right reserve prices r_i that make bidders incentive compatible to bid according to thresholds a_i. The following equations connects a_i and r_i:

$$r_k = a_k = 0 \text{ and } \forall i\ (1 \le i < k),$$
$$P(a_i)(a_i - r_i) = \int_{a_{i+1}}^{a_i} (a_i - x)p(x)dx + P(a_{i+1})(a_i - r_{i+1}) \tag{1}$$

where for convenience we denote:

$$P(x) := F(x)^{n-1}$$
$$p(x) := P'(x) = (n-1)F(x)^{n-2}f(x)$$

The equation 1 says that the bidder with valuation a_i should be indifferent from bidding in round i (the left hand side) and bidding in round $i-1$(the right hand side). The following theorem describes the equilibrium of MVAs determined by equations above.

THEOREM 1. *If we make reserve prices r_i to be:*

$$r_k = a_k = 0$$
$$r_i = \left(\int_0^{a_i} x\,p(x)dx\right)/P(a_i) \qquad (i < k) \tag{2}$$

Such MVA will have a pure strategy Bayesian Nash Equibibrium characterized by thresholds $a_1, a_2, \ldots, a_k$ where the bidder with valuation greater than a_i (but not greater than a_{i-1}) will bid in round i

PROOF. By equation 2, we have $r_i P(a_i) = \int_0^{a_i} x\,p(x)dx$ for all i. Thus the right hand side of equation 1 is:

$$\int_{a_{i+1}}^{a_i} a_i p(x)dx - \int_{a_{i+1}}^{a_i} x\,p(x)dx + P(a_{i+1})(a_i - r_{i+1})$$
$$= a_i P(a_i) - \cancel{a_i P(a_{i+1})} - r_i P(a_i) + \cancel{r_{i+1} P(a_{i+1})}$$
$$+ \cancel{P(a_{i+1}) a_i} - \cancel{P(a_{i+1}) r_{i+1}}$$
$$= \text{left hand side of equation 1}$$

□

This tells us that a bidder will bid in a round of MVA if and only if the expected second highest bidding conditional on this bidder's valuation is the highest is greater than the reserve price of that round. For example, if the distribution is uniform, i.e. $F(x) = x$, $r_i = \frac{n-1}{n} a_i$ for $i > 0$. [4] has given some more discussions and proof (e.g. once a bidder choose to bid, bid truthfully is a unique weekly dominant strategy) about the equilibrium of this kind of sequential auctions. That paper, however, aims on the optimal auctions for a different cost model with time discount but without broadcast or bidding cost. We will discuss this difference later when we introduces bidding cost for bidders.

3.2 Optimality of MVAs

Since the mechanism is required to be efficient, revenue equivalence theorem [6] told us that the seller's gross revenue without substracting costs is fixed. Thus to maximize the profit is equivalent to minimize the cost.

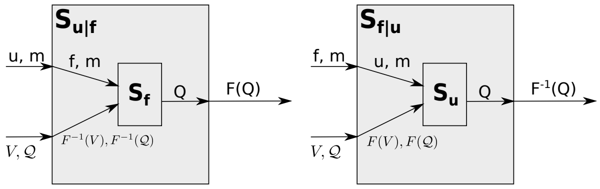
By efficiency constraint, the mechanism should at least find out the bidder with highest valuation. The best case is that every reply contains the exact and truthful valuation of the corresponding bidder since every reply (or bidding) has a cost c. By doing that, we never need someone to reply twice. The optimizing problem in this best case is defined as definition 4. This best case minimum cost provides us a lower bound for minimum cost of our mechanisms. We will then prove that MVAs can achieve this lower bound so they are optimal.

DEFINITION 4. *Assume there are n values v_i $(1 \le i \le n)$ independently and identically distributed over $[0, 1)$ with PDF $f(x)$ and CDF $F(x)$. A query strategy is to find the maximum value by asking queries $Q_1, Q_2, \ldots$ sequentially where $Q_i \subset [0, 1)$ and $\forall i \ne j$, $Q_i \cap Q_j = \emptyset$. After a query Q_i, all numbers within Q_i will be reported. Note that Q_i may depend on results of $Q_1, Q_2, \ldots, Q_{i-1}$. Thus a strategy can be denoted as a function:*

$$S\big(f, m, V, \mathcal{Q} = \{Q_1, Q_2, \ldots, Q_{i-1}\}\big) = Q_i$$

which means, suppose there are still m i.i.d. unknown values left whose PDF is $f(x)$, the reported values set is V and the set of queries asked is $\mathcal{Q}$, the strategy S will make Q_i as the next query.

The cost of each query is equal to $b + j \cdot c$ where j is the number of reported values from that query. The cost of a strategy for a given set of unknown values v_i is equal to the sum of all queries' costs



(a) Construct $S_{u|f}$ from S_f (b) Construct $S_{f|u}$ from S_u

Figure 1: These two figures illustrates how to construct a strategy for uniform PDF u from another strategy for another arbitrary PDF f and vice versa. Here we depict a strategy as a box which takes four inputs $f, m, V, \mathcal{Q}$ (PDF, number of unknown values, reported values set, set of asked queries) and make an output Q (the next query)

it has to make before identifying the maximum value. The optimal query strategy is the one that has minimum expected cost. We will write such minimum expected cost as $C^*(f, n)$, a function of PDF $f(x)$ and number of unknown values in the beginning n.

To find out the optimal query strategy, we first introduce a lemma saying that the minimum cost is independent of PDF $f(x)$.

LEMMA 1. *Assume uniform PDF $u(x) = 1$ and define $C^*(n) = C^*(u, n)$. For any other PDF $f(x)$, we have*

$$C^*(f, n) = C^*(u, n) = C^*(n)$$

PROOF. Define $F^{-1}(x) = \sup\{y \mid F(y) = x\}$. For any strategy S_f that works for PDF f, we can come up with a strategy $S_{u|f}$ for uniform PDF u:

$S_{u|f}\big(u, m, V, \mathcal{Q}\big) = F\Big(S_f\big(f, m, F^{-1}(V), F^{-1}(\mathcal{Q})\big)\Big)$

Thus $C^*(u, n) \leq C^*(f, n)$ since we can adopt any strategy for f to run under u with the same cost. Similarly, for any strategy S_u that works for PDF u, we can make strategy $S_{f|u}$ for PDF f:

$S_{f|u}\big(f, m, V, \mathcal{Q}\big) = F^{-1}\Big(S_u\big(u, m, F(V), F(\mathcal{Q})\big)\Big)$

Therefore $C^*(f, n) \leq C^*(u, n)$. Combining with $C^*(u, n) \leq C^*(f, n)$ we have $C^*(f, n) = C^*(u, n) = C^*(n)$. Figure 1 illustrates those two constructions we used for this proof.

□

Then we prove that descending query strategies are optimal:

LEMMA 2. *There exists an optimal strategy with only descending queries $Q_1 = [a_1, 1],\ Q_2 = [a_2, a_1),\ Q_3 = [a_3, a_2) \ldots$*

PROOF. If not, there must be an optimal strategy where none of its non-descending queries can be changed to descending queries without increasing the cost. In that strategy S, there must be a first non-descending query $Q_{i+1} = S\big(F, m, V, \mathcal{Q} = \{Q_1, Q_2, \ldots, Q_i\}\big)$ where Q_1 to Q_i are all descending. We can make another descending query $Q'_{i+1} = [a'_{i+1}, a_i)$ (or $Q'_{i+1} = [a'_{i+1}, 1]$ if $i = 0$) such that

$$\Pr(v \in Q_{i+1}) = \int\limits_{Q_{i+1}} f(x)\mathrm{d}x = \int\limits_{Q'_{i+1}} f(x)\mathrm{d}x = \Pr(v \in Q'_{i+1})$$

After Q'_{i+1}, we'll use as optimal query as possible.

Since Q_1 to Q_i are all descending, we have $m = n$ and $V = \emptyset$ (otherwise the strategy should terminate without asking Q_{i+1}). Define C to be the expected cost of using Q_{i+1} and later queries. Similarly we define C' for Q'_{i+1}:

$C = b + \sum_{j=0}^{n} p_j(j \cdot c + C_j)$ and $C' = b + \sum_{j=0}^{n} p'_j(j \cdot c + C'_j)$ where: p_j (or p'_j) is the probability that there are j reported values within Q_{i+1} (or Q'_{i+1}); C_j (or C'_j) is the expected cost of later queries given that j values have been found in Q_{i+1} (or Q'_{i+1}).

As $\Pr(v \in Q_{i+1}) = \Pr(v \in Q'_{i+1})$, we have $p'_j = p_j$. Since Q'_{i+1} is a descending query, $\forall j > 0,\ C'_j = 0 \leq C_j$. And by lemma 1, $C'_0 = C_0 = C^*(n)$ because knowing no value is in $Q_1, Q_2, \ldots, Q_{i+1}$ is equivalent to revise PDF $f(x)$ to a refined PDF

$$f_{i+1}(x) = \begin{cases} \lambda f(x), & x \notin Q_1 \cup Q_2 \cup \ldots \cup Q_{i+1} \\ 0, & x \in Q_1 \cup Q_2 \cup \ldots \cup Q_{i+1} \end{cases}$$

where λ is a constant to make $\int_0^1 f_{i+1}(x)dx = 1$.

Thus $C' \leq C$ which contradicts to that no non-descending query can be changed to descending query without increasing the cost.

□

Finally, we conclude the optimality of MVAs.

THEOREM 2. *Among all mechanisms that can include multiple rounds of broadcasts and are required to be efficient (allocate the item to the bidder with highest valuation), Multi-round Vickrey Auctions (MVAs) are of minimum cost.*

PROOF. The best case optimizing problem defined in definition 4 provides us a lower bound of minimum cost we can achieve by any mechanisms. By lemma 2, such lower bound minimum cost can be achieved by descending query strategy $Q_1 = [a_1, 1], Q_2 = [a_2, a_1), Q_3 = [a_3, a_2) \ldots$. And by theorem 2 we are able to design such an MVA with reserve prices $r_1, r_2, \ldots$ whose Bayesian Nash Equilibrium achieves this best case descending query strategy. Thus, MVAs are of minimum cost.

□

And by revenue equivalence theorem, MVAs are optimal:

COROLLARY 1. *If all broadcast costs and bidding costs are charged to sellers, MVAs are optimal if efficiency is required. Such optimal MVA is the one that minimizes the overall cost.*

3.3 Cost Minimized α-MVA

Now let's try to calculate the parameters (thresholds a_i or equivalently reserve prices r_i) of the optimal MVA for a given settings F, n, b, c (valuation CDF, number of bidders, broadcast cost, bidding cost). By lemma 1, the cost is indifferent with F and we can always derive an optimal mechanism for any F from a uniform distribution. Thus we will focus on uniform cases below. We will also introduce $\rho = \frac{b}{c}$ to simplify our analysis by normalize bidding cost c to 1 and thus broadcast cost b to ρ.

An optimal MVA must be an α-MVA where each round only $(1-\alpha)n$ bidders are expected to bid, i.e. $a_1 = \alpha, a_{i+1} = \alpha \cdot a_i$ for uniform cases. Then the expected overall cost C satisfies: $C = \rho + (1-\alpha)n + \alpha^n C$. In the right hand side, the first term ρ is the broadcast we have to use in the first round, the second term $(1-\alpha)n$ is the expected bidding cost for the first round, the third term $\alpha^n C$ is a recursive term, the probability that no one bids times if that happens the same cost C should be expected in later rounds.

From that equation, we get $C = \frac{\rho + (1-a)n}{1-\alpha^n}$. To minimize cost C, we either choose boundary cases $\alpha = 0, 1$ or we have:

$$\frac{\partial C}{\partial \alpha} = \frac{\alpha^{n-1}\, n\, (\rho + (1-\alpha)\, n)}{(1-\alpha^n)^2} - \frac{n}{1-\alpha^n} = 0$$

$$\Updownarrow$$

$$\alpha^{n-1}(\rho + (1-\alpha)n) - (1-\alpha^n) = 0 \qquad (3)$$

The boundary case $\alpha = 0$ can be ruled out because $\frac{\partial C}{\partial \alpha}|_{\alpha=0} < 0$. When $\alpha = 1$, α-MVA becomes Dutch Auction thus ρ must be 0 (otherwise total broadcast cost would be infinity). And this is indeed a solution of equation 3 when $\rho = 0$. Thus equation 3 characterize the optimal α in all cases. [7] has a proof for why α-MVA is optimal among MVAs and how to determine α as well. Thus if you find them not intuitive you may reference more details there. Our simplified cost model (with efficiency constraint and no buyer's cost) is equivalent to their cost model where learning cost is linear to the number of replied agents. That paper makes descending query as a contraint [2] and proves that α-MVA is optimal among descending query mechanisms. Our focus in this section, however, is to have a preliminary introduction for MVA and proves optimality of descending queries and eventually MVAs. Thus we omit the proof about α-MVA. Even for the α, we will be more interested in its relation with larger n so we will give approximations for α.

3.4 Approximation of α

It's difficult to get an exact closed formula for optimal α by equation 3. Thus we are going to use some simpler formulas to approximate α. We'll conduct experiments to compare our approximation with the optimal α that's computed numerically.

Firstly, $\alpha = 1 - 1/n$ is a natural guess which means each round the expected number of biddings is equal to 1. Thus we have $C(\alpha = 1 - 1/n) = (\rho + 1)/(1 - (1 - 1/n)^n)$. Because $(1-1/n)^n \le e^{-1}$, we have $C(\alpha = 1-1/n) \le (\rho+1)(1-e^{-1})$. It's obvious that at least one broadcast and one bidding is required to terminate so $C \ge \rho + 1$. Thus we have

THEOREM 3. *$\alpha = 1 - 1/n$ is a $1/(1-e^{-1})$ approximation of optimal α-MVA. That means, by simply choosing $\alpha = 1 - 1/n$, we would at most get about 1.582 times of optimal cost. Another obersavation of this approximation is that no matter how large n is, the cost of this simple approximation is at most $(\rho+1)/(1-e^{-1}) = O(1)$. Thus the optimal cost is bounded by constant $O(1)$ no matter how large n is.*

A better approximation when n is large is to observe that $\alpha \to 1$ when n grows large. Thus we guess that $(1-a)n \approx A$ (for some constant A) and $\alpha^n \approx \alpha^{n-1}$. Then we have:

$$n(\rho + A) \cdot \alpha^n - n(1 - \alpha^n) = 0$$

which gives us $\alpha = (1+\rho+A)^{-1/n}$. Put this back to $\lim_{n\to\infty}(1-a)n = A$ we have $\ln(1+\rho+A) = A$ which gives us

$$A = -1 - \rho - W(-1-\rho)$$
$$\alpha = (-W(-1-\rho))^{-1/n} \qquad (4)$$

where $W(x)$ is the Lambert W function [cite wikipedia?] defined by $W(x)e^{W(x)} = x$. Actually, $w\, e^w = x$ has two solutions for w when $-1 < x < 0$. Here our $W(x)$ refers to the lower[3] branch $W_{-1}(x) < -1$. This second approximation that converges to the optimal one when n is large:

[2]In their model they want to find the minimum value so increasing threshold search is equivalent to descending query

[3]The upper branch is $W_0(x) > -1$ when $-1 < x < 0$

THEOREM 4. *Suppose that the optimal α is α^* which satisfies equation 3. Then $\alpha = (-W(-1-\rho))^{-1/n}$ satisfies*

$$\lim_{n\to\infty} C(\alpha^*) = \lim_{n\to\infty} C(\alpha = (-W(-1-\rho))^{-1/n})$$

That is, our approximation's cost will converge to optimal cost when n grows to infinity.

PROOF. Define sequence α_n^*, C_n^* where $n = 1, 2, 3, \ldots$ to be sequences of optimal α^* and corresponding optimal cost C^* when there are n bidders. We first show that C_n^* is increasing: if we make $(\alpha_{n-1})^{n-1} = (\alpha_n^*)^n$, then we have 1) The expected broadcast cost of α_{n-1}-MVA with $n-1$ bidders is equal to that of α_n^*-MVA with n bidders as the probability that one round will terminate is the same; 2) $\alpha_{n-1} < \alpha_n^*$ thus the expected bidding cost of α_{n-1}-MVA with $n-1$ bidders should be less than that of α_n^*-MVA with n bidders. Therefore, $C_{n-1}^* \le C_{n-1}(\alpha_{n-1}) < C_n^*$. Thus sequence C^* is indeed strictly increasing.

Secondly, theorem 4 says C_n^* is bounded. Therefore $(1-\alpha^*)n$ must also be bounded otherwise $C = \frac{\rho+(1-\alpha)n}{1-\alpha^n}$ cannot be bounded. Thus according to Bolzano-Weierstrass theorem [cite wikipedia?], there must be a subsequence $\alpha_{n_i}^*$ such that $(1-\alpha_{n_i}^*)n$ converges to some constant A. Recall that α^* satisfies equation 3 and obviously $\lim_{n\to\infty} \alpha^* = 1$, we could use calculations similar to what we used for equation 4 to derive

$$\lim_{n_i\to\infty}(1-\alpha_{n_i}^*)n_i = A = -1 - \rho - W(-1-\rho)$$
$$\lim_{n_i\to\infty}(\alpha_{n_i}^*)^{n_i} = \lim_{n_i\to\infty}(\alpha_{n_i}^*)^{n_i-1} = (-W(-1-\rho))^{-1}$$

This proves that

$$\lim_{n_i\to\infty} C(\alpha^*) = \lim_{n_i\to\infty} C(\alpha = (-W(-1-\rho))^{-1/n_i})$$

Then using the fact that C_n^* is strictly increasing and bounded completes the proof. $\square$

Experiments in figure 2 compare the optimal α, our first approximation of $\alpha = 1 - 1/n$ and our second approximation $\alpha = (-W(-1-\rho))^{-1/n}$ together with their corresponding cost under settings $\rho = 0.2, 1, 5$.

As you can see from figure 2, the first approximation $1 - 1/n$ is bounded to be a constant time of optimal cost while the second approximation converges to optimal cost when n grows large. When ρ is close to 1, both two approximations are very close to the optimal one. But the second approximation is much better when ρ is much smaller or greater than 1. Anyway, the second approximation isn't always better than the first approximation, as the case $\rho = 1, n = 2$ shows.

3.5 Experiments

Now let's compare optimal α-MVA with other kinds of MVA. In all following experiments, the valuation distribution is always uniform over $[0, 1]$. According to lemma 1, other distributions can always be adapted to uniform distribution so this won't be a problem. Also recall that under this simpler model, the revenue is fixed thus the profit is completely determined by cost. Thus we'll only compare cost.

α-MVA can potentially have infinite many rounds. But in reality, it's more naturaly to come up with an MVA that has finite many, say k rounds at most. Let's call them k-MVA where k is some positive integers. One particualr k- MVA is uniform k-MVA where the k thresholds is uniformly distributed over $[0, 1]$. It's also known as fixed-step search strategy in [7, 2]. An optimal uniform MVA is the uniform k-MVA that minimize the cost by choosing the best k.

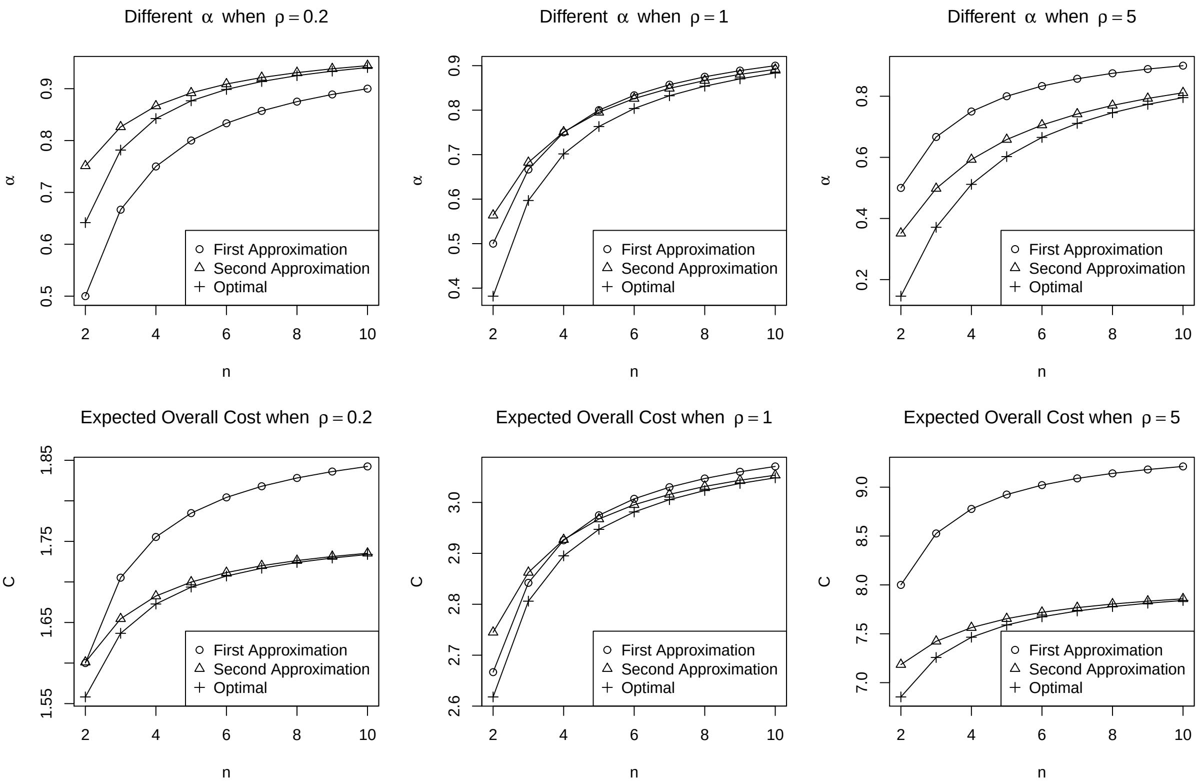


Figure 2: Comparisons for optimal α and its approximations. The first approximation is $\alpha = 1 - 1/n$, the second is $\alpha = (-W(-1-\rho))^{-1/n}$. The second row is the corresponding cost for different α

In practice, such k might also be very limited. For example, if someone need to selll something in 3 days before moving out, the k might be limited to 3 since it's too annoying to send out two broadcast messages per day (e.g. it might be labeled as spam by selling platform). With this limitation, one can still use uniform thresholds as a baseline. Or we may formulate this as another optimzing problem and solve the best k thresholds under this constraint (this problem is defined and solved in section 4.4 and equation 6). Between the baseline k uniform thresholds and the optimal k thresholds, another heurestic way to get k thresholds is to use $\alpha, \alpha^2, \ldots, \alpha^{k-1}$ as thresholds. That means, in first $k-1$ rounds, we query as if we have infinite many rounds, and we query all the left in last round. We call this mechanism as α-cutoff-k-MVA.

The comparison results achieved from simulation experiments are shown in figure 3. Here are some observations:

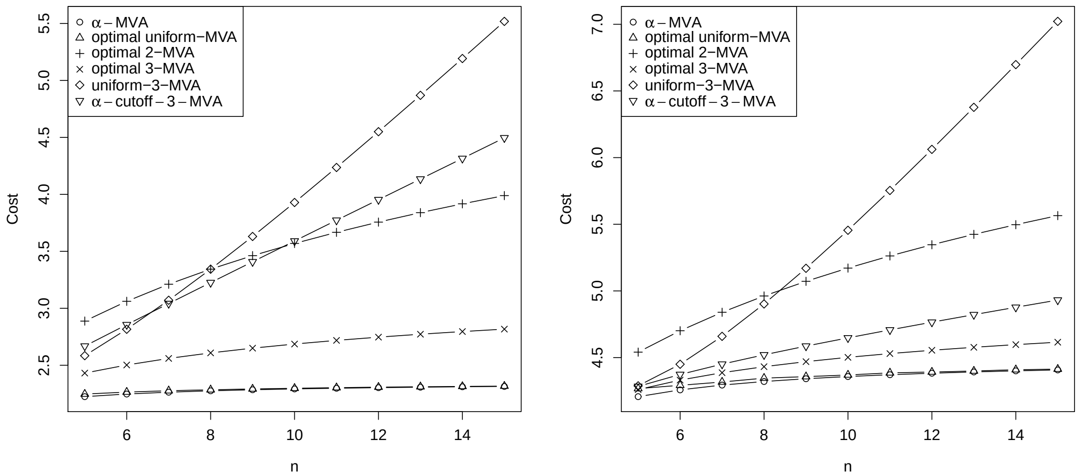
- The optimal uniform-MVA's cost is very close to optimal α-MVA's, especially when n is large. That's probably because
  1. α approaching 1 when n grows large, which makes the first k thresholds $\alpha, \alpha^2, \alpha^3, \ldots, \alpha^k$ close to uniform thresholds $1-(1-\alpha), 1-2(1-\alpha), \ldots, 1-k(1-\alpha)$;
  2. the probability that the highest value falls out of first k thresholds, α^{nk}, becomes negligible for large n.
- The optimal k-MVA's cost decreases and approaches to optimal cost quickly when k grows (check optimal 2-MVA and 3-MVA).
- When k is small, uniform thresholds has significant higher cost than optimal k-MVA, and the heuristic α-cutoff-k-MVA.
- The heuristic α-cutoff-k-MVA works pretty well especially when ρ is large as shown in figure 3(a). But it's not as good as optimal 2-MVA when ρ is small and n is large as shown in figure 3(b). Thus find the right thresholds is even more important than adding one more round in those cases.

4. OPTIMAL MECHANISMS WITH BOTH SELLER'S AND BIDDER'S COST

In this section, we take out the constraints and prove that MVAs are optimal in general. We will also try to find the specific MVA to achieve such optimality, which turns out to be significantly more complex than previous simplified case.

The first constraint we are going to remove is seller's cost only. It's exciting to introduce bidder's bidding cost since it occurs very often in real cases and it plays an important role. Sending emails, making phone calls, enterring credit card nubmers, depositing money and clicking buttons are all costly for bidders, though sometimes very tiny. Bidders may not bid when this cost is greater than their expected utility. Note that even if the valuation is very high, the expected utility can be very small because of tense competition, which is very common on the Internet as n, the number of potential bidders, is very large.

This behaviour (bidders won't bid because of competitions) is very different compared to that in previous model [4] of sequential auctions. In that model, there's a time discount which makes bidders eager to bid in early rounds with high reserve prices to avoid waiting lost. That makes a lot sense in some cases but sometimes it may not. For example, if the seller posts an auction with a very low reserve price in the first round, most bidders with high valuation



(a) $\rho = 0.5$

(b) $\rho = 2$

Figure 3: Cost comparison for different MVAs over n, number of bidders, and ρ, broadcast/bidding cost ratio.

must be happy to bid according to that time discount cost model. But this may not be true. For example, when I encounter such an auction online [4], I might be very reluctant to bid because there's a big probability that my bidding will be over taken by someone else's so it's just a waste of effort. Our cost model can describe this behaviour very well.

Assume an extreme case where the broadcast cost is 0, the bidder's bidding cost is 0.1 and there are $n \to \infty$ many $[0, 1)$-uniform distributed bidders. In a Dutch auction (a Dutch auction has infinite many rounds of broadcasts so we have to set braodcast cost to 0), only one bidder is expected to bid (no competition), thus every bidder with a valuation $v_i > 0.1$ should be benifitable to bid when the reserve price drops to a little bit below $v_i - 0.1$ (recall that $n \to \infty$). In a Vickrey auction, however, the competition is very tense. Only bidders with valuation greater than t can accept such intense competition where t satisfies $t^{n-1}t - 0.1 = 0$ (the expected utility for a bidder with valuation t is 0). Thus $t = \sqrt[n]{0.1}$ which is arbitrary close to 1 as n grows to infinity. Thus almost all bidders can't bare this competition when n is really large.

So a bad mechanism (e.g. a Vickrey auction) with too much cost will have less participations and therefore may decreases seller's profit significantly. To see this, look at previous extreme case again. The revenue of Dutch auction will converge to 0.9 (someone with valuation very close to 1 will bid for price very close to 0.9) when n becomes infinity. The revenue of a Vickrey auction, however, is only $\int_t^1 x\,(n-1)\,n\,(1-x)\,x^{n-2}dx$ which converges to about 0.67 when n grows to infinity.

Another problem caused by bidder's cost is that revenue equivalence theorem seems to be no longer applicable. That's not strange as the revenue equivalence theorem assumes that the utility of a bidder is equal to the valuation minus the payment. This assumption is no longer true as now the utility is also influenced by the cost charged to this bidder.

Finally, because of allowing bidding costs, it seems that we also have to remove efficiency constraint. Let's consider the case where the highest valuation is below the bidding cost. Without compensation to bidders, enforcing efficiency means bidders with highest valuation have to bid (so we can allocate the item to them) which would make their expected utility negative.

In summary, we now introduce bidder's bidding cost and drops efficiency constraint for our mechanisms. The first issue we are going to solve is to make revenue equivalence theorem, or a very similar theorem, applicable to our model again.

4.1 Spending Equivalence Theorem and Revenue Optimization Strategy

THEOREM 5. *The expected overall spendings from all bidders (including their bidding costs and payments to the seller) in a feasible mechanism (with our cost model) is completely determined by the expected utility of lowest type bidders and allocation probability function*

$$p : (v_1, v_2, \ldots, v_n) \to (p_1, p_2, \ldots, p_n)$$

where p_i is the probability that bidder i will get the item.

PROOF. This theorem is exactly the same as revenue equivalence theorem except that we exchange revenue with spending. To prove it, let's construct another mechanism M' without bidding cost from our mechanism M with bidding cost so M' fits into the original revenue equivalence theorem's model. Suppose there's a virtual seller in M', who collects valuations from all bidders at no cost (a direct revelation mechanism). Then this virtual seller will make n virtual bidders delegating bidders to communicate with the true seller in our mechanism M. When our mechanism ends by allocating the item to virtual bidder i, the virtual seller also allocate the item to the real bidder i. The payment from each bidder i to this virtual seller will be equal to the payment that virtual bidder i pays to our real seller plus all the bidding costs charged to virtual bidder i. Thus, from the real bidders' aspects, this mechanism M' is just a direct revelation mechanism which will satisfy revenue equivalence theorem. The only difference is that the payment from real bidder i to the virtual seller actually has two parts, one is payed to the real seller, another is payed to bidding costs, which sum up to the total spending. $\square$

Thanks to theorem 5, our profit maximization problem is now greatly simplified:

[4] For example, when I see a very good item in the Auction House of Diablo III with a very low current bidding

COROLLARY 2. *To maximize profit for a given allocation rule $p : (v_1, v_2, \ldots, v_n) \to (p_1, p_2, \ldots, p_n)$, we only need find the minimum total cost (including both seller's cost and bidders' cost).*

PROOF. The total spending, substracts cost charged to bidders, will be the revenue that the seller receives. This revenue, substracts cost charged to the seller, will be profit. Thus profit is total spending minus total cost. As total spending is fixed by allocation rule, we only need to find the minimum cost to maximze profit. $\square$

The highlight here is that we won't have to differentiate cost charged to bidders and cost charged to sellers if we just want to maximize seller's profit. The difference of them may make revenue different, but as long as their sum doesn't change, the profit won't change. This not only helps us simplify our analysis, but also helps us simplify the optimal mechanism: the seller could just design a mechanism with compensations to help bidders pay all the bidding cost. Thus the bidders won't bother the cost.

There's one more challenge, however: the optimal allocation rule here isn't as simple as the one that's discovered by Myerson [ref:Optimal Auction]: allocate the item to the bidder with highest virtural value if it's positive. This rule, though optimal in Myerson's setting, may not be optimal here. Theorem 5 tells us that this rule will maximize the total spending. But we must substract the cost from the spending to get the profit. Therefore, there might be another weird allocation rule that has less total spending but even much less minimum cost.

It's not hard to find one example of this. We know that for bidders with uniform valuation over $[0, 1)$, the spending maximizing allocation rule is allocate the item to the bidder with highest valuation that's greater than $1/2$. However, if the broadcast cost is too large, for example 1 (which is equal to the highest possible valuation), the cost to find out whether there's any bidder with valuation greater than $1/2$ is at least 1. Thus if we use this allocation rule, the final profit would be negative since the total spending must be less than the minimum cost. However, the allocation rule that never allocate the item will have 0 cost and 0 spending, which achieves 0 profit, better than previous allocation rule.

Thus, the revenue optimal mechianism will depend on how minimum cost is defined. We previously defined how cost is charged and proved that a specific mechanism (MVA) has minimum cost when allocation rule is allocate efficiently. But unfortunately, that's not enough to give us a well defined minimum cost for any allocation rule. For example, one allocation rule might be always allocate the item to each bidder with the same probability $1/n$, i.e. $p(v_1, v_2, \ldots) = (1/n, 1/n, \ldots)$. You might think that the minimum cost for this is 0 since we don't have to know anyone's valuation. But that cost isn't realistic: how can you ever allocate the item to someone that you have never communicated with? Thus the minimum cost seems to be at least b, the cost for one round of broadcast, if we ever allocate the item to some bidder. In order to make a realistic constraint and get a well defined minimum cost for any allocation rule, we define

DEFINITION 5. *For an allocation rule*

$$p : (v_1, v_2, \ldots, v_n) \to (p_1, p_2, \ldots, p_n)$$

if $p_i > 0$ for some valuation profile $(v_1, v_2, \ldots, v_n)$, there must be a broadcast query that the i-th bidder with valuation v_i reply to the seller under that profile setting.

With this definition, we have

THEOREM 6. *The optimal mechanism should always allocate the item to the bidder with highest virtual valuation $v_i - \frac{1-F(v_i)}{f(v_i)}$ if it decides to allocate the item. In regular cases when the virtual valuation is monotone strictly increasing, the optimal mechanism should always allocate the item to the bidder with highest valuation if it decides to allocate the item.*

To see why this theorem holds, firstly notice that in our cost model, the minimum cost to let any bidder reply is equal to the minimum cost to let the max-valuation bidder or max-virtual-valuation bidder to reply. This is because that the mechanism is only allowed to ask broadcast queries and it's equivalent to specify a range $Q \subseteq [0, 1)$ and ask all bidders whose valuations are within that range to reply. That means, by properly mappings of valuations and query ranges, we can always adapt a strategy that lets any bidder reply to another strategy that lets the max-virtual-valuation bidder reply and vise versa (just like what we did to prove lemma 2). This means that finding the max-valuation bidder won't cost more than any other cases except doing nothing (which costs 0). Thus the optimal mechanism either do nothing (no one replies and thus $p_i = 0$ by definition 5) which has minimum cost or ask queries to find the max-virtual-valuation bidder and allocate the item to that bidder which has maximum total spending. The flexibility of an optimal mechanism is that it can choose the cases in which it won't allocate the item (in any other cases it will allocate the item to the max-virtual-valuation bidder). It's also intuitive to see that the no allocation cases can be described by a single parameter l: no allocation if every bidder's valuation is below l. For limit of space, we won't list the detailed proof for the above theorem and arguments.

For simplicity, we won't mention virtual valuation below because they are indifferent in terms of minimizing cost. For convenience, we will use valuation instead of virtual valuation even if we are later talking about profit because our mechanism are just going to find the maximum value and it's equivalent to finding the maximum virtual value in regular cases.

Now we can narrow our optimal mechanisms with relaxed efficiency constraint:

DEFINITION 6. *We say mechanisms satisfy relaxed efficiency constraint with low value l if:*

1. *They only allocate the item to bidders whose valuation are at least l (the low value is l)*
2. *If they will allocate the item, they will always allocate the item to the bidder with highest valuation.*

When we say a mechanism has a low value l, we imply that this mechanism satisfy relatex efficiency constraint with low value l.

4.2 MVAs' Optimality in General

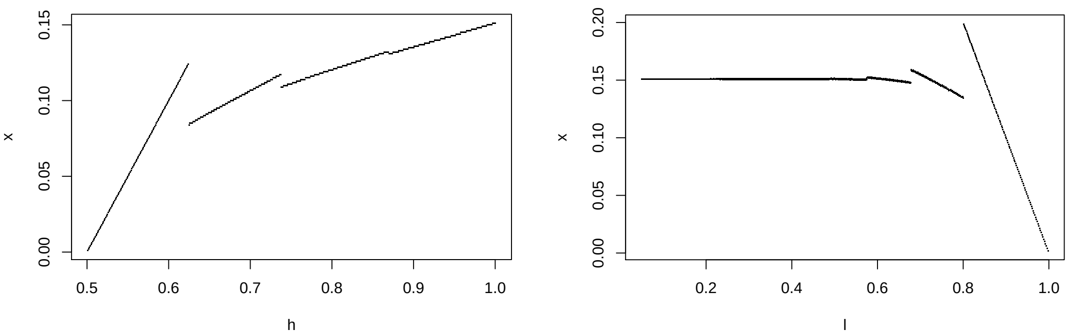
We have already narrowed down optimal mechanism to relaxed efficient mechanisms and by theorem 5, it's straightforward to see

COROLLARY 3. *For mechanisms with a fixed low value l, the maximum profit is achieved when the mechanism minizes the cost.*

Our next question is naturally: what's the cost minimized mechanism given a low value l. As expected:

THEOREM 7. *MVAs have the minimum cost among all mechanisms with a low value l*

PROOF. A special case of this theorem when $l = 0$ is theorem 2. We proved that special case by introducing lemma 1 and 2. To prove the general cases with arbitrary l, we just need to revise lemma 1 a little as following lemma 3. All other part of the proof remains similar. For space limit, the detailed proof is omitted. $\square$



(a) Best query length x over h with fixed l (b) Normalized best query length x over low value l

Figure 4: In the first subfigure, we plot the best query length $x = \hat{x}/D$ over $h = \hat{h}/D$ (the highest undiscovered value) with discretized low value $\hat{l} = 500$, broadcast bidding cost ratio $\rho = 2$, number of bidders $n = 10$ and maximum discretized valuation $D = 1000$. In the second subfigure, we normalize x to be $x = \hat{x}/\hat{h}$ and l to be $l = \hat{l}/\hat{h}$, as if h is always 1.

LEMMA 3. *Suppose that there are two cases n, F_1, f_1, l_1 and n, F_2, f_2, l_2 where n is the number of values for both cases, F_i, f_i are CDF and PDF of the n i.i.d. values in case i, l_i is the low value for case i. If $F_1(l_1) = F_2(l_2)$, then these two cases have the same minimum cost to find the maximum value above the low value l_i.*

The proof of this revised lemma is almost identical to the one for original lemma. Thus for space limit, we won't elaborate it again. Finally, we conclude that MVAs are optimal in general.

COROLLARY 4. *MVAs are optimal. The only parameters we are going to determine for the specific optimal MVA are 1) the low value l; 2) the descending query thresholds $a_1, a_2, a_3, \ldots$.*

When we later investigate such parameters that minimizes the cost, we will also assume uniform distribution $F(x) = x$ in default because distribution won't change this minimum cost and we can always adapt an optimal MVA for uniform distribution to an optimal MVA for any distribution easily.

4.3 Experiments to Discover Optimal MVA with a Given Low Value

To discover the specific MVA that's optimal, we first try to identify the optimal thresholds a_i given a low value l (recall that in i-th round, MVA will ask all bidders whose valuation is within $[a_i, a_{i-1})$ to bid). If we can write the minimum cost C^* as a function of n, ρ, l (recall that $\rho = b/c$ is the ratio between broadcast and bidding cost), we may then determine the optimal l using this function.

In the special case that we studied in previous section where $l = 0$ (efficiency is enforced), the optimal thresholds can be easily described as a single parameter α. This strategy won't work when $l > 0$ (we assume uniform distribution in default). Obviously that $a_i \geq l$, thus we can't let $a_i = \alpha a_{i-1}$ since $\lim_{i\to\infty} a_i = 0 < l$. Additionally, if we revise the equation by letting $a_i - l = \alpha(a_{i-1} - l)$, we will get a positive possibility $F(l)$ that we would ask infinite many broadcast queries, which is even worse.

As it's not immediately clear what optimal thresholds should be like, we present a simple algorithm to calculate such thresholds numerically. We firstly discretize the continuous valuation $[0, 1)$ to D discrete values $\{0, 1, \ldots, D-1\}$. That means, the original valuation v will be transformed to integer value $\hat{v} = \lfloor vD \rfloor$. Then we use dynamic programming to inductively calculate $\hat{x}_{\hat{h}}$, the length of next optimal descending query $[\hat{h} - \hat{x}_{\hat{h}}, \hat{h})$ to ask, conditional on that we have already queried $[\hat{h}, D)$ and no one replies.

Algorithm 1 Calculate discretized best query lengths

Require: $\hat{l}$ is the discretized low value, ρ is the ratio between broadcast and bidding cost, n is the number of bidders, D is the maximum discretized valuation

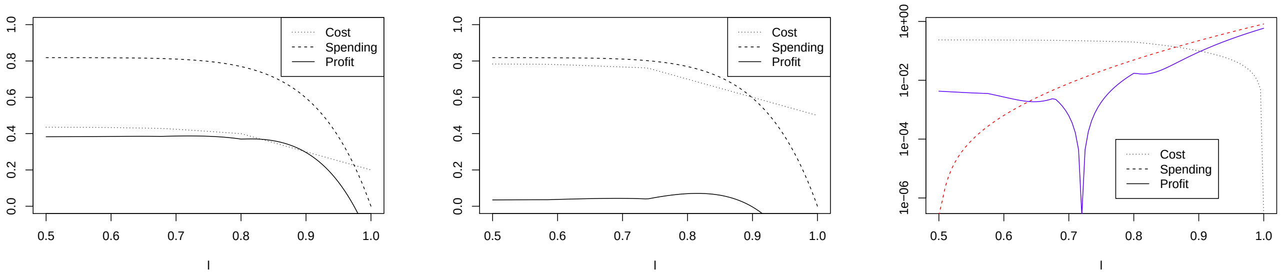
function BESTQUERYLENGTHS($\hat{l}$, ρ, n, D)
  $\hat{x}_{\hat{l}} \leftarrow 0$
  $C_{\hat{l}} \leftarrow 0$
  for $\hat{h} = \hat{l} + 1$ to D **do**
    $\hat{x}_{\hat{h}} \leftarrow \underset{1 \leq \hat{x} \leq \hat{h} - l}{\arg\min}\ \rho + n\frac{\hat{x}}{\hat{h}} + (\frac{\hat{h} - x}{\hat{h}})^n C_{\hat{h} - \hat{x}}$
    $C_{\hat{h}} \leftarrow \rho + n\frac{\hat{x}_{\hat{h}}}{\hat{h}} + (\frac{\hat{h} - \hat{x}_{\hat{h}}}{\hat{h}})^n C_{\hat{h} - \hat{x}_{\hat{h}}}$
  end for
  return $\hat{x}$
end function

Algorithm 1 runs in time $O(D^2)$. Having $\hat{x}$, we can then infer the best strategy x for original continuous problem by converting $\hat{h}, \hat{x}$ back to h, x and using them to interpolate continuous strategy. The larger D is the more accurate it will be. But it will also require more running time. Running this for case $l = .5, \rho = 2, n = 10, D = 1000$ we get x showed in figure 4(a). It seems that x is a piecewise linear function over h. For those h which is close to l, obviously that the optimal strategy should be $x = h - l$, which means using only one query to explore all potential bidders. But it's unclear why x is linear when the best strategy is using multiple queries to explore the valuation range.

Another way to plot the graph is to normalize x and l so that h becomes 1 since our model is $v_i \in [0, 1)$. It's showed in figure 4(b). This again looks like a piecewise linear function. A more interesting observation is that the left-most piece is quite a long straight line $x = 1 - \alpha$. Thus it seems that $x = 1 - \alpha$ is optimal for quite a lot l which is not far from 0. Here the α is the the optimal α of α-MVA if we set $l = 0$.

4.4 Analysis of Optimal MVA with a Positive Low Value

Figure 4(a) shows that the optimal MVA with a low value $l > 0$ is much more complicated but there might still be hope to get a nice



(a) $n = 10, b = 0.2, c = 0.1$ (b) $n = 10, b = 0.5, c = 0.1$ (c) Suboptimality, $n = 10, b = 0.2, c = 0.1$

Figure 5: Optimal MVA's cost, spending, profit over l, n i.i.d. uniform distributed bidders, broadcast b and bidding cost c

analytical result: piecewise linear function. That is, we can't use a single α to describe such optimal MVA, but perhaps we can use a sequence of α to describe it. Now let's take an analytical treatment.

The first key to analyze the optimal MVA with $l > 0$ is to utilize the fact that there's a maximal number of rounds to exploit the whole reportable valuation range $[l, 1)$. Let's call that number k.[5]

For convenience, define $\vec{a} = (a_0, a_1, a_2, \ldots, a_k)$, the vector of k thresholds in such optimal MVA. We make $a_0 = h, a_k = l$ so in i-th round the query would be $[a_i, a_{i-1})$. Now define cost $C(\vec{a}, k, \rho, n)$ to be the expected cost for MVA defined by $k, \vec{a}$ when there are n i.i.d. $[0, 1)$-uniform bidders (note that l, h are implicitly defined by a_k, a_0). If we can get a neat form of C, we can use $\frac{\partial C}{\partial a_i} = 0$ to characterize optimal MVA, as we did in section 3.3.[6]

Thus the second key is to represent this C. Rather than considering one round after another recursively as we did before, we now consider all rounds together. Let CR_i denote the cost that occur in round i (the broadcast of that round and the bidding cost charged in that round) and $v^* = \max_i v_i$. Then expected cost would be sum of expected cost of each round:

$$\begin{aligned} C(\vec{a}, k, \rho, n) &= \sum_{i=1}^{k} E[CR_i] \\ &= \sum_{i=1}^{k} \Pr\left(v^* < a_{i-1}\right) E[CR_i \mid v^* < a_{i-1}] \\ &= \sum_{i=1}^{k} \frac{a_{i-1}^n}{a_0^n} \left(\rho + \frac{a_{i-1} - a_i}{a_{i-1}} n\right) \end{aligned} \tag{5}$$

Taking derivative we get

$$\frac{\partial C}{\partial a_i} = \frac{n(n+\rho)a_i^{n-1} - n(n-1)a_{i+1}a_i^{n-2} - na_{i-1}^{n-1}}{a_0^n} \tag{6}$$

Unfortunately, equation 6 isn't neat enough to get a piecewise linear query length. Recall that in previous subsection, the experiment seems to show that query length x is piecewise linear over h, which means $a_1 = a_0 - x = h - x$ must also be piecewise linear over $a_0 = h$. One counter example is the simple case $k = 2, n = 3$. We have: $a_1 = (\sqrt{a_0{}^2\,\rho + a_2{}^2 + 3\,a_0{}^2} + a_2)/(\rho + 3)$. Anyway, by definition we have $a_1 \geq a_2$. And the former equation indeed looks very linear when $a_1 \geq a_2$.

Though we failed characterizing the optimal MVA using piecewise linear functions, equation 5 gives us a better way to calculate thresholds a_i. We use an R package called BB [10] to solve these non-linear equation systems. Before throw those equations to that package, we have to first decide k and an initial guess of $\vec{a}$.

One obvious upper upbound of optimal k is $k_\alpha = \lceil \log_\alpha (a_k/a_0) \rceil$ where α is the optimal α for α-MVA when $l = 0$. The intuition of this bound is that the query length of any round except the last round couldn't decrease if we increase l from 0 to some positive value. Having this upper bound, we can eigther bruteforcely search all k between 1 and k_α, or use ternary search [cite wiki?] to get optimal k in $O(\log k_\alpha)$ time if we can prove it's unimodal.[7]

The initial thresholds $\vec{a}$ are very important for solving optimal a_i. A bad choice may lead to much more computation and even non-convergence. For example, the trivial uniform $a_i = (1 - i/k)a_0 - (i/k)a_k$ is a bad initial guess. We find the initial guess $a_i = \alpha^i$ (the same α of k_α) to be very efficient. Let's call this MVA with thresholds $\alpha, \alpha^1, \ldots, \alpha^{k-1}$ where $\alpha^{k-1} \geq l, \alpha^k < l$ an α-cutoff-MVA, similar to what we did in previous section 3.5. It's suggested by experiments showed in figure 4(b) where query length x seems to be $1 - \alpha$ for a lot of l. In experiments, the actual optimal k is very close to our upper bound k_α: it's mostly either k_α or $k_\alpha - 1$. This might be a direct consequence of α-cutoff-MVA's good performance shown in figure 7. As you can see, in most cases their profit are very close. The significant difference only occur when low value l is a little below α and this isn't likely to occur when n is large where α is much closer to 1 than l.

4.5 Choosing Low Value

Having k and a_i, we are still one step away from optimal MVA: choosing the low value l. It's obvious to see that optimal cost C is non-increasing as l increases. By Myerson's optimal auction theory and theorem 5, setting virtual value of l to be 0 will yield the maximum total spending. Name such l as l_0. We then conclude that optimal l must satisfy $l \geq l_0$. The question is, how to search or determine optimal l. Let's first check the simple case when the value distribution is uniform.

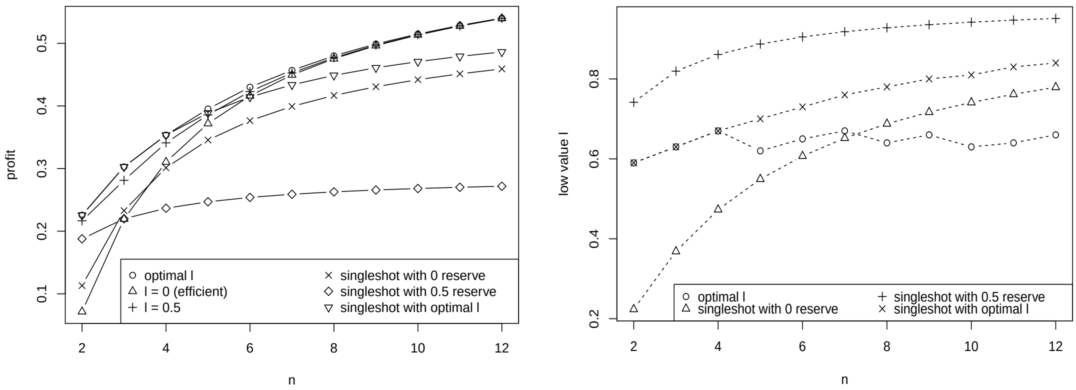
Figure 5(a) and 5(b) plot cost, spending and profit over l. It's clear in figure 5(b) that increasing l from 0.5 to about 0.8 will get a significant profit increase. However, it's not so clear in these two plots whether profit is unimodal and whether we can do ternary search.[8] To see it more clear, we plot figure 5(c), which transforms the cost, spending and profit to their corresponding suboptimality, i.e. the distance between the specific value and the optimal value (to make the log-scale work for distance 0, we add some small constant to that suboptimality). Thus, the new plot will preserve

[5] We have argued this before: if such k doesn't exist, or equivalently the maximum number of rounds is unbounded, the cost will be infinite as the possibility that no values lie in $[l, h)$ is positive.

[6] It's easy to see that boundary cases $a_i = a_{i-1}, a_{i+1}$ are not optimal too.

[7] We didn't prove that it's unimodal but experiments seems to support this property.

[8] Unlike k which is an integer with a fairly small upper bound, l is continuous over $[0.5, 1)$, thus a ternary search is more needed.



(a) profit (b) low value

Figure 6: Compare profit and corresponding low value l over n. The broadcast cost $b = 0.1$. Bidding cost for seller and buyer are $\beta_1 = \beta_2 = 0.05$ (thus $c = \beta_1 + \beta_2 = 0.1$)

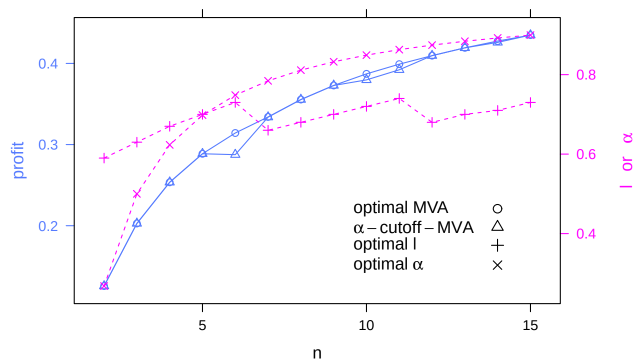


Figure 7: The profit of optimal MVA and α-cutoff-MVA are plotted in solid line. The low value l is chosen to maximze optimal MVA's profit. We also plot l and α in dashed lines to see how they affect the relative profit difference. Note that α does not depend on l.

the peaks and global optimal point as the lowest point.

As you can see from figure 5(c), the cost isn't unimodal over l and it's quite wierd. Thus in later experiments, we are just going to brute forcely search over all possible l from 0.5 to 1, with a search increment of 0.01.

4.6 Experiments

Finally we are going to compare profit of different mechanisms in general settings. To keep it simple, we still use a uniform valuation distribution for all experiments. As theorem 5 shows, the profit is simply spending minus cost where spending is solely determined by l. In fact, the total spending won't be very sensitive to this l when n is large (except that l is very close to 1) and we will see this in later comparisons. Thus most experimental comparisons that we have done for cost in section 3.5 would also give us a lot information for profit comparison. As a result, we will compare mechanisms very different from those in section 3.5. Specifically, we won't be interested in k-MVA where k is very limited. Instead, we are going to compare how l is going to affect the profit and how much profit we lose if we ignore the cost.

If we ignore the cost, Myerson has already proved that a singleshot Vickrey auction with reserve price 0.5 would be optimal for uniform i.i.d. bidders. Thus we will compare this singleshot mechanism, as well as its variants, the singleshot Vickrey auction with 0 reserve price. But be aware of that reserve price isn't equivalent with low value l when cost exists. To make it more fair, we add one more singleshot Vickrey auction whose reserve is set to be optimal among all singleshot Vickrey auctions (i.e. we optimize l for this particular singleshot mechanism). One may also suspect whether it's good enough to set low value l to be 0.5 in such singleshot Vickrey auction, since it will bring us maximal total spending. The answer is no, because its bidding cost is $0.5n \times c$, which grows way too large when n is large.

The experimental result is shown in figure 6. As shown in profit comparison figure 6(a), the profit of optimal l, $l = 0$ and $l = 0.5$ become close very quickly when n grows and they are almost identical for $n \geq 8$ in this experiment setting. Thus choosing l won't be a critical issue for large n. However, if we use singleshot mechanism (which is optimal if cost doesn't exist), the gap is significant. This is because the bidding cost will drive low value l very close to 1 thus lower the total spending significantly. For example, even if we set reserve price to 0, when $n = 10$, l will be between 0.6 to 0.8. The optimal reserve price 0.5 becomes the worst as its l is too high. Even if we adapt our reserve price to optimal one, the singleshot mechanism is still not so good because it can't balance the total spending and bidding cost, i.e. it either makes l close to 1 to lose a lot total spending, or makes l close to 0.5 to cause a big bidding cost.

5. CONCLUSION

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