Optimal Broadcast Auctions with Costly Actions

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1. INTRODUCTION

2. COST MODEL AND SETTINGS

In this section, we define our cost model and explain why they make sense. Our cost model is inspired from online second-hand item transactions, including examples like eBay, craigslist and universities' mailing-lists.

DEFINITION 1. *In our settings, there's only one seller selling one item to n i.i.d. (indenpendent and identically distributed) bidders whose valuation distribution (CDF) is continuous over* $[0, 1]$ *(i.e. no mass point). The seller can broadcast a message to all bidders with a cost b (the broadcast cost). A bidder may remain silent after that broadcast with no cost. If not, it costs c_0 if the bidder's bidding(reply) doesn't imply positive valuation. Otherwise, if the bidding implies a positive valuation, a greater bidding cost $c_1 > c_0$ is charged. Such bidding cost c(c_0 or c_1) main contain two parts, β_1, β_2 where $\beta_1 + \beta_2 = c$. The first part β_1 is charged to the seller while the second is charged to the corresponding bidder. The bidder's reply should be deterministic with respect to the seller's broadcast (for simplicity, we only consider pure strategy equilibrium).*

(shall we insert a graph to show our cost model?)

Most settings in definition 1 are quite standard except for the cost and broadcast capability.

We introduce broadcast capability because it's shared by many auctions. For example, a Vickrey auction or a first-price auction with reserve price can be described as a broadcast auction with only one broadcast: telling every bidder the reserve price. The bisection auction [cite bisection auctions] is another example which has many rounds of broadcasts. In each round , it will broadcast a price and ask bidders to reply whether his valuation is beyond or below that price. In real world, sellers make such broadcasts via sending emails to a bunch of receivers (typically a mailing-list), listing items on a platform such as craigslist and eBay, or even making ads on Internet/TV. Such broadcast activities costs either money (e.g. a list or ads fee) or time/effort (e.g. writing and sending an email).

Note that in our model, we only give sellers broadcast capability so they cannot find or communicate with each bidder one by one. The first reason to have this constraint is there are too many potential bidders on the Internet (our model focus on online item transactions) and it's hard to explicitly find them one by one. On the other hand, in offline cases where the set of bidders are small and explicit (e.g. the government want to sell a land to one of three companies), it might be helpful to let the seller communicate with bidders one by one [cite search mechanisms]. The second reason is that we want to focus on mechanisms that avoids time consuming bargaining. Such feature is very important as one of the most vital advantages of online transactions are their convenience and the time consuming bargainning can ruin it.

We distinguish bidding costs c, c_0 because a trustworthy system need to verify a bidding which implies positive valuation. For example, the bidder may have to input his credit card number and prepay an amount of money. Without such verification for the bidder, a bidder may bid very high and refuse to pay in the end. A verification for the seller might also be needed. For example, a bidder may want to set up an appointment with the seller to check the item. Setting up such an appointment might be costly because they need to dicuss time and place via emails or phone calls. Attending that appointment may also cost travel fees and time. Therefore biddings that imply positive valuations cost more. Biddings that won't imply positive valuation seem to be useless, but sometimes they do exist and do have cost. For example, the no-bid in bisection auctions (i.e. the reply that indicate someone's valuation is below a price) cost 1-bit for communication.

The cost of bidding may differ from other cases (e.g. the higher valuation a bidding implies, the higher cost there might be). However we only distinguish two cases above since we believe that the difference between other cases are not so significant. For example, entering credit card number and prepaying \$100 should cost almost the same effort as entering credit card number and prepaying \$10.

Finally, we define optimal mechanisms to be the ones that maximze seller's utility since when facing many different auction mechanisms, a rational seller will choose the one that gives him maximum utility.

DEFINITION 2. *We say a mechanism is optimal if it gives the seller maximum utility (revenue) which equals to all the value payed to this seller minus all the cost charged to this seller. A class of mechanisms are optimal if they contain one optimal mechanism.*

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3. OPTIMAL MECHANISMS WITH EFFICIENCY AND ONLY SELLER'S COST

In this section, we consider a simplified optimizing problem with efficiency constraint and the cost is only charged to the seller. Though these two constraints simplify our problem a lot, they are very common in real cases such as craigslist or moving sales in mailing-lists. (more to explain efficiency and no bidder's cost).

First of all, we introduce a mechanism called Multi-round Vickrey Auction (MVA) based on what's been using in real-world online second-hand item transactions. Then we prove that MVAs are optimal. After that we'll try to find the specific MVA that achieves the optimality. Finally, we conduct some experiments to compare the optimal MVA with other mechanisms.

3.1 Multi-round Vickrey Auctions

A Multi-round Vickrey Auction (MVA) has multiple rounds of Vickrey auctions with progressively decreasing reserve prices. This kind of auction effectively occurs on eBay. The seller may set up a reserve price and let buyers bid for this item. The proxy bidding functionality makes such an auction equivalent to a Vickrey auction with a reserve price. If no buyers bid for a given reserve price, the seller may lower the reserve price, which makes the whole process equivalent to an MVA.

DEFINITION 3. *(Multi-round Vickrey Auction, MVA)* *In a Multi-round Vickrey Auction (MVA), there's a sequence of reserve prices $r_k, r_{k-1}, \ldots, r_0$ where $r_k > r_{k-1}$. The seller creates a Vickrey auction with a reserve price r_{k-i} at time i (or round $k-i$). In each Vickrey auction, if only one buyer bids, he/she gets the item and pays reserve price. Otherwise, the buyer with the highest bidding gets the item and pays the second highest bidding.*

MVAs require Vickrey auctions (or equivalent English auctions) as basic steps. In reality, however, such functionality won't always be provided by online platforms such as craigslist. Thus a simplified version of MVA occur very often in those platforms. People call it first-come first served which means for every reserve price r_i, the first one who accept that price wins the item and pays r_i directly. This mechanism may loose revenue and social efficiency as the person with lower valuation p may get the item for r_i while there's someone else who is willing to pay a higher amount of q where $r_i \le p < q < r_{i+1}$. We won't focus on this first-come first served mechanism because it's harder to analyze analytically and it's inferior than MVAs in terms of both sellers' utility and social welfare.

Since there's no cost charged to buyers, it's obvious to see that whenever a bidder decides to bid, he/she must bid truthfully. Thus the Bayesian Nash Equilibria (BNE) for MVAs can be described as k thresholds $a_k, a_{k-1}, \ldots, a_0$ where $a_i > a_{i-1}$. Whenever a bidder's valuation for the item is greater than a_i, he/she is going to bid in round i whose reserve price is r_i. Because of efficiency constraint we also have $r_0 = a_0 = 0$.

So the equations to determine a_i are:

$r_0 = a_0 = 0$ and $\forall i$ $(1 \le i \le k)$,

$$P(a_i)(a_i - r_i) = \int_{a_{i-1}}^{a_i} (a_i - x)p(x)dx + P(a_{i-1})(a_i - r_{i-1}) \tag{1}$$

assuming

$$P(x) := Pr(v_1, v_2, \ldots, v_{n-1} \le x)$$
$$p(x) := Pr(\max_{1 \le j \le n-1} v_j = x) = P'(x)$$

and $v_1, v_2, \ldots, v_{n-1}$ are valuations of the remaining $n-1$ bidders. The equation 1 says that the bidder with valuation a_i should be indifferent from bidding in round i (the left hand side) and bidding in round $i-1$(the right hand side). The following theorem describes the equilibrium of MVAs determined by equations above.

THEOREM 1. *In a MVA with reserve prices $r_k, r_{k-1}, \ldots, r_0$, there's a pure strategy Bayesian Nash Equibibrium characterized by thresholds $a_k, a_{k-1}, \ldots, a_0$. The bidder with valuation greater than a_i (but not greater than a_{i+1}) will bid in round i with reserve price r_i. The relation between a_i and r_i is:*

$$r_0 = a_0 = 0$$
$$r_i = \left(\int_0^{a_i} x\, p(x)dx \right) / P(a_i) \qquad (i > 0) \tag{2}$$

PROOF. By equation 2, we have $r_i P(a_i) = \int_0^{a_i} x\, p(x)dx$ for all i (TOO BAD this won't hold if $a_0 > 0$). Thus the right hand side of equation 1 is:

$$\int_{a_{i-1}}^{a_i} a_i p(x)dx - \int_{a_{i-1}}^{a_i} x\, p(x)dx + P(a_{i-1})(a_i - r_{i-1})$$
$$= a_i P(a_i) - \cancel{a_i P(a_{i-1})} - r_i P(a_i) + \cancel{r_{i-1} P(a_{i-1})}$$
$$+ \cancel{P(a_{i-1}) a_i} - \cancel{P(a_{i-1}) r_{i-1}}$$
$$= \text{left hand side of equation1}$$

□

This tells us that a bidder will bid in a round of MVA if and only if the expected second highest bidding conditional on this bidder's valuation is the highest is greater than the reserve price of that round. For example, if the distribution is uniform, i.e. $P(x) = x^{n-1}$, $r_i = \frac{n-1}{n} a_i$ for $i > 0$.

3.2 Optimality of MVAs

THEOREM 2. *Among all mechanisms that can include multiple rounds of broadcasts and are required to be efficient (allocate the item to the bidder with highest valuation), Multi-round Vickrey Auctions (MVAs) are of minimum cost.*

COROLLARY 1. *If all broadcast costs and bidding costs are charged to sellers, MVAs are optimal if efficiency is required. Such optimal MVA is the one that minimizes the overall cost.*

3.3 Cost Minimized α-MVA

3.4 Approximation of α and Experiments

It's difficult to get an exact closed formula for optimal α. Thus we are going to use a closed formula to approximate this α. We'll conduct experiments to compare our approximation with the optimal α that's computed numerically. We are also going to show comparisons between optimal MVA, approximate optimal MVA and other conventional mechanisms such as Vickrey auctions.

4. OPTIMAL MECHANISMS WITH BOTH SELLER'S AND BIDDER'S COST

In this section, we take out the constraint and prove that MVAs are optimal in general. We will also try to find the specific MVA to achieve such optimality and it turns out to be significantly more complicated than our previous simplified case.

4.1 Spending Equivalence Theorem and Revenue Optimization Strategy

DEFINITION 4. *We say mechanisms satisfy relaxed efficiency constraint with lowest type l if:*

1. *They only allocate the item to bidders whose valuation are at least l (the lowest type is l)*
2. *If they will allocate the item, they will always allocate the item to the bidder with highest valuation.*

When we say a mechanism with a lowest type l, we imply that this mechanism satisfy relatex efficiency constraint with lowest l.

THEOREM 3. *For all mechanisms with the same lowest type l, they will have the same overall spending from all bidders (including bidders' bidding costs and payments to sellers).*

COROLLARY 2. *For mechanisms with a fixed lowest type l, the maximum utility for sellers is achieved when the mechanism minizes the cost.*

THEOREM 4. *MVAs have the minimum cost among all mechanisms with a lowest type l*

THEOREM 5. *MVAs are optimal. (watch out for relaxed efficiency constraint)*

COROLLARY 3. *Shall we increase lowest type l a little above Myerson's optimal lowest type to trade payment with cost? Or is it optimal to use Myerson's l and then minimize cost according to that?*

4.2 Experiments to Discover Optimal MVA with a Given Lowest Type

4.3 Analysis of Optimal MVA with Lowest Type

4.4 Using Piecewise Linear MVA to Approximate Optimal MVA

4.5 Choosing Lowest Type?

4.6 Experiments

Now we are going to compare revenue in general cases. We not only compare our approximate optimal MVAs to optimal MVAs computed numerically, but also compare MVAs to other conventional mechanisms.

5. CONCLUSION